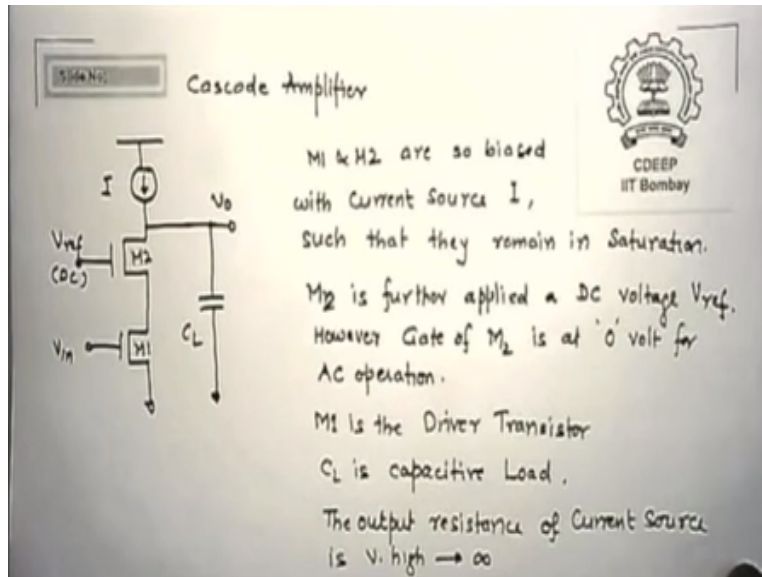


Analog Circuits
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Lecture – 09
Cascode Amplifier

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Here is a Cascode amplifier which is similar to a normal amplifier this is your driver amplifier and one which are all standard value of W/L V_T etc. series to that M_1 there is another transistor M_2 , please look at it this is very relevant and good concept to understand there is a another transistor in M_2 please remember this is common source M_1 is a common source system inputted at the gate source is grounded wherever M_2 .

Actually has a DC potential at the gate what AC what is it ground for AC it is ground so what kind of this amplifier or circuit will be common gate it is a common gate we already done common source common gate common drain. so now I say this is a common source amplifier over which we are sitting a common gate amplifier and it is biased by a constant current source I this is I told you the other day there are different way of biasing circuits.

One is my resistance network the other of course is by active load like a diode connected loads or a current sources as shown here they can be also taken from Miller sources ok, Miller that

mirrors a normally see mirrors current mirrors. so it is biased in such a way that both M1 and M2 are in saturation okay is that point clear so, the first problem one can see from here.

If you are drawing the circuit rest you do not write just read listen and then think if M1 and M2 are in series as shown here in normal case if M2 would have been absent only M1 would have been there so how much would have been supply this is our VDD. Let us say source requires some drop but the most of the VDS would have come in M1 is that layer most of the drop would have come in M1, so VDS would have been larger to make it in saturation.

Now I want 2 transistors M1 M2 also to be in saturation if they are to be in saturation this voltage should be sufficient enough that $V_{DS2}+V_{DS1}$ should be such that both makes it insanity is that clear that means power supply needs to be slightly boosted because otherwise I most of the voltage goes here then you have nothing left but to make them both them in saturation, so one of the major problem in Cascode Sharpe you may require larger power supply voltages okay.

It is not very true what I am making but roughly larger than the single stage amplifiers I will not say large or something because also made in the chip so it cannot be more than 1.5 also we can adjust VDS we can adjust VT and therefore $V_{gs}-V_T$ still can be smaller than VDS but irrespective of other number I chose I need larger drops for M1 M2 to be saturation.

This is one is problem one you should understand now right now assume this current source is ideal and we say it is output resistance is infinite, infinite is that okay this is a Cascode amplifier is that clear this sentences have written just for the sake of it you need not because I already explained you what I am talking these are some okay.

Basically I am saying this is driver this is the gate common gate transistor this is the essentially current shows biased.

Now I want to analyze what is the condition what is the criteria I said last time to make cascode better than cascade that normally I say ft or unity gain amplifier frequency or the gain bandwidth product as we called GBW that is constant for a normal common source or any amplifier.

So if I increase gain bandwidth goes down if I increase bandwidth n goes is that clear and the maximum frequency is you need again frequency is that correct you cannot go beyond that because why because beyond again is $-DB$ means >1 , so we have been now worried that how can I increase gain without actually losing on the bandwidth or which out getting that cutoff point to be any different if I remain there and still boost the gain.

Then I achieved something which was not possible in a single stage that first case gain bandwidth constant or UFT is constant is essentially called the figure of merit you cannot go beyond that now I want to say even at that frequency gain is higher if I do that and I have achieved something is that clear and this is exactly what Cascode is trying to do in this circuit.

If you see carefully the gain there is no additional load resistance I have put here the load resistance is essentially due to $M2$ $M1$ okay series some combination of this will lead to R_o and this R is also parallel to this R of this source is that clear why because this is ground for AC so this R is also parallel to R_o of these combinations but that we treated as how much infinite.

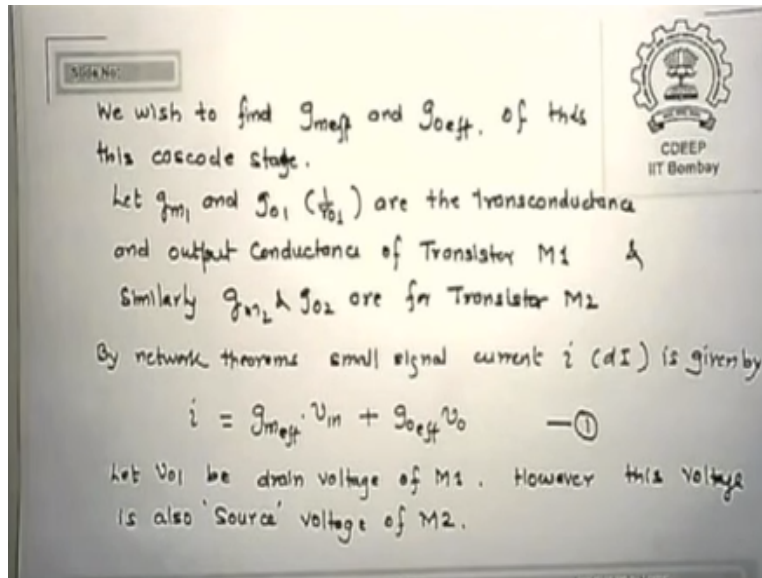
So, essentially I said this arrow equal and what I am going to see here is the only load resistance I have and therefore gain is $g_m \times R_o$ and this R_o will call R_o effective because without this whatever is out over this with 2 of them. Whatever we will call it say we will call it R_o effective so if I can make $g_m R_o$ effective larger than $g_m R_o$ with and what is the bandwidth frequency and just show you g_m/C say g_m/C is not touched and I increase R_o .

