Microwave Integrated Circuits
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Mod 03, Lec 12
Properties of scattering parameters

Hello! welcome to another module of this NPTEL mock course ‘Microwave Integrated Circuits’. In the previous module we had discussed about scattering parameters and we saw that they are related to the power transmission in a network that is then voltages and currents. In this module we shall be studying the properties of the scattering parameters just like we studied the properties of the impedance and admittance parameters.

So for the impedance and admittance parameters we saw what the properties those matrices take when we have the condition that the network is reciprocal or the network is lostless. Similarly let us try to see what happens for the scattering parameters.

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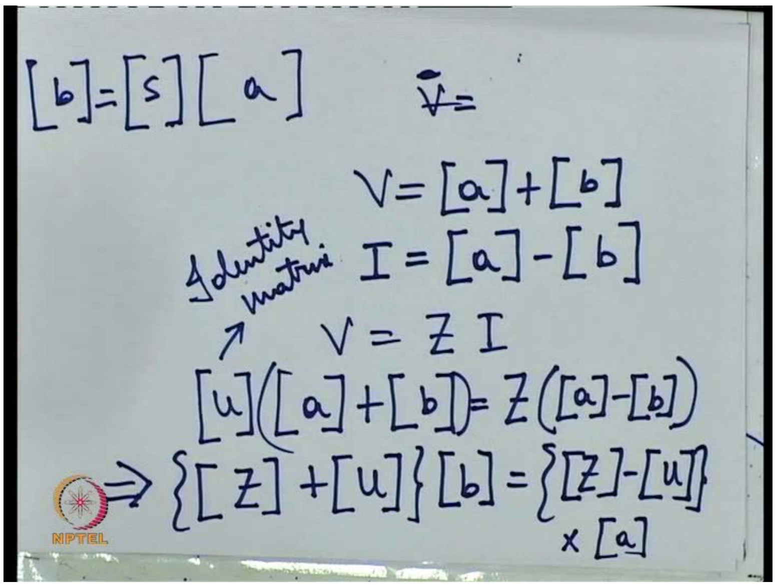

So we saw that our scattering parameters relate the incident to the reflected normalized reflected or normalized reflected voltages or currents suit this equation so with in mind let us for a moment and we also saw that the voltages, the normalized voltage, let us remove the bar here and just assume that V and I hence port means the normalized voltages and current.

So then V we saw is equal to the sum of the incident and the reflected waves and current is equal to the difference between them. Now you might ask me why again I came back to the concept of voltage and current because I had said that voltage and current cannot be measured so easily, well here we are not measuring voltages and currents we are simply using them as a mathematical tool.

Just so that you can arrive at a different result the final results will not have any of these voltage or current. So then with you know we can write this V so if we know that voltage and current are related to the impedance matrix where the Z represents the impedance matrix and the voltages and currents are given by these 2 relationships so I can simply write this as…

And then this if I just take the identity matrix outside, usually the identity matrix is represented by the symbol capital I but since I have already used that symbol for current I cannot, just to separate between the 2 this I representing the current I use the symbol U.

So now from here I can write down this expression this is expression that I can get and then we know that S parameter matrix is a relationship between B and A. If I just take this whole matrix the inverse of this matrix on both the L H S and left hand side and the right hand side then we can simply write this equation as.

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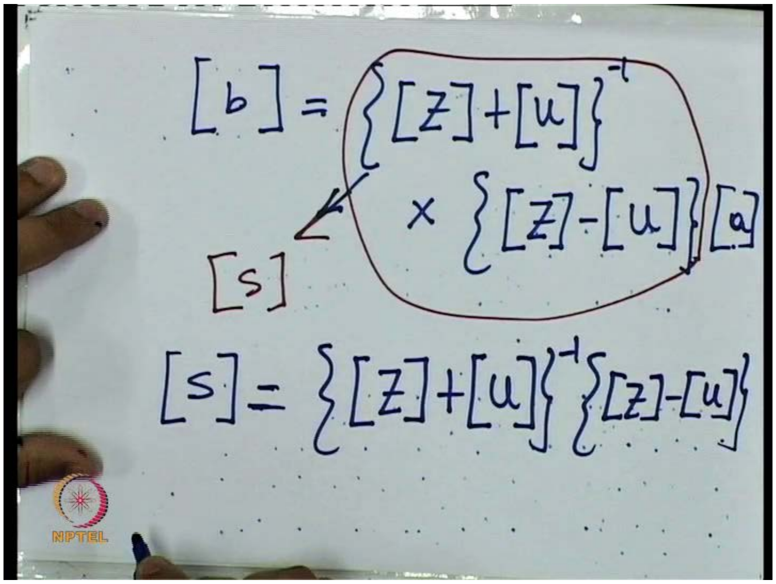

