Microwave Integrated Circuits
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Mod 03, Lec 11
Properties of Scattering Parameters (contd.)

Welcome to another module in this course 'Microwave integrated circuits'. In the previous module we had seen the basic properties of the S parameters; in this module we will continue our discussion on the properties of S parameters especially as applied to 2 port networks.

So we saw that in the previous module we had seen that for 2 special cases, 2 special properties of networks 1 is the reciprocity and the other is the lostlessness, we can derive some special conditions for the, that the S parameters of such networks should [Sa] should [Sat] satisfy. So let us continue the discussion for 2 port network so…

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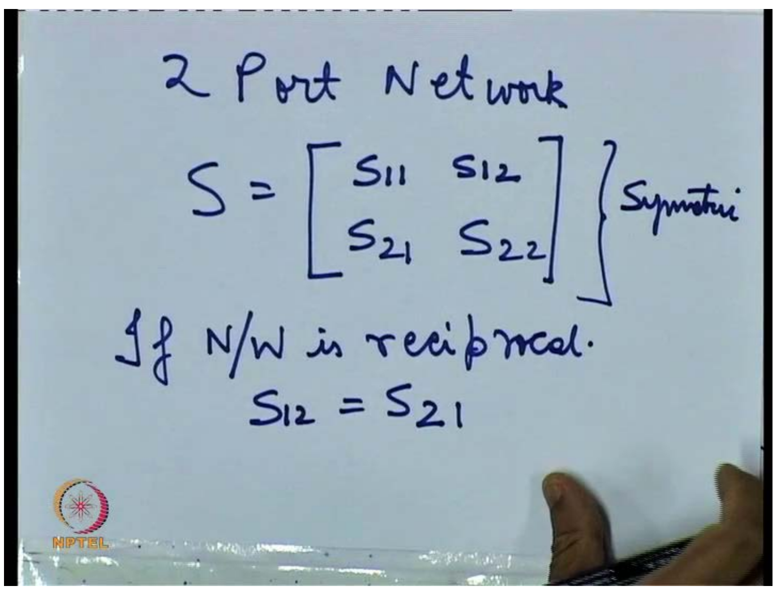

For a 2 port network our S parameter matrix will be something like this. If the device is lostless, if the, lets first start with the condition for reciprocity and this matrix should be symmetric, okay. If it is lostless as well in addition to being reciprocal, now here I must say something that all passive networks are reciprocal. So if I say, if I am referring to a reciprocal and lostless network when it is like a network which contains only passive reactive elements like inductances and say some capacitates there are no resistances in that network.

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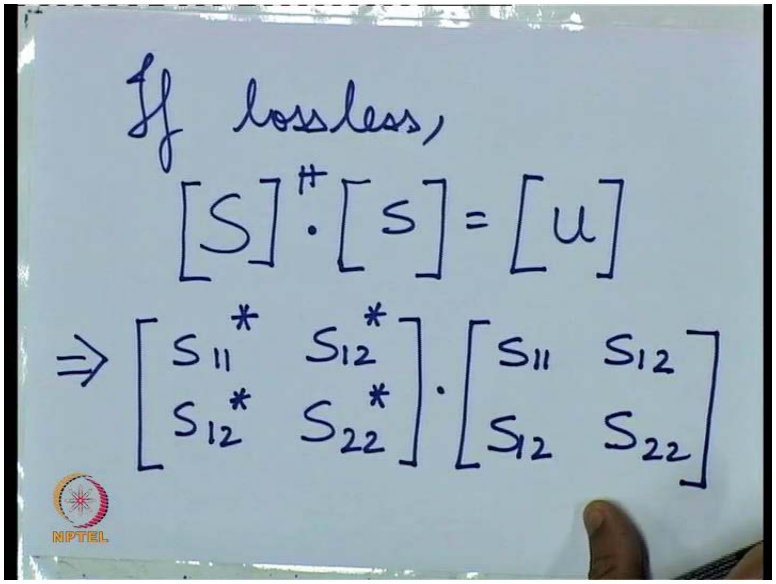

So if it is lostless then we know the condition that S hermitian where U is the identity matrix. So if I expand my S parameter matrix then I should get this S 2 1 is equal to S 1 2 because of the reciprocity as we have already discussed.

