# Microwave Integrated Circuits

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**Mod 03, Lec 11**

## Scattering Parameters

Hello! Welcome to another session of this NPTEL course 'Microwave integrated circuits'. Today we are in deep three of this course. In the last class we have talked a lot about the various microwave theories, the techniques used in this module we are going to talk about a special class of parameters, circuit parameters that are frequently used in microwave engineering and they are called scattering parameters.

Now we all know that there are various circuit parameters that are already well known for example, the impedance parameters, the admittance parameters, but then all these parameters are dependent on the presence of voltage and current and microwave engineering we don't… let us say that voltage and current do not make much sense.

One reason is because in single conductor waveguides you cannot have a proper definition of voltages and currents and in two conductor wave guides they are very difficult to measure. So instead of voltages and currents we tend to talk in terms if incident and reflected parts and s parameters are those parameters which relate the incident and reflected wave at the various port of a microwave device. So let us so let us see what these parameters are.

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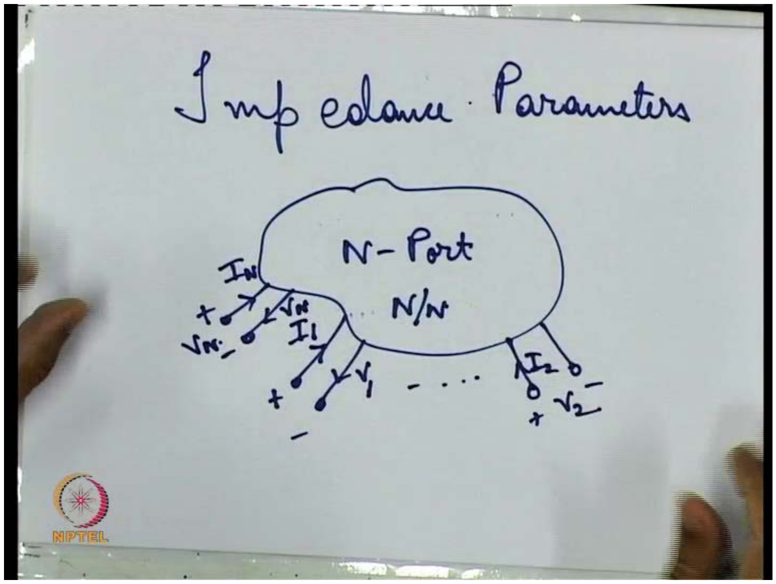


Now before going on moving on to the scattering parameters or S parameters. Let us see what impedance parameters are so I am sure this is well know impedance parameters. If we say have an N port network with various ports as shown like this, then the matrix that relates all these voltages and currents to each other is called a impedance matrix.

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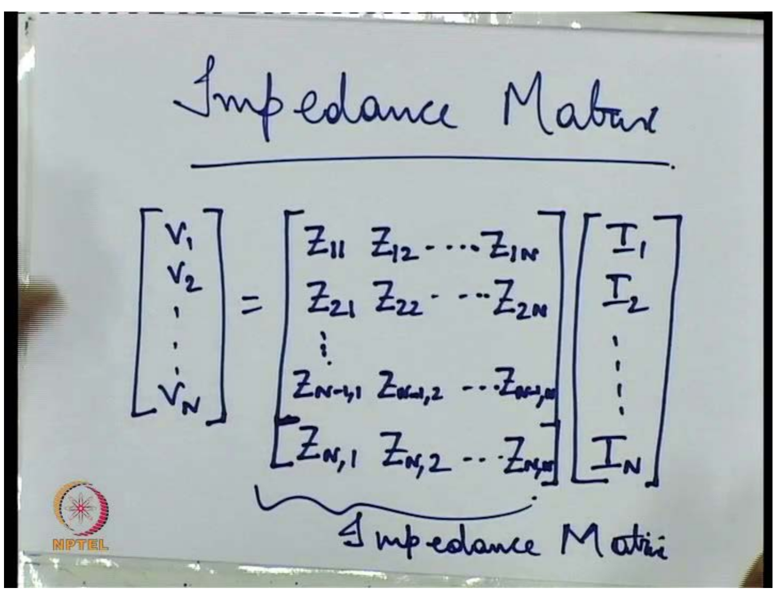


So if we can if we can write down the impedance matrix. Suppose we have this column matrix for all the voltages at the various ports and say this is my impedance matrix and suppose this column matrix is the currents presents and the various ports then this matrix is my impedance matrix.