Then, I am increasing the gain with not losing bandwidths is that clear so essentially what Cascode is trying to do is to boost output resistance without losing anywhere in g_m factors is that that is exactly what we are trying and this is an achievement which is actually not the device is not able to tell us directly it is we say device is always cut off cut or you can go beyond.

Now I am circuit where I am showing you I can beat the system this is the trick in analog how to beat the system some cost will play somewhere which we are not actually telling but at that

cause I will boost this gain without losing the bad news is that clear this is why Cascode are used where do you use goals you want gain boosting okay without losing the bandwidth okay.

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Here is that, we wish to find g_m effective and R_0 I have a tendency to earlier because this is my old papers and this since I used to use all instead of R_0 conductance G_0 is nothing but 1 upon R_0 okay G_0 is nothing but 1 upon or so I want to calculate for this Cascode stage a g_m effective.

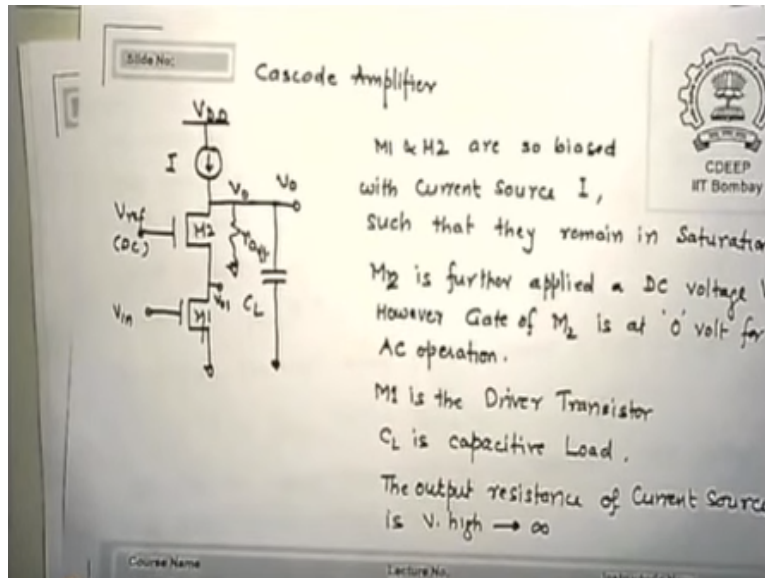
What is g_m effective if the M_2 would not have been there would have been only g_{m1} is that correct with empty present the if a few g_m is we call g_m effective we divide that left g_m and g_{m1} are the trans conductance and output conductance of transistor into and g_{m2} and g_{m2} and g_{o2} are similar numbers for M_2 means g_{m1} 1 is for M_1 2 is for M_2 is that ok.

So if I define that g_{m1} and g_{o1} and g_{m2} are the patterns for M_1 and M_2 by a network Theory the small signal current flowing in this transistor is I which is equal to g_m effective $\times V_L + g_m \times V_O$ this is a simple 2 port network.

If you are not doing done so far in your network course do it again otherwise the easiest way to represent there to put outputs okay $I = g_m + g_m$ effective V_0 okay, if you cannot I will show you

how but other I should be able to give it I to any 2 port Network in and out V_{in} and V_{in} and you will get these expressions ok.

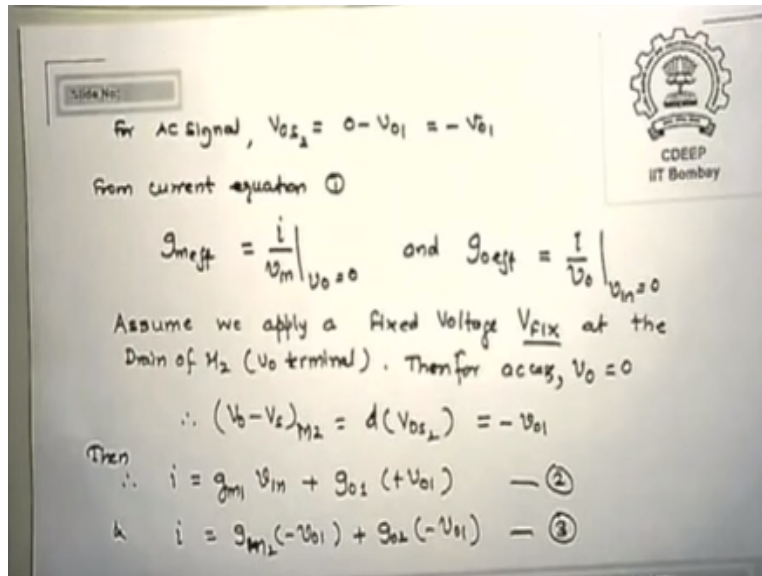
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Now in this figure I call this voltage as V_{O1} this of course I still call it V_0 output of we are not taking it but the drain voltage of M_1 we defined as output 1 V_{O1} is that okay different definition nothing great about however, if you see carefully the V_{O1} which is the drain voltage of M_1 is also acting like a source voltage of M_2 is that point clear.

V_{O1} is the drain voltage of M_1 but it also source voltage of M_2 now do you get the point, so V_{gs} is still available if this is your 0 for example for AC also $0 - V_{O1}$ which is good enough because then $V_{gs} - V_T$ still will be possible is that clear, so this is a trick which I have played I said okay I put a series there the drain voltage of this is same as source voltage of burst and therefore I can say my V_{gs} is still plus V_{O1} and if that is larger than this it is still conduct ok.

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So, this is what I did I took on that for AC signal that is what I say V_{gs2} is $0 - V_{01}$ is $-V_{01}$ my now from the current equation which I wrote I is equal to g_m effective or $V_{in} + g_o$ effective times V_0 if I make V_0 of 0 do you get the point what I am saying if I make $V_0 = 0$ I upon V_{in} is g_m effective is that clear I upon V is $-g_o$ if V_0 is tends to 0 I by V in is g_m effective.

