Microwave Integrated Circuits
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Mod 02, Lec 10
Non uniform transmission line (Tapers)

Hello! Welcome to another module of this course 'Microwave Integrated Circuit'. In the previous module we have covered on binomial and Chebyshev impedance transformation network, in this module we shall be covering another special category of impedance matching network known as Tapers.

Now we saw that when we have multi section transformers, both for the binomial or the Chebyshev polynomials that we have considered the bandwidth increases when we increased the section that is the general trend that we saw.

Then logically we should have an intuition that if we keep on increasing the number of sections then our bandwidth should go on progressively becoming larger and then when we make the number of sections as infinite we should have infinitely large band width. That is the intuition off course but is it true in reality. So for that to find out whether such thing is possible let us try to analyse these special classes of circuit where infinitely large number of sections are present and these classes of circuit are known as Tapers.

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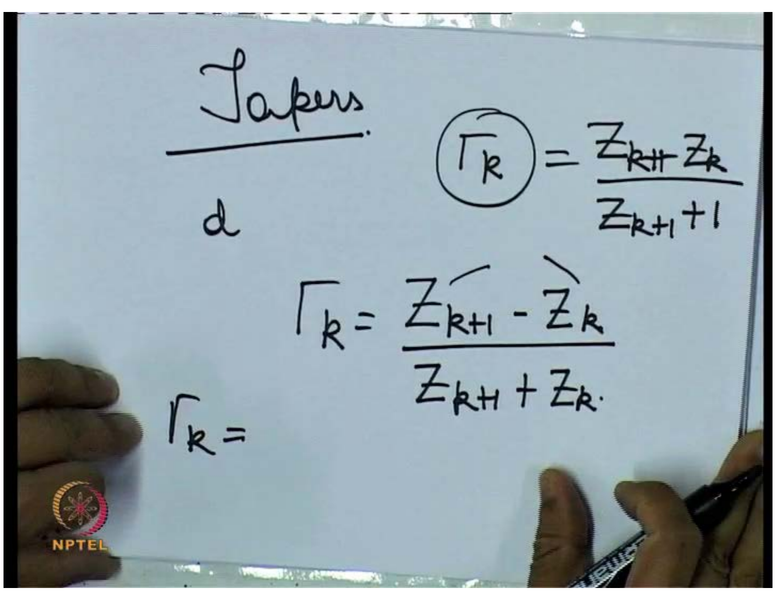

So to analyse, so as I said that the common intuition might suggest that if we go on increasing the number of sections as in the case in Tapers then we should obtain infinitely large bandwidth. So first thing is if we write Gamma K values that we have been writing in the previous sections as given like this, ok. Let me write it properly. So this is the value of Gamma K or the mismatch factor between adjacent transmission line segments that we have stated earlier.

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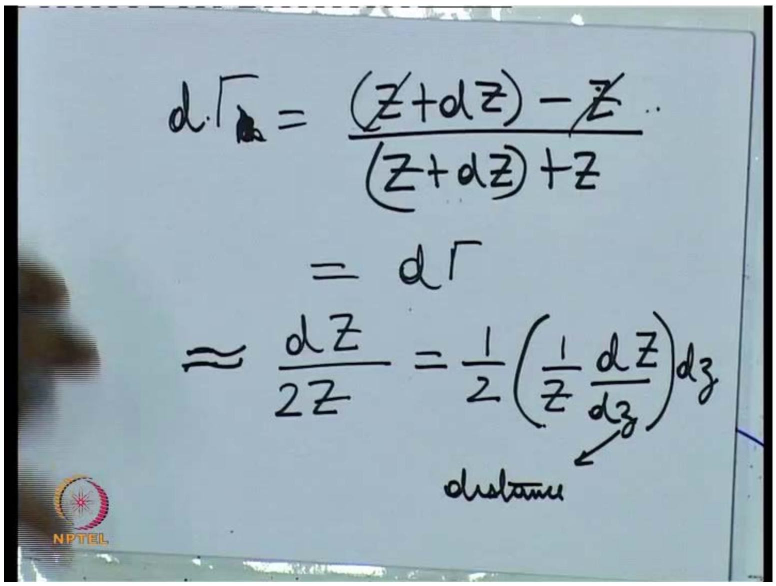