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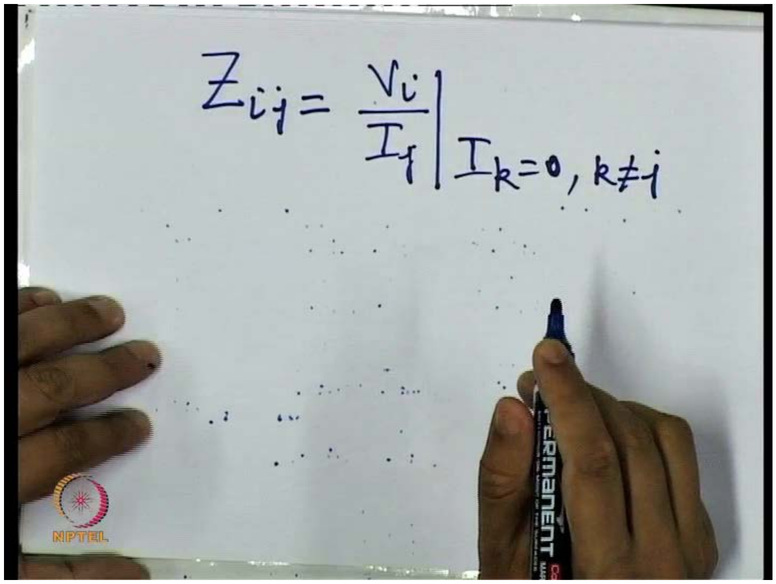


So any impedance parameter J I J is defined as V I upon I J with all other I Ks equal to 0, k not equal to J. Now similarly in the same way that I defined impedance matrix I should also be able to define what I call the admittance matrix.

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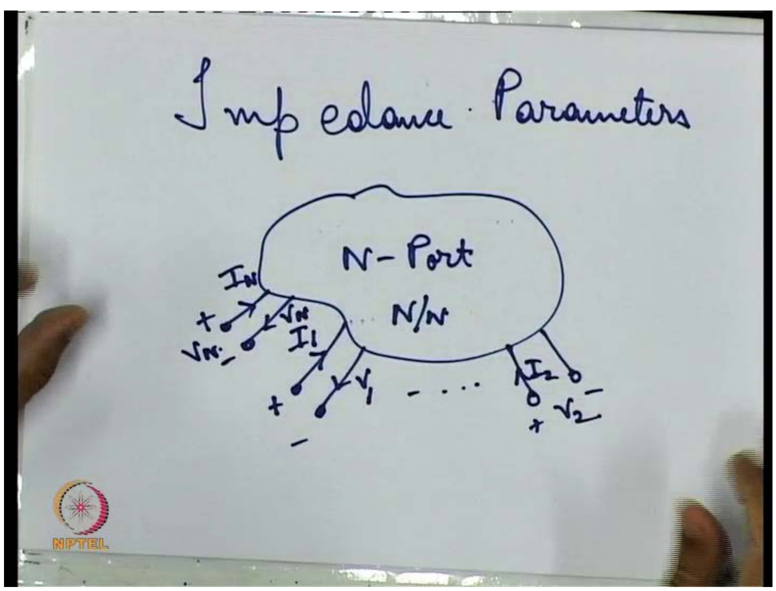

So once again if we can go back to the slide this is our N port network with currents and voltages for the various ports as shown.

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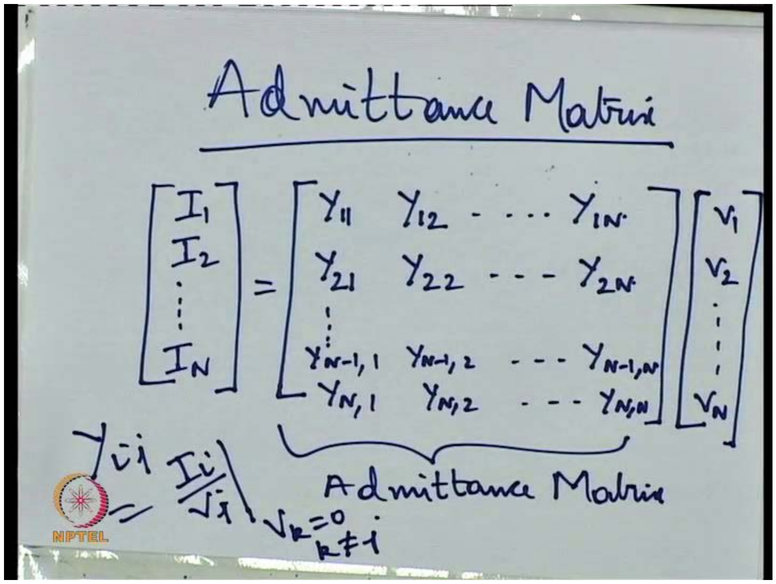


Then admittance matrix is defined as and any admittance parameter Y I J is defined as where V K is equal to 0, k not equal to J.

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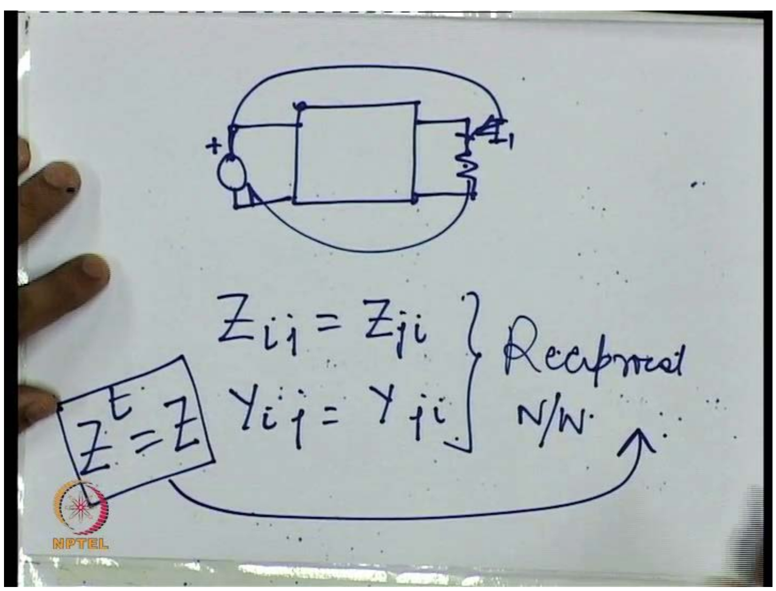