So then this must represent my S so this whole multiplied by A, so this whole term must represent this whole term must represent my S matrix. So then S matrix is defined like this in terms of this equation I can define my S parameter matrix as.

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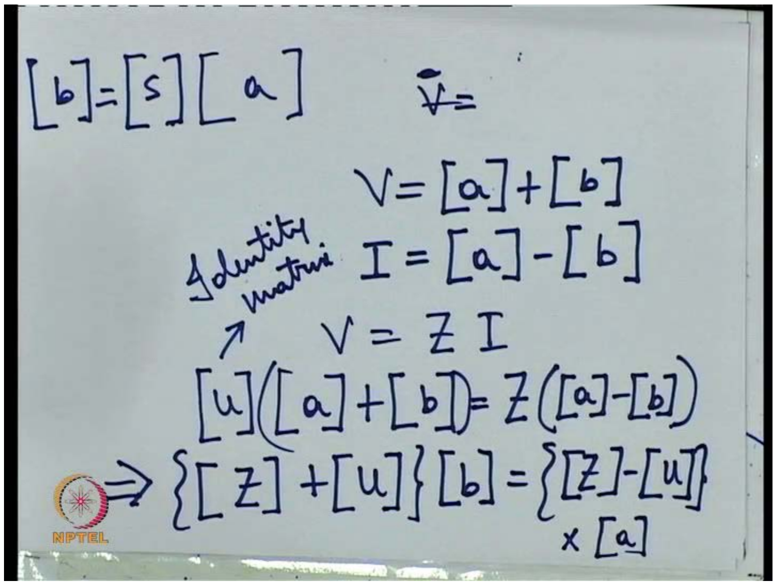

Now using those previous 2 relationships that I just described earlier that is my expression for V and I using these 2 expressions.

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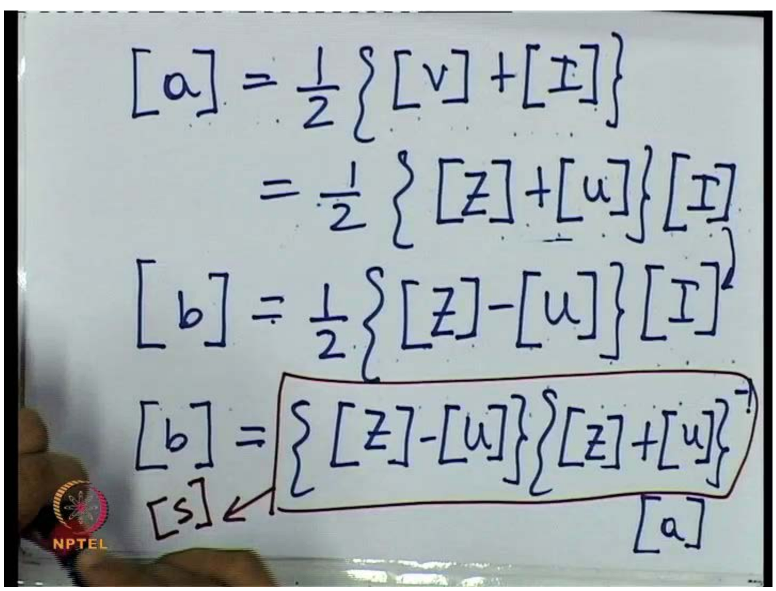

I can also derive an express by, I can derive an expression for my A and B individually, so A in terms of v and I is VN like this and this is equal to…Here we can write this because I have resolved V as equal to Z times I and so then I can take I common from both the 2 if I take I common from V then I had Z inside and if take I common from this I will say you have the identity matrix resolved.

