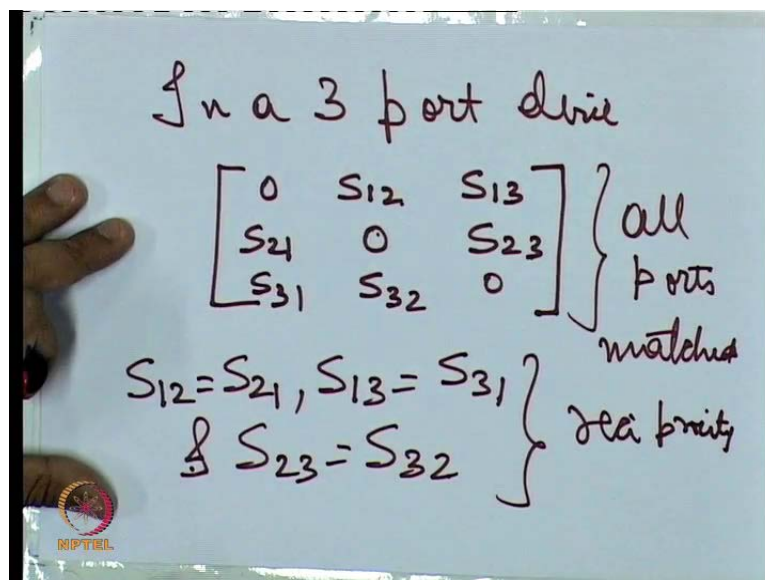


Microwave Integrated Circuits.
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Department of Electrical Engineering.
Indian Institute of Technology Bombay.
Lecture -16.
3 Port Microwave Components.

Hello, welcome to another module of this course microwave integrated circuits. In the previous module we had discussed about 1 and 2 port microwave devices, in this module we will continue our discussion on the same lines. And we shall be discussing about 3 port devices. So, while we have derived the conditions of the S parameters for 1 and 2 port networks, let us try to do the same thing for 3 port devices.

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So, in a 3 port device, if all the ports are matched, then our S parameter matrix looks something like this.

Now, in addition you saw, this is the condition for all ports are matched. Then if it is reciprocal, then we have S₁₂ is equal to S₂₁, S₁₃ equals to S₃₁ and S₂₃ is equal to S₃₂. This is the condition for reciprocity. In addition to this, if it is also a lossless, we have to have and...

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For lossless $SS^H = U$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{23}|^2 = 1$$
$$|S_{13}|^2 + |S_{23}|^2 = 1$$
$$S_{13}^* S_{23} = S_{23}^* S_{12} = S_{12}^* S_{13} = 0$$

The whiteboard contains handwritten mathematical derivations for a lossless system. It starts with the condition $SS^H = U$ and derives three equations for the magnitudes of the scattering parameters: $|S_{12}|^2 + |S_{13}|^2 = 1$, $|S_{12}|^2 + |S_{23}|^2 = 1$, and $|S_{13}|^2 + |S_{23}|^2 = 1$. Finally, it shows that $S_{13}^* S_{23} = S_{23}^* S_{12} = S_{12}^* S_{13} = 0$.

This by the way follows from this condition that we had started earlier. This whole thing will be equal to 0.

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In a 3 port device

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \left. \begin{array}{l} \text{all} \\ \text{ports} \end{array} \right\} \text{matched}$$
$$\left. \begin{array}{l} S_{12} = S_{21}, S_{13} = S_{31} \\ S_{23} = S_{32} \end{array} \right\} \text{reciprocity}$$

The whiteboard notes describe a 3-port device. It shows a scattering matrix with zeros on the diagonal and off-diagonal elements $S_{12}, S_{13}, S_{21}, S_{23}, S_{31}, S_{32}$. A bracket indicates that all ports are matched. Another set of equations, $S_{12} = S_{21}, S_{13} = S_{31}$ and $S_{23} = S_{32}$, is indicated as reciprocity conditions.

Now the problem with these 2 sets of equations, this set that I derived previously, that I had stated previously and this another set, so these were all ports matched, these were for reciprocity

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For losslessness

$$|S_{12}|^2 + |S_{13}|^2 = \underline{S S^H = 0}$$
$$|S_{12}|^2 + |S_{23}|^2 = 1$$
$$|S_{13}|^2 + |S_{23}|^2 = 1$$
$$S_{13}^* S_{23} = S_{23}^* S_{12} = S_{12}^* S_{13} = 0$$

The whiteboard also features the NPTEL logo in the bottom left corner.

and this was the condition for losslessness. The problem with all these sets of equation is that they do not have any solution, that is a problem.

