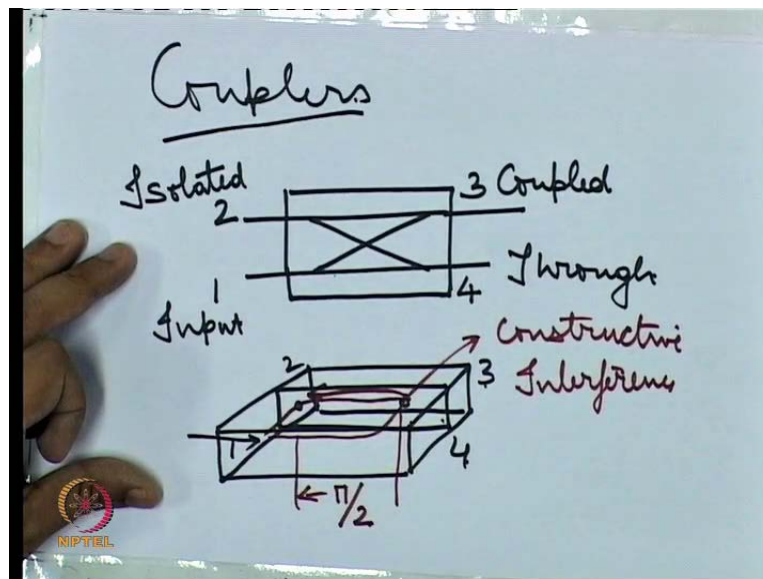


Microwave Integrated Circuits.
Professor Jayanta Mukherjee.
Department of Electrical Engineering.
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Lecture -17.
Couplers.

Hello, welcome to another module of this course microwave integrated circuits. In the previous module we had covered the topics related to 3 port microwave devices like circulators, Tees and power divider. In this module we are going to cover 4 port devices, now one thing about 4 port devices is that they can all be matched at all ports, they can also be reciprocal and they can also be lossless. Excuse me. So, the mathematics related to the 4 port devices permit them to have all the 3 properties which was not the case for 3 port devices.

And all 4 port devices are known by the general term couplers. So, unlike 3 different types of devices for 3 port devices like circulators, or tees or power dividers, all 4 port microwave devices are known by the general term couplers. So, let us discuss a little bit more about couplers.

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So, couplers are represented by this symbol. There is one port which is called the input port there is another port which is called the coupled port, there is another which is called the through port and one which is called the isolated port.

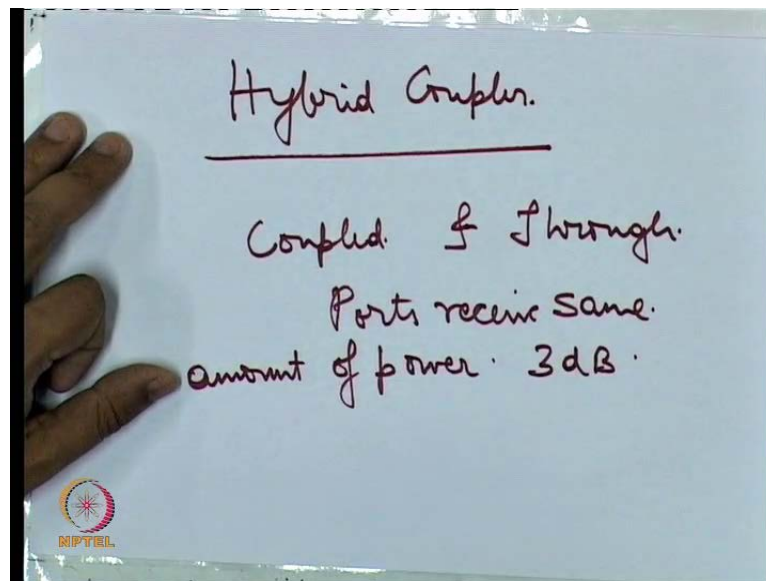
Now the basic point is that there will be a signal reaching the input port and the part of that will travel to the coupled port and a part of that will travel to the through port and no part of

the input signal should reach the isolated port. So, in olden days the reason we have, you might ask me what is the difference between the coupled and the through port? There is actually no difference in modern-day couplers but when couplers 1st began to be manufactured or designed, it was that there was this this coupled port, the energy appearing at the coupled port was kind of coupled to it rather than direct connection as that existed in through port, the energy would have to travel over a gap or something like that.

So, one example of a coupler, a very simple waveguide-based coupler is suppose, suppose we have a waveguide with the partition in between and suppose this is our input port, let us number this, isolated port is usually represented by the symbol 2, coupled port is represented by the symbol 3 and throughput by the symbol 4. Now, if here suppose here 2, this is 3 and this is 4, now if there is a signal coming at port 1 then a part of that signal will travel like this and another part will travel like this and the distance between these 2 holes or coupling holes are so adjusted that when these a signal passing through a hole coming here and another single passing this way and then coming here, when they meet, they add constructively.

