Microwave Integrated Circuits.
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Lecture -14.
Signal Flow Graph, ABCD Parameters.

Welcome to another module of this course microwave integrated circuits, we are in week 3, module 4. So, in the previous module, we had discussed about the properties of S parameters of 2 ports, properties of the S parameters for 2 port network and then we have derived the expression for the input and output reflection coefficients and then I said that there is a method to make this computation of reflection coefficients or for that matter, any ratio term that involves, you know, there is a ratio of either the incident wave or the reflected wave, not just at one port but say the incident wave is at one port and reflected wave is that port 5 or port 6, port 4.

And if you want to find out the ratio of these 2 quantities, then there is a simpler method and that is called the signal flow graph method.

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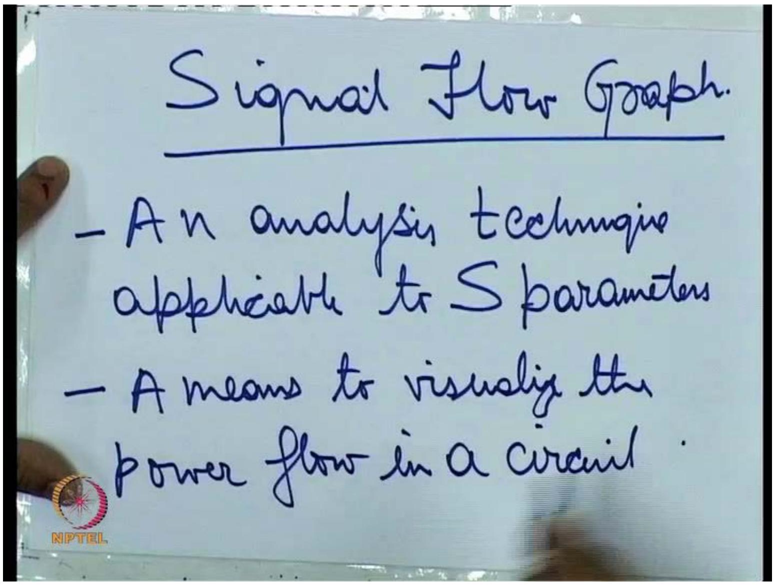

Now, let us see what is the signal flow graph method. So, signal flow graphs, what is it, it is an analysis technique applicable to S parameters a means to and also it is a means to visualize the power flow. These are the 2 things.

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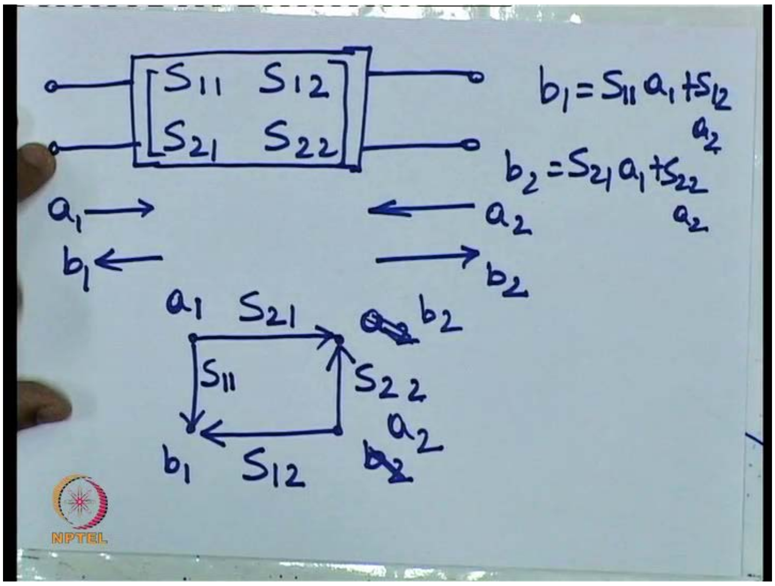

So, let us go back to our 2 port network. 1st of all, every incident or reflected wave is represented by a node and the note is represented by a small bubble.

We have 4 incident and reflected waves, so we have 4 nodes this way, B2 and A2. Now, we know that A1 and A2 give rise to B1 and B2 as described in our 2 port network, is not it because if we write down the equations as applicable to just this 2 port network, the equations are as B1 is equal to S11 A1 + S12 A2 and B2 is equal to S21 A1 + S22 A2. So, we can see that if we consider just this network, then, it is that B1 and B2 are caused by A1 and A2. So, it is like A1 and A2 contribute to B1 and B2. In other words, power is flowing from A1 because A1, the power flowing in A1 is giving rise to B1.

So, when we have one variable or one node giving or contributing to another node, we draw a line or an arrow between the 2 so here from this equation, A1 and A2 contribute to B2, similarly A1 and A2 contribute to B1 and the magnitude by which of the contribution for each, that is A1 Times S 11, S 11 times A1 is contributed to B1. So that S11 will be the weight of this branch joining the 2 nodes, similarly this will be S21, this will be S22, this will be S12, fine. So, as we shall see, this is, from here it might appear as if only A1 can give rise to B1 but that is not the case, it might be that through another process.

For example, if we connect a, there is a generator load present here, then B1 will also contribute to A1 in a recursive manner. So let us see a simple example, a simple 2 port network with both its input and input connected to a generator and output connected to a load.

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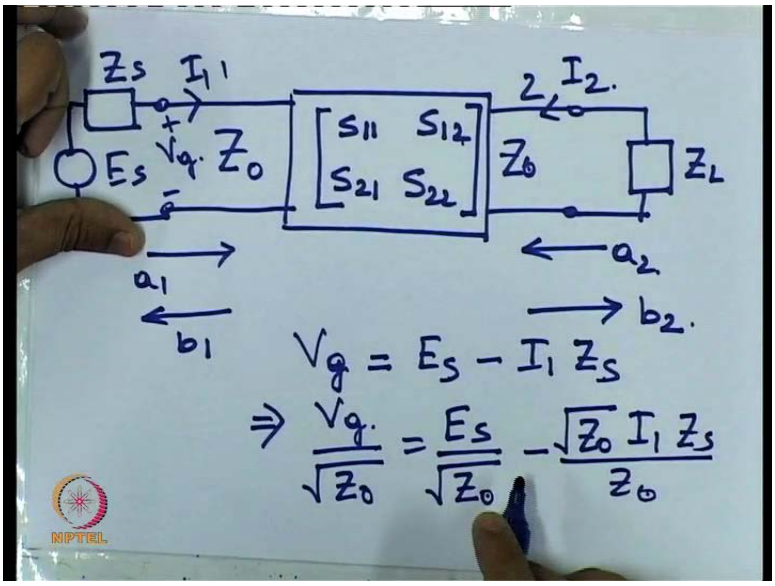

So, the circuit diagram for that, let us say Z0 is the characteristic impedance at both port 1 and port 2 of this network. ES is the source voltage. Now suppose VG is the voltage that is applied to the 2 port network, then we can write an expression for VG as equal to ES - I1 times ZS. Where I1 is the current flowing into port 1 and I2 is the current flowing to… From here we can write this as,

Now this is the circuit equation in terms of voltages and currents, if we now convert this in terms of normalised voltages and currents, normalised incident voltages and currents, what we get is following from the previous equation. And simplifying this, what we get is.