So, I write g_m effective is I/V_{in} when V_0 is 0 by same argument effective is I/V_0 when V_{in} it is in equation do you understand you make one codes output 0 next time you put input for 0 is that correct and then you get g_m effective and g_o effective I/V is both cases is I/V_{in} is g_m effective I/V_0 is G effective is that okay as shown we apply if now there is a game we are playing ok why I say it is this is DC.

So, not easy I add this node, I apply V fixed DC1 at the output where or V_0 its I apply fixed DC value V fixed so what is the AC value applying 0 so V_0 is 0 is that point clear V_0 is 0 when I say I applied fixed DC bias cannot change DV/DZ is 0, so we say it is V_0 there is 0, so we say assume a apply effects we fix at the m_2 terminal therefore bias that is the condition what is the condition I was asking for g_m effective that I buy V_{in} is G_m effective and V_0 is 0.

So, I made that condition true and then I write $V_0 - V_s$ which is nothing but variation of V_{DS} is 2 is nothing but 0-please remember this is 0 this is V_{01} so how much is change in V_{01} AC value $0 - V_{01}$ which is $-V_{01}$ so V_{DS} is how small v_{ds} that is AC is $-V_{01}$, now look at the 2 for the 2

transistors I like to current though I same in both transistors is that clear I cannot have 2 in a circuit there can be only one current which is flowing through M2 as well as through M1.

So for a transistor 1 $I = g_{m1} V_{in}$ say 2 port Network so I is equal to this is what we say intrinsic values external resistances will put light on if whenever we need in actual circuit we want to know intrinsically whether we can break that gm that is gain bandwidth limits okay, we can know so $I = g_{m1} V_{in} + g_{o1} V_{O1}$.

What is V_{O1} V_{DS} R_{M1} V_{DS} F_{M1} is V_{O1} so V_{O1} for the transistor 2 $g_{m2} V_{O1}$ because that is the input now $0 - V_{O1}$ so this is $V_{DS} + g_{o2}$ again $V_{DS} - V_{O1}$ is that okay just substitute values for both transistors M1 and M2 and currents are same please take it in a circuit same current can only flow okay, so the at the drain of the M2.

I am only still looking at this that is why it is $0 - V_{O1}$ is V_{O1} for the V_{DS} for this is that correct that is what I put-view is that correct okay.

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from eq 3 $V_{O1} = \frac{-i}{g_{m2} + g_{o2}}$

substituting this in eq 4 as

$$i = g_{m1} V_{in} + \frac{-g_{o1} i}{g_{m2} + g_{o2}}$$

$$\therefore i \left(1 + \frac{g_{o1}}{g_{m2} + g_{o2}} \right) = g_{m1} V_{in}$$

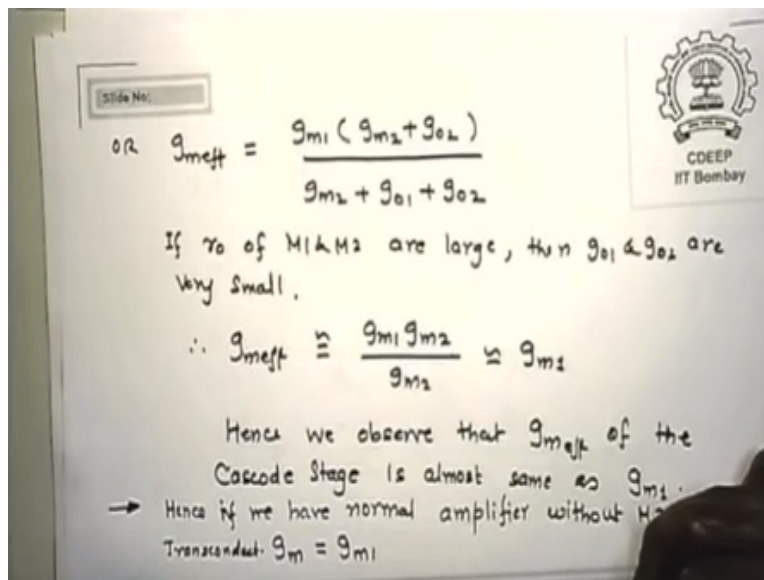
$$\therefore g_{m_{eff}} = \frac{i}{V_{in}} = \frac{g_{m1}}{\left(1 + \frac{g_{o1}}{g_{m2} + g_{o2}} \right)}$$

If I do this I can now find out from the equation 3 V_{O1} 3 or 4 I do not know which one where 3 V_{O1} is I upon $g_{o1} + g_{o2}$ simple math okay.

VO1 is -I upon gm2+go2 please remember G is R conductance 1 upon G R whenever you wish it you can convert to back to R now this VO1 value which I got from equation 3 can be substituted where in equation 2 in that case I=gm1Vin please remember what was there -go1 VO1 so it is 0 1 x I upon gm -go2 so if I collect items I=1+0.1 upon gm go2=gm Vin under what condition.

I derived this VO is external VO big DC value therefore ACVO it is 0 and under that case III/VN is gm, gm effective so gm effective is I/gm1 upon this quantity is that correct so I are now derived expression for GM effective is that unconditional I have made what was the condition they wanted me to say that VO is 0.

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So I have made it VO and I solve the equation and I got my gm effective now I want to see all these values which I put if I do little extensions of this I can bring this denominator back to numerator so the gm effectively gm1 x gm2+go2 upon gm1 gm2+0.1 2 just remove that one part and just put the denominator to this now typically lambdas of the transistor is how much close to 0.00100.2 or +2.0 as people is saying.

So R0 are normally infinite or even if it is higher tens of mega ohms okay or at least a mega ohm is that correct unless your current is very high R0 cannot be reduced to kilo ohms if it is very high current and the power will actually take care of your device failures you do not worry on that okay.

So if R_0 very large what does conductance is smaller or larger very small normally g_m are the order of ten to millions per volt g_m are typically of the order of millions this g_o the order of microns g_o are the order of microns is that clear, so I say g_{m2} is much larger than either g_{o1} or g_{o2} so I need love g_{o1} and g_{o2} come the denominator same way I neglect g_o from the top is that correct.