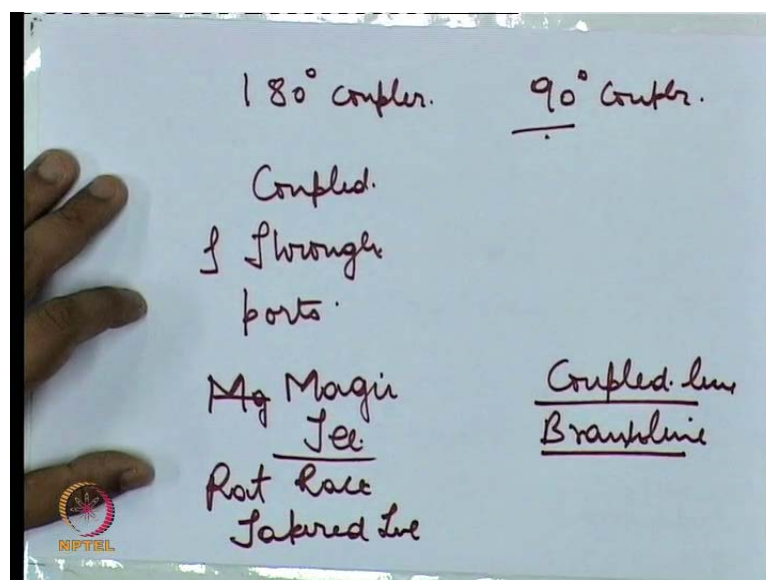
So, constructive interference. And now a part of this signal entering this hole will also come back to this point and here the phase differences are such, this is basically π by 2 phase difference, so the signal travels undergoes a phase change of π by 2 while travelling from here to here and then again a phase change of π by 2 and finally when it reaches back at this point, there is destructive interference and hence port 2 is isolated or decoupled. But then at port 3, where constructive interference happens, there there is addition of power. So, some amount of this input power will reach port 3.

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Now, with this let us see what other various types of couplers. One term that is frequently associated with couplers is what is known as a hybrid coupler. Now, in a hybrid coupler, the division between the coupled and through ports is equal, so coupled and through ports receive same amount of power. So, in other words, that is why this hybrid coupler is also known as the 3 dB coupler. That is equal division or half of the power goes input power goes to the coupled port and half of the input power goes to the through port.

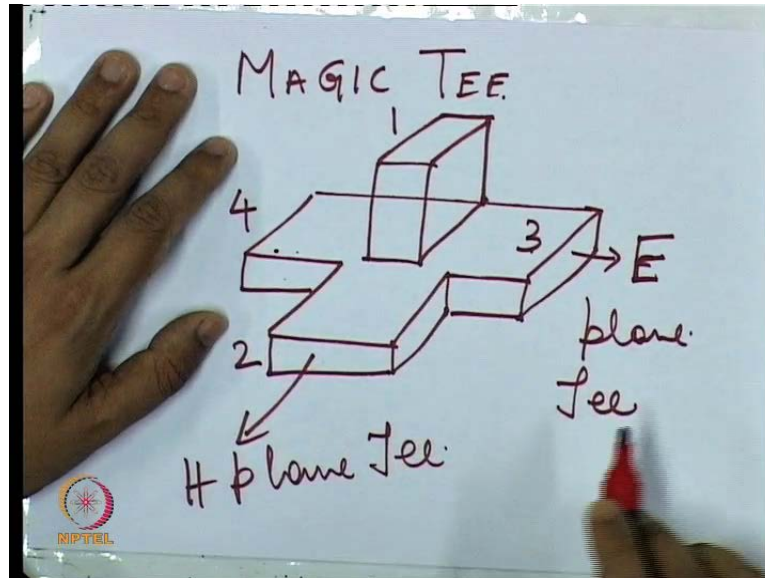
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The other way of classifying couplers is based on a property in the S parameter matrix and page on this we can have 2 different types of couplers, one is called 180° coupler and the other is the 90° coupler. Now, in a 180° coupler, the phase difference between the coupled

and through ports is 180° and in 90° , the coupled and through ports have a 90° phase shift, that is the difference between them. Some examples of 180° couplers are magic tee, then what we call rat race coupler, tapered line and then some examples of 90° couplers are coupled line, Blanch line, of this we will study in detail this magic Tee coupled line and branch line.

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So, let us start with the magic tee. Now the construction of a magic Tee is something like this. So, it is like E plane tee superimposed with a H plane tee. Now we knew that for an E plane tee, suppose this is port 1, 2, 4 and 3, we knew that for E plane tee the phase shift between the port for and port 3 will always be 180° . But when power is inputting say port 2, there will be equal 3 shifts. So, basically by a combination of this and by suitably arranging the suitably calculate designing length of these tee branches, we can obtain the S parameter matrix for the Magic Tee as shown here.