And B similarly can be written as here once again I got these 2 expressions by solving for A and B in terms of V and I comes these 2 equations so once I get equations I can I can write in terms B and A I can I can write it the expression for B as I am getting this expression by finding an expression for I in terms of A. So here if I take the inverse of this matrix and both the left hand and right hand side then I get an expression only for I and if I substitute that expression for I in this equation then I get this equation.

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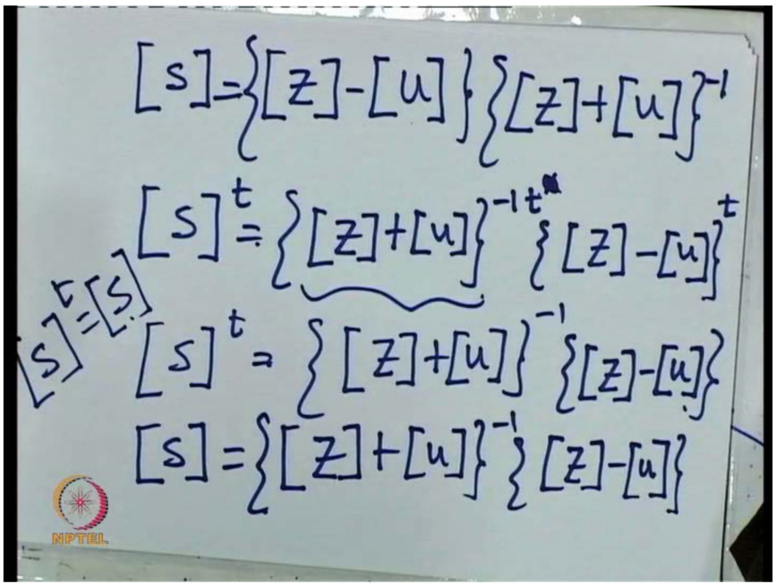

Now this is again relating B to A so then this part must be the S matrix so if that is the S matrix then let me write down the for the expression so from the equation that I just derived S parameter matrix than the given like this. If I take the transpose of this equation then what I get is…

Now we know that U and Z are symmetric for a reciprocal network if both U and Z are symmetric that is the case for reciprocal network then this whole expression must also be symmetric, so then in that case we can simply write S transpose as equal to Z plus You, the transpose term this term gets transit so only we will have the inverse.

Similarly this transposed term will also vanish. Now since this expression we call this same as the expression that we had earlier derived for our S parameter matrix if we recall that expression for S parameter matrix that we had earlier derived was like this both, these 2 are same but the left hand sides are not the same.

So that means for a reciprocal network we can simply write that S transpose is equal to S so this is the condition for reciprocity.

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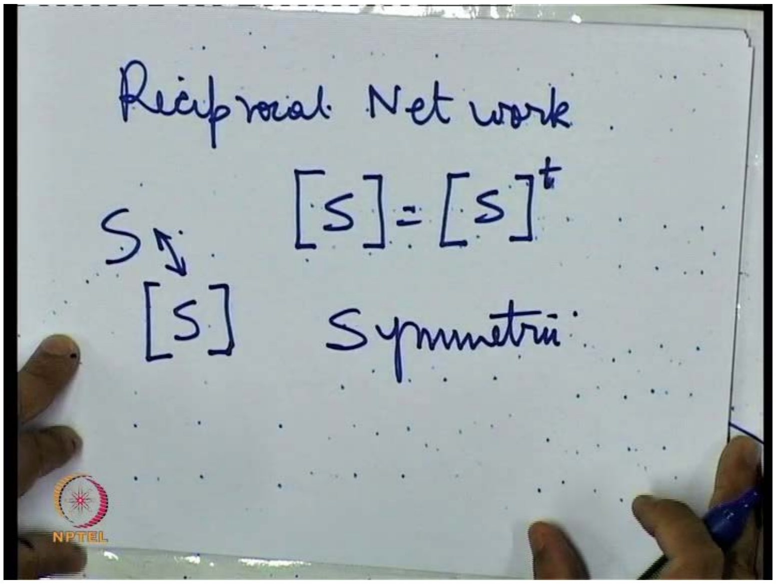

Reciprocal network S is equal to S transpose now I just want to give a short short description of my of my naming convention in some K places I am using S the capital S symbol to represent the S parameter matrix and some places I am using a bracket now both these 2 are equivalent sometimes due to the J, I am familiar with I these both these symbols interchangbly.