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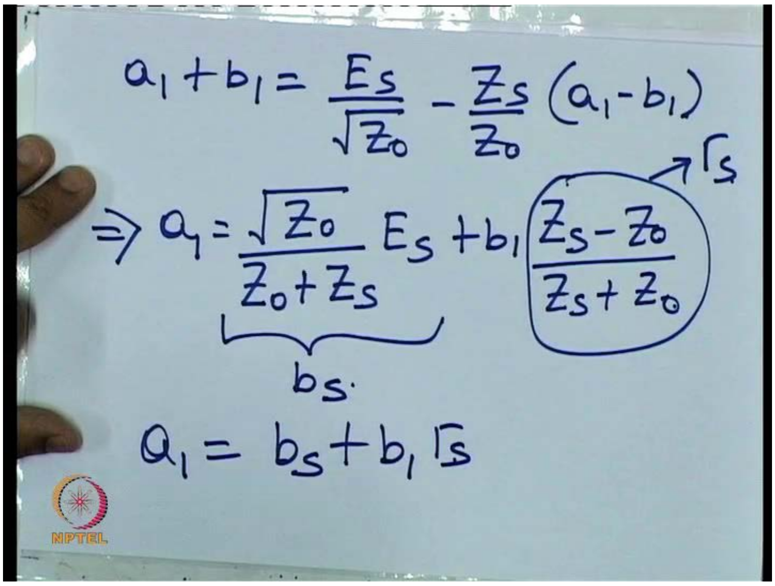

Now, this term if I represented by, is actually the source reflection coefficient or generator reflection coefficient and this term, this whole term if I represent it by term BS, then I can simply write my A1 is equal to BS + B1 gamma S. And if I draw the signal flow graph diagram of this entire circuit, so we can start with the 1st, just for a 2 port network as we have seen.

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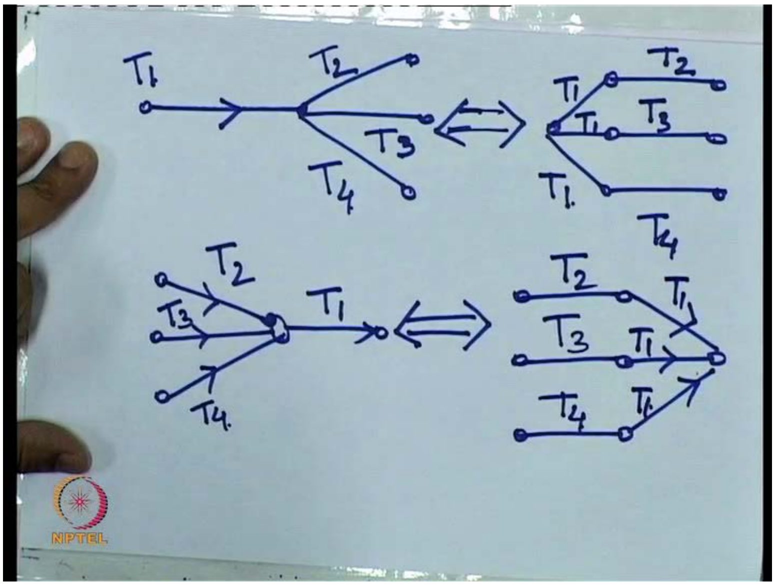

This is the basic signal flow graph diagram and in addition to this, we also have the equation for our load, so we know that A2 is equal to B2 times gamma-L, so then what that means is that we have B2 contributing to A2 with the multiplicative factor gamma-L and we also have seen that there is another equation A1 equal to BS + B1 gamma S. To do that, what I do is I take another node as as the wave BS and it contributes to A1 with the magnitude 1 because BS is not multiplied by any term here and this gamma S is contributing back, so B1 is also contributing to A1 with the multiplicative factor gamma S.

Now, using this, as you know, 1st thing that we have to see is if we want to find out the value of gamma in, gamma in in the previous module we saw was equal to, was given by this relationship.

Now the question is can we use this signal flow graph diagram to simplify our equation to find the same relationship. So, in order to do that, there are some rules for simplification of signal flow graphs. Suppose you have a signal flow graph diagram like this, then this is equivalent to this.

If you have a signal flow graph diagram like this. Then it is equivalent to this. So, this is like the distributed property of signal flow graph diagram. Similarly if you have a signal flow graph diagram like this.

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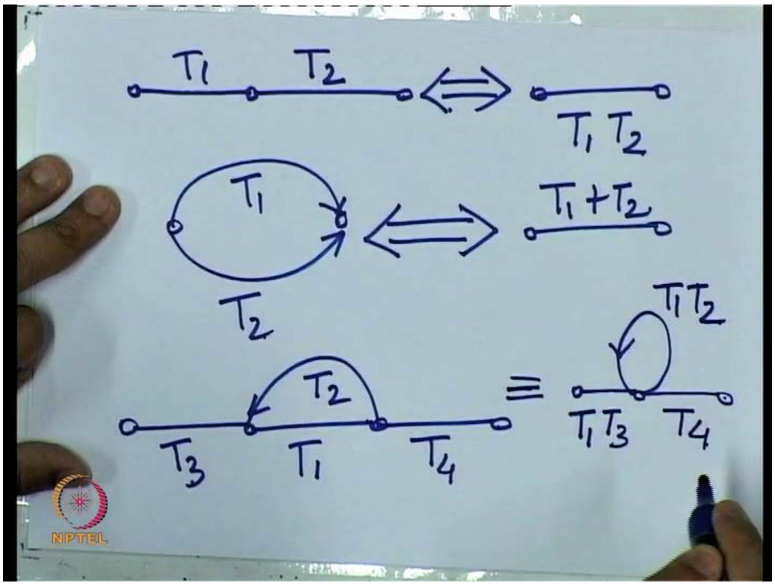

That is to signal flow graph following each other, then that is equivalent to this. If they are in shunt like this. Then that is equivalent to this structure. So, here we have a loop with magnitude T1, T2 and finally if we have a loop, if we have a loop like this.