What is the condition I said since R_0 are very hard g_m g_o are very small compared to trans conductance s which is typically one order or 3 to 3 order higher therefore equivalently saying it is g_m g_{m2} upon g_{m2} , which is equal to how much g_{m1} is that correct, so even by using your cost code how much trans conductance.

I obtain same as equivalent of their normal single stage amplifier transistor $M1$ if I were used only among $R_{g_{m1}}$ with 2 in this also I got equivalent of g_m , so what is that if g_m months are equal you can say g_m is $g_{m1}=g_{m2}$ also if you wish and still it is valid for it if the transistor identically can always use.

Otherwise you can always get accurate value please remember this is the net value I have, but in your life number y g_m will be very close to or sorry g_m effective will be very close to g_{m1} so if you see this expression again okay, I will come back first let us keep this and we will use it on bandwidth part later, so what value we have calculated g_m effective.

What is the next value I must calculate Q effective what is the condition in g_o effective I said range should be 0, so if I say V_{in} it 0 that is the condition you wondered G_N should be 0, then I said you are effective is $1/V_0$ when V_{in} is 0 is that circuit okay input its grounded that is what you said being a 0.

So I grounded it I use the same equations of current again for this as well as for this for the second transistor I is equal to $-g_m$ to V_{O1} okay what is videos for this $0-V_{O1}$ please remember V_{gs} is how much $0-V_{O1}$ that is this $-V_{O1}$ is appearing here, so the first is $-g_m$ to $V_{O1}+g_{m2} \times V_{DS}$ how much is V_{DS} $V_{O1}-V_{O1}$ so g_{o2} is okay I made a mistake so I rewrote V_{O1} is V_{O2} .

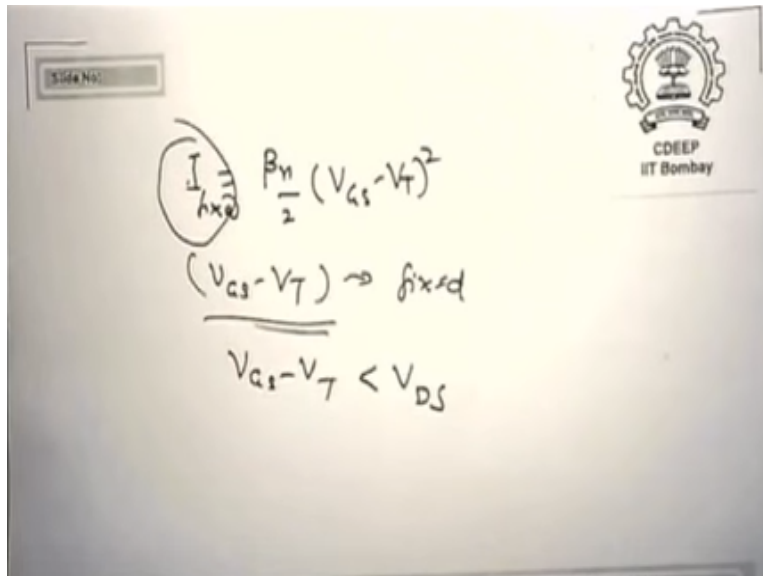
Which is actually so $V_O - V_{O1}$ this is another equation now if I write for M1 I had it on it for M2 now I write for M1 current are in this is $g_{m1} \times 0 V_{in}$ is grounded so $g_{m1} \times 0 + g_{o1} \times V_{DS}$, which is V_{O1} , ok yes input signal ground small signal every single parameter I am using right now a small signal.

So from this equation V_{O1} is R/g_{o1} from this equation V_{O1} is I/g_{o1} substitute this V_{O1} in this expression like yeah from five this time I by V_{O1} R/g_{o1} is V_O one use this V_O one value here and give new equations. Now is that okay all that I note from the circuit five equation I calculate V_{O1} and sorry substitute back in equal s the biasing DC bias is coming from I source and nothing to do with AC signals is that clear to you M1 M2 are biased.

I am not saying this is V_{gs} is biasing greater than V_T small which have nothing to do with bias is that correct this is signal what is input to this signal is for him to $0 - V_{O1}$ is the V_{gs} power that is the signal it is seen it is that clear it is a AC signal is that clear 10 month AC signal is grounded but bias is not removed bias is coming from I.

I have a little told you if $I = \beta \times \beta / 2 V_{gs} - V_T$ square if I keep fixed I am keeping $V_{gs} - V_T$ fixed which is biasing it for saturation is that clear I repeat you are not that my issue anytime I say I is fixed please look at it the way I say it knowing once for all capital I is if the device is in saturation assuming $\lambda = 0$ right.

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Now so if I is fixed $V_{GS} - V_T$ is fixed we make i sufficiently such that $V_{GS} - V_T$ is always less than V_{DS} choice of i is such that $V_{GS} - V_T$ is smaller than V_{DS} so when I fixed active load which is current source I am actually biasing the device in saturation the choice of I is mine is that correct choice of I is mine which will fix my bias for any transistor is that correct same current is passing in and one same current is passing in $M2$.

So both will be up till saturation is that correct $V_{GS} - V_T$ for both should be smaller than V_{DS} for each transistor is that clear that I am adjusting externally which is called bias current okay so all that analysis is clear all that we are talking is AC analysis and not DC and all DC we are assuming that I am keeping both b of the first one.

I showed you is so just hope that $M1$ and $M2$ remains in saturation that I externally I did it is that correct I am NOT saying that if the voltage goes to a fixed value it does not pass a current who said you will do-the effect still I can be adjusted all let us say as if pure AC is made I am only looking for VOA see now your question is if I make ground itself there then the whole of all current will actually go to the ground.

So I must put some potential bear so that current can keep flowing is that correct otherwise all of current will be grounded I want a she ground but I do not want DC ground there is that correct and that is why I said I put some voltage there so that the whole current does not short-circuit out

it should it will not go through M1 M2 then is that clear that is why that condition was me is that clear to you why that voltages are good otherwise.

If I physically ground that then all of I will actually go to the ground physical ground so no m_1 m_2 will be on is that correct so these conditions have to be made so assumptions enthralled we are saying which arrow I do M1 M2 will remain in saturation this is DC biasing nothing to do with AC signals okay AC I am calculating but that is DC that decides g_m to beta I under root of that decides g_m .