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These eqns do not have a solution

matching + reciprocity + losslessness

↓ cannot achieve all together

The whiteboard also features the NPTEL logo in the bottom left corner.

So these equations do not have a solution, so therefore we cannot have all the 3, that is matching + reciprocity + losslessness, all together cannot achieve.

So, then what is the solution? The solution is that if we relax some of these criterion, then however it is possible. So, while we cannot have matching at all ports, we can have matching at one port, not more than 1 and along with that we can also have losslessness and reciprocity.

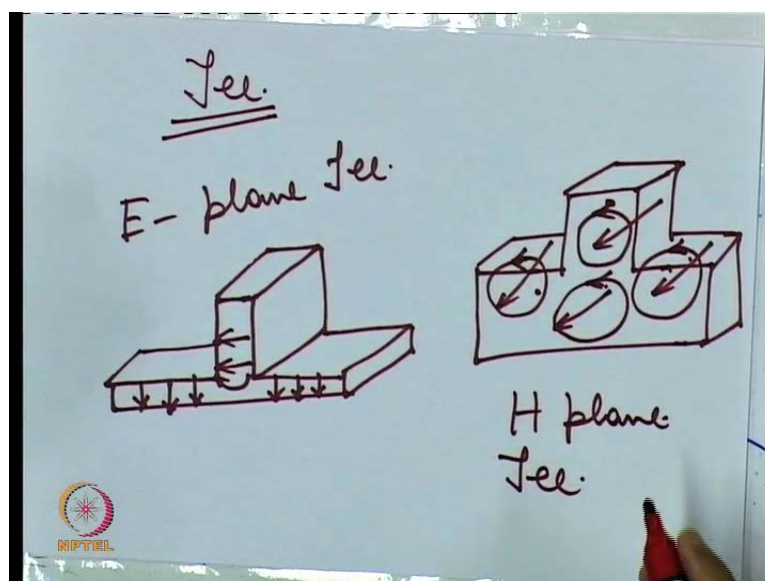
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TEE		Power divider	
	Tee.	Circulator	Power divider.
Lossless	X	X	
Reciprocal	X		X
Matched		X	X

So, if I make a small table like this, suppose I want lossless and reciprocal and the other property that is desirable is matched. Then a device that is both lossless and reciprocal, such a 3 port device is called Tee, device which is lossless and matched is called a circulator and a device which is reciprocal and matched is called a power divider.

So, from this discussion, this is a, let me write it down, power divider. So, we will discuss this later on, all this in much detail. But if we can go back here once, see this only this T device called Tee, only this device does not provide matching at all ports, however it can be matched at a single port. So, let us now discuss one by one all these devices.

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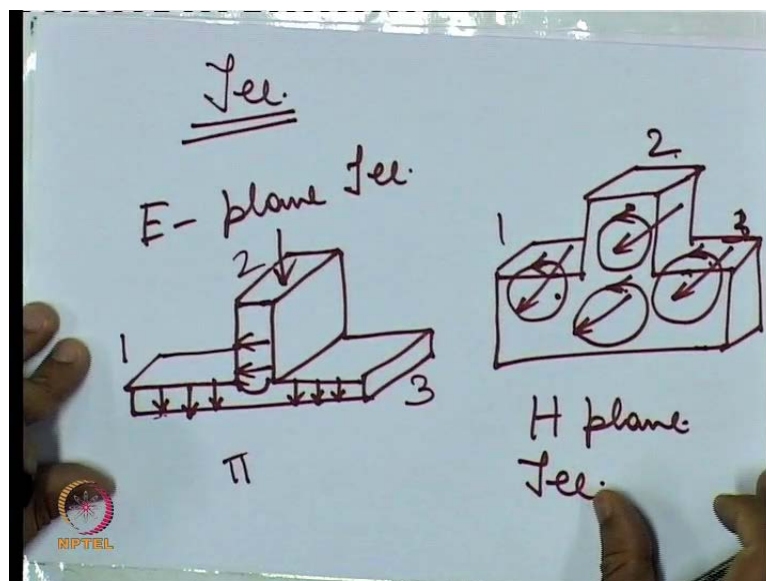


The 1st device that I would like to discuss is the Tee. There were 2 common types of Tees, one is what is called as E plane Tee. Now, E plane tee is a single conductor waveguide whose shape is given like this.