Now there is a concept of reciprocity various materials or various circuits are reciprocal, what it means is that if we have a network we have some network and if we apply some voltage at one part of the network and say in another part we get a certain I I, I 1 then reciprocity means that if we reverse this that if we… no in place of this if I suppose connect this resistor at this end, and this source at this end then I should be able to get the same current at this at this branch so that is the concept of reciprocity.

Now in simple terms reciprocity in terms of the Z and Y parameters that we just defined is that Z I J is equal to Z J I or Y I J is equal to Y J I. So these are the similar, they are the same expression so it's not both these have to be satisfied any one of these with any one of these condition is satisfied then the network is said to be a reciprocal network.

And since this is the condition I can also write it in a different way that of my matrix that is either my admittance or impedance matrix that I had defined suppose this Z matrix, I call that represent it by the capital Z symbol the if this condition is satisfied that is the transpose of Z is equal to Z then again that implies that the circuit is or the network is reciprocal.

So with this let us try to derive some properties of reciprocal network so first net the first property that we try to, I beg your pardon let us try to derive some of the properties of  network for various cases like reciprocity and lostlessness. So for reciprocity we already saw that the Z or Y matrix should be equal to its transplacement. Let us see the condition for lostness.

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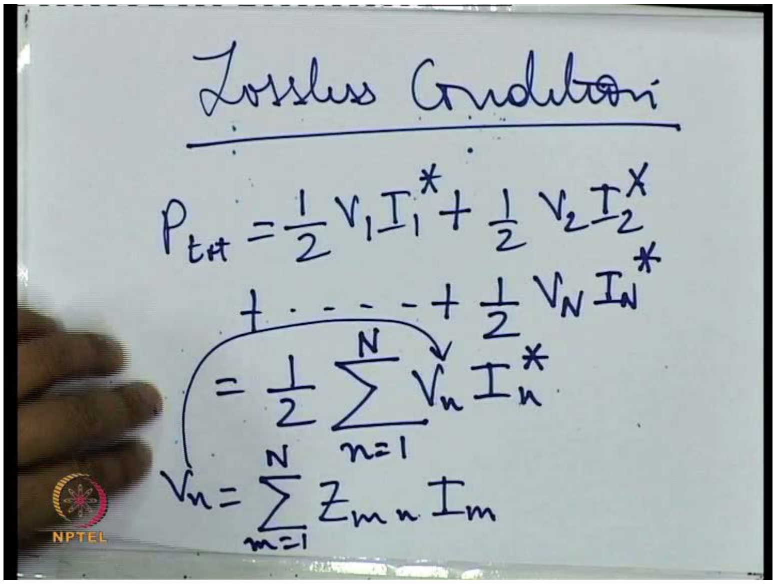

Now the total complex power decipated by an N port network can be given by this relationship. So this is the total complex power decipated at port 1 V 1, I 1 conjugate plus half V 2 I 2 here of course we are assuming that the V 2 and I 2 are the peak values not the R M S values that's the reason we have half symbol here. So this how we can represent it. Now if we suppose write an expression for V N like this from the Z matrix then and then if you flag it bag here…

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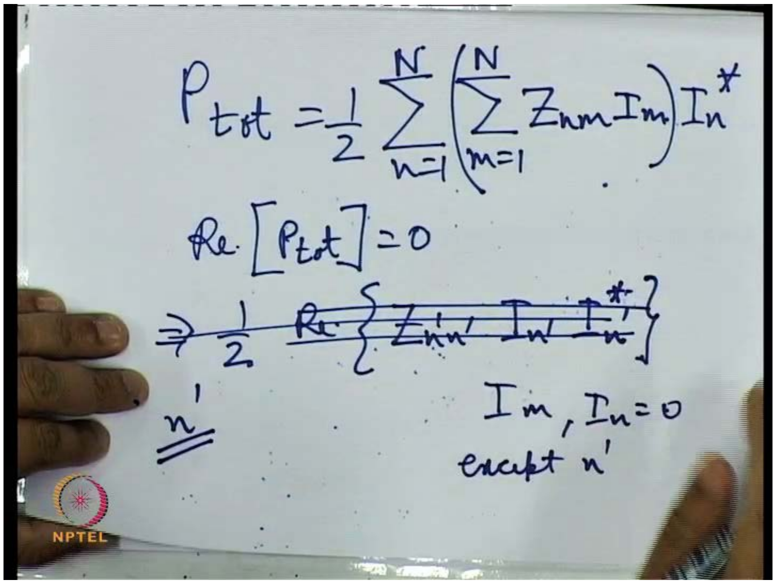


…then what we get is that the P total is given by… in the previous expression I by mistake had written this as Mm, this should be Nm. I want to correct this. So then we know for a lostless network the real part of this P total is equal to 0 so then this implies that half of V L. So now let me explain it in a different way.