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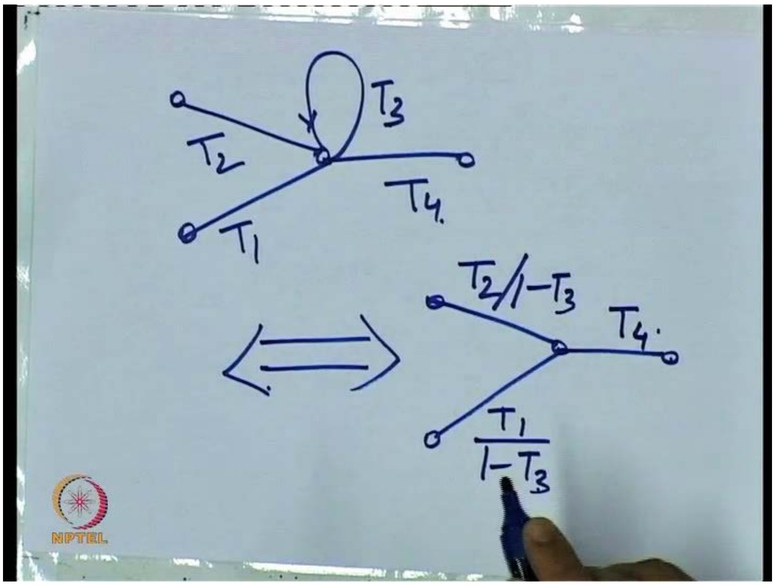

Then that is equivalent, we can remove that loop and have an equivalent circuit like this. So, now let us go back to the original, to our, to the signal flow graphs diagram for the 2 port network that we had discussed a few moments ago and see whether we can get a similar, we can decompose the signal flow graph diagram and obtain the simplified version of it.

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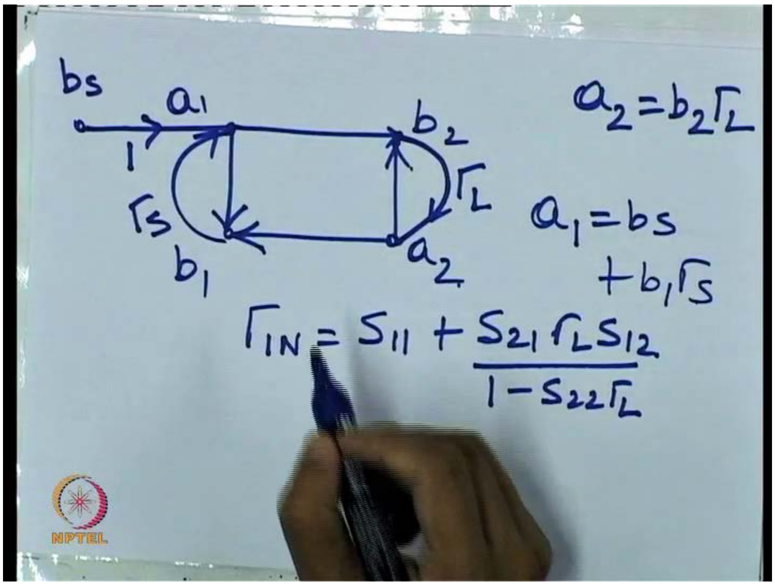

So, this was the signal flow graph diagram that I had discussed a few minutes ago, this is for a 2 port network with both the loads connected to ZL and source connected to a generator with internal impedance ZS.

Now for this, let us just see the load since we are trying to find out gamma in, let us just draw the, let us just draw the circuit with the load in. So, the signal flow graph diagram with just the load will be.

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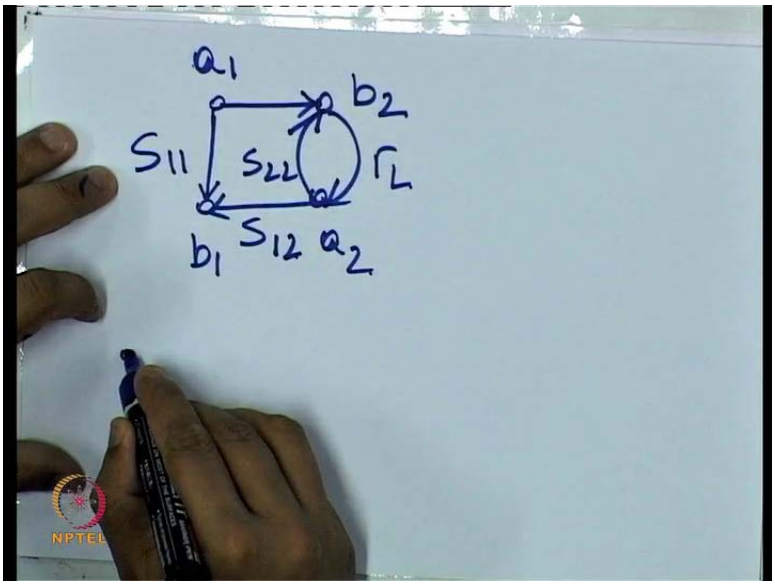

Now look at this loop that is there, this is the loop and what I do is instead of having a single point A2, let me split this loop. What I mean is, we now have gamma-L here, S22 here, I also have gamma-L here.

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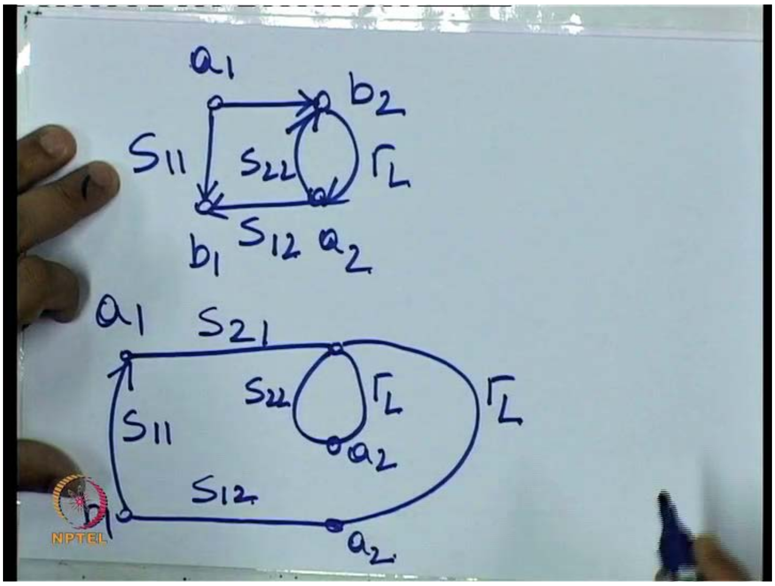

So, what I did was I took 2 points, I split this node A2 and I have now multiplied both the A2s with… I have provided a path with magnitude with weight gamma-L to both these nodes

A2. So, then… What I have done this way that I have now removed this loop and made it a loop, single look like this.