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S parameter Matrix

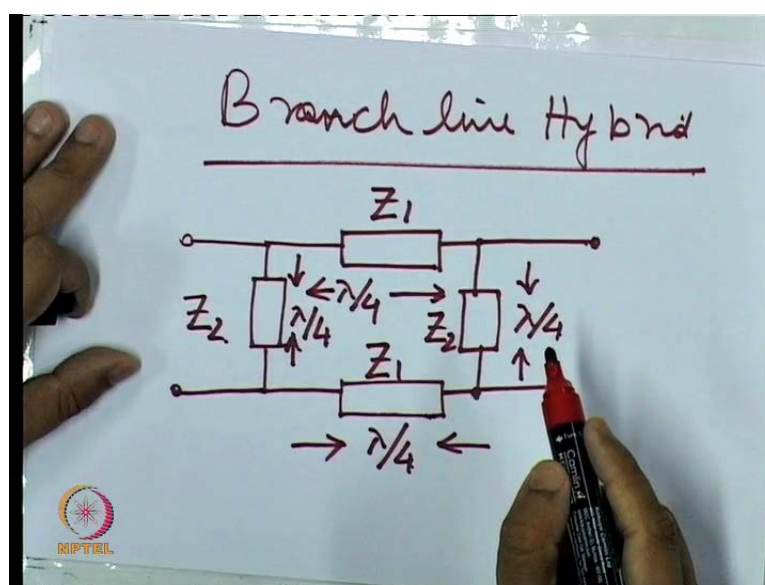
$$[S] = \begin{bmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

$S_{41} = -S_{31} = \frac{1}{\sqrt{2}}$
 $S_{42} = S_{32} = \frac{1}{\sqrt{2}}$

E plane Tee
 H plane Tee

So, the S parameter matrix for a magic tee. So, from the E plane tee, basically what we got was S_{41} is equal to $-S_{31}$ which is equal to 1 upon square root of 2 , so this is from the E plane tee. And from the H plane tee we got S_{42} is equal to S_{32} which is equal to 1 upon square root of 2 , so this is from the H plane tee. And that is why when combining both these, they are getting this kind of S parameter matrix and as we can see port 1 and 2 are isolated and similarly port 3 and 4 they are also isolated, port 3 and 4, this value and this value is also equal to 0 . So, that is why port 3 and 4 are isolated and port 1 and 2 are isolated and the remaining are either through or coupled.

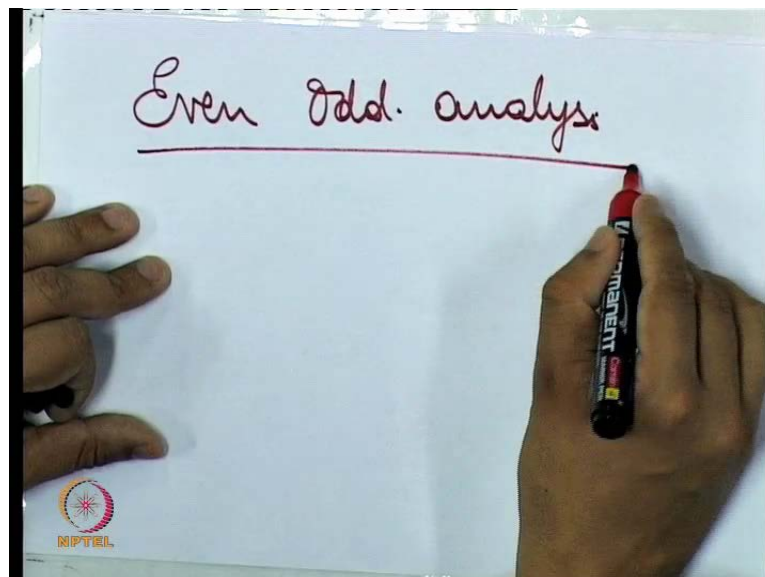
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Similarly if we would like to discuss the branch line hybrid, so that is an example of a, so this magic tee was an example of a, it was an example of a 180° coupler and this branch line hybrid coupler is an example of a 90° coupler. The construction of branch line hybrid is we just go via schematic diagram is like this. So, it consists of 4 transmission lines connected in this format, each having length $\lambda/4$ where λ corresponds to a particular frequency and characteristic impedances of the task mission lines along the vertical transmission line is Z_2 .

And the characteristic impedances of this horizontal transmission line is equal to Z_1 . Now we can what we call an even-odd analysis to analyse this hybrid, to just give you a hint of how to do this analysis, let us see the what is this even odd analysis.