In the limit when say the difference between ZK plus1 and ZK is differential or very small. Let us say that Gamma K I define as… So if we write the Gamma K once again as the given by Z plus DZ minus Z upon Z plus DZ plus Z, here Z plus DZ and Z are the characteristic impedances of subsequent and proceeding transmission line segment respectively

And since this is the mismatch between two sections which have infinitely small differences in characteristics impedances, I can write, instead of writing Gamma K, let me write D Gamma K or I can all together remove this sub script and straight away write it as D Gamma.

Now this expression can be further simplified or rather I should say approximates to DZ upon Z, here the denominator contains two Z terms, Z and this Z this DZ being very small compared to the Z values, I can simply write two in the denominator. And then in the numerator this Z cancels this Z so this then becomes equal to 1 upon Z, D capital Z upon small Z and this whole multiplied by the small Z. Now here the small Z is the distance factor. This expression shows the variation of the characteristic impedance with respect to the distance along the Taper.

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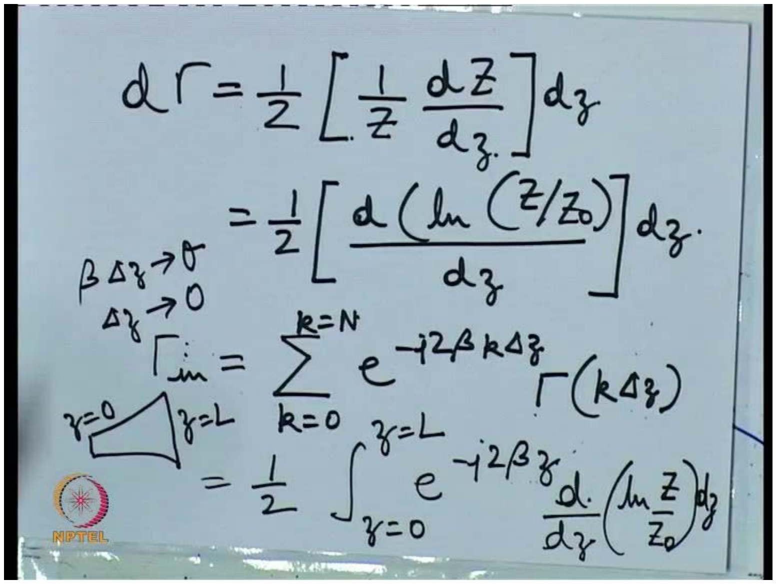

So then my complete expression for D Gamma becomes this. This is my complete expression for D Gamma. Now depending upon what kind of relationship I have between this Z and this is my small Z. That will give rise to various kind of Taper. Now if I come back to the original expression for Gamma in, now this using the Fourier series expansion that I had described earlier can be given like this. Now here instead of taking this as a individual values of K the limit the delta Z becomes equal to 0 or approaches 0, because this B delta Z now represents our theta.

And since this delta Z is approaching in the limit that it is approaching 0 I can write this instead of submission expression, I can write this expression as an integral whose limits are from Z to Z equal to 0 Z equal to capital L capital L is one end of the paper then the beginning of the taper is Z equal to 0 then the end is at Z equal to Capital L.

Something like this I hope it is visible. Instead of this integral expression, instead of this submission expression I have an integral from Gamma in. Now depending on as I said what the impedance profile or this capital Z as a function of Z is I get various types of tapers. Now this is the mathematical description of taper. Taper is an impedance matching network which has infinite sections, the question is how we achieve or what does it physically look like?