Ok so. So this is the condition for a reciprocal network that is its transpose is equal to its original in another words the matrix is symmetric. So this condition is very similar to the condition of reciprocity for the Z and Y matrices let us see the condition for lostlessness.

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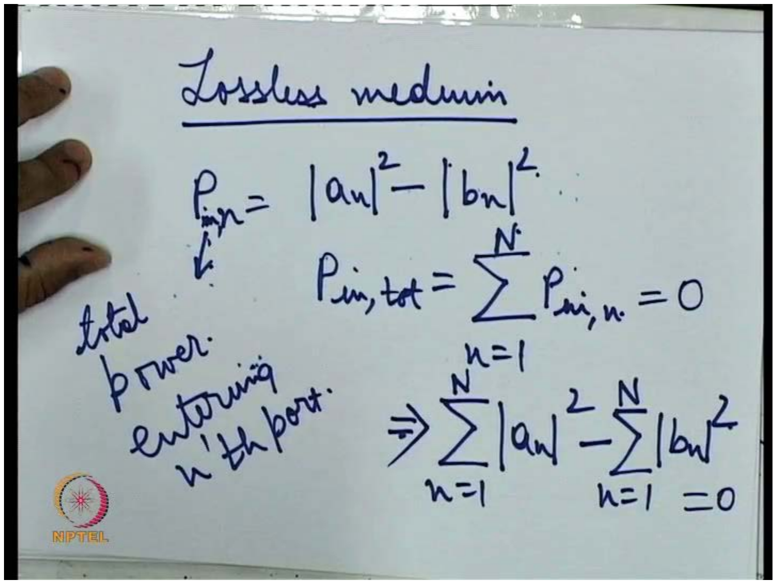

Now in a lostless network the portal power entering the nth port is equal to the sum is equal to the difference of the power carried by the incident wave and the power returned by the reflected way, so that way I can say the portal power entering is given like this so then the total input power for all ports or the total power entering the whole network is nothing but the summation of the powers entering all the ports.

So this p in just to be clear, and this in turn should be equal to the summation of An square minus the summation of Bn square. Now this total input power is equal to 0 for a lostless medium so if this is 0 then implies that this is also equal to 0, in other words we have the summation of An square is equal to the summation of Bn square.

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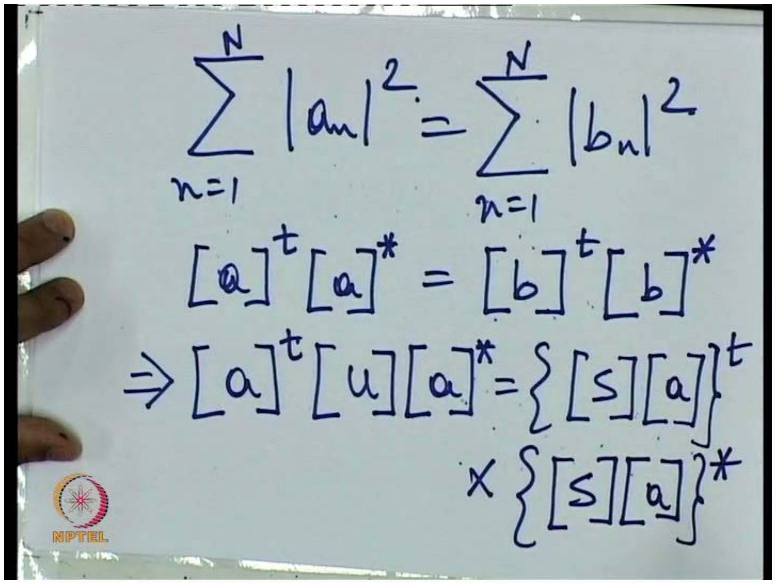

What I mean is we have summation of An square is equal to the summation of Bn square. I can write this same expression in matrix form as A transpose multiplied by A conjugate is equal to B transpose times B conjugate. This in turn can be written as, if I now express in terms of the S matrix then this simply becomes…