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IIT Bombay
Even Odd Analysis

$$0 = \frac{a_1}{2} + \frac{-a_1}{2}$$

$$a_1 = \frac{a_1}{2} + \frac{a_1}{2}$$

• The even odd mode circuits are then as shown above.

You see, this is a schematic diagram that I have just drawn and this line represents these 2, this is Z_1 and these 2 lines represent the transmission lines having characteristic impedance Z_1 and these 2 lines represent the transmission lines having characteristic impedance Z_2 .

Now assume that there is an imaginary line which is cutting through these circuits along a horizontally. Now this is the imaginary line which is cutting through the vertical transmission lines horizontally. Now suppose there is a signal A_1 appearing at port 1 then this A_1 can be considered as the sum of 2 sub signals A_1 upon 2 and $+ A_1$ upon 2 and at port 2, where no signal is appearing, that can be considered, basically that is 0, 0 input at port 2. Now this can be considered as a summation of 2 equal and opposite signals A_1 upon 2 and $- A_1$ upon 2.

So, when they sum up, the net input appearing at port 2 is 0. So, port 2 is the isolated port, so anyway it should be isolated, that is no signal should either come in or come out from the spot. Now, because of this cemetery that is present in this circuit, when we cut it half along this line assume the case when we have only A_1 upon 2 at port 2 and A_1 upon 2 at port 1, that we call is even mode. And the case when we have A_1 upon 2 as the input at port 1 and $- A_1$ upon 2 as the input at port 2, that we call the odd case. And since this is a linear circuit, the net output or any parameter that we obtained in this circuit will be the superposition superposition theorem will be valid and hence the net output at any port will be the superposition of the outputs obtained due to the even mode or the odd mode.

Now consider when only the odd mode is present, that is port 1 is fed by A_1 upon 2 and port 2 is fed by $- A_1$ upon 2, then because of this equal and opposite nature of the input signals, we can assume that the voltage along this horizontal line is 0 because this is $+$, this is $-$ this is $+$ and both are equal in magnitude, then the voltage along this horizontal line should be 0 and based on this we have drawn this equivalent circuit for the odd mode. So, here we have a short which is represented by this line and this short is represented by this line. This short is now $\lambda/8$ length since we have cut it in half and Z_1 remains the same but this $\lambda/8$ length has a characteristic impedance Z_2 .

And there is a load Z_2 Z_0 connected. Similarly in the even mode, when the inputs are equal at both the port 1 and port 2, this line along the can assume the voltage along this horizontal line is this horizontal line is basically open, here it was a short because the voltage was 0 and because the voltages of these 2 points are same for the even mode, hence this point is basically floating. There is no current passing through it. And based on this, we have this

equivalent model of for the even mode which is exactly the same as odd mode, except that instead of short, there is an open here and there is an open here.

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Even Odd Analysis

$$0 = \frac{a_1}{2} + \frac{-a_1}{2}$$

$$a_1 = \frac{a_1}{2} + \frac{a_1}{2}$$

Port 1 Z_0 Z_1 Z_2 Port 4
 Even Mode Open Open

Port 1 Z_0 Z_1 Z_2 Port 4
 Odd Mode Short Short

• Given a_1 at port 1 the reflected waves of the 4 port network are given by :

$$b_1 = S'_{11e} \frac{a_1}{2} + S'_{11o} \frac{a_1}{2} \Rightarrow S_{11} = \frac{1}{2} S'_{11e} + \frac{1}{2} S'_{11o}$$

$$b_2 = S'_{12e} \frac{a_1}{2} - S'_{12o} \frac{a_1}{2} \Rightarrow S_{21} = \frac{1}{2} S'_{12e} - \frac{1}{2} S'_{12o}$$

$$b_3 = S'_{41e} \frac{a_1}{2} + S'_{41o} \frac{a_1}{2} \Rightarrow S_{41} = \frac{1}{2} S'_{41e} + \frac{1}{2} S'_{41o}$$

$$b_4 = S'_{42e} \frac{a_1}{2} - S'_{42o} \frac{a_1}{2} \Rightarrow S_{41} = \frac{1}{2} S'_{41e} - \frac{1}{2} S'_{41o}$$

Because these circuits are composed of cascaded lines (two ports), it is easiest to analyze these networks using ABCD matrices.

Now, how do we analyse this circuit? As I said, any, let us consider that A1 is the input at port 1 and B1 is the reflected wave at port 1 and suppose S11E and S11O are the S11 parameters for the even mode circuit and the odd mode circuit. Then the total reflected wave at port 1 will be the combination of the contribution of the even mode and the odd mode and hence we can write B1 by this equation which in turn translates to that the overall S11 parameter that is B1 upon A1 is equal to this equation. Similarly for B2, now in the case of B2, we can assume that B2 is the reflected wave at port 2, so B2 is the combination of the input of the even mode and the odd mode.