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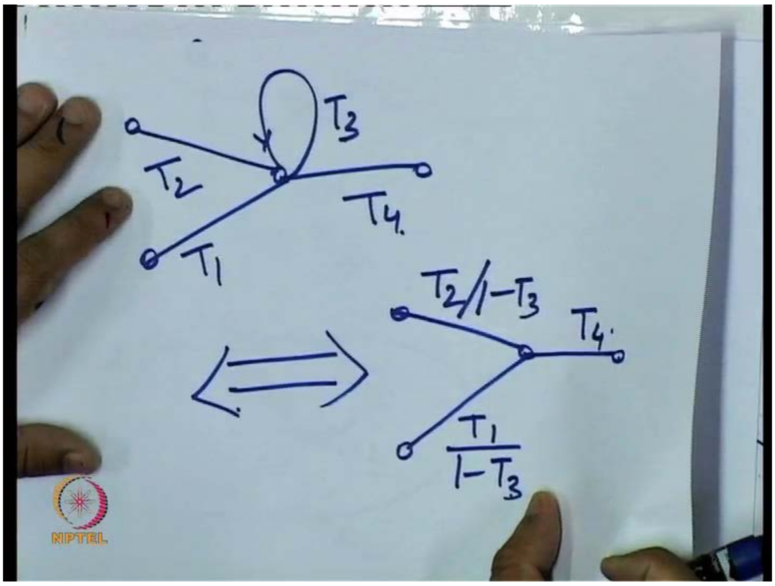

So, then this is, this from the identity that I just discussed previously, this identity, that when a loop like this is present, I can equivalent circuit, the equivalent signal flow graph diagram will become like this. So, I simply apply that this identity and my signal flow graph now simplifies. So, from here I have come to this point, now I am going to this point.

So, now only this path remains, gamma-L. This loop will be absorbed inside this path with a weightage of,

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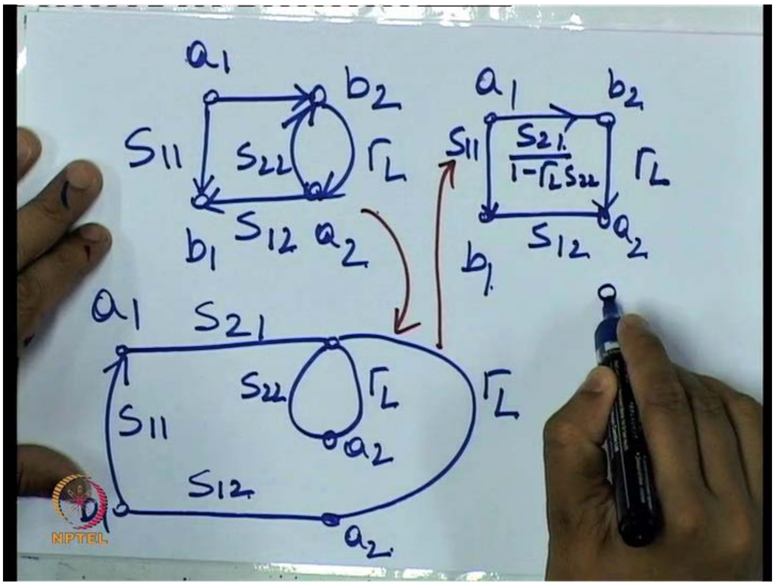

Now I have, now this becomes a simple thing, this is just 2 parallel paths, one path is here and so this whole, since I have to go from A1 to B1, this whole thing can be simplified to this where gamma in is just the addition of the total path product along this path, along this path and the total product of the total weight along this path.

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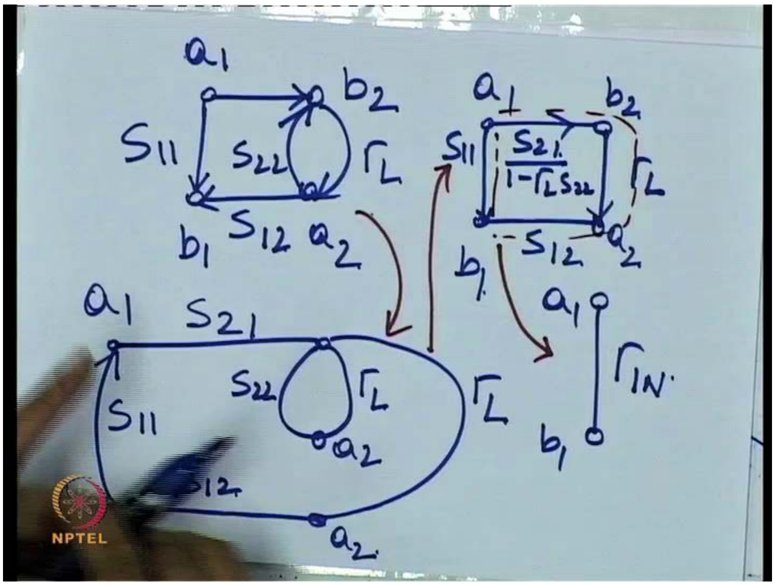

So, along this path, the total product of weights is one gamma-L times S21 1 - gamma-L S22 multiplied by S12. And along this path, the total weight is this.

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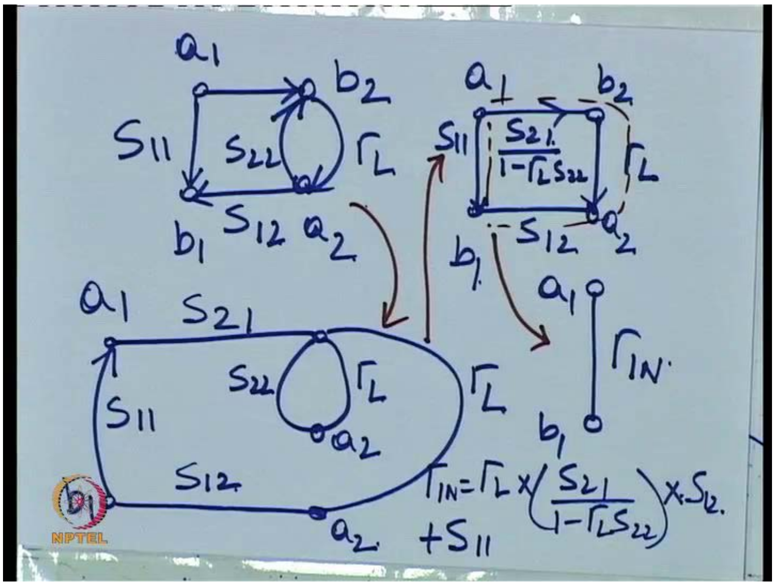

So, this expression is equal to this, which is same as the expression that I derived earlier. Now, I would like to add some more, so eh, this is, this is more or less how signal flow graph are analysed, there are of course some specialised rules called Masons rules that if we probably in some future module, we will be covering that. But I would like to now, in addition to the Z and Y parameter that I described in the past few lectures, I also said that there are S parameters that is what we had been discussing extensively but another class of circuit parameters called Cascade parameters or ABCD parameters are also quite useful while describing microwave circuits.