So whenever I fix I I am fixing GM is that clear to you I am as far as please get confused on this AC signals theory is based on values of DC you fix is that correct that is what we keep saying $R_A=1$ upon $\lambda \times I$ whatever I fix $g_m = \sqrt{\beta \times I}$ so once fixed I I fix both g_m and R_{OS} but that is a DC current yeah why but that is what is he what is there is he at any point is the slope if we calculate what is the area of small signal.

We discussed all these days that at that point the variation is V near enough that we do not have to say major value changes DC value does not change every does $I_{DC} + \text{small } i_{dc}$ will come but that current is how much so it is I am putting 5 milliamps of 1 million and this is microns thousand times less or even lower is that clear that on any small signal analysis.

This value is always going to change you cannot say star but how do I calculate slopes if anything I hope this value and I get a slope here and I say yeah within the small change I hope yes 2 values are actually being changing there is nothing I can do about but this value being what are the change here and here is negligible that is what a Cs are all about correct it is always possible power supply will adjust to that small AC variation will over it all the time on DC okay always that is what where and when we want to get output.

We remove that DC part by putting a capacitor so that the DC does not go to the output is that clear that is what we have been doing all through so far is that okay okay okay sorry he is right and I think I must thank him for getting clarifications otherwise I will be doing and doing and then everyone will confuse myself and yourself okay.

So if I substitute V_O one into our second whatever number of that equation I have 3 or 2 equation 2 then I rewrite is $g_{m2} I_{y01} = g_{m2} V_O - I_{y02}$ by is that clear mam that just substitute V_O 1 in upper equation and you get this equation I forgot to I forgot this term so I do you wrote again.

So I is equal to collect items collect items on the left the other terms is 0 to V_O so I some of this is $1 + g_{m2} r_{o2} + g_{m2} r_{o1}$ and then what is the definition of g_{o} effective I said I/V_O v NH this equations were derived based on $V_{NH} = 0$ so g_{o} effective is I/V_O which is $g_{m1} + g_{m2} / (1 + g_{m2} r_{o1} + g_{m2} r_{o2})$ again if I linear it j 1 so I get $g_{o1} + g_{o2} + g_{m2} r_{o1} + g_{m2} r_{o2}$ —ok.

Then if you wish I can write our effective is $g_{o1} + g_{o2} + g_{m2} r_{o1} + g_{m2} r_{o2}$ upon $g_{o1} r_{o1} + g_{o2} r_{o2} + g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} r_{o1}$ is that ok what did I get I get auto effective is that correct I got R_{oH} effect is $1 / (g_{o1} r_{o1} + g_{o2} r_{o2} + g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} r_{o1})$ effective so I just got other effective which is $1 / (g_{o1} r_{o1} + g_{o2} r_{o2} + g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} r_{o1})$ is R_{o1} .

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$$\begin{aligned} \therefore Y_{\text{eff}} &= Y_{o1} + Y_{o2} + Y_{o1} \left(\frac{g_{m2}}{g_{o2}} \right) \\ &= Y_{o1} + Y_{o2} + A_{v2} Y_{o1} \\ Y_{\text{eff}} &= Y_{o2} + (1 + A_{v2}) Y_{o1} \end{aligned}$$

Voltage Gain A_{v2} of the Cascode Stage Amplifier is

$$\begin{aligned} A_{v_{\text{eff}}} &= - \frac{g_{m_{\text{eff}}}}{g_{o_{\text{eff}}}} = - g_{m1} [Y_{o2} + (1 + A_{v2}) Y_{o1}] \\ &= + A_{v2} + A_{v1} + A_{v1} A_{v2} \\ &= A_{v1} A_{v2} \quad \text{if } A_{v1} = A_{v2} \\ &\quad \text{then } A_{v_{\text{eff}}} = A_{v1}^2 \end{aligned}$$

Now if I do this you can see from here our effective is $r_{o1} + r_{o2}$ this $1 / (g_{o1} r_{o1} + g_{o2} r_{o2} + g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} r_{o1})$ I made it $r_{o1} \times g_{m2} / g_{o2}$ or $g_{m2} r_{o2}$ also you can write okay but what is g_{m2} / g_{o2} to r_{o2} $g_{m2} r_{o2} = g_{m2} r_{o2} = g_{m2} r_{o2} = g_{m2} r_{o2}$ so I write our effect it is $r_{o1} + r_{o2} + A_{v1} r_{o1}$ is that correct if A_V is large enough.

How I make a V launch by increasing the W pile of those transistors or by increasing the currents okay if I increase sufficiently a V which I can then r_o effective eight times how much $1+AV$ times R_{1+r_o2} if the gain is say 50 or 100 how much is other effective 100 times r_{a1} if I do not put $M1$ $M2$ how much would have been output resistance r_{o1} if I put M -how much I got gain times r_{o1} got is that correct $+r_o$ of course this plus term is there but this is much higher.

So what I have boosted the output resistance got boosted gain times in this case code then we say a V effective is g_m effective r_o effect essentially so I get $g_{m1} \times r_{o2} + 1 + AV_1$ if I expand this well please note down this AV effect in is g_m effective upon go effective is $-g_{m1}$ I wrote this I put here $r_{o2} + 1 + AV_2 R_1$.

Then I say this can be written as $g_{m1} r_{o1}$ is $AV_1 g_m$ this is our route assuming $g_{m1} = g_m$ -everything is same then the effect will be equal to AV^2 square if G ends are equal that is sizes are equal same current is flowing will be equal so any effect you will be equal to AV_1 square so what did I move gain is that clear gain I boosted how much square AV square.

How can I have boosted my gain up any amplifier let us say I have 2 such amplifiers A_1 and A_2 then what would have been if this is my way in this would have been also output of this VO_1 would I become input for this and next gain times it would have come like this okay.

If they are equal I still could have got the NEB gain is a square this is called cascade output of first is given to the input of next stage so gain boosting is not very difficult I could have put doing series like this cascade them I have got the boost of the grids okay.

So what did I achieve out of that that is what the cost code is different what is that we ask you please remember gain boosting is not the only thing we achieved one day to be held constant in that g_m we even with cascade a cascade we maintain same g_m g_{m1} g_m effective okay.

Because of that thing one can see the gain-bandwidth product is g_n effective/CL g_m/C because g_m is same g_{m1} is same is that correct, so since if all g_m are equal one can say gain of a cascade

is gain single stage square but the bandwidth remains same is this one impossible in cascade no because full system would have gain-bandwidth constant.