Now for B2 for the even mode, the voltage that appears at port 2 is A1 upon 2 and for odd mode the voltage that, the signal, I beg your pardon, I should not say voltage, it is a normalised voltage, the normalised voltage that appears at port 2 is - A1 upon 2. So then the net B2 should be the summation of the contribution of A1 upon 2 and - A1 upon 2. Hence B2 is written like this and S21 that is B2 upon A1 is given by this equation. Now we can similarly derived the condition the equation for S41 and S31 in terms of the S41 and S11 parameters for the even and odd mode circuits.

The other thing that we note is that these even and odd mode circuits consist of 3 different circuits, one is this branch, then the length of the transmission line and again an open branch,

so these 3 are in cascade and because the 3 are in cascade, we can apply the cascade parameters to find out the equations.

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IIT Bombay
ABCD Matrices

- In Chapter 4 we introduced the ABCD matrix of a transmission line of arbitrary electrical length. For $\theta = \pi/2$ ($\lambda/4$) we obtain :

$$\begin{bmatrix} \cos \theta & jZ_1 \sin \theta \\ jY_1 \sin \theta & \cos \theta \end{bmatrix}_{\theta=\pi/2} = \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix}$$
- The open or short lines are connected in shunt, so we can use the ABCD matrix for a shunt lumped element Y with Y the input admittance of a $\lambda/8$ length open or shorted line :

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \text{ with } \frac{1}{Y} = Z = \begin{cases} -jZ_2 \cot \theta = -jZ_2 \text{ open stub (even mode)} \\ jZ_2 \tan \theta = +jZ_2 \text{ short stub (odd mode)} \end{cases}$$
- The cascaded ABCD matrix $[M]_{e/o}$ for the even/odd mode is then :

$$\begin{bmatrix} 1 & 0 \\ \pm jY_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \pm jY_2 & 1 \end{bmatrix} = \begin{bmatrix} \mp Z_1 / Z_2 & jZ_1 \\ jZ_1 \left(\frac{1}{Z_1^2} - \frac{1}{Z_2^2} \right) & \mp Z_1 / Z_2 \end{bmatrix}$$

So, these are the cascade parameters. Now for a simple length of transmission line which we had earlier seen that the cascade parameters are given by this matrix.

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IIT Bombay
Even Odd Analysis

$$0 = \frac{a_1}{2} + \frac{-a_1}{2}$$

$$a_1 = \frac{a_1}{2} + \frac{a_1}{2}$$

- Given a_1 at port 1 the reflected waves of the 4 port network are given by :

$$b_1 = S'_{11e} \frac{a_1}{2} + S'_{11o} \frac{a_1}{2} \Rightarrow S'_{11} = \frac{1}{2} S'_{11e} + \frac{1}{2} S'_{11o}$$

$$b_2 = S'_{11e} \frac{a_1}{2} - S'_{11o} \frac{a_1}{2} \Rightarrow S'_{21} = \frac{1}{2} S'_{11e} - \frac{1}{2} S'_{11o}$$

$$b_4 = S'_{41e} \frac{a_1}{2} + S'_{41o} \frac{a_1}{2} \Rightarrow S'_{41} = \frac{1}{2} S'_{41e} + \frac{1}{2} S'_{41o}$$

$$b_3 = S'_{41e} \frac{a_1}{2} - S'_{41o} \frac{a_1}{2} \Rightarrow S'_{31} = \frac{1}{2} S'_{41e} - \frac{1}{2} S'_{41o}$$
- Because these circuits are composed of cascaded lines (two ports), it is easiest to analyze these networks using ABCD matrices.

And since the length of this transmission line is lambda by 4, so we should be able to deduce from this matrix go from this matrix to this matrix.

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IIT Bombay
ABCD Matrices

- In Chapter 4 we introduced the ABCD matrix of a transmission line of arbitrary electrical length. For $\theta = \pi/2$ ($\lambda/4$) we obtain :

$$\begin{bmatrix} \cos \theta & jZ_1 \sin \theta \\ jY_1 \sin \theta & \cos \theta \end{bmatrix}_{\theta=\pi/2} = \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix}$$
- The open or short lines are connected in shunt, so we can use the ABCD matrix for a shunt lumped element Y with Y the input admittance of a $\lambda/8$ length open or shorted line :

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \text{ with } \frac{1}{Y} = Z = \begin{cases} -jZ_2 \cot \theta = -jZ_2 \text{ open stub (even mode)} \\ jZ_2 \tan \theta = +jZ_2 \text{ short stub (odd mode)} \end{cases}$$
- The cascaded ABCD matrix $[M]_{e/o}$ for the even/odd mode is then :

$$\begin{bmatrix} 1 & 0 \\ \pm jY_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \pm jY_2 & 1 \end{bmatrix} = \begin{bmatrix} \mp Z_1 / Z_2 & jZ_1 \\ jZ_1 \left(\frac{1}{Z_1^2} - \frac{1}{Z_2^2} \right) & \mp Z_1 / Z_2 \end{bmatrix}$$