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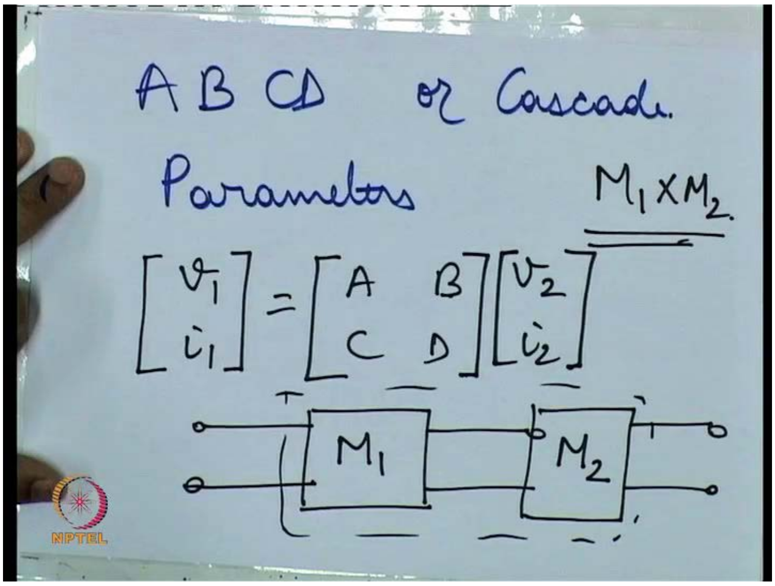

So, let us see what is ABCD or Cascade parameters. So, ABCD or Cascade parameters relate the input and output voltages and currents, input voltages and currents to the output voltages and currents. The advantage of an ABCD parameter is that if you have say 2 networks connected in Cascade like this and say the ABCD parameter matrix all the 1st M1 and 2nd network is M2 and the overall ABCD parameter of this Cascade will be M1 times M2, that is the biggest advantage.

And in passing, I might also refer that there are others that parameters that we described for the circuit, okay, we have seen the application of the S parameters but is there a situation where we have 2 networks and analysis of those 2 networks using Z parameters or Y parameters might actually be more, more convenient as compared to the S parameters themselves. So, such a situation actually exists, for example, if 2 networks are connected in series or in shunt, then actually, we just discussed it that Z and Y parameters might actually be more convenient than S parameters.

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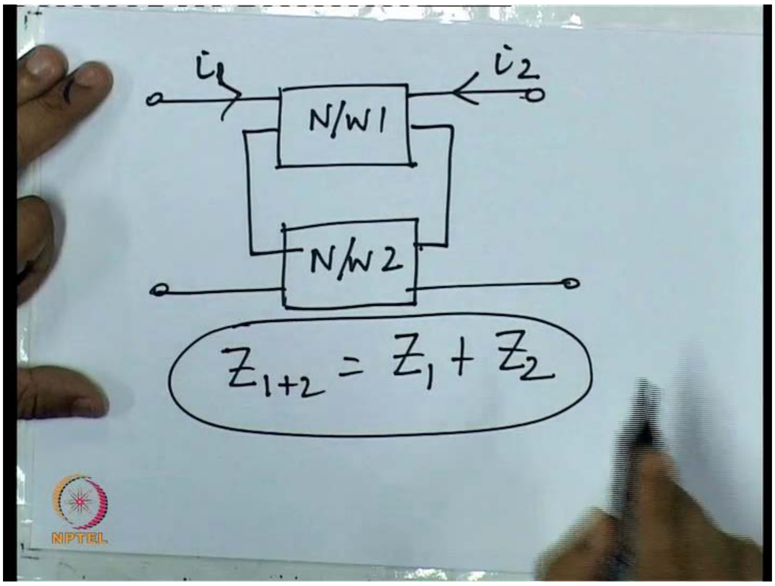

Let us see how. Say we have 2 networks in series, I am calling these 2 networks to be in series because they have the same currents flowing through them, so this is network 1 and this is network 2. So, this 2 combination, the overall Z parameters, Z1+2 is simply the addition of the individual Z parameters matrix. So, when 2 networks are in series, Z parameters might actually be more convenient.

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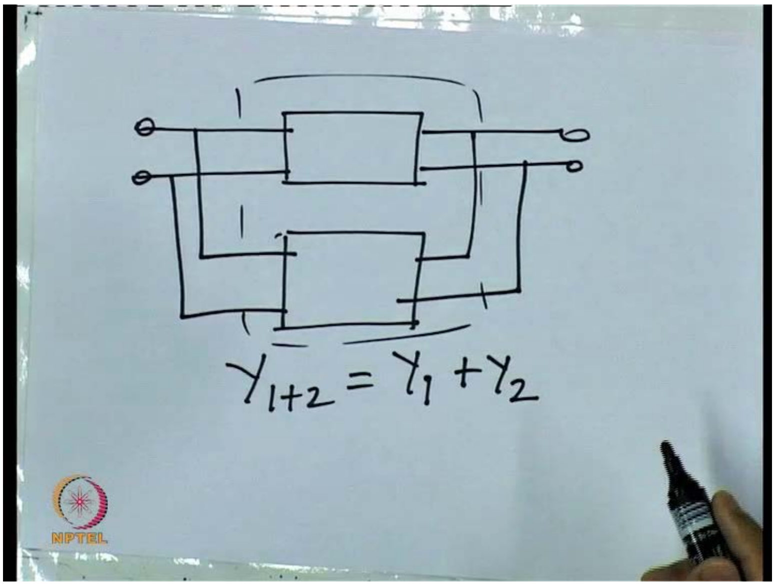

Similarly say if 2 networks are in shunt like this then actually the overall Y parameters of these 2 networks of the combination is simply the addition of the individual Y parameter matrix.

So, this was the 2 cases where Z and Y parameters might actually be more convenient than the S parameters. Now, we were discussing about the ABCD parameters, let us try to find out the ABCD parameters of some common circuit elements that we have been discussing, especially the distributed circuit element. So, what is say the**,** let us, if we consider a simple transmission line.

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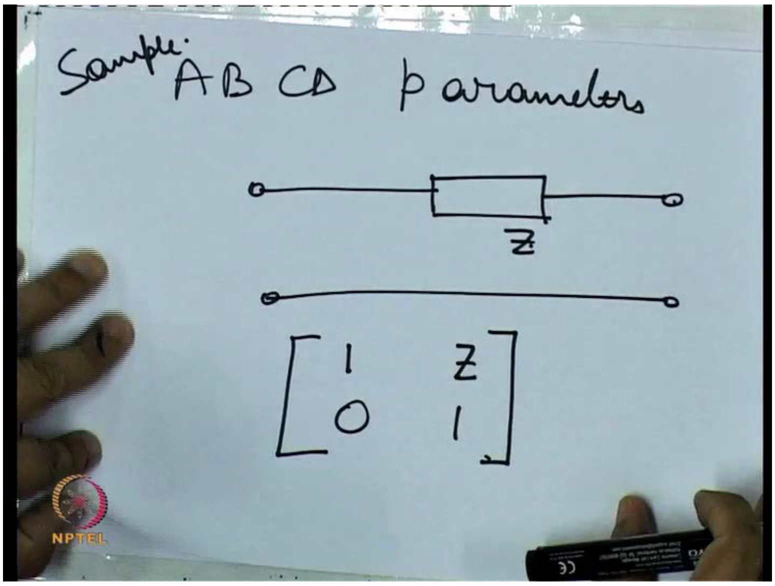

Sample ABCD parameters of some common circuits. Say we have a lumped impedance in series, then it is ABCD matrix is given like this.