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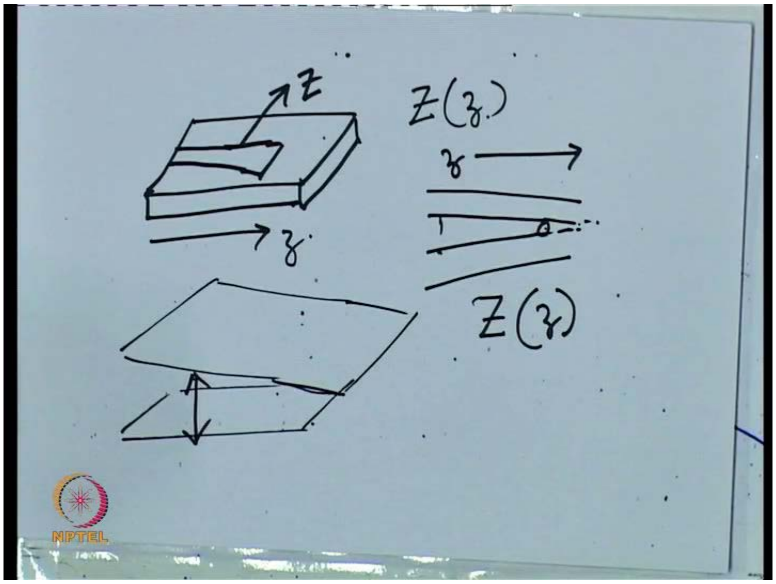

Now if we consider a micro strip line which is as we said in the previous class that they are the PVC boards so this is our PCV boards without top metal line like this then if the width of this line gradually varies then the impedance or the characteristic impedance of this line will also gradually vary. And thus we will have impedance profile Z of Z whereas Z is in this direction. For a coaxial cable it might be the way we can achieve is by having diameter of the cable slowly changing this part will have a lower impedance as compared to this part because of this lower thickness.

Here also we will have a variation of the characteristic impedance with distance and thus we can achieve the taper or if we consider a parallel plate wave guide where in the previous class previous module I said that parallel plate wave guide is nothing but two infinitely long plates at constant distant from each other.

If you want to implement taper then the plates will not be infinite anymore and the distance between them will keep vary. So the various kind of tapers that we have, the first most common type of paper from mathematical point of view the most commonly described paper is what we call exponential paper.

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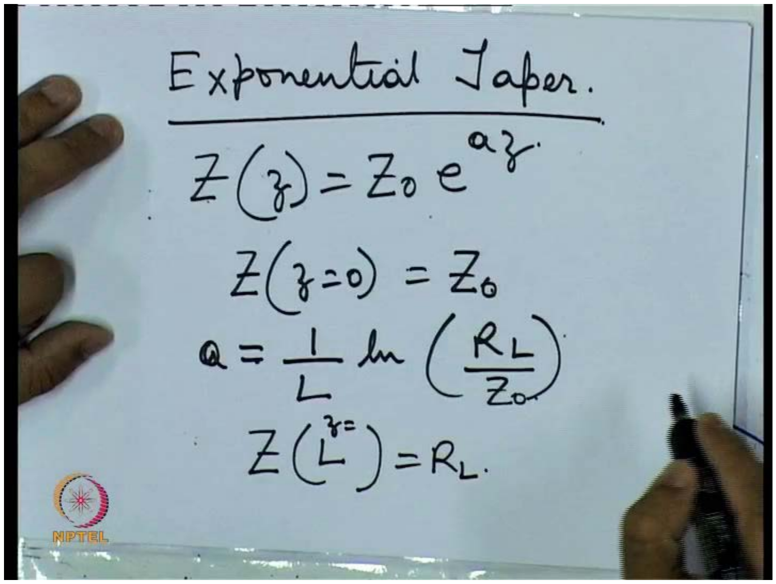

So exponential paper has an impedance profile like this where A is a constant, Z 0 is also constant, Z capital is the characteristic impedance small Z is the distance along the taper. And so from here we see that Z equal to 0 is equal to Z 0 and A can be given by this expression where this capital L is defined as that distance where Z or small z is equal to L becomes equal to RL. So it is the point it is that distance along the taper where we connect the load resistance.