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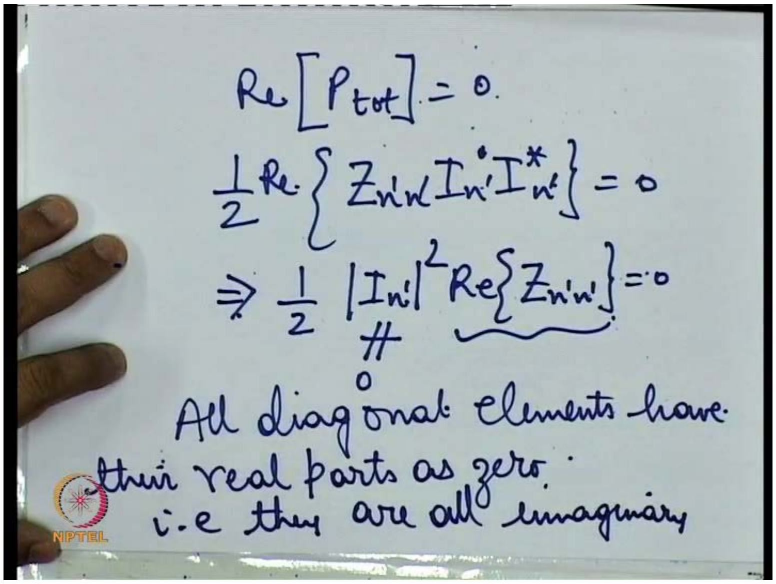


Suppose considered that only one of the one of the ports currents are non 0 and let that port be called as N dash. So only the current in N dash is non zero. So if we now so all other I M or I Ns equal to 0 except N dash. So now if that condition is, if we if we plug that condition in this equation then what we get is that for real value of P total to be equal to 0 we should have half of real N dash N dash, I conjugate N dash should be equal to 0.

And this implies that half of I N dash whole square. Now since I n dash is non 0 so then only this thing should be 0 in other now look this N dash can be any value. Now here this element is a diagonal element Z N dash N dash so what it means is that this particular diagonal element is 0, that is the real part of this particular diagonal element is 0 but then since N dash can be any value any one of those diagonal elements, we have to conclude that all diagonal elements have their real parts as 0.

That is they are all imaginary see for only the network is lostless then this is the condition then all the diagonal elements are purely imaginary.

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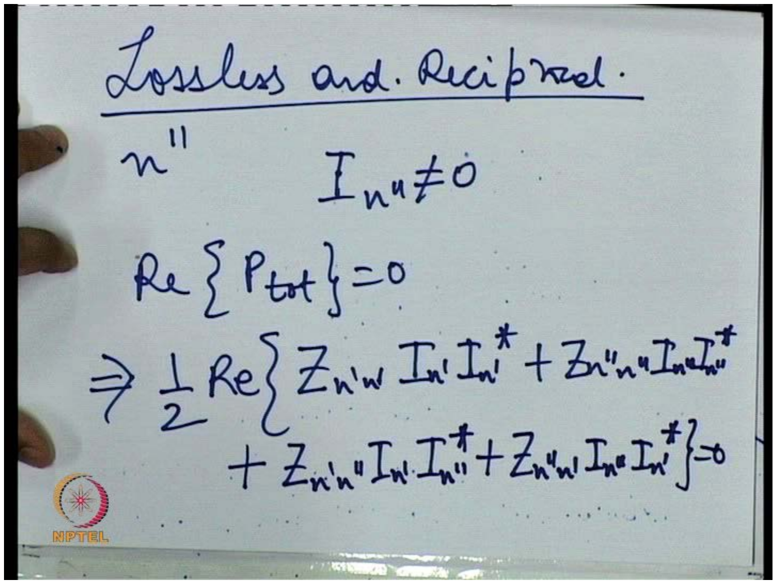


Now if we if we consider that this network is both lostless and reciprocal then what happens, so lostless and reciprocal. Now consider that in addition to N dash the port N dash we also have another port N double dash where the current is non zero.