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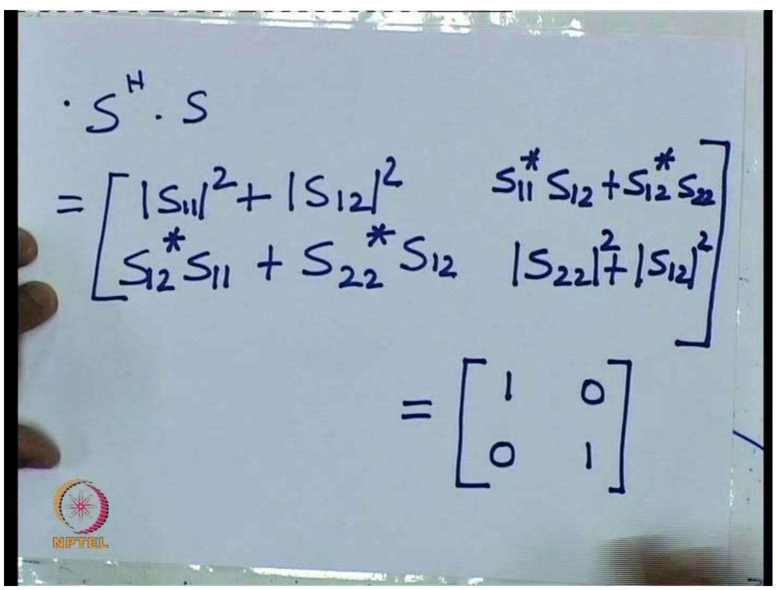

Further doing within the derivation even further so this S hermitian S times S we can following, so basically if we expand this this matrix multiplication then what we will get is… And then this should be equal to the identity matrix so from the first equation and this equation so we have 4 sets of equation so if label this 1 2 3 4 then from this equation and this equation you can straight away write…

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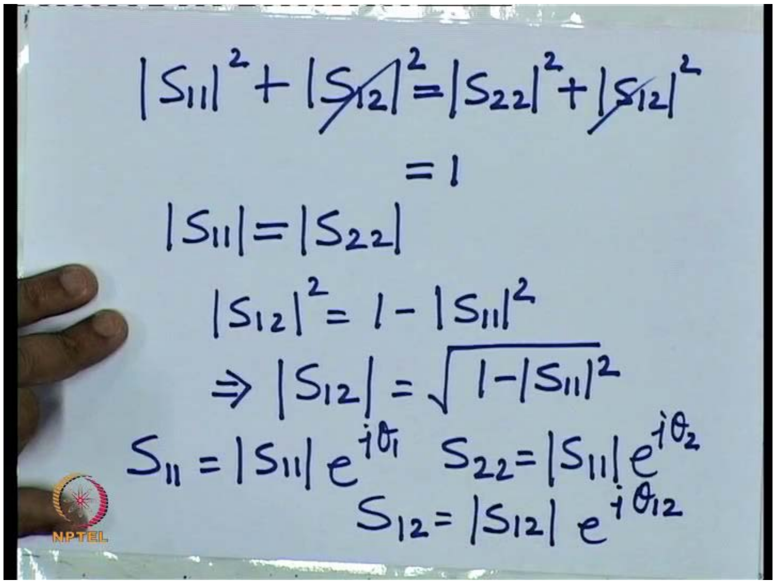

Now from these equating the 2 because this is common for them so basically this gets eliminated this term so from here we can straight away write modulus of S 1 1 is equal to modulus of S 22 in other words the S 1 1 and S 2 2 will have to have the same magnitude. And S 1 2 from this equation, S 1 2 square is equal to 1 minus S 1 1 square so this in turn implies that modules of S 1 2 is equal to square root of 1 minus S 1 1 square.

Now suppose we define the since the S parameters themselves are complex quantities, if we define the S parameters in terms of phases that is a magnitude and an argument S 2 2 is equal to S 1 1, since S 2 2 magnitude is same as S 1 1 magnitude, but then they may have different phase different argument terms so that is why I have chosen theta 1 for S 1 1 and theta 2 for S 2 2.

And S 1 2, if I choose it like this and let its Fazer be theta 1. Now note if I know the magnitude of S 1 1 then I know also the magnitude of S 2 2 and I also know the magnitude of S 1 2 from this relationship from this relationship so with these equations so let us see what we have to…

So if know S 1 1 magnitude we know the S 2 2 magnitude S 1 2 magnitude but we still have to find out the values of theta 1 theta 2 theta 1 2, so at this stage we have 4 unknowns theta 1 theta 2 theta 1 2 and magnitude of S 1 1. So to completely characterize the 2 port network mean at this stage we still mean 4 parameters, let us see whether we can further reduce the number of parameters.