The electric fields, rule of thumb for electric fields is that they are always, they always exist between the surfaces which are closest to each other. So, for an E plane tee, since these 2 surfaces are closest to each other, they E plane tee, the electric fields will be between these 2 surfaces. The magnetic fields however are concentric to these electric fields. So, there is the plane of this tee. So, for example for these electric field lines, the magnetic fields will be rotating about these electric field lines.

Now this is an E-plane tee, a kind of complimentary structure where the shape is same but the widths are different is what is called an H-plane tee. In H plane tee again, the same principle will be followed that the electric field lines are between the surfaces which are closest to each other and the magnetic field lines will be like this, concentric to the electric fields. So, this structure is known as H plane tee.

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Now the S parameter matrix, we can show by a derivation but I can just simply state it that for an E plane tee, if we go back to this figure, the phase difference between say this is port 1, this is port, this is port 2, this is port 1 and this is port 3, the phase difference between any signal exiting suppose the input power at port 2, then the phase difference of the exiting signals at port 1 and port 3 will be π phase shifted. Whereas for an H plane tee, if we have power entering port 2, then there will be no phase shift between Port 1 and port 3.

This is port 3 for a H plane tee this is port 1, if you have power entering port 2, there will be no phase shift between Port 1 and port 3 of the power exiting through them. So, with this knowledge, we can, I am just simply stating the S parameter matrix of the E plane tee.

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The image shows two handwritten S parameter matrices on a whiteboard. The first matrix is for an E plane tee and the second is for an H plane tee. Both matrices are 3x3. The E plane tee matrix has a negative sign in the S₂₃ element, while the H plane tee matrix has a positive sign in the S₂₃ element. The NPTEL logo is visible in the bottom left corner of the whiteboard.

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \begin{matrix} E \\ \text{plane} \\ \text{tee.} \end{matrix}$$

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \begin{matrix} H \text{ plane} \\ \text{tee.} \end{matrix}$$

It is given like this. So, this is the S parameter matrix for an E plane tee. As parameter matrix for an H plane tee will be similar except that these negative signs will not be here. So, this shows that there is no phase shift between say, this is S₂₃ and this is S₂₁, the presence of a negative sign shows that the phase shift between ports 1 and 3 for power entering port 2 is 180°. For an H plane tee there will be exactly the same, except these negative signs will not be there.

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The image shows handwritten text on a whiteboard. The word "Circulator" is underlined. To its right, the words "lossless" and "+ matched" are written. A hand holding a red marker is visible on the right side of the whiteboard. The NPTEL logo is visible in the bottom left corner.

Circulator lossless
+ matched

The next class of 3 port devices that I stated was circulators. So for a tee we saw that it was a reciprocal and lossless device, for circulator, it is a lossless and matched but not reciprocal. So, tee was lossless and reciprocal but not matched at all ports. A circulator is matched and lossless but not reciprocal, so it is the 1st non-reciprocal material that we are seeing and there are some special materials which have this property, which are passive materials but they kind of because of the different polarisations introduced when wave travelling in different directions through the same material, because of that, because of that when wave travels in one direction, we get a different power transfer as compared to the power transfer happening when the wave travels in the other direction.

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Circulator lossless
+ matched.

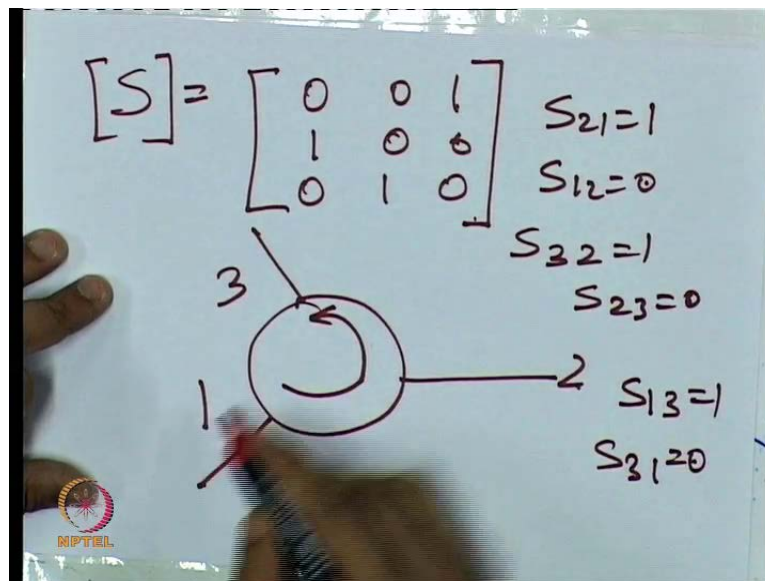
$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \Rightarrow |S_{21}|^2 + |S_{31}|^2 = |S_{12}|^2 + |S_{32}|^2$$