Okay, so then if N double dash I N double dash is also not zero then the condition of P real part of the P total being equal to 0 that is the lostless condition equates to once we have derived this equation from there, we can further because of that condition that all diagonal elements should be purely imaginary from that we can simply extend that equation as. This we could do because if we go back to our expression for this…

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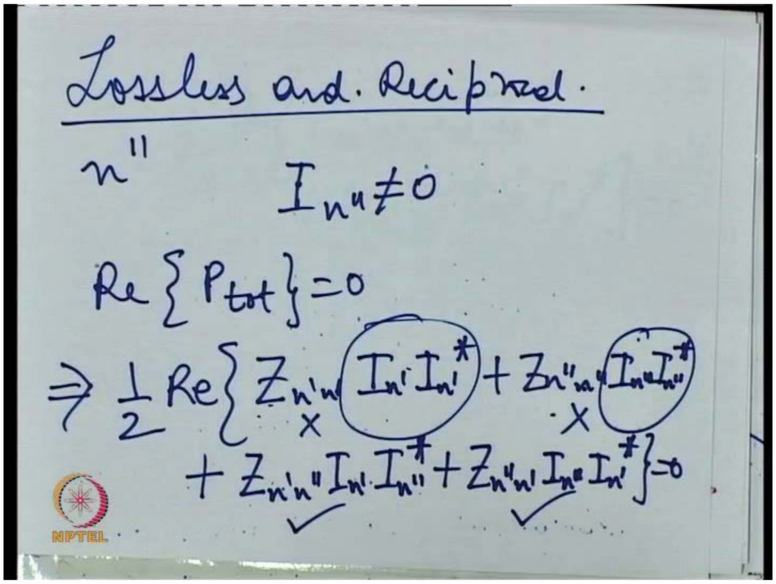


We saw that Z N dash N dash is purely imaginary so is Z N Double dash N double dash and these 2 terms that I N dash I N dash conjugate and I N double dash I N double dash conjugate these are purely real so then this whole term is purely imaginary hence the real part of these sum of these 2 terms that is this term and this term will be 0.

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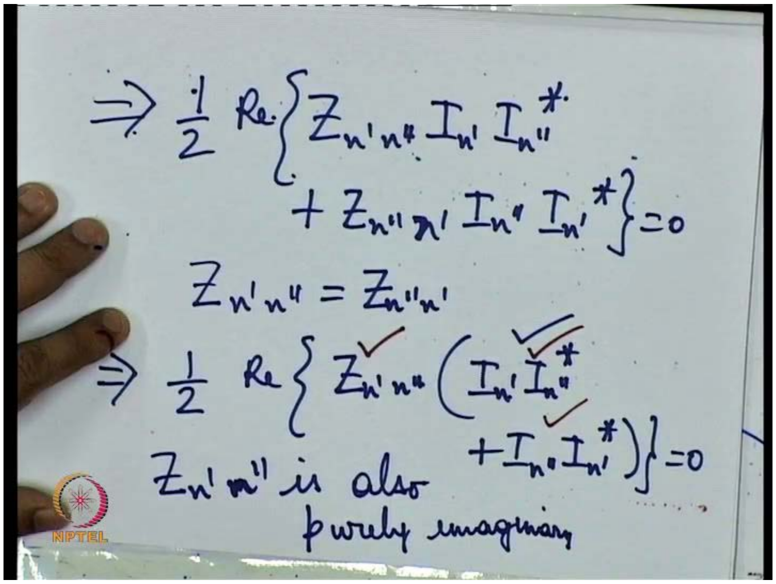


And only this term and this term will be remaining, so that is what we have kept here only the remaining 2 terms, so since this is also a reciprocal network we have defined our network like that. Hence Z of N dash N double dash should be equal to Z of N double dash N dash and this condition if we apply to our circuit translates to… Now this term is purely real just like the other 2 terms these 2 terms are purely real since the real part of this whole term is equal to 0, hence this term must be purely imaginary.

If this term is purely real and the whole [thin] whole real part of this expression is 0 then term has to be imaginary if it is real then it cannot be 0, so then we conclude that Z of N dash N double dash is also purely imaginary therefore for lostless and reciprocal network we have that all the elements in the Z or Y matrix are purely imaginary.

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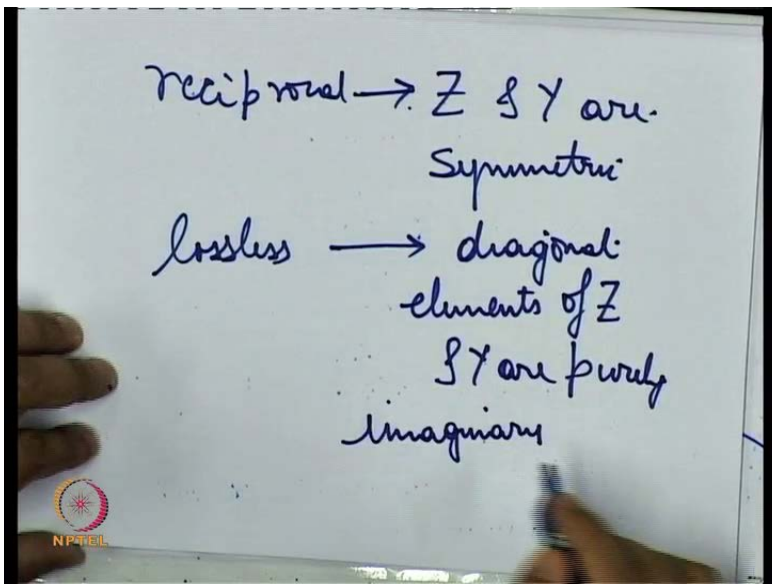


So in summary we can write it like this.