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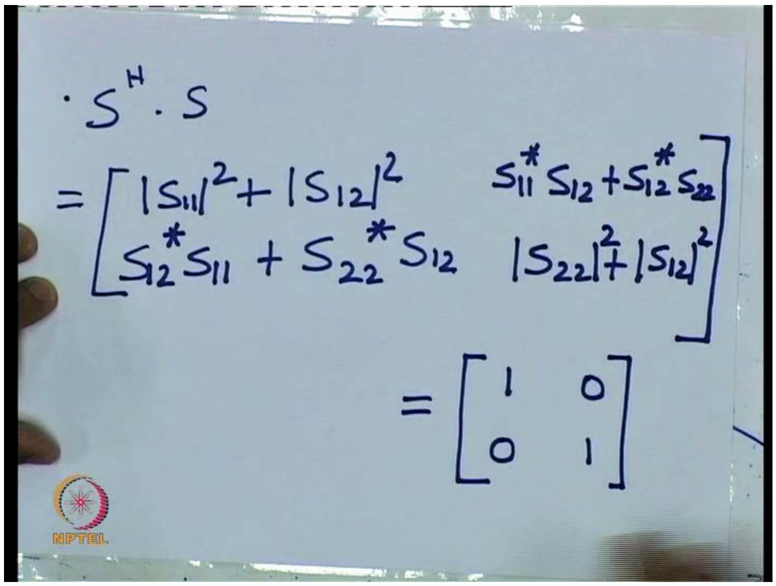

Now we have another 2 equations as we saw so 1 other set of equation 1 another equation from this matrix if we go back to this matrix we still have this equation that is S 1 2 conjugate times S 1 1 plus S 2 2 conjugate times S 1 2 is equal to 0, so if we equate this let me rub out these these terms these might create some confusion, this is just a label for those equations.

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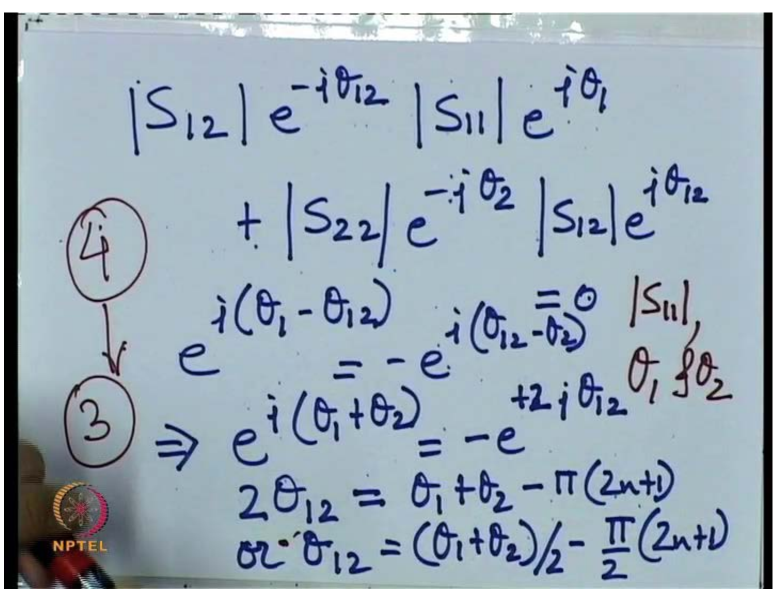

Okay from that equation what we can find is, now I am writing the s parameters in terms of the phasers, now simplifying this will give us because S 2 2 is same as S 1 1 so what we will get all these magnitude terms will be cancelled and what we will end up at the end is…

So equating, so finally simplifying this we mean this equation now comparing the left hand side and right hand side we can simply write or so this is the final expression so what we have done here is we have eliminated the need for another variable. So we at the beginning made a why in the previous night I said we need 4 more you need 4 more variables to completely characterize the system so if we needed 4, so now with this relationship we have reduced the requirement to 3.

So if we know S 1 1 magnitude theta 1 and theta 2 then we can completely characterize a lostless and reciprocal 2 port network. Now let us see whether how we can directly calculate the S parameters. Now 1 way that we saw in the previous lecture was there was a formula relating the Z parameters to the S parameters we can use that so for any network we can first find out the Z parameters or Y parameters and then use that formula to find out the S parameters or if you want to know the S parameters and we want to find out the Y parameters we can do the reverse.