Similarly the cascade parameters of the open or shorted lines can be given by these equations and finally as I said, the overall cascade parameters will be the multiplications of the 3 cascade parameters of the 3 circuits, the 1st one being either an open or shorted stub, the 2nd one is a length of transmission line, and the 3rd one is again either an open or shorted transmission line. And after doing this multiplication, the overall S the overall cascade parameters or ABCD matrix of the circuit for both the even or odd mode is given by this equation. So, - here represents the even mode, + here represents the odd mode, similarly here.

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IIT Bombay
ABCD Matrices

- We can now convert the ABCD matrix for the even/odd mode into a two-port S matrix between ports 1 and 2 in the even or odd modes (Chap 4)

$$S'_{11e/o} = \frac{A_{e/o} + B_{e/o}/Z_0 - C_{e/o}Z_0 - D_{e/o}}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}} = \frac{B_{e/o}/Z_0 - C_{e/o}Z_0}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$

$$S'_{21e/o} = \frac{2}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$
 where we used the fact that $A_{e/o} = D_{e/o}$.
- To obtain a coupler port 1 needs to be matched and port 1 and 2 isolated :

$$S_{11} = \frac{1}{2}(S'_{11e} + S'_{11o}) = 0 \text{ and } S_{21} = \frac{1}{2}(S'_{11e} - S'_{11o}) = 0$$
- This is achieved if $S'_{11e} = S'_{11o} = 0$:

$$\Rightarrow \frac{B_{e/o}}{Z_0} = C_{e/o}Z_0 \Rightarrow \frac{1}{Z_1^2} - \frac{1}{Z_2^2} = \frac{1}{Z_0^2} \Rightarrow Z_2 = \frac{Z_1}{\sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}}$$
- For a 3 dB coupler we will find that $Z_1 = Z_0/\sqrt{2}$ and $Z_2 = Z_0$.

Now, once we get the cascade or ABCD matrix, we can convert those ABCD parameters to the S parameters from this equation, this equation is given in the book by Pozart, the

derivation of this conversion is also given and then once we do that, we will get the, once we do that, we find out the S parameters for the individual even and odd modes, from that we can find out the overall S parameters of the circuit using this equation that I just described.

Now, 1st thing, we saw that couplers needs to be matched at port 1 and we know we have just seen that S11 is given by this equation and also port 2 needs to be isolated from port 1 from which we have got this equation, so if both S11 and S21 have to be 0, then S11 E and S 11 O have to be individually equal to 0. And on performing this equation on solving this equation we get a relationship between Z1 and Z 2 like this. Now, for a 3 dB couplers, here I should mention that Z0 is the characteristic impedance or the matching impedance, that is the impotence to which the coupler is assumed to be connected all the ports.

Now, for a 3 dB couplers, we should get Z1 equal to Z0 upon square root of root 2 and Z2 equal to Z0 if we plug-in these values then we will see that the 3 dB condition that is equal power division between the coupled and through ports is satisfied.

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IIT Bombay
ABCD Matrices

- We can now convert the ABCD matrix for the even/odd mode into a two -port S matrix between ports 1 and 2 in the even or odd modes (Chap 4)

$$S'_{11eo} = \frac{A_{e/o} + B_{e/o}/Z_0 - C_{e/o}Z_0 - D_{e/o}}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}} = \frac{B_{e/o}/Z_0 - C_{e/o}Z_0}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$

$$S'_{41eo} = \frac{2}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$

where we used the fact that $A_{eo} = D_{eo}$.

- To obtain a coupler port 1 needs to be matched and port 1 and 2 isolated :

$$S_{11} = \frac{1}{2}(S'_{11e} + S'_{11o}) = 0 \quad \text{and} \quad S_{21} = \frac{1}{2}(S'_{11e} - S'_{11o}) = 0$$

- This is achieved if $S'_{11e} = S'_{11o} = 0$:

$$\Rightarrow \frac{B_{eo}}{Z_0} = C_{e/o}Z_0 \Rightarrow \frac{1}{Z_1^2} - \frac{1}{Z_2^2} = \frac{1}{Z_0^2} \Rightarrow Z_2 = \frac{Z_1}{\sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}}$$

- For a 3 dB coupler we will find that $Z_1 = Z_0/\sqrt{2}$ and $Z_2 = Z_0$.