$$|S_{13}|^2 + |S_{23}|^2 = 1$$

$$S_{31}^* S_{32} = S_{23}^* S_{21} = S_{12}^* S_{13}$$

So, for a circulator, suppose since I said all the ports are matched, I might be able to write the S parameter matrix like this. Now since it is lossless, it should satisfy the unitary condition and from this equation we get the following equations. Now, with some suitable scaling of the various arms of the circulator that is by scaling the arms and we might change the phase relationships at the different ports.

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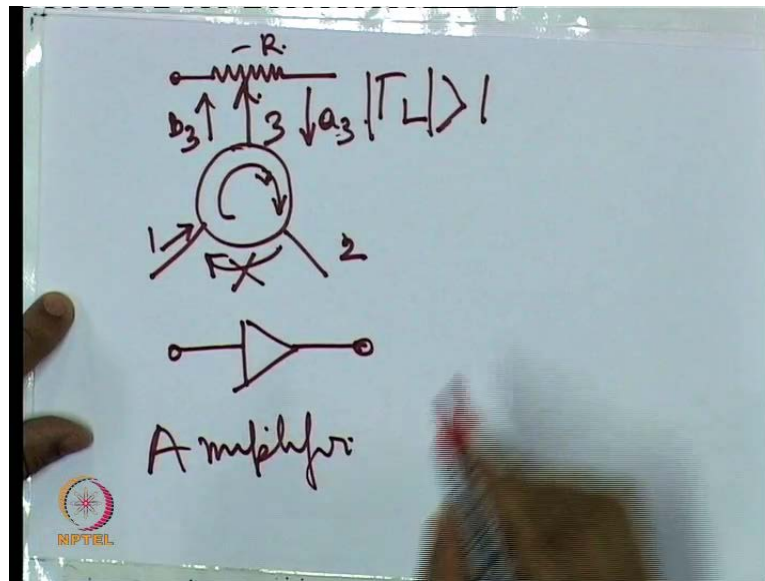


the final S parameter matrix that I get for this circulator can be given as... The symbol of a circulator is like this, this is port 3, this is port 1, this is port 2 we use an arrow to show the direction in which power transfer is allowed, so in this case, power as also seen from this S parameter matrix, we see that S_{21} equal to 1 but S_{12} equal to 0.

What this means is power can transfer from port 1 to port 2 but not from port 2 to port 1 and also we see S_{32} is equal to 1 but S_{23} is equal to 0. So, again what it means is power can transfer from port 2 to port 3 but not back and again S_{13} is equal to 1 but S_{31} is equal to 0 which again means that power can transfer from port 3 to port 1 but not in the reverse direction. Now what are some applications of a circulator?

Circulator, even though it might appear to be a very novel device, it is actually quite extensively used because sometimes, it is not that these nonreciprocal devices are always undesirable, there are some very specific uses of these devices. Let us see some of the uses.

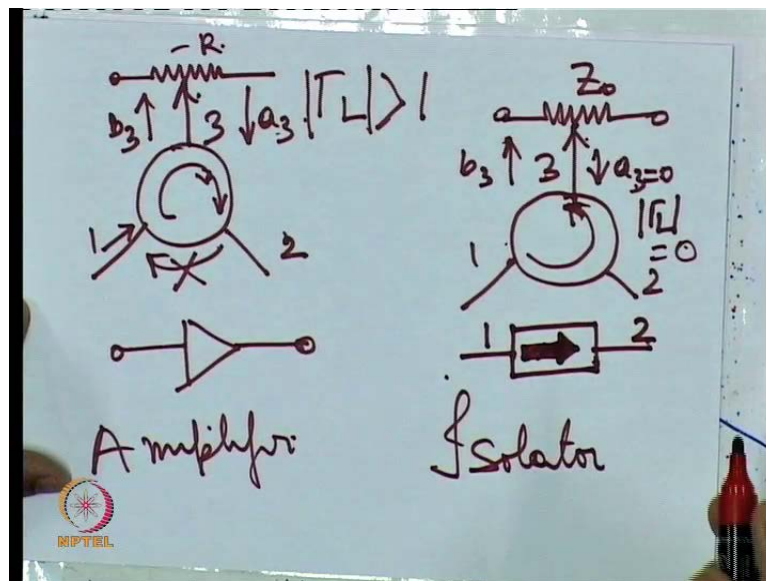
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One is if you say this is your circulator and you connect a negative resistance to say port 3, port 2. Now negative resistance will always cause input say if this is my say B_3 that is entering this negative resistance, then A_3 the wave incident at port 3 but reflected from the load will be at the magnitude of this wave will actually be greater than the magnitude of B_3 because Γ_L magnitude will be greater than 1.