So if I increase gain what could I lost in that the bandwidth so cascade stages actually improves gain but lose on okay, so this has have you got the point advantage of cascade or cascade that the cascade amplifiers boost gains at the cost of bandits COS coves boost the gain without losing the bandwidth is that but what is the penalty I am paying out of all of it I may require larger power supply voltages is that correct at the cost of VDD.

I achieved this is that clear so is that point clear so what is the advantage of COS forward cross coal is a very interesting device that it allows you to boost output resistance without changing the trans conductance okay and that is something, which no one else can achieve this is normally not done so much in discrete amplifiers this is always done in integrated circuits because that is much easier to adjust that gm_1 um^2 identical others everything is possible on single chip to different devices.

If you take on a breadboard they will never be identical try any number take thousand of such devices whether RVs and series of them nothing will be equal okay, so any discrete such experiments are very difficult to through sometimes but in case of ICS since the areas are very small everything is almost identical therefore divided circuit used almost all such tricks more.

So in analog I see the on mixed signal ICS since future is only on I see it is not on discrete so why do we discrete because discrete tell you which is the variable part IC okay unless you know discrete you can do I see today I showed you same methods can be extended for IC designs is that here okay.

The next chapter for our course is very important frequency response of amplifiers now R assumption, so far that there is a frequency of signal which device does not bother I had amplified output at the same frequency please take my word what I am saying let us take an amplifier and I have an input signal at 20 kilohertz okay and again off 300.

So I know one minute of 20 kilohertz will become 100 milli volts at 20 kilohertz at the output let us say I change that 20 kilo Hertz to 50 kilo Hertz I still believe that it may still come, so many milli volts at 50 kilohertz if I increase to mega Hertz, then I am not very sure whether the same hundred gain will remain for that on amplifier even at 100 one mega order about which means the gain is also not only a function of v_0 of a normal magnitude wise but also is a function of frequency.


So I must for an amplifier what is the condition I must have I must know the region of frequency band as we call say f_1 to f_2 in which gain is constant so that I know any frequency in this range will give amplified normally but beyond that the gain may not be same or before that f_1 also cannot be safe, what do you want is that clear this is what we want to know this is essentially called the bandwidth or mid band frequencies of an amplifier.

I want to know where is the mid band what is mid band where the gain remains constant, so first thing to notice I must now first do a mathematics to some extent and see how do I evaluate the frequency response of any okay amplifier is a system which has some input some output any system has input and an output why I am saying this generalized because this theory.

Which we are talking has nothing to do with electrical networks it can be applied to any energy mechanical or any kind okay this is what control system is all about is that correct we are only looking electrical signals but this is true for any other signals as well okay so let us start with our electrical for example I hope that in your math course if not so far must have done Laplace transforms if you are not done it do it.

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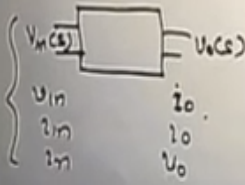
Slide No. Frequency Response of Amplifier


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S-Domain Analysis

We know $j\omega = s$

Any Transfer Function of a Network is given as



$$H(s) = H(j\omega) = \frac{V_o(s)}{V_m(s)}$$

$$= K \frac{(s-z_1)(s-z_2) \dots (s-z_m)}{(s-p_1)(s-p_2) \dots (s-p_n)}$$

where z is called 'zero' of the function
 p is called 'pole' of the T. function

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So we do $j\omega$ is what we call s which is called s domain analysis and if you are a system which is shown here any typical system which can be elliptical in our case then it has then which is a function of frequency that means frequency can be $j\omega$ so ω changes, please remember anything which is return is only right.

Now imaginary quantity but there will be also real quantity s is equal to $\sigma + j\omega$ that clear there will be a real quantity plus imaginary right now I am assuming everything is imagined but in future in their log we will use that real quantities also there will be some output and please remember there can be many possibilities with voltage input voltage output voltage input current output current input current output current input voltage for possible outputs for possible inputs.

This is called ratio of output to input function is called transfer function ratio of output to input is called transfer function so how many kinds of transfer function in normal this system could be four kinds voltage by voltage will be voltage gain voltage gain and current outdistance conduct gain current is input and output is current gains current is input a output is trans resistance gains is that correct.

So for possible gains can be possible and there can be for therefore different we are right now using voltage transfer function why because in most cases amplifiers we are using r voltage amplifiers but later we will see it can be also any one of them as well the typical transfer function

of a network shown here is $H(s)$ or which is a $V(s)$ upon $V_{in}(s)$ and can be written as arbitrary in this form K is constant independent of frequency in numerator.

You have a series of terms which is $s^{-Z_1} \times s^{-Z_2} \dots$ into $s^{-M} / s^{-P_1} \dots s^{-P_N}$ these Z is called 0 of the function jets are called zeros of the function and peas are called poles of the function is that correct this is nothing to do it network per se as I repeat this is true for any transfer functions why are we looking into this because let us say okay.

Maybe there is another slide here or just from here if s is equal to $H(s)$ how much is a change if s is equal to $H(s)$ how much is it is infinite it just goes to infinity if s is equal to Z_1 or there toward it how much is a $j\omega$ so plane is ascension is saying the transfer function value goes to infinite and 0 means in both transmission value goes to 0 this is magnitude wise what we are trying to say is that layer.

So, poles and zeros are those values of frequencies at which the transfer function may become 0 or may become or turn to infinite is that clear that is exactly what mathematically we are saying how does it circuit wise we see is very important for us because there is where we are worried about is that general idea of transfer function here any and please remember for there in our system.

We may use I either of the four in actual amplifiers I may use a current amplifier then what is the output will be I_O and input will become I_n so there will be A_I A_V r_I R and G both possible in this case is that clear so we only write now look for voltage but do not worry it is true for all kinds of transformations here is a simple circuit okay there is an input being a see there is a city search series R_S .

Of course, since there is no mass transistor R_S with the source resistance there is a capacitance of C which has an impedance of $1 / (sC)$ and there is a load which is R_L and let us say by Kirchhoff's law if the mesh current is i is $R_S \times R_L$ is v_0 is a function of s is that correct V_0 is the current in this mesh or loop so $i \times R_L$ is v_0 but how much is there is only one voltage source then divided by the net impedance.