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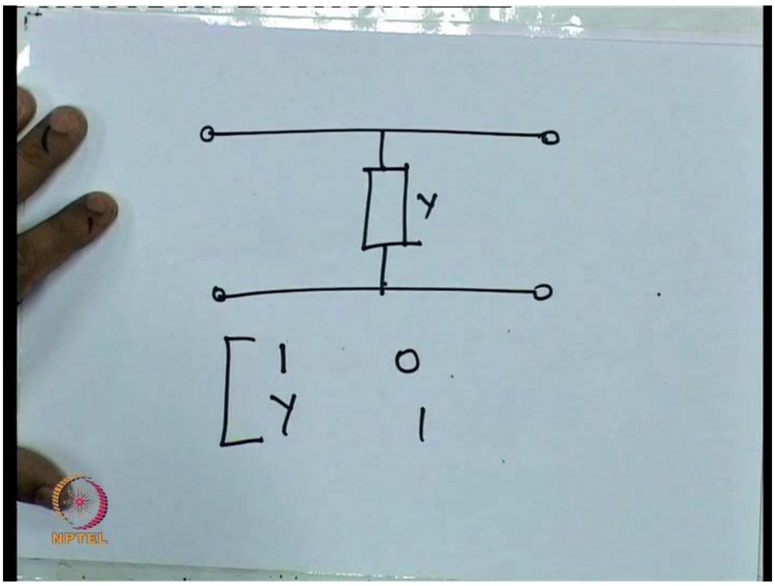

If we have say a shunt admittance, an admittance connected in shunt like this, then it is ABCD matrix is given like this. If we have a of a certain length say L or beta L or theta, it is ABCD parameters are given by say Z0 is the characteristic impedance of this transmission line. Now, there are some relations relating the S parameters to the ABCD parameters as well just like the relations we have between the Z parameters and the S parameters.

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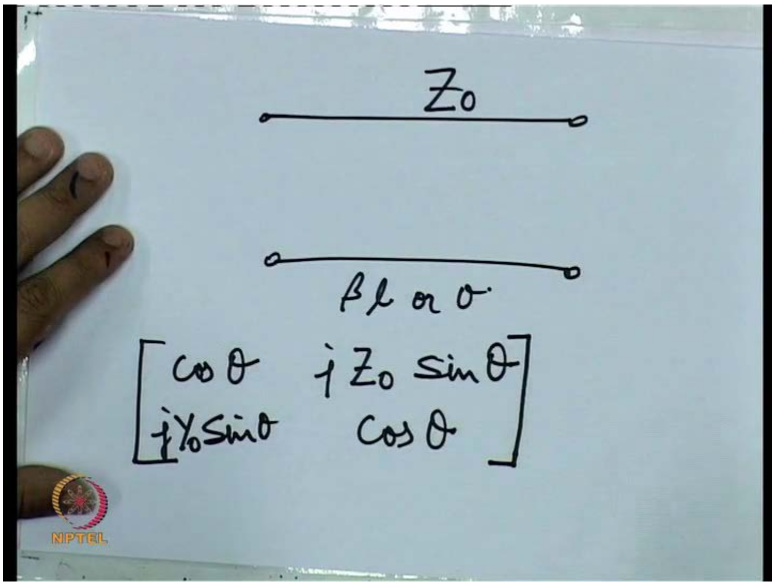

ABCD to S parameters.

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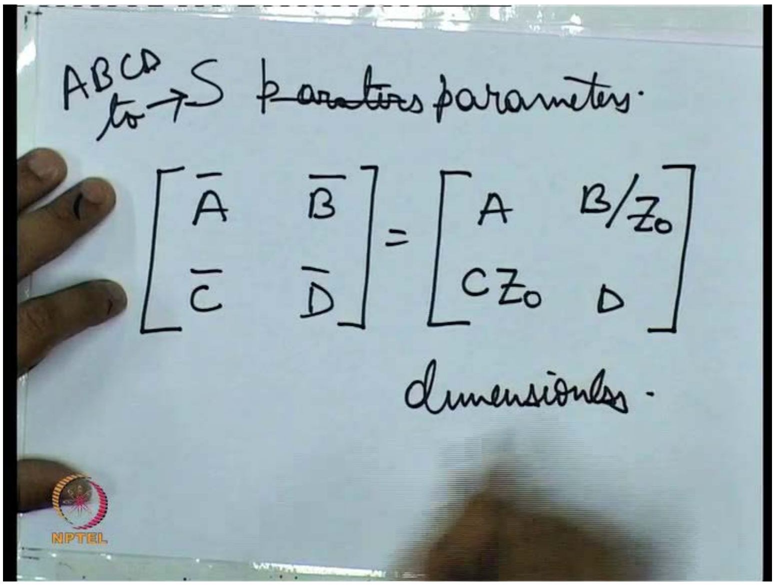

1st of all if we have a transmission line, then we can define what is known as normalised ABCD parameters. So, normalised ABCD parameters are given like this, so A and B remain the same, C and D are scaled by the characteristic impedances and all these parameters in the normalised ABCD parameters are dimensionless.

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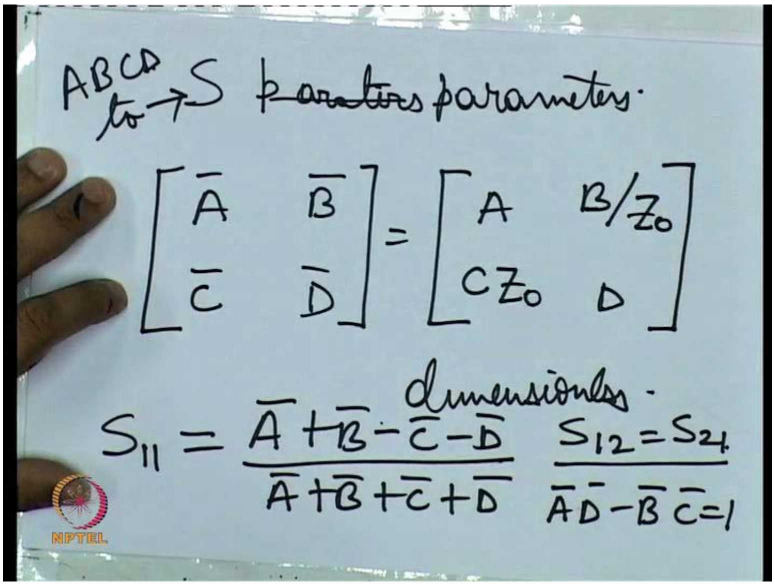

Now, proceeding this way, we have S 11 for the network is given by this, so I will just give one parameter conversion and you can maybe find out for the other parameters.

A condition of reciprocity that we saw for reciprocal networks for S parameter matrix for A2, in an S parameter matrix for a 2 port reciprocal network that boils down to this relationship. Now, this it can be shown that for in terms of the ABCD parameters, this boils down to this relationship. Here of course, now this is A bar, B bar, C bar and D bar are the normalised ABCD parameters. Proceedings similarly in, proceeding similarly and trying to find out the ABCD parameters say of a quarter wave transformer.