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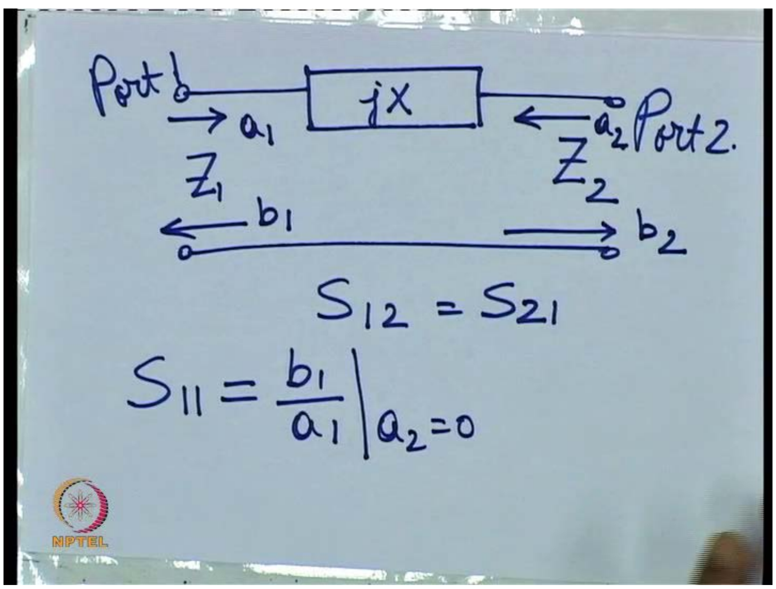

But suppose we want to find out the S parameters from first principles then how do we go so let us take a very simple simple example. Suppose let me use a new slide for this suppose I have a simple network like this it's a simple series reactants, let's say Z1 and Z2 being the characteristic impedances at port 1 and port 2 respectively.

Now we have to find out all the S parameters of this system, now first thing that we have to see is this is that this is the lostless system, this is also reciprocal system since this is a reciprocal system we should have S 1 2 is equal to S 2 1, ok. Now to find out say S 1 1 we have to so S 1 1 is defined like this so if we have A 1 as the incident wave and B 1 as the reflective wave, similarly say A 2 is incident wave at port 2 and B 2 is the reflective wave at port 2. So then this is with A 2 equal to 0 and the question is how do we make A 2 equal to 0 let us see whether we can make A 2 equal to 0.

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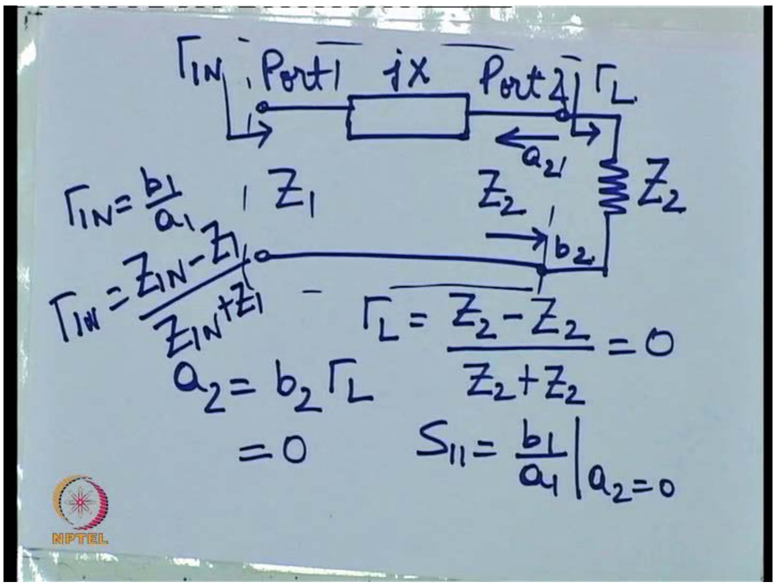

Suppose consider we add a load Z 2 at port 2 now what will be the reflection coefficient for a wave hitting the load Z 2, gamma l will be equal to Z 2 minus the characteristic impedance. We know that the characteristic impedance at Z port 2 this is port is also Z 2 and the characteristic impedance at port 1 is Z 1.