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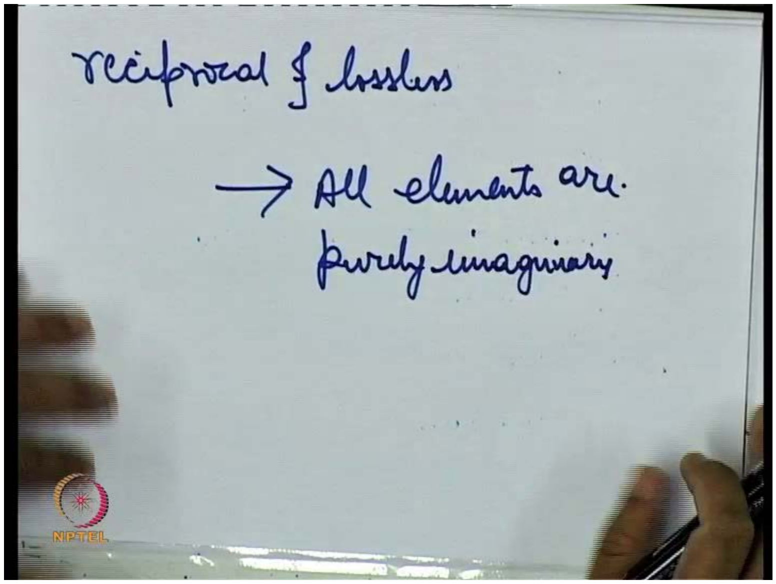


And if the network is both reciprocal and lostless so now we have analyzed our network for with respect to the concepts of impedance and admittance parameters.

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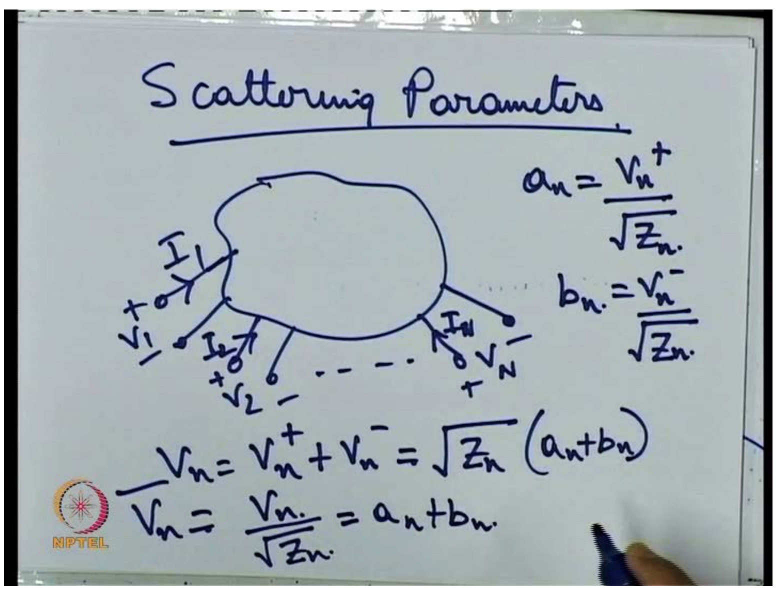


Let us now go to the next and main topic of our discussion here which is the scattering parameters. Now scattering parameters as I said have several advantages, 1 is that they have the concept of power so let us go back to that N port network which we had talked about earlier. Now we also know that the normalized voltages or currents at the various ports are given in terms of the incident voltages as follows.

Where V N and V N minus sum of them is equal to the total voltage at the port and this we can also write in terms of a N and b N and then we also know that normalized voltages at V port is given in terms of the actual voltages as shown, so this can simple be witnessed AN plus BN.

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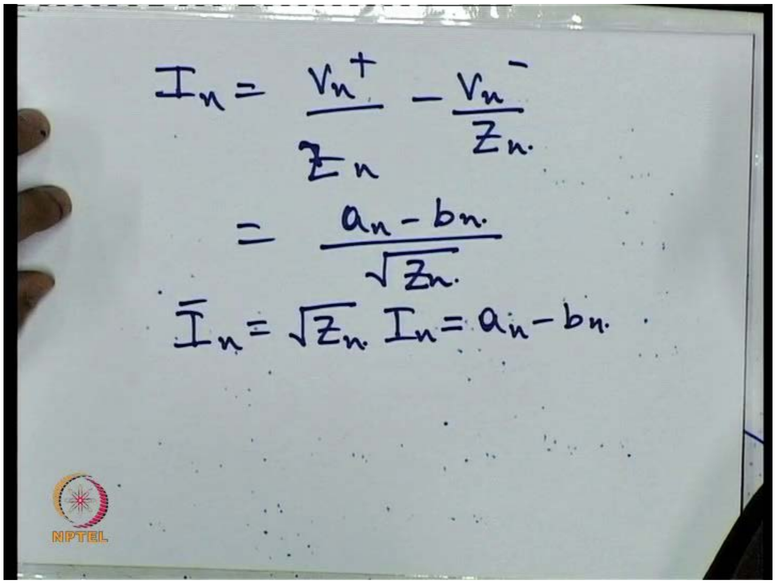

So then with this in mind we can also then jot down the formulae for the current the current as you know is given by… And then the normalized current is given as this.