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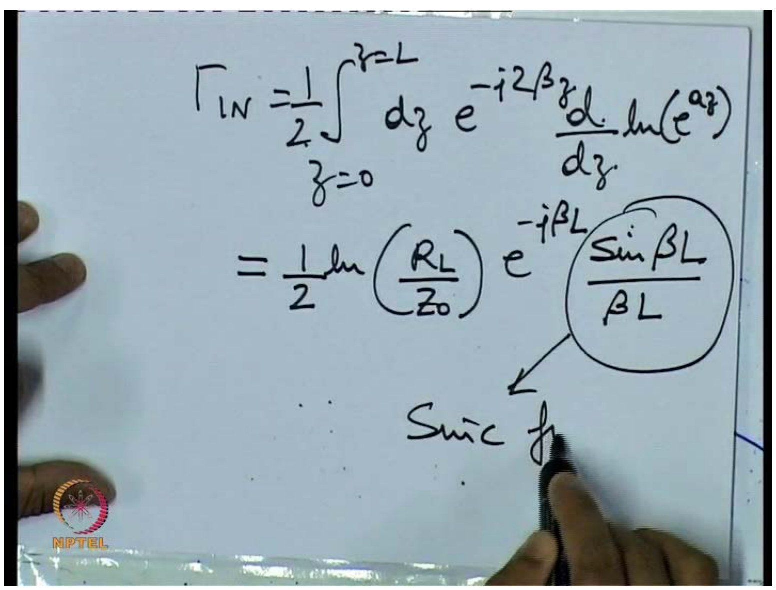

So now using this formula if we try to find out the expression for Gamma in. So Gamma in value is found using the integral expression that we have described earlier just a few moments ago. So this is the expression for the input reflection coefficient of a of a of a exponential taper. And as we as we know the various value of theta or this or as this distance from the input changes,

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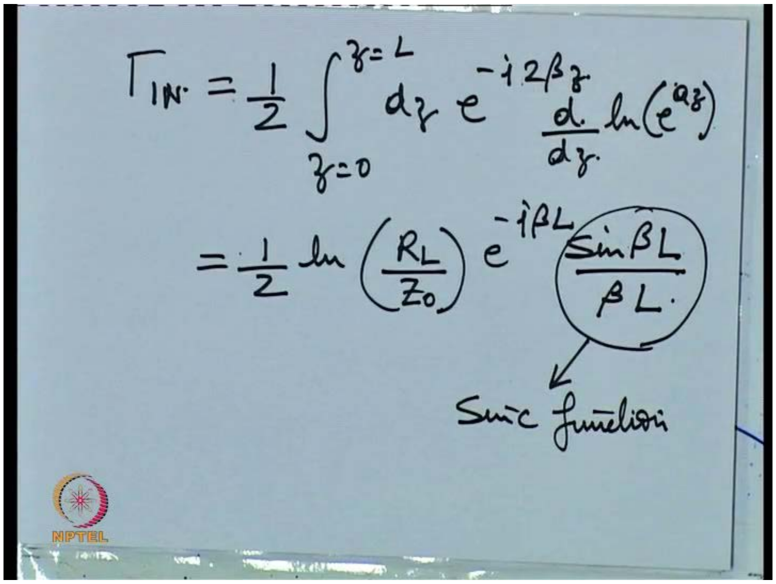

The value try to find out the value of Gamma in using the expression for exponential paper that we just derived… the Gamma in value can be written as like this and then on simplification we see it like this. So we see that Gamma In for this exponential taper is proportional to the sync function. There are other kinds of tapers that we see that we commonly use, one of these tapers if we go back to the monitor slides is what we call triangular paper function.