So gamma L will be Z 2 which is the load minus the characteristic impedance which is also equal to Z 2 upon Z 2 plus Z 2 and this is equal to 0. Now if I write A 2 in terms of B 2 the way they are related is A 2 is equal to B 2 times, gamma L because gamma L is the reflection coefficient at this point so this relationship will be valid but then since gamma L is equal to 0 so this is also equal to 0.

So we have by connecting a match load Z 2 at the at port 2 we have eliminated A 2, so then we can now if we find out so the first condition for S 1 1 we call that we had to satisfy was B 1, S 1 1 is equal to B 1 upon A 1 with A 2 equal to 0, so we have done that. Now what is B 1?

What is the b 1 upon A 1 for this circle, that we can say it is simply equal to the gamma in, isn't it. gamma in is defined as B 1 upon A 1 with no restriction the difference between gamma in and S 1 1 is that for gamma S 1 1 we have to have A 2 equal to 0 but for gamma in it is not necessary for A to equal to 0, but then the way we have made this circuit by connecting this load we have eliminated A 2 so now A 2 will always be 0 and therefore S 1 1, will S 1 1 for this circuit will be equal to the gamma in for this whole circuit.

The whole circuit, I mean the this circuit plus the load Z 2 so now gamma in itself we know is equal to Z in minus the characteristic impedance at port 1 upon Z in plus Z 1, so then we if we can find out what is Z 1 then we can find out what is gamma in and then hence we can find out what is S 1 1, so let us see what is S Z in for this circuit.

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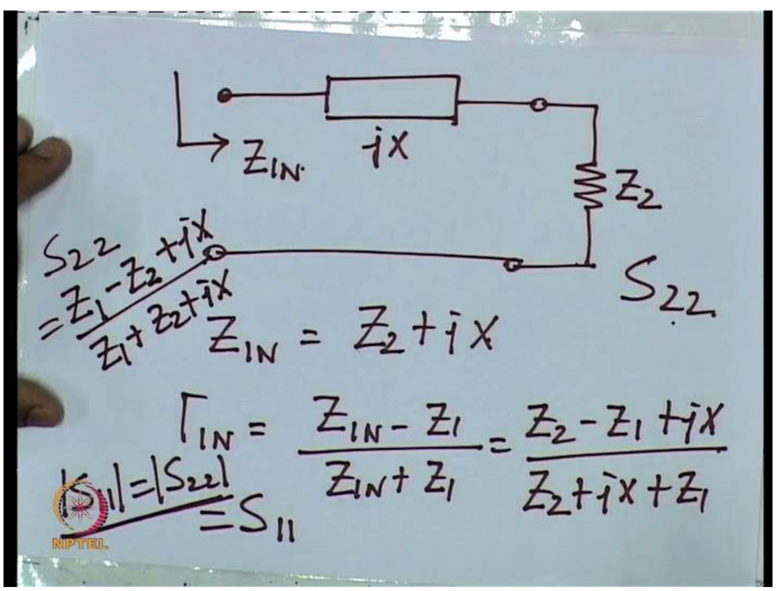

This the value of Z in because Z is in series combination with J X therefore gamma in will be equal to Z in minus Z 1 upon Z in plus Z 1 which is equal to Z 2 minus Z 1 plus J X upon Z 2 plus J X plus Z 1. From this we can also verify you know that if you want you know you can verify that since this is the lostless and this is a reciprocal network if you by the way this whole thing will be equal to S 1 1. Now see if we proceed similarly we can also find out S 2 2 and we can verify that mod S 1 1 will be equal to S 2 2.

And this similarity is because of this the lostless and reciprocal nature of this circuit and due to the relationship that we had give it a condition for 2 port network just we that had derived earlier a few slides early you can verify them that this modulus of this will be same as the modulus of S 2.

In fact just from symmetry we can write down directly what will be S 2 2 if S 1 1 is given by this expression S 2 2 will be something like this. So Z 1 and Z 2 will be interchanged and from this we can easily verify that S 1 1 [mag] magnitude will be same as S 2 2 magnitude.