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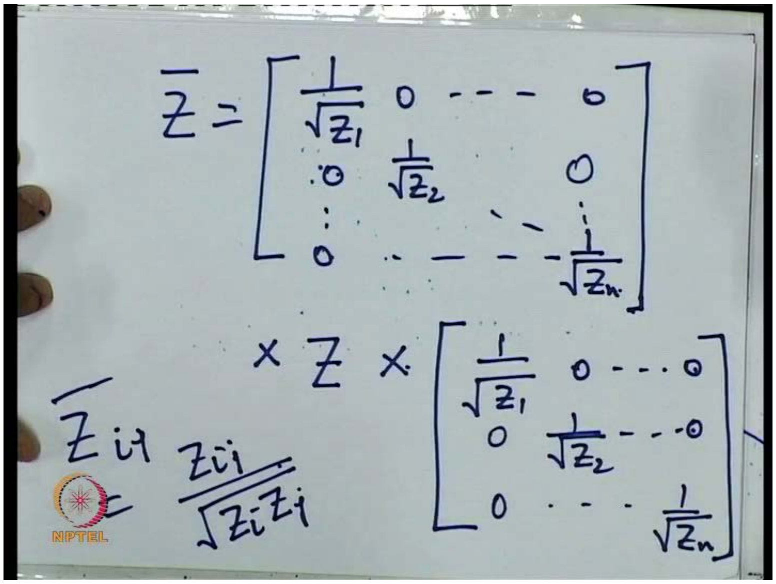

We can also have an expression for the normalized impedance matrix just like we have normalized voltages and currents the normalized impedance matrix will be given like this, so any particular element in this normalized matrix is given like this.

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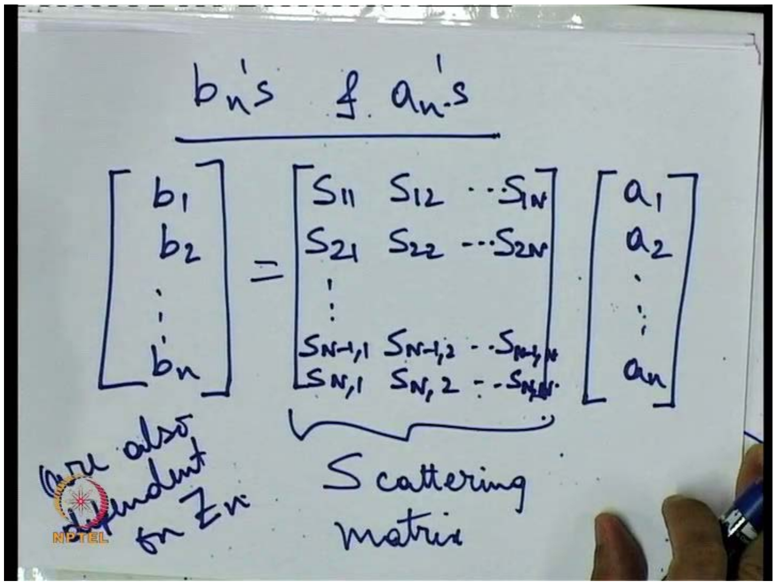


So then you know we can see we the input and output the incident or reflected waves are given as B Ns and A Ns. Now if we write a matrix relating the Bs to the As like this so these are the value this column matrix represents the reflect a normalized reflective voltages or normalized reflected currents at the various ports and say at the other end we have another column matrix representing the incident normalized voltages and currents or currents in the matrix that relates the 2 is called a S matrix.

So we see that just like the impedance and admittance matrix this is also a matrix that relates to circuit elements or 2 circuit parameters of course here the circuit parameters are defined in terms of power rather than voltages or currents and the other difference that we will see in a moment is that these parameters, the circuit parameters this S [Mat] parameters or scattering parameters are also dependent on the characteristic impedances Z M.

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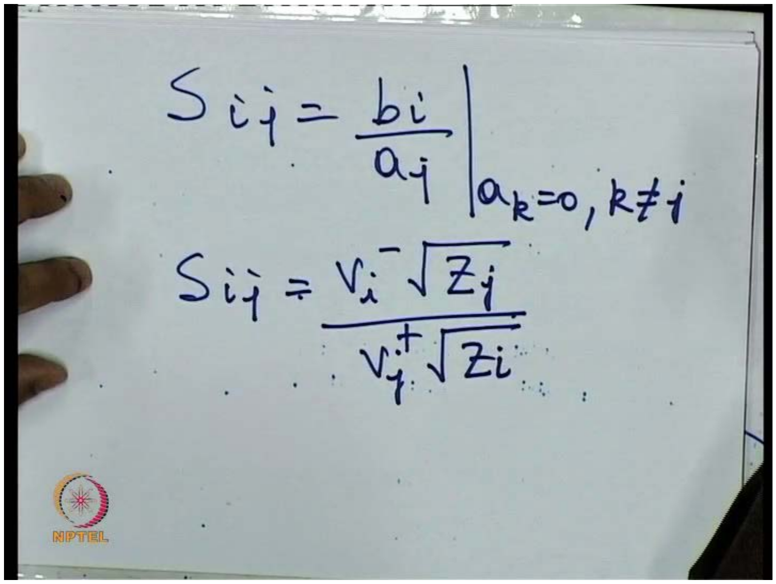


So if we want to define a particular S parameter then the definition the formal definition of S I J is with A K equal to 0 or K not equal to J, alternatively we can also define it as.