Now, similarly proceedings similarly we can also find out the S 31 and S 41 parameters. So the S 31 and S 41 parameters using the relation,

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IIT Bombay
ABCD Matrices

- We can now convert the ABCD matrix for the even/odd mode into a two-port S matrix between ports 1 and 2 in the even or odd modes (Chap 4)

$$S'_{11e/o} = \frac{A_{e/o} + B_{e/o}/Z_0 - C_{e/o}Z_0 - D_{e/o}}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}} = \frac{B_{e/o}/Z_0 - C_{e/o}Z_0}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$

$$S'_{41e/o} = \frac{2}{A_{e/o} + B_{e/o}/Z_0 + C_{e/o}Z_0 + D_{e/o}}$$

where we used the fact that $A_{e/o} = D_{e/o}$.

- To obtain a coupler port 1 needs to be matched and port 1 and 2 isolated :

$$S_{11} = \frac{1}{2}(S'_{11e} + S'_{11o}) = 0 \quad \text{and} \quad S_{21} = \frac{1}{2}(S'_{11e} - S'_{11o}) = 0$$

- This is achieved if $S'_{11e} = S'_{11o} = 0$:

$$\Rightarrow \frac{B_{e/o}}{Z_0} = C_{e/o}Z_0 \Rightarrow \frac{1}{Z_1^2} - \frac{1}{Z_2^2} = \frac{1}{Z_0^2} \Rightarrow Z_2 = \frac{Z_1}{\sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}}$$

- For a 3 dB coupler we will find that $Z_1 = Z_0/\sqrt{2}$ and $Z_2 = Z_0$.

here we have of course substituted in place of Z2, we have plugged in this value and,

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IIT Bombay
Coupling and Insertion Loss

- Using $\frac{Z_1}{Z_2} = \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}$ the normalized 2-port ABCD matrix $[M]_{e/o}$ reduces to :

$$[M]_{e/o} = \begin{bmatrix} A_{e/o} & B_{e/o} \\ C_{e/o} & D_{e/o} \end{bmatrix} = \begin{bmatrix} \mp \frac{Z_1}{Z_2} & j \frac{Z_1}{Z_0} \\ jZ_1Z_0\left(\frac{1}{Z_1^2} - \frac{1}{Z_2^2}\right) & \mp \frac{Z_1}{Z_2} \end{bmatrix} = \begin{bmatrix} \mp \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} & j \frac{Z_1}{Z_0} \\ j \frac{Z_1}{Z_0} & \mp \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} \end{bmatrix}$$

- The insertion gain is then :

$$S_{41} = \frac{1}{2}S'_{41e} + \frac{1}{2}S'_{41o} = \frac{1}{A_e + B_e + C_e + D_e} + \frac{1}{A_o + B_o + C_o + D_o}$$

$$= \frac{1}{j2 \frac{Z_1}{Z_0} - 2 \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}} + \frac{1}{j2 \frac{Z_1}{Z_0} + 2 \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}} = -j \frac{Z_1}{Z_0}$$

- The coupling gain is then :

$$S_{31} = \frac{1}{2}S'_{41e} - \frac{1}{2}S'_{41o} = \frac{1}{A_e + B_e + C_e + D_e} - \frac{1}{A_o + B_o + C_o + D_o}$$

$$= \frac{1}{j2 \frac{Z_1}{Z_0} - 2 \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}} - \frac{1}{j2 \frac{Z_1}{Z_0} + 2 \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}} = -\sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2}$$

on solving S 41 and S 31 using these equations, we get the values of S 41 and S 41 is equal to - JZ1 upon Z0 and S 31 is equal to - whole square root of -1 - Z1 upon Z0 whole square.

Now these equations and these derivations will be found in the in the notes accompanying this presentation, accompanying this course, so in case you have missed these in the lectures, you will be able to find it in the accompanying notes or PPT slides.


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IIT Bombay
General Branch Line Coupler

- The rest of the scattering matrix follows from symmetry and reciprocity to give

$$[S] = \begin{bmatrix} 0 & 0 & \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} & j\frac{Z_1}{Z_0} \\ 0 & 0 & j\frac{Z_1}{Z_0} & \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} \\ \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} & j\frac{Z_1}{Z_0} & 0 & 0 \\ j\frac{Z_1}{Z_0} & \sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} & 0 & 0 \end{bmatrix}$$

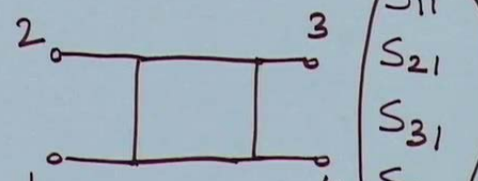

- The bandwidth of a branch line coupler is limited by the use of quarter wave lines. We can improve bandwidth by using multiple sections.



So, if we go back to the slides on the monitor, so the overall, so we have found out all the parameters that is S11, S21, S31 and S41 and then using those parameters, we can find out the complete S parameter matrix of a coupler, now I would like to mention something here at this point.