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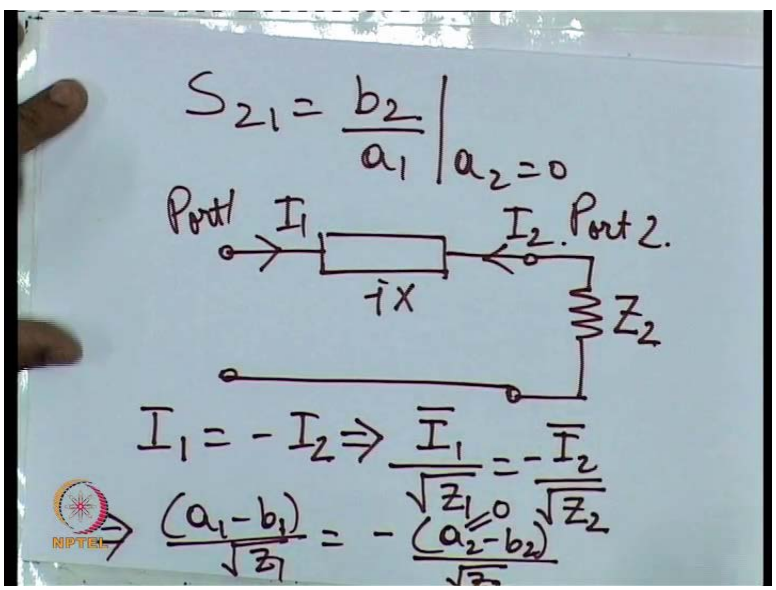

If we want to find out the S 2 1 parameter or S 1 2 parameter how do I do that let’s see now S 2 1 is defined as B 2 upon A 1 with A 2 equal to 0 so again we need to have A 2 equal to 0. Now coming back to our circuit we saw that in order to make A 2 equal to 0 we simply connect a load Z 2 like this but then what how what do we do next.

Suppose I 1 and I 2 are the currents flowing in to port 1 and port 2 respectively then we know that I 1 is equal to the negative of I 2 if we write the same equation in terms of normalized [inped] currents then we know we can write it like this, I 1 magnitude we know is equal to, the normalized I 1 is simply A 1 minus B 1 that upon Z 1 is equal to A 2 minus I 2 normalized is equal given by this. We know this A 2 has been made equal to 0 by this connection.

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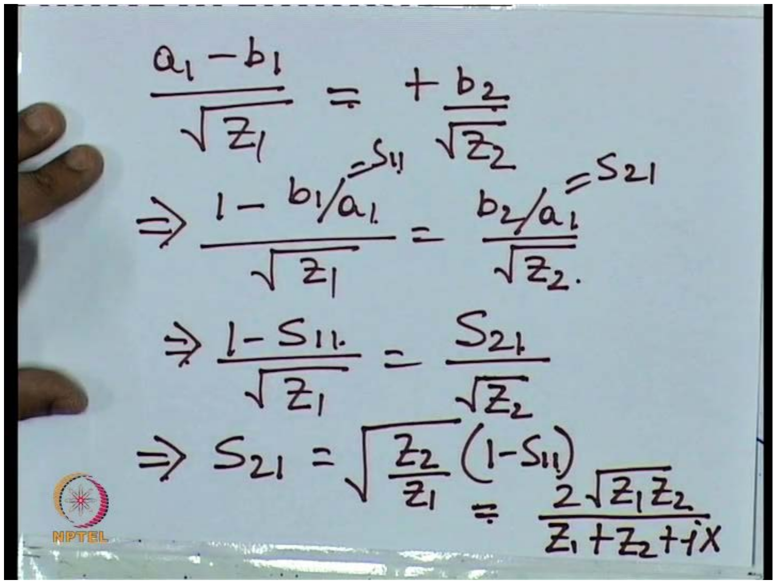

So from here we can simply write this down. As A 1 minus B 1 upon square root of Z 1 is equal to plus B2 upon square root of Z 2 so from here if we divide both sides by A 1 then what we get is 1 minus B 1 upon A 1 upon square root of Z 1 is equal to B 2 upon A 1 upon square root of Z 2, so we know that this is equal to S 1 1. So then this implies that 1 minus S 1 1 upon square root of Z 1 is equal to S 2 1 and this is equal to as we know is equal to S 2 1 from here we can derive an expression for S 2 1 will be equal to Z 2 upon Z 1 square root. And then the final expression for S 2 1 that we should get is equal to, so this is the final expression for S 2 1.

Now this is the you know this the basic property of 2 port network we discussed we discussed very simple case here and for more complicated circuits we will need a more extensive network analysis to find a relationship between I 1 and I 2, but the basic proceed here is the same that you first find the relationship between I 1 and I 2 or between B 1 and B 2 and then express these Is and Bs in terms of the normalized voltages and the normalized current.