So, we saw this for passive devices Γ_L is always has a magnitude less than 1 that means the resistance is negative, the magnitude of the reflection coefficient will be greater than 1. So, with such a setup, any signal entering port 1 will be allowed to pass port 3 only at which point it will get amplified and transmit back to port 2 and any transmission back from port 2 to port 1 is not allowed. So, this will act like an amplifier.

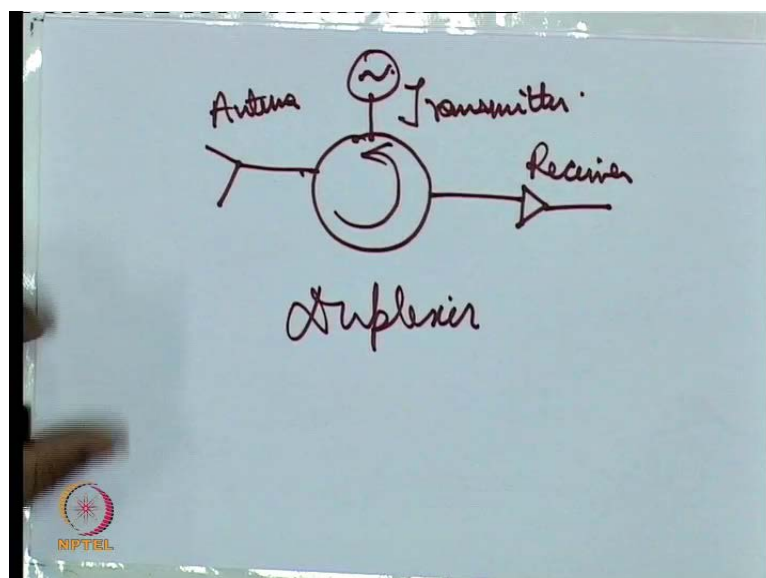
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Then one other use could be, suppose I connect my matched load to my circulator at the port 3 once again and so what is happening is that power will transfer from port 1 to port 2 and in the reflected part, power, any signal that is reflected from port 2 will be allowed to transmit only to port 3 where it gets absorbed at this matched load. So, since here the A_3 reflected wave will be 0 and only B_3 will... So, any signal reaching port 3, once it encounters Z_0 , it will not be, no component of that B_3 will be reflected back.

So, we have here Γ_L is equal to 0, because of that A_3 is 0 and so power will transfer only from port 1 to port 2, any reflection happening at port 2 will never come back to port 1. So, this is like an isolator. Symbol for isolator is like this, it is a solid arrow.

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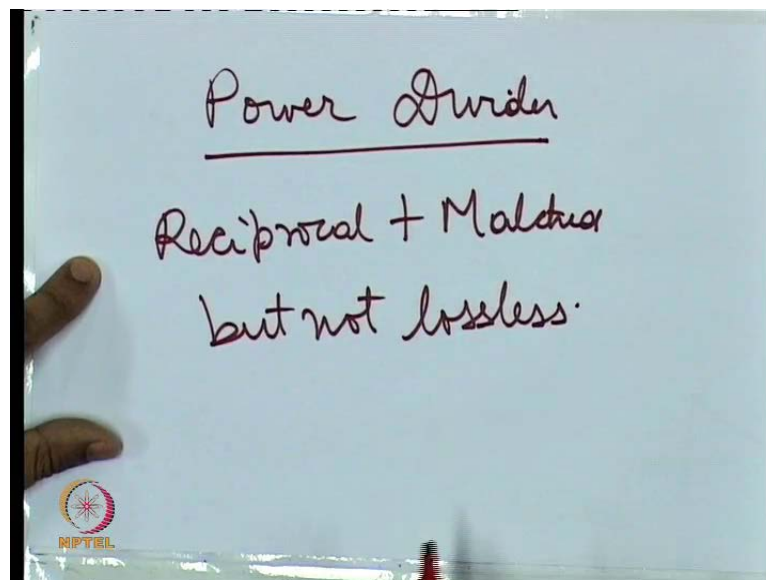


And one other use of circulator is what is known as a shared antenna. So, a shared antenna, you know in communication we have to use the same antenna often use the same antenna for both the transmitter as well as the receiver but then to do that, we have to allocate different timeslots for the transmitter and the receiver.