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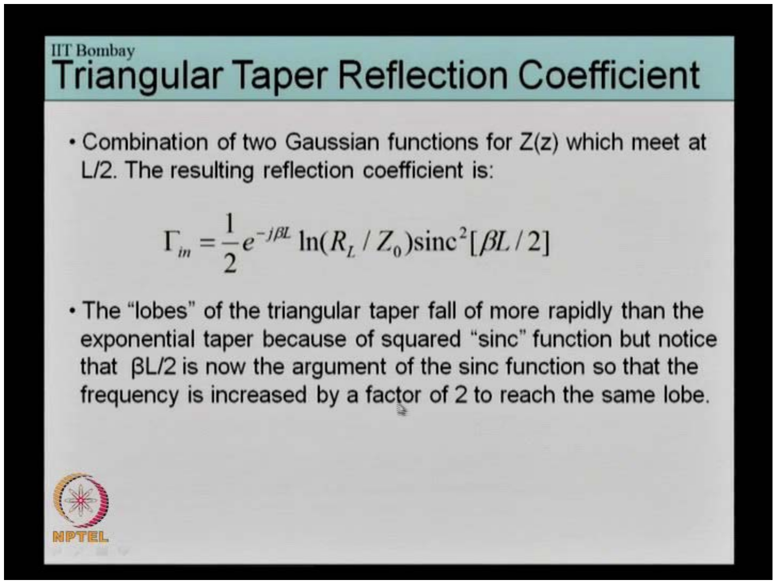

Now derivation of the triangular paper function is not that simple it is actually the combination of two Gaussian functions, in that case the Gamma in value is given like this which is actually proportional to square of the sync function. So impedance that we can derive from the triangular taper function or this triangular taper is that the frequency…

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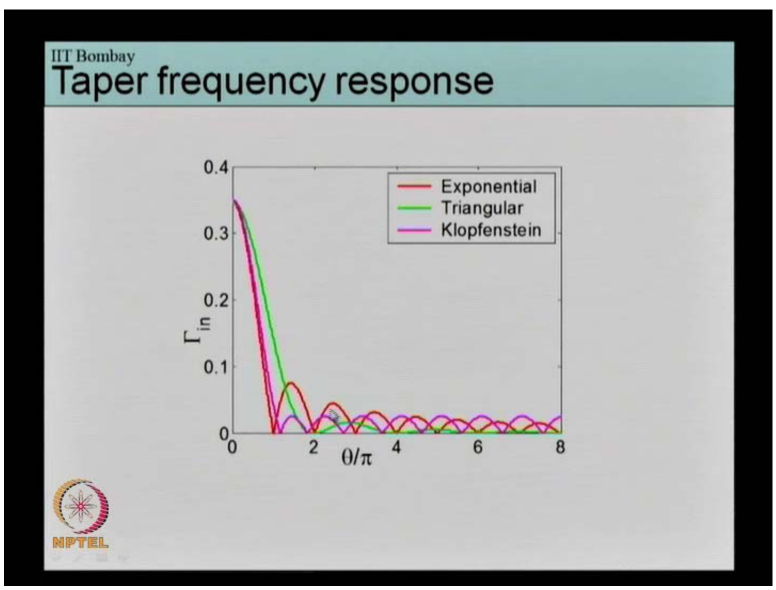

…if we go back to the slide that this green line represents the input reflection coefficient of triangular taper and this red line represents the input reflection coefficient of exponential taper. And we see that the frequency of the ripples for the triangular taper is actually half that of the exponential taper and this is because of the presence of the sync square term for this expression Gamma in of the triangular taper. Whereas for the exponential taper we have a sync function only, there is no power of sync.

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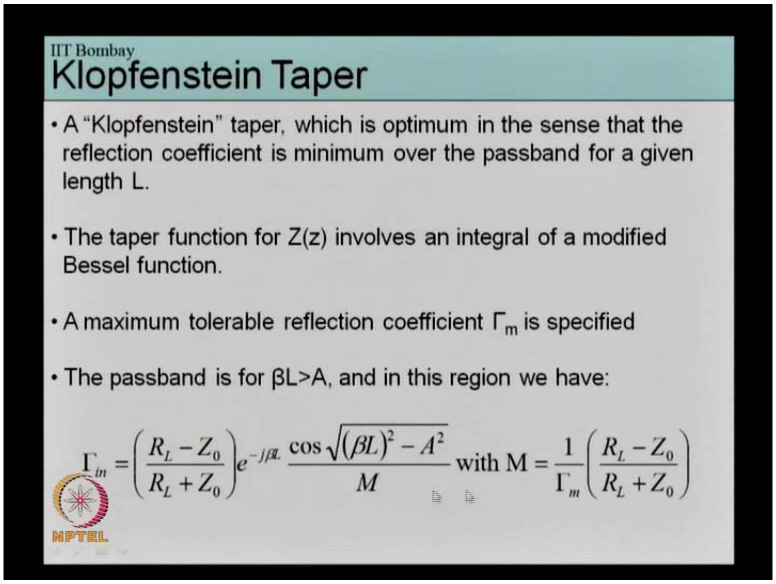