Now 2 port networks are very popular one says 2 port networks are the most widely used microwave components both for passive as well as active circuits. Some examples of 2 passage circuits are printers then alternators and for active circuits we have amplifiers which are also 2 port circuits so popular at these networks that we have some special names for their 2 port S parameters matrixes of the elements of the 2 port S [Pa] S matrix…

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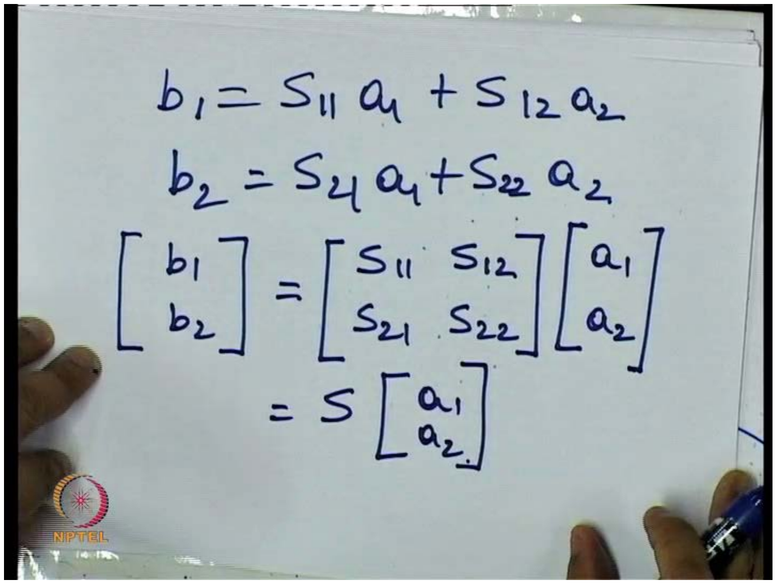


So for 2 a port network we can write the Bs and as B 1 is equal to S 1 1 A 1plus S 1 2 A 2 and B 2 as S 2 1 A 1 plus S 2 2 A2 and these B 1 and B 2, I write it in the form of a matrix.

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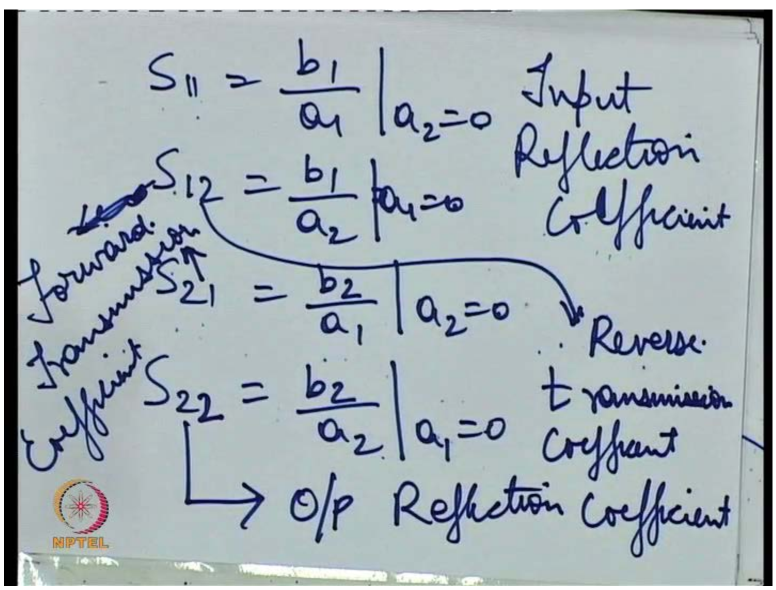


We have a total of 4 S parameters for a 2 port network these 2 port networks these parameters I can just write down like this is equal to B 1 on A 1 with A2, so this is B 1 on with A 1 equal to 0 then this is equal to B 2 upon A 1 with A 2 and this is S 2 2 B 2 upon A 2 with A 1 equal to 0.

At this parameter this S 1 1 is known as the input reflection coefficient, this is now S 1 2 is known as the forward transmission coefficient, S 2 1 sorry this S 2 1, I beg your pardon is known as the forward transmission coefficient. S 1 2 is known as the reverse transmission coefficient and S 2 2 is known as the output reflection coefficient.

Now we have certain special properties for these S parameters matrices just like you derive the conditions the 2 port for the Z parameters and the Y parameters that is under when they are lostless or reciprocal or both lostless and reciprocal. Similarly for these S parameters also we can derive certain conditions and that we shall derive in the next module.

Thank you.