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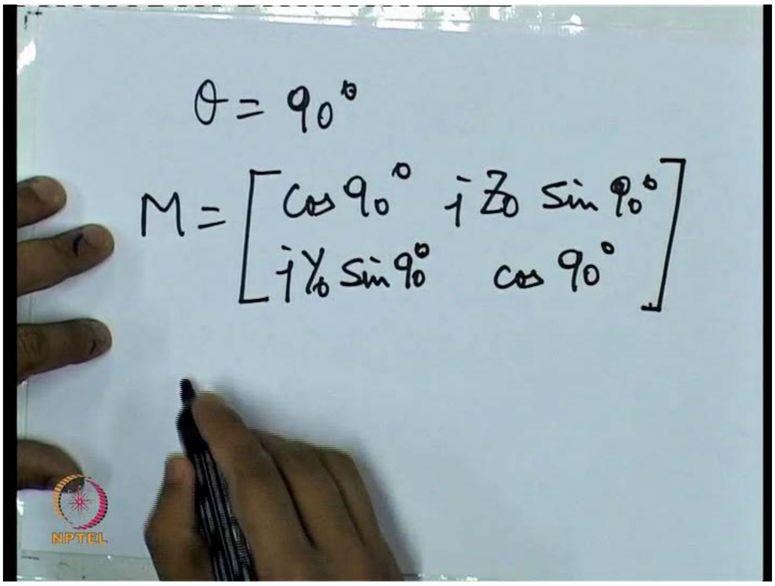

Then we know for a quarter wave transformer, theta is equal to 90° and therefore the ABCD matrix will be cos 90 J Z0 sin 90°, then JY0 sin 90° cos 90° and this simply turns out…

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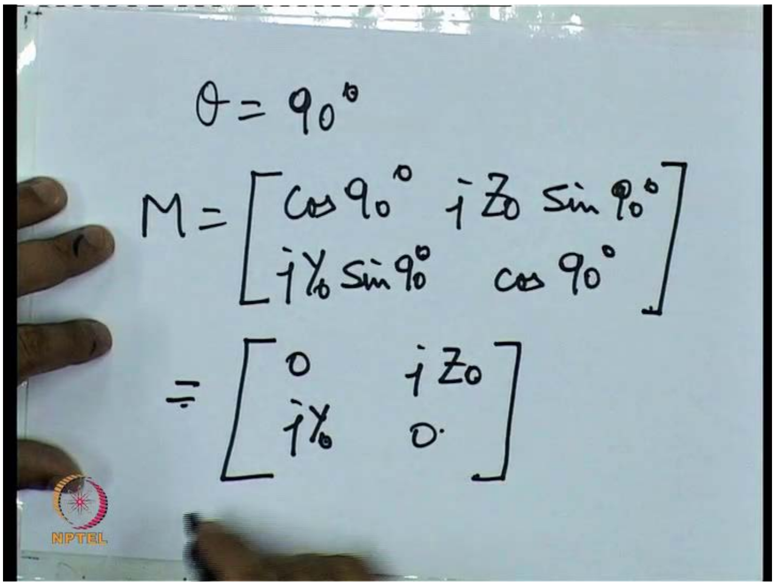

And if we choose to find out the normalised ABCD parameters then this would come out to,

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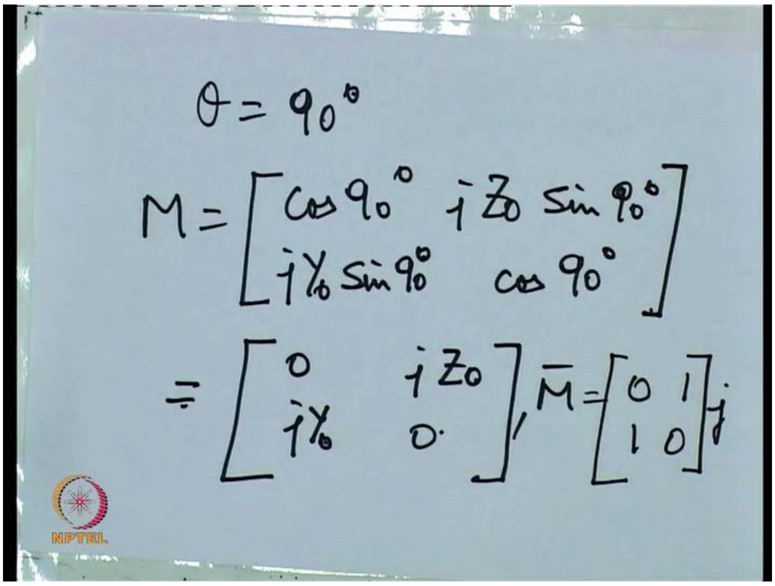

with J on the outside. So, in summary, I would like to mention that in this module, we covered 2 very important concepts in microwave engineering, one was the signal flow graph diagram, which is extensively used when they want to analyse very complicated circuits with many ports or many circuit elements and also the ABCD parameters. So ABCD parameters, they might look like any other circuit parameters like like the impedance or the admittance parameters, they are used quite extensively, especially when, since in microwave circuit analysis, it is convenient to analyse a circuit and terms of Cascade elements rather than considering the whole element as one.

So, when we have such a uh scenario where we have many elements in cascade, we do use the ABCD parameters to analyse such circuits. Thank you.