That often becomes a problem because such allocation needs good switch and there should also be no leakage between the signals of the transmitter and receiver, so there the circulator comes handy because if this is our antenna and suppose this is the path along which signal transmission is allowed and any signal received by the antenna will be transmitted only along this path will be received by the antenna and suppose the receiver is well matched, then no component of the received signal will be transmitted back to the transmitter.

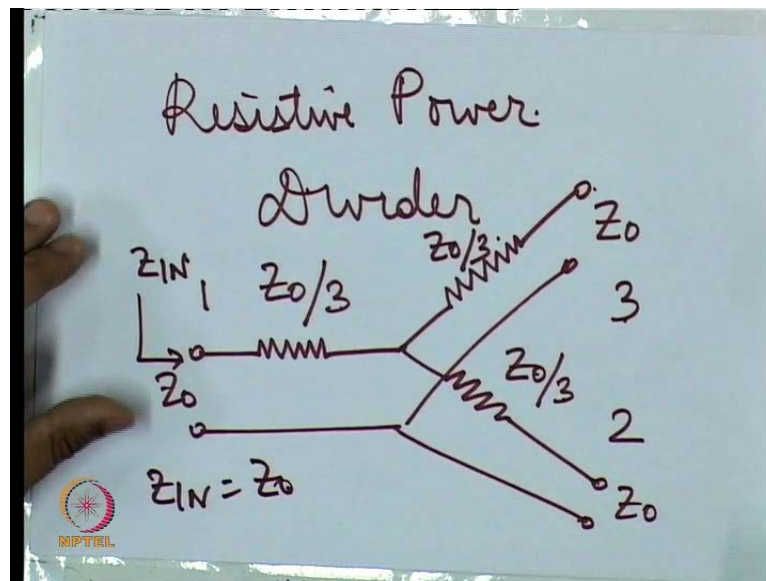
The transmitter on the other hand, any signal that it sends will reach the antenna 1st and if the antenna is well matched, then it will not allow any reflected wave, it will allow no reflection back, so no component of the transmitted signal will be received back by the receiver. So, this is a duplexer implementation. And finally the next, the 3rd category of 3 port devices that I mentioned which is reciprocal and matched at all ports but not lossless. That kind of device is called a power divider.

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So, it is reciprocal + matched but not lossless. Now, power divider is a well-known circuit, simple resistive network can provide power division. So, let us see a simple resistive power divider.

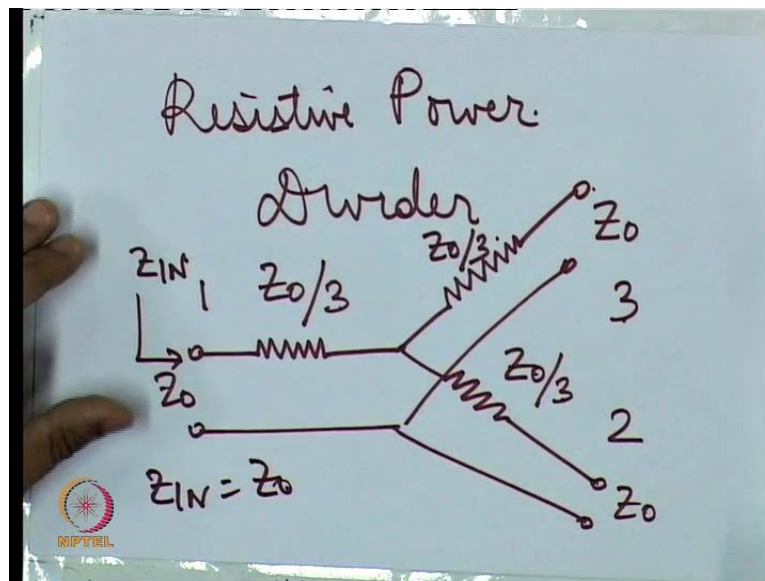
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Say Z_0 is the characteristic impedance at all 3 ports and say we have 3 resistances of value Z_0 by 3, Z_0 by 3 and Z_0 by 3, now you can verify that if I connect a load Z_0 to each of these ports, say I call this port 2, this as port 3 and this is port 1. Suppose I connect impedance Z_0 at ports 2 and 3 and then I try to find out the input impedance Z_{in} , then Z_{in} will be equal to Z_0 and that will be true for all the other ports.