And then further you expand a normalized currents and normalized voltages in terms of the normalized incident voltages and the normalized reflective voltages and then from that we get the normalized the incident and normalized incident and reflected voltages and currents to find out the required S parameters.

Now there are some other applications of since then as I said the 2 port network is very extensively used it is the most common form of microwave network, there are some unique properties or unique results or unique unique parameters associated with 2 port network that we would that I would like to show you how to derive it and what is the what is the necessity of that…

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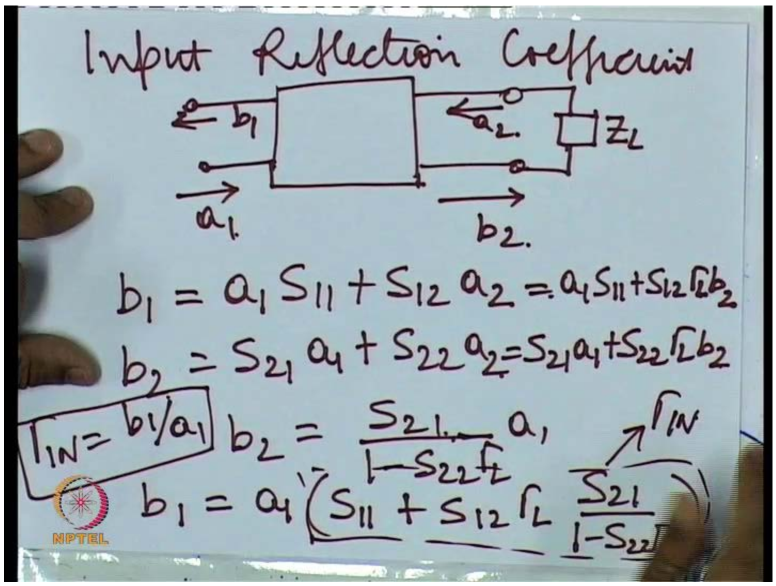

So 1 into important important parameter that is frequently associated with all 2 port networks is what is known as the input reflection coefficient. For this 2 port network we can write down the S B relationship between the As and Bs like this. And then A 2 this incident at port 2 they way that is incident at port 2 as I describe while we are is actually the reflect can also be considered to be reflected from the load.

So then I should be able to write this equation as and this as… Now solving this first equation as, I beg your pardon solving the second equation will get B 2 is equal to S 2 1 upon 1 minus S 2 2 gamma l times this whole times A 1 and then if you substitute this value in this equation we get an expression for B 1 in terms of A 1 as S 1 1 plus S l 2 gamma l times S 2 1 upon 1 minus S 2 2 gamma l.

Now from here we can find out an expression for gamma in because gamma in input reflection coefficient is simply given by this gamma in is equal to b 1 upon A 1. So then this whole thing should be my gamma in…

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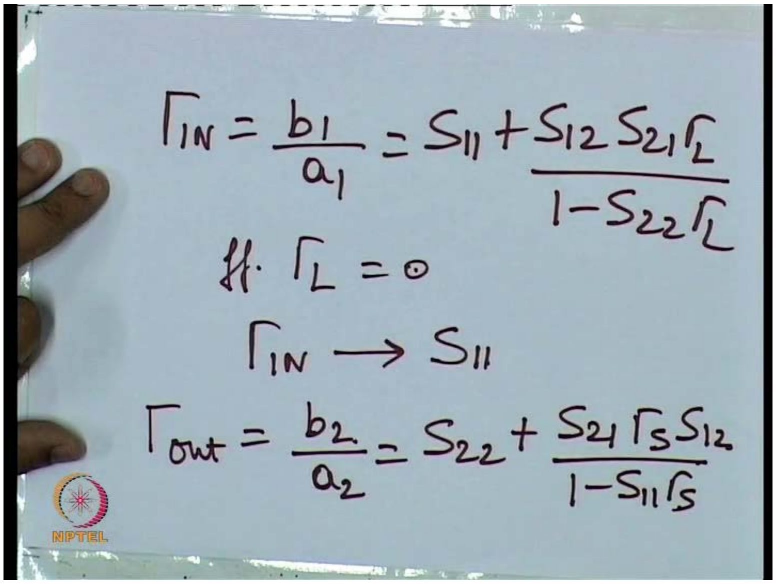

Or I can write down properly as gamma in is equal to B 1 upon A 1 is equal to S 1 1 plus S 1 2 S 2 1 gamma l on 1 minus S 2 2 gamma l. Now this is a important parameter for all 2 port networks because as we saw that for good input matching this gamma m should be as low as possible preferably 0. Now usually we don't have any control over the S 1 1 suppose we are given an amplifier the S 1 1 is kind of constant but we do have some control over say the gamma l.