So one other type of taper that is commonly used is what is called Klopfenstein taper. If we go back on the slides of monitor it again involves a complicated impedance profile, it is actually the integral of modified Bessel function and the expression for Gamma in you get is something like this.

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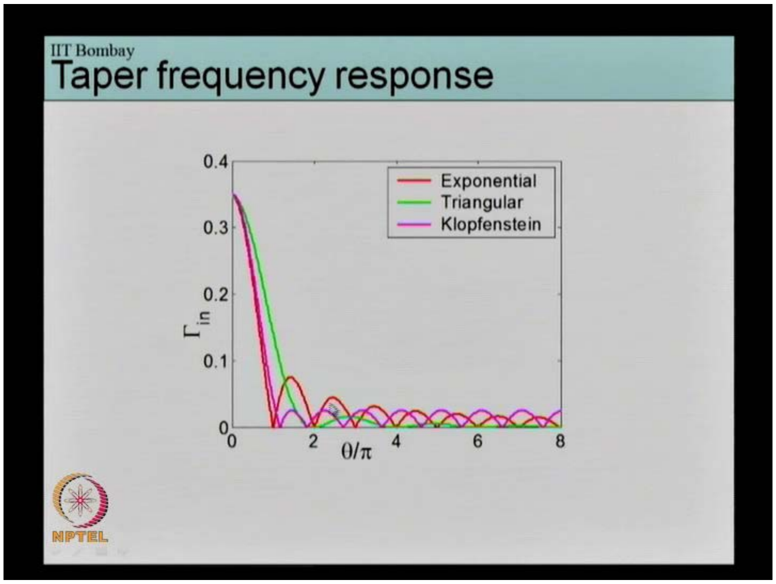

And if we again see the Klopfenstein taper this purple or violet line represents the taper of the impedance frequency characteristics of the input reflection coefficient various kind of tapers and this purple line shows the characteristics of Klopfenstein taper. One thing we note for Klopfenstein taper is that the ripples are all of same height. For the exponential taper the ripples nearer to the DC have higher heights as compared to the ripples far away from DC, where as for the Klopfenstein taper all the ripples are of same height.

For the triangular taper the ripples are of minimum height but then the bandwidth is much lower as compared both the Klopfenstein as well as exponential taper. That is why this Klopfenstein taper is often a good compromised between bandwidth ripple heights. Because for the exponential taper we see that we get the widest possible bandwidth but then we get the highest ripples. For the triangular taper we get the lowest height of the ripples but we also get the lowest bandwidth and Klopfenstein taper is somewhere between the two.

So in summary I would like to mention that tapers are in commonly used in microwave engineering. In fact at any time one reflection at any surface or device to be less, tapers are commonly used. And tapers also prevent the formation of spurious reflections; they also prevent the formation of what are called evanescent modes. Because any sharp corner or any sharp transition is always on undesirable modes. Tapers solve that problem they help to gradually move from one impedance to another impedance and in that way they enable us to have smooth impedance transition as well as tapers are also often used in antenna structure.

In fact many of the commonly used antennas are tapers for example horn antenna, TEM horn antenna or pyramidal horn antenna or the corner antenna they all are examples of tapers that we used in commonly. And they solve important purpose in microwave engineering as well as antenna engineering.

Thank You.