This Plus this Plus this are s upon $R_S/1$ upon C_S+R_L is the net impedance in the loop series implements so being upon this is rewrite in $V_N \times C_S$ upon $R_S+R_L C_S+1$ knots nothing great just take C_F like this and bring C_S about okay please remember the not seized up as if something see small as is the Laplace transform s J Omega term okay.

You can now J Omega C as well if you wish so V_0S upon V in H is $R_L C$ h upon $R_L+R_S \times C s+1$ this is the transfer function of this simple what is come where is the capacitor in this circuit it is in series to the source and the output where could have been otherwise across R_L also I could have a capacitor it would be called parallel capacitance.

This is called series capacitance for namesake okay is that okay obviously if the frequency 0 s is 0 you can see what is the transmission R_C s in the numerator if s is 0 HS it is 0 so there is A_0 at frequencies you know okay so A_0 exists at Omega equal to 0 or $F=0$ however if you look at the transmission again okay this to become 0 R_L+SC+1 .

Then, we can say $R_L+R_S CSE$ Quilly-1 at that time HS will become infinite and this-sign is avoided in character is always on the left half plane but just for the sake of it is 1 upon R_L plus $R_S \times C$ and this is it is called is the pole for this transfer function please take it what is RE called RM to see time constant is that correct.

So I define R_L plus R_S times C as the time constant tau s as the time constant this then is that okay zero exists at zero and pole exists at one upon RC of this value of n upon tau s however we can rewrite the transfer function again H is zero please look at your function V zero is upon Venus I use that R_L plus R_S I bring it outside.

So I get R_L upon R_L+R_S and I am multiplied by R_L+R_S these are symptoms okay I just multiply and you do not divide so I get R_L upon R_L+R_S is R_L+R_S upon $1+$ and use R_L+R_S see as tau and this R_L upon R_L+R is SK then HS is $K \times \tau$ is $S+1+\tau s$ is that correct all that I did because on the numerator I do not have a cow getting.

So I multiplied R_L plus R_S and divide R_L by $R_L + R_S$ so this also becomes τs this is a constant why it is called constant it is independent of frequency so it is called $K \tau S$ upon $1 + \tau S$ so we now see if it is a voltage gain transfer function is a voltage gain then abs is our L upon $R_L + R_S$ if I write $J \Omega J \Omega \tau s$ upon $1 + J \Omega \tau s$ is that correct.

Now one can see now you are 3 terms here what are the 3 terms you see one is this term the second is this term and third is 3 terms are you sure your 3 terms independently can be seen one 2 and 3 what I am going to do I will actually see response of each function with frequency initial and then what I say if they are all together that is this one how will they look is that correct.

I want to find variation of gain with frequency that is my aim frequency response you said I want frequency response so I want a V as a function of Ω I want to find so I made a trick to understand future I may not do it but initially I say okay I see now 3 functions one of this the other is this is the numerator this is in denominator.

So I use my tricks and I say ok first I get the magnitude I want how many quantities I should really find for any complex function magnitude and phase so first derive the phase I want to know the magnitude of the gain as a function of frequency it is all L upon order plus $R_S \Omega \tau s$ upon $1 + \Omega^2$ this is just a plus $J B$ the magnitude is under root $A^2 + d^2$ squared is that clear.

So I just substituted the magnitude values where Ω of course is $2 \pi F$ ok so if I take a $V F$ I can write R_L upon on it plus $R_S 2 \pi F \tau s$ $1 + 4 \pi^2 F^2 \tau^2$ to the power half same function rewritten in the form of frequency is that clear the function which I wrote here is same as this only thing now written in $2 \pi F$ forms ok.

Ω is 2π now I start like taking this is gain and gain can be expressed in some numbers which is called decibels is what is the why we use decibel as a world because essentially the decibel was found for method kind of noise or measurement of audio signals initially but they define when $T \log p_2/p_1$ or rather if I put one meter is some function power in DB.

But since in our case power can be IV and I being same we can write oh sorry 10 I am sorry this will be v square by odd this will be v square bar so if I say gain voltage kind of this it is this is 10 square means $20 \text{ D log } V_2/V_1$ so AV in DB is $20 \text{ log } V_2/V_1$ which is gain okay is that correct so if I want to find any DB values all that I do is $20 \text{ log } 10$ of magnitude of this value.

Which is AV J Ω is that clear $20 \text{ log } V_2/V_1$ is the voltage in voltage gain in DB it is okay so am but V_2 by V_1 is the gain actually what we have found so I actually write this function and star you know in long what is the log a into B/C will be $\text{log } a$ plus $\text{log } B$ here long if lake yep yeah theme Transco will occur there so the first term is $20 \text{ log } RL$ upon $RL+RF$ the second term will be $20 \text{ log } 2 \pi \tau s$ and the third term is $-20 \text{ log } 1+2 \pi F \tau a$ square or 4π square τR square.

So let us start plotting individual terms please remember if I want this total what is the theory I am applying when I say like this like this what is the theorem called this Plus this Plus this with a-what is it called so for positions so I actually do individuals and then superimpose them to get the net AVF Indy beads.

So the first term which is $20 \text{ log } RL$ by RS I plot as a variation of frequency whether it will be less than 1 or more than 1 because this is RL and the denominator is $RD+RS$ so it will be less than 1 therefore-so I say from 0 dB it will be some-value which is $20 \text{ log } RL+RS$ some value depending on the ratio of RL and $RL+RS$.

Please remember why because denominator is higher than the numerator then this is always constant because no there is no term in this which is frequency dependent so this remains constant is that correct why I do all this simple things pause because I want to make it very clear that things are trivial in real life okay.

Unless I really do not get you know excess or a analog for both bar here what is the value I repeat $20 \text{ log } R$ l upon $RL+RS$ has a fixed value throughout any frequency it remains constant is that okay that okay now use the second term the second term is $20 \text{ log } 5$ powers ok $20 \text{ log } 10$ power towers.