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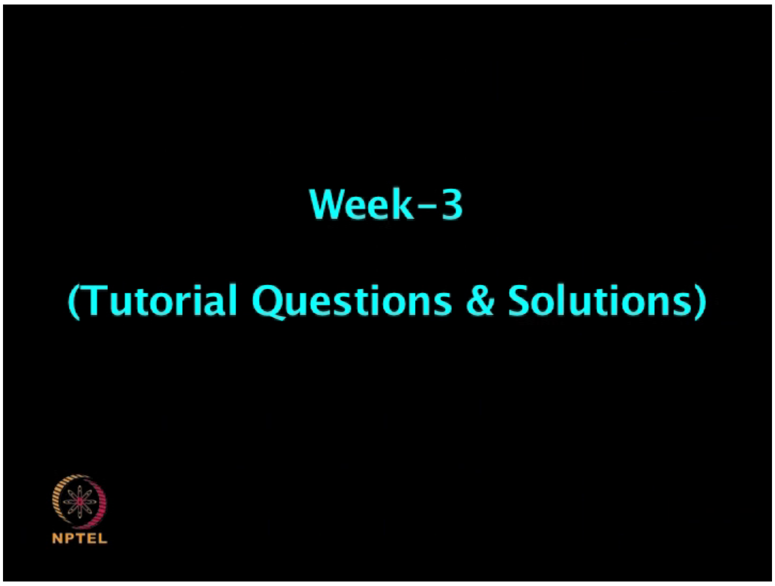
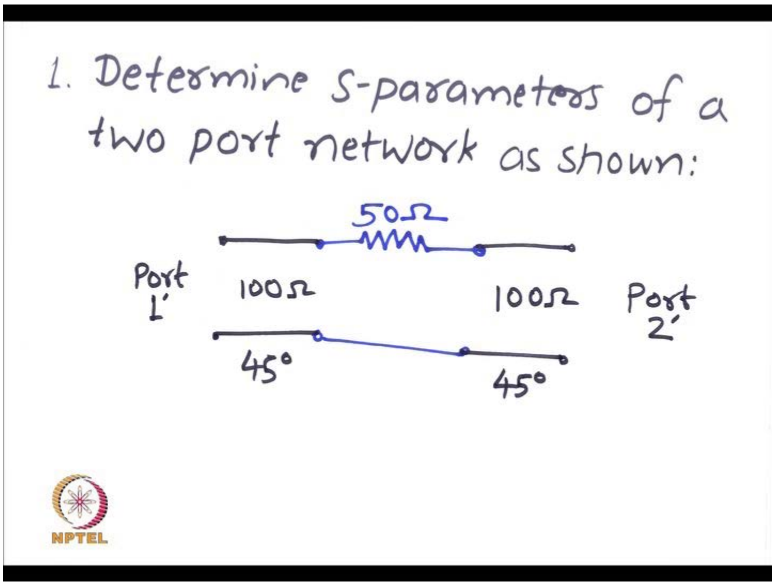
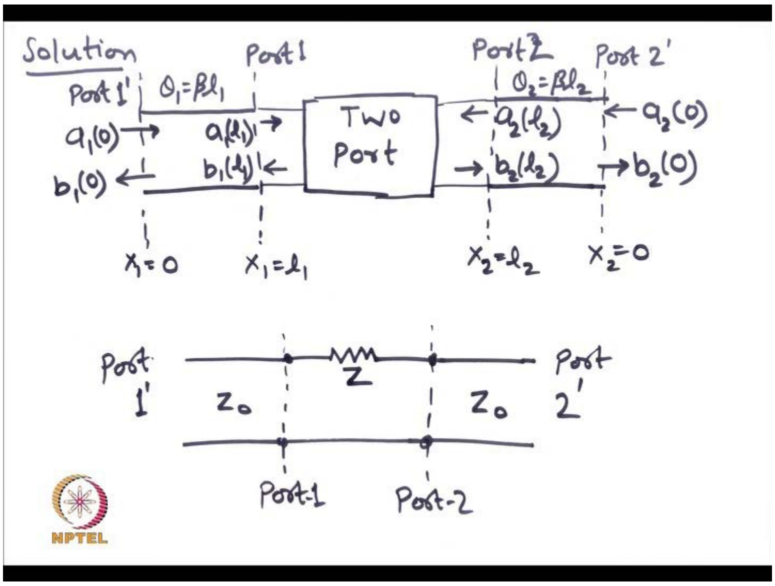






At reference planes at port-1 and port-2 scattering matrix

$$\begin{bmatrix} b_1(l_1) \\ b_2(l_2) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(l_1) \\ a_2(l_2) \end{bmatrix} \quad \text{—— (1)}$$

and at port-1' and port-2'

$$\begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \quad \text{—— (2)}$$

For lossless transmission line we can write

$$b_1(l_1) = b_1(0)\, e^{j\theta_1}$$

$$a_1(l_1) = a_1(0)\, e^{-j\theta_1}$$

$$b_2(l_2) = b_2(0)\, e^{j\theta_2}$$

and $a_2(l_2) = a_2(0)\, e^{-j\theta_2}$

substituting in eq^n (1) gives

$$\begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\theta_1} & S_{12} e^{-j(\theta_1+\theta_2)} \\ S_{21} e^{-j(\theta_1+\theta_2)} & S_{22} e^{-j2\theta_2} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \quad \text{—— (3)}$$

Comparing equation (2) & (3) gives

$$\begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\theta_1} & S_{12} e^{-j(\theta_1+\theta_2)} \\ S_{21} e^{-j(\theta_1+\theta_2)} & S_{22} e^{-j2\theta_2} \end{bmatrix} \quad \text{—— (4)}$$

and

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S'_{11} e^{j2\theta_1} & S'_{12} e^{j(\theta_1+\theta_2)} \\ S'_{21} e^{j(\theta_1+\theta_2)} & S'_{22} e^{j2\theta_2} \end{bmatrix} \quad \text{—— (5)}$$

Using (4), S-parameters for given network can be calculated.

For 

$$T = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \Rightarrow S = \begin{bmatrix} \dfrac{Z/Z_0}{2+Z/Z_0} & \dfrac{2}{2+Z/Z_0} \\ \dfrac{2}{2+Z/Z_0} & \dfrac{Z/Z_0}{2+Z/Z_0} \end{bmatrix}$$

We have

$\theta_1 = \beta l_1 = 45^\circ$, $\theta_2 = \beta l_2 = 45^\circ$, $Z = 50\Omega$, $Z_0 = 100\Omega$

Using (4) we can write

$$\begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\frac{Z}{Z_0} e^{-j2\theta_1}}{2+Z/Z_0} & \dfrac{2e^{-j(\theta_1+\theta_2)}}{2+Z/Z_0} \\ \dfrac{2e^{-j(\theta_1+\theta_2)}}{2+Z/Z_0} & \dfrac{\frac{Z}{Z_0} e^{-j2\theta_2}}{2+Z/Z_0} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{-j0.5}{2+0.5} & \dfrac{-2j}{2+0.5} \\ \dfrac{-2j}{2+0.5} & \dfrac{-j0.5}{2+0.5} \end{bmatrix}$$

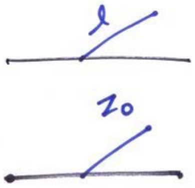
$$= \begin{bmatrix} -j0.2 & -j0.8 \\ -j0.8 & -j0.2 \end{bmatrix}$$

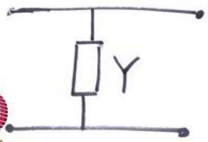
These are the s-parameters for given network between port 1' and 2'.

2. Find the scattering parameters of an open circuited stub of length l and characteristic impedance Z_0.

Solution.

Given network can be compared with





ABCD-parameters for admittance Y are

$$T = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

For stub of length l, $Y = \frac{1}{Z} = \frac{1}{-jZ_0 \cot\beta l} = \frac{j\tan\beta l}{Z_0}$

Therefore equivalent S-parameters can be calculated

$$S = \begin{bmatrix} \frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D} & \frac{2(AD-BC)}{A+B/Z_0+CZ_0+D} \\ \frac{2}{A+B/Z_0+CZ_0+D} & \frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D} \end{bmatrix} = \begin{bmatrix} \frac{-j\tan\beta l}{2+j\tan\beta l} & \frac{2}{2+j\tan\beta l} \\ \frac{2}{2+j\tan\beta l} & \frac{-j\tan\beta l}{2+j\tan\beta l} \end{bmatrix}$$