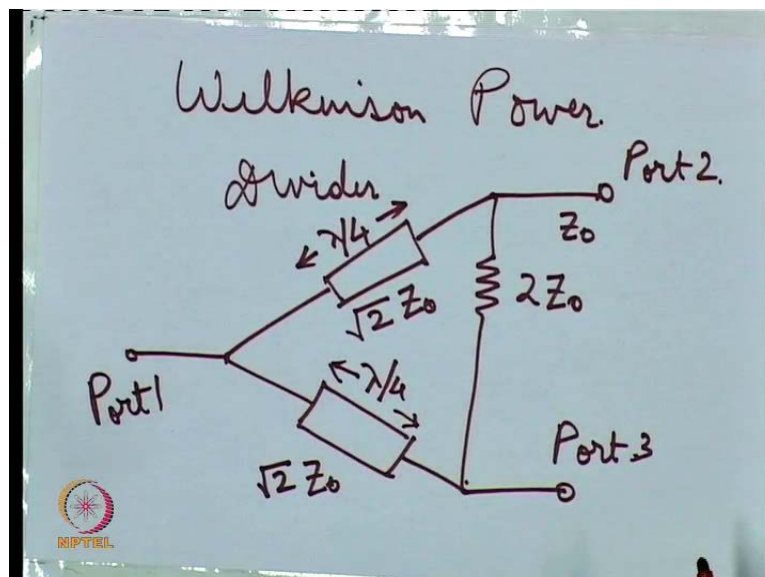
So, such a common of this kind of implementation **is** achieves power division and it is very simple. But the problem with this implementation is that if we go back to our slide, any single entering port 1 will be equally divided between ports 2 and 3 or by suitably adjusting these resistances we can have a division ratio between ports 2 and 3. But any signal entering port 3 will also have a component appearing at port 2 in addition to the component appearing at port 1.

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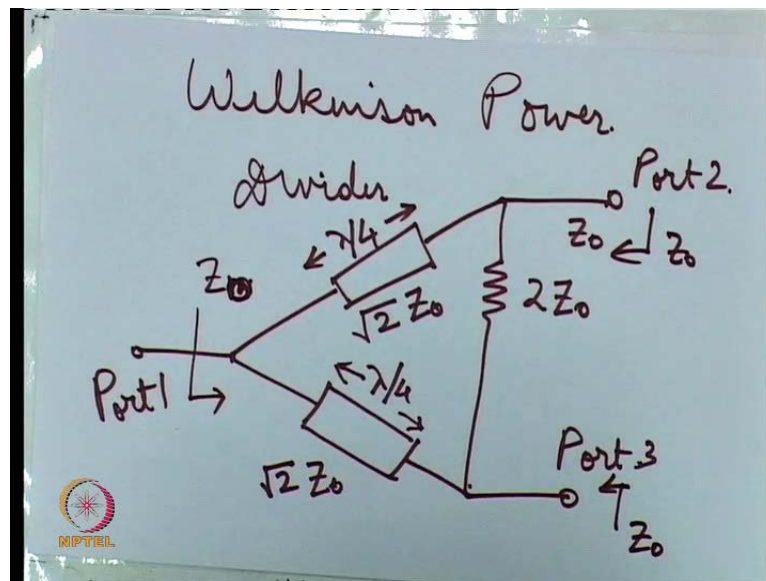
In applications, what happens is it is desirable to isolate these ports 2 and 3 that is only power division is possible only when signal enters port 1 but when signal enters port 3, no component of that will appear at port 2 or vice versa. So, one such implementation is using a circuit known as Wilkinson power divider.

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So, the circuit for this Wilkinson power divider is like this. The circuit is like this and any power entering port 1 will be equally divided between port 2 and port 3 but no component of power entering port 2 will appear at port 3.