So if you see if gamma l is equal to 0 then gamma in reduces to S 1 1. Now similarly in the same way that we have written expression for gamma in we can also write an expression for gamma out which is called output reflection coefficient and just following symmetry you should get the value of this gamma out as.

So again just like the same case as the gamma in the value of gamma out also is preferred in many applications we prefer the gamma out to be as low as possible and so that's how no, both these gamma in and gamma out if they are low then it's considered to be a good matching.

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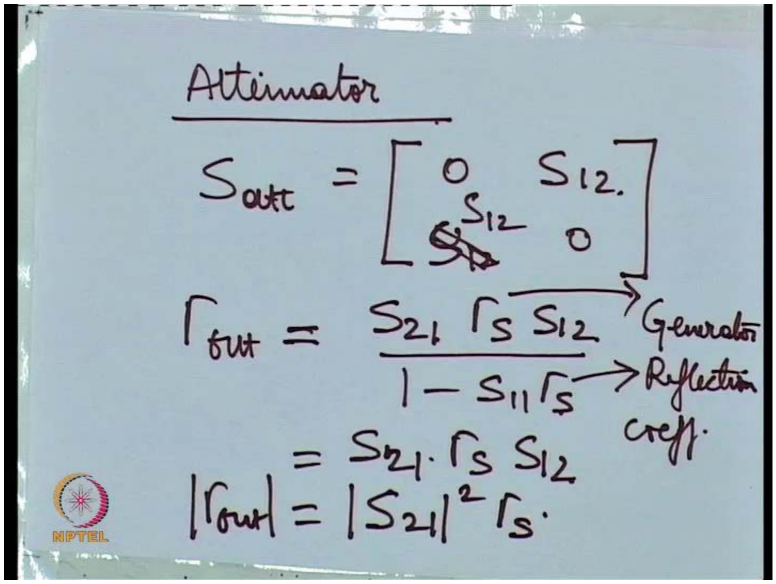

Now one simple example that, I would like to show here is that of an attenuator an attenuator has S parameter matrix a simple, attenuator is a 2 port device whose S parameter matrix is given by this matrix this is S 1 2 also it is 0 so both S 1 1 and S 2 2 are 0. And gamma out therefore should be equal to S 2 1 gamma S S 1 2 okay. Now since S upon 1 minus S 1 1 gamma S that this gamma S is the generator reflection coefficient just like the load reflection coefficient we also have the generator reflection coefficient.

Now since S 1 1 is 0 so this should be reduced to S 2 1 gamma S S 1 2 and then since it is a reciprocal device so we have S 2 1 square if I directly write the magnitude like this. So in order to reduce gamma out we see that either we make gamma s equal to 0 or we design our systems such that the S [ga] 2 1 magnitude square is as low as possible.

So in summary for this lecture I would like to say that in this lecture we have, we have covered the basics of not the basics we have covered gone little more in depth in to the properties of the S parameters for 2 port networks. Whereas in the previous module we had only considered the properties of any port of any general network here we are specifically seen the properties of 2 port networks since as I said 2 port networks are very widely used and 2 of the major parameters that are associated with incidence matching are the gamma in and the gamma out.

Now as we saw writing down the equations for even a 2 port network is quite involved, there are lots of equations involved so now imagine if we have a number of 2 port networks connected with each other or 2 port network connected with higher number of with a device which has at the say 3 port or 4 port so then keeping on writing such equations becomes very complicated and it might take a lot time and its difficult to analyze generally.

So in order to make life simpler when we have a large number of ports we there is a packing called signal flow graph, and as we shall see in the next module the signal flow graph significantly reduce the complexity of the signal flow especially when we have incident and reflected waves to consider and there are some methods to analyze such signal flow graph diagrams, so that we can find out these say gamma in or gamma out the input and output reflection coefficient which we have derived here using equations you can use simply use a signal flow graph to find it much quickly so that is something we will discuss in the next module.

Thank you.