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Even Odd. analysis



S₁₁
 S₂₁
 S₃₁
 S₄₁

$S_{41} = S_{32}$ $S_{21} = S_{34} = S_{43} = S_{12}$
 $= S_{23}$ $S_{11} = S_{22} = S_{33} = S_{44}$
 $= S_{14}$ $S_{31} = S_{24} = S_{42} = S_{13}$



Say you have a coupler which is symmetric, so you might say that we have 16 parameters in a coupler because it is a 4 port network and we found out S11, S21, S31 and S41, then how come I can find out all the other parameters? So, here my notation is like this because of this symmetry, I can say that S21 is equal to S34 and then because it is a reciprocal device, they

should also be equal to S 43 which in turn should be equal to S12. So, just by knowing S 21, I find out 3 more S parameters. Similarly S 11, since it is matched at all ports, S 11 is equal to S 22 is equal to S 33 is equal to S 44.

Further, S 31 is equal to S 24 is equal to S 42 is equal to S 13. And finally S 41 is equal to S 32 which is equal to S 23 which is equal to S14, so you see just by knowing these 4 S parameters of a symmetric of a symmetric coupler, we can find out all the remaining S parameters. Now, coming back to our discussion on couplers, so we have studied the basic construction of the coupler of a branch line hybrid and we have also found out the S parameter matrix of that branch line coupler. Let us see what happens for a 3 dB case, that is when the branch line coupler provides equal power division between the coupled and the through ports.

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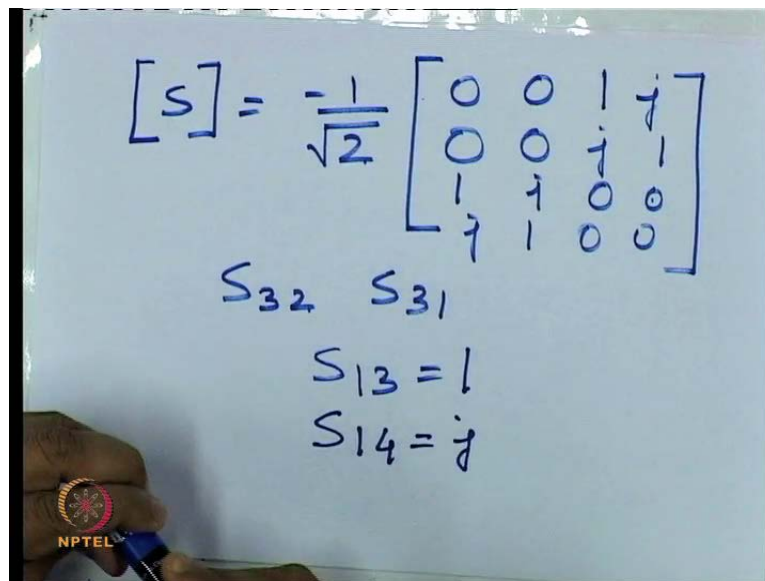
$$Z_1 = \frac{Z_0}{\sqrt{2}} \quad Z_2 = Z_0$$

$$S_{41} = -j/\sqrt{2} = -jZ_1/Z_0$$

$$S_{31} = -\sqrt{1 - \left(\frac{Z_1}{Z_0}\right)^2} = -\frac{1}{\sqrt{2}}$$

So, there we saw that Z1 is equal to Z0 upon root 2 and Z2 is equal to Z0, so S 41 becomes equal to - J upon root 2, S 31 becomes equal to, so this comes from this equation, S 31 becomes equal to -1 -, so these are the values of S 41 and will S 31, so the S parameter matrix should become like this.

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A hand-drawn S-parameter matrix on a whiteboard. The matrix is written as $[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix}$. Below the matrix, the elements S_{32} and S_{31} are written, followed by $S_{13} = 1$ and $S_{14} = j$. A hand holding a blue pen is visible at the bottom left of the whiteboard.

A complete S parameter matrix for a 3 dB hybrid should become like this. As we can see the phase shift between this S_{32} and say S_{31} , so between these ports, when suppose we assume an input at port 3, then the outputs at port 2 and port 1 are 90° phase shifted.

Or say there is an input at port 1, then the S_{13} is equal to 1 and S_{14} is equal to j , so we can again see that if we have an input at port 1, then the outputs at port 3 and port 4 are phase shifted by 90° . So, in this lecture, we covered 2 types of couplers, the 1st was the example of a 180° coupler which we call the Magic Tee and the 2nd was the example of the 90° coupler called the branch line coupler or if there is equal power division between the through and coupled ports, then it is called branch line hybrid. In the next lecture we will cover another type of coupler, another type of 90° coupler called coupled line couplers. So, thank you very much.