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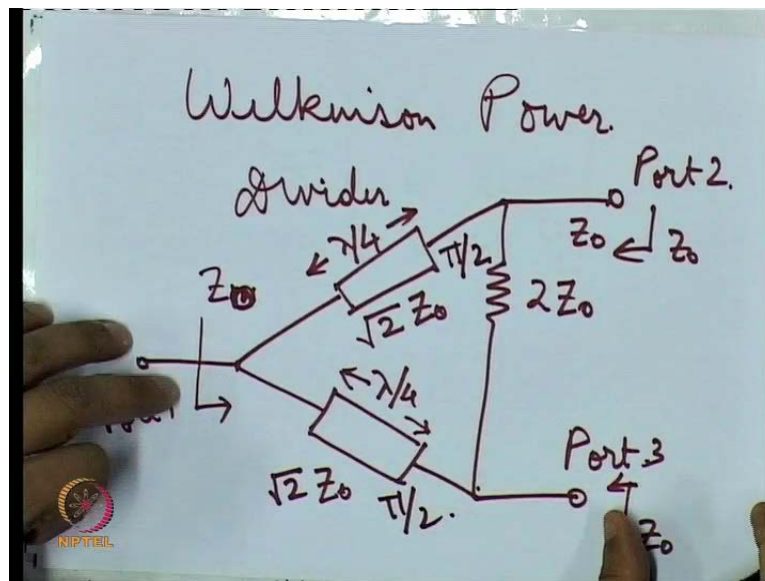
Now you can verify that the input impedance is $Z_{in} = Z_0$ for all the ports. The methods have been shown in the slides associated with this analysis. The S parameter matrix of this Wilkinson power divider can be given by this equation.

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$$S = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

If we can go back to our slides please, the S parameter matrix is given by this matrix. We see that it is a reciprocal device because it is symmetric. It is also matched because the diagonal elements are 0.

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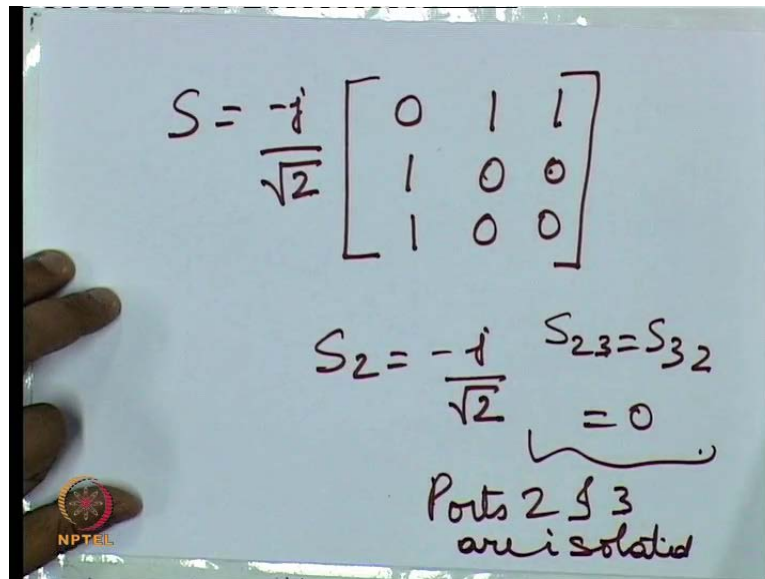
And this J is introduced because of the presence of these quarter wavelength pieces of transmission line. Now everytime, what these quarter wavelengths, as we know total electrical angle of a quarter wavelength transmission line is $\pi/2$.

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$$S_{11} = -\frac{j}{\sqrt{2}}$$
$$S_{21} = -\frac{j}{\sqrt{2}}$$

That is why we have say S_{21} is actually equal to $-j$ upon root 2. That means anytime power enters, signal enters port 1 by the time it reaches port 2, it is phase shifted by $\pi/2$ and same is the case for signal appearing at port 3.

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A photograph of a whiteboard with handwritten mathematical expressions. The top expression is a 3x3 S-parameter matrix: $S = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Below this, the text reads: $S_{22} = \frac{-j}{\sqrt{2}}$, $S_{23} = S_{32} = 0$. A bracket underlines the zero values, and the text below says "Ports 2 & 3 are isolated". In the bottom left corner of the whiteboard, there is a small circular logo with the text "NPTEL" below it.

Of course we see one thing that S_{23} is equal to S_{32} is equal to 0 which means ports 2 and 3 are isolated.

Ha so, in this lecture we cover the 3 port devices as I said, there are 3 types of 3 port devices, the 1st one being reciprocal and also lossless but not matched at all ports, it can be have a match at one port and that device is called a tee. The 2nd device which we discussed was nonreciprocal, lossless and matched at all ports, that device was called a circulator. And the 3rd device that we studied was reciprocal, matched at all ports but not lossless, and that device we call it as power dividers. In the next lecture, we shall be covering 4 port microwave devices and 4 port microwave devices are known by the general term of coupler. Thank you.