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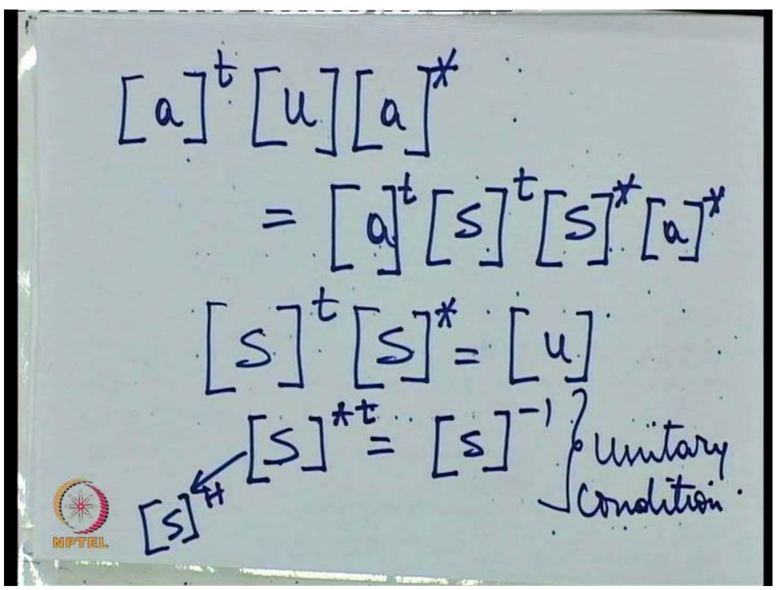

Further if we proceed then we get A transpose A multiplied by A conjugate is equal to A transpose S transpose S conjugate so from here we see that the left hand side and right hand side are similar so then for a lostless network the condition of lostlessness in terms of the S parameters matrix is this or we often sometimes write this as S conjugate transpose is equal to S inverse. Now this conjugate transpose is often represented by this symbol S hermatian SH, then S hermatian being equal to S inverse is also mathematically this is known as the unitary condition.

So for a network to be lostless the unitary condition that is S hermatian or S conjugate transpose should equal to S inverse now [mos] more conveniently this is expressed like this…

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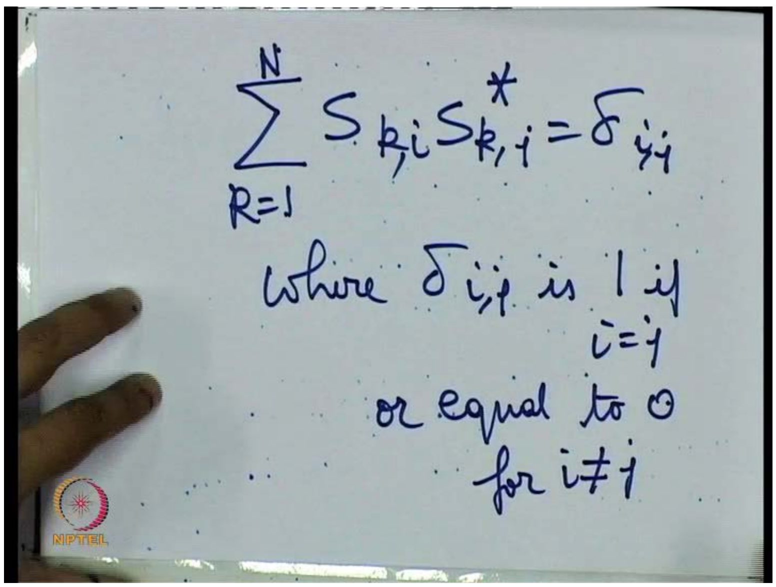

S K I S K J conjugate K equal to 1 is equal to delta I J where delta I J is 1 if I equal to J or equal to 0 for I not equal to 0 so this is the condition for listlessness in terms of the S parameter for a for the Z and Y parameter matrices we had seen that if the network is just lostless then only the diagonal elements should be purely imaginary if it is but lostless and reciprocal then all the elements should be reciprocal and imaginary.

So basically we have covered some of the basic properties of S mass matrix as applicable for reciprocity and listlessness in the next module we will be covering some property some special properties of the S matrices or for 2 port networks so.

Thank you.