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$$\therefore |A_v(jf)|_{dB} = 20 \log \left(\frac{R_L}{R_L + R_S} \right) + 20 \log (2\pi f \tau_s) - 20 \log \left[\sqrt{1 + (2\pi f \tau_s)^2} \right]$$

1st Term

+ dB

-ve

$20 \log \left(\frac{R_L}{R_L + R_S} \right)$

$\omega (f)$

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So I say at $F=1$ upon 2π hours please take the frequency $F=2\pi$ tau s what is the value of bracketed term $2\pi F$ tau s is how much 1 how much is value of $\log 1$ $0 \log 1$ is 0 so it is at 0 dB is that clear F is equal-1 upon 2π hours if that is equal to 1 so at F is equal to this frequency the gain is 0 dB this second term only I am saying not the net one only second term if I multiply it by ten frequency that is if this becomes 10 upon 2π tau s okay.

Then what will be the value 2π tau s is 10 how much is locked in drop 10 is 1, so how many DBs 20 DBs so at 10 upon 2π powers how much is the value of gain dysfunction 20 dB is it okay 20 lakh 10h 20 DB by I take another other side balance point 1 upon 2 part of it that is 1/10 of that value so what is the again 1/10 means how much means how many d beads-20 db is that man clear if 0.1 means one more 10 1/10 means log.

So $\log 10$ is 1/4-20 DB will occur at one tenth of this frequency and 20 DB will occur when it is plus 10 times the frequency 10 times this frequency you are 20 DB 10 times less is-20 DB so if you go from 110 to ten how many slow how many how much gain DVD I increase 40 DB in how many orders of frequency how many orders to orders 10 and 10 okay so pearl 10 which is called decayed how much TV we are increasing 20 DB.

So the slope of this function gain for the second term is 20 DB per decade slopes are called 20 DB per decade is that kind of eyes because they are very 10 times this 20 DB will come into picture okay 100 if you do it how much it will be 40 DB if it is thousand it will be 60 DB is that clear so every times you increase ten times another 20 DB will appear is that correct.

So 20 DB per decade is the slope of this is that correct by same logic if I do the third function which is $20 - 20 \log \sqrt{1 + 2.5 \tau^2 \omega^2}$ up to when that F is equal to $2 \pi \tau \omega$ is equal to 1 how much is the value of that value should be at this frequency $-20 \log$ off no no not 2 root 2 $1+1$ under root is that correct so under group which in numbers it is 3 DB-3 dB is that correct this is -3 dB.

So what is it showing that at $F \tau$ is equal to 1 upon $2 \pi \tau \omega$ the gain has fallen from 0 to 3 DB down at this this frequency please remember initially how much of the game and f is smaller than 1 what is this spot I am showing you when f upon 1 upon 2π is a bachata so 1 plus 0 as a for I so I am getting a 0 DB gain as I start increasing the frequency that 1 upon 2π is term coming into picture and then we may start saying the gain will start reducing at at that frequency the gain would have fallen by 3 DB-3 DB.

Essentially 0 to-now this further down if you say what will happen one may be negligible or this and again 20 log decade it will start if you increase F 10 times again it will come the same way 20 DB per day so now you can say 3 functions which ones first one is this second this one is this and this one is this.

So I have 3 functions which I created and I now plotted 3 different frequency response term for 3 terms but what is the importance for us I want addition for this okay so if I add all of them together and this value is 0 dB please take it for a sigh but we will save a function deck tonight for please remember this is less than zero sum value okay from this frequency onward the gain is going towards this is 0.

Please remember if I I am adding now all 3 of them this Plus this and this is 0 okay so if I add all 3 there then what do I get initially of course this may be our L upon -20 this and it starts rising up

to what value it was rising towards 0 DB/2 pi towers but actually that value is not this is zero but this value from the third one is how much 3 DB down at this point it is not 0 but us-3 dB.

So it is somewhere-3 DB is that point clear up to this frequency at this frequency how much is actual gain-3 dB one is zero the other is-3 dB so this is-3 dB okay then I if you see further the gain starts falling and the other one is gain starts please remember this is 20 DB per decade this is 20 DB per decade down.

So what is the sum total of the 3/2 this is 20 DB this is Plus this is-how much is constant zero so afterwards the function becomes constant plus 20 you can see in math and geometry whites plus 20-20 is 0 0 ok per decade 20 debate or lies 20 degree it will decrease so it essentially no its opposite yeah you are right and I should have put instead of 0 but I showing that term is 0 small ok.

You are right what is saying is correct I should this term is not 0 what is saying is this term right now I lifted that you are right I fully appreciate so what is it trying to show you this means please remember this is the crux of it now this is the name which I am now going to give you this is the frequency response of the a transfer function I used this is the 3 DB point and what is this point is offering at where $F=1$ upon 2π hours.

Beyond this frequency gain is becoming constant gain is becoming constant if I draw 2 such lines one line straight array and the other from the 3 DB point straight way like this this point essentially represent 1 upon 2π tau s this point ok this is called corner frequency this is called corner frequency and actual curve is I some critically following these 2 curves is that correct I some totally.

So this is your real curve this is the curve which I am now modeling-is that clear this is the real value is that correct this is what I am now saying equivalent to lines like this here it may be so this frequency is also called 3 DB frequency or-in DB frequency see, but this is called corner frequency this thinking that such 3d waypoints can be put sharply are like this was first

suggested in 1940 by the famous mathematician or other control system man all control system term as people okay vice-a-versa all masked people can do controls.

What is the name of that person beret okay boy they are bored some people say I do not know I call it Buddha and yes I will call first time suggested that to look at the response I must know the corner frequency because I am interested in below which frequency gain is higher or lower or above it and actually I may calculate by actual function okay 3 DB down but I am NOT interested in the value of that I am interested which is that frequency.

Which is called the corner frequency therefore that is substituting that essentially was first suggested by Boni and therefore all frequency response plots were called bode plots of course this is only amplitude but will also see next time frequency phase part of the frequency okay is that clear to you is that clear.

So why it was called Buddha this first part is not what is people even now called this Buddha they kill, Buddha only suggested they should have a corner frequency concept which was first time suggested by him because in real life I want to know what is the frequency up to which gain I should not use or beyond which I should use or a vice versa I only want that value the actual magnitude may be 3d.

We know who cared I want that frequency is that and I will be always away from this corner frequencies okay to be guaranteed Li on this side on this side is that clear that is why these plots were called border so in future we may not draw the actual cars we made always border plots okay.

But, when you ask you to real you must do as some tours on that both side for example something like this occurs so you know it will be something like this is that clear it will be awesome to dig here and showing there here okay that is actual values but they will only draw like this is that correct this is how bad I thought and that is what the Buddhist diagrams are all about, see you next time you.