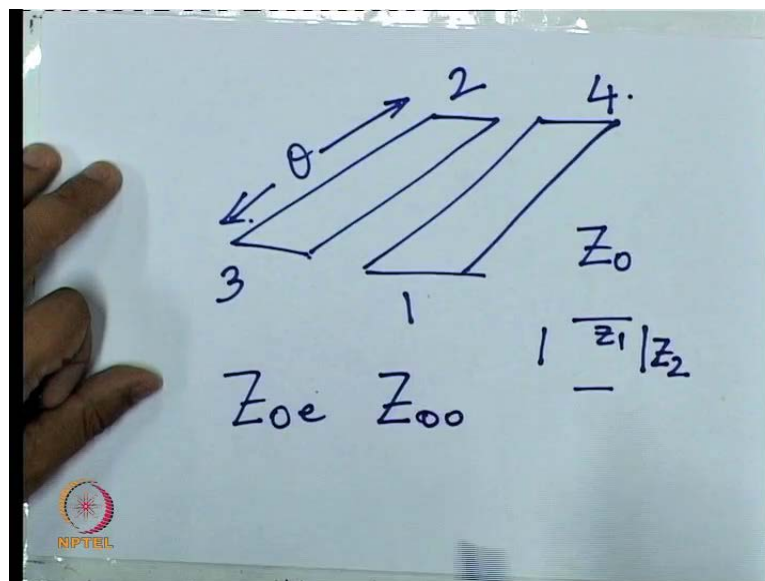


Microwave integrated circuits.
Professor Jayanta Mukherjee.
Department of Electrical engineering.
Indian Institute of Technology Bombay.
Lecture -18.
Coupled line couplers.

Hello, welcome to another module of this course, microwave integrated circuits. In the last module we introduced we discussed about couplers and we also discussed about 2 specific types of couplers, the branch line couplers and the rat race couplers. In this module we shall be discussing the coupled line couplers. So, let us see what is a coupled line coupler.

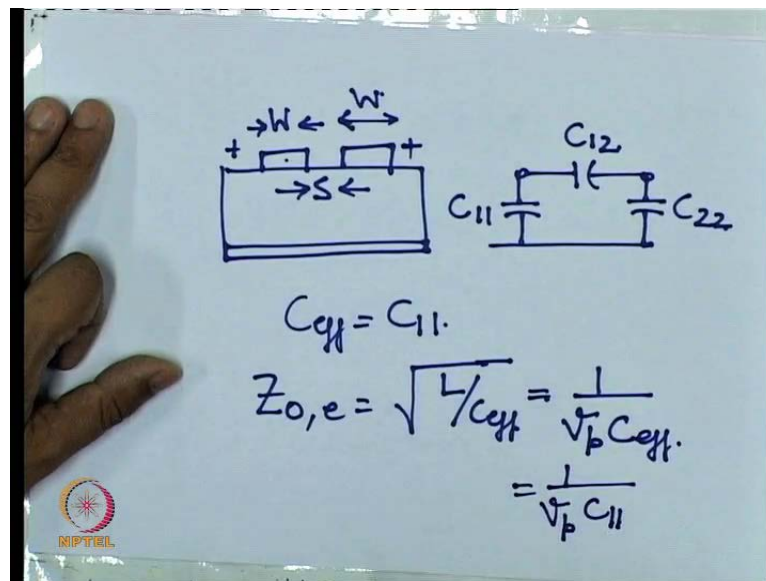
(Refer Slide Time: 00:45)



A coupled line coupler consists of 2 pieces of transmission line which are side-by-side. So, the construction of a coupled line coupler is quite simple.

The ports are labelled like this, if this is the input port, this is the isolated port, this is the coupled port and this is the throughput. Now, the difference between a coupled line coupler and the other couplers that we have discussed so far is that we cannot take a single value of this characteristic impedance for both the systems. Even in a branch line coupler, for example the vertical lines had Z_1 , sorry the horizontal lines had the characteristic impedance Z_2 whereas the vertical lines had a characteristic impedance Z_1 , Z_1 and Z_2 . But here because of the physics involved, there is the concept of the even mode even mode characteristic impedance and the odd mode characteristic impedance. So, let us see what are these even mode and odd mode characteristic impedances.

(Refer Slide Time: 2:05)

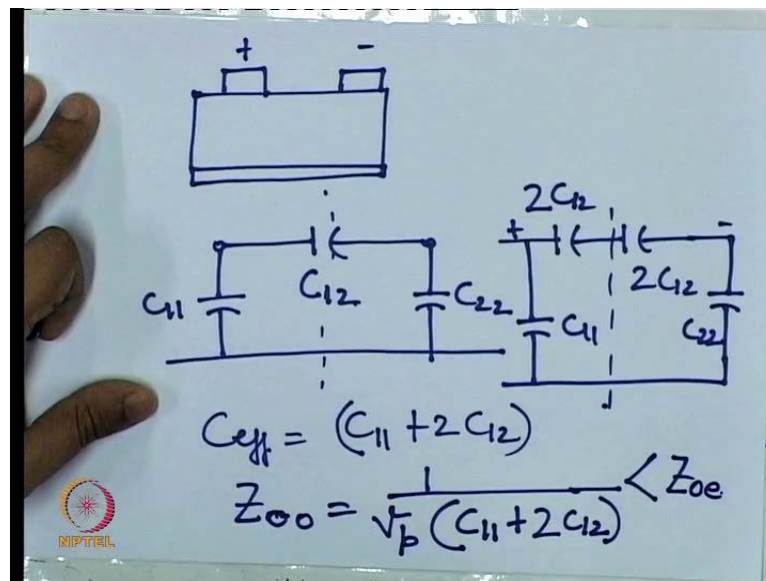


Consider a cross-section of a coupled line, 2 coupled lines, so we have one couple line here, another couple line here. Suppose the width of this is W , spacing is S , now there will be capacitance between the 2. So, there will be in into line capacitance C_{12} and there will be capacitance between the top metal and the bottom metal which is represented by C_{11} .

Now, consider that when the voltages on both are the same, if the voltages on both the lines are same, then this central capacitor does not play any role, so the total capacitance between the top metal on the bottom metal is simply $C_{\text{effective}}$ is equal to C_{11} . Now we know that Z_0 for the even mode should be given by L upon $C_{\text{effective}}$, we call $C_{\text{effective}}$ is a capacitance per unit length and this equation we had discussed while discussing the equation for the transmission line. And this can also be written in this way.

So, in this case for the even mode, when the voltages on both the lines are the same, total $C_{\text{effective}}$ we saw is equal to C_{11} . So, the Z_0 is or the characteristic impedance for the E even mode is simply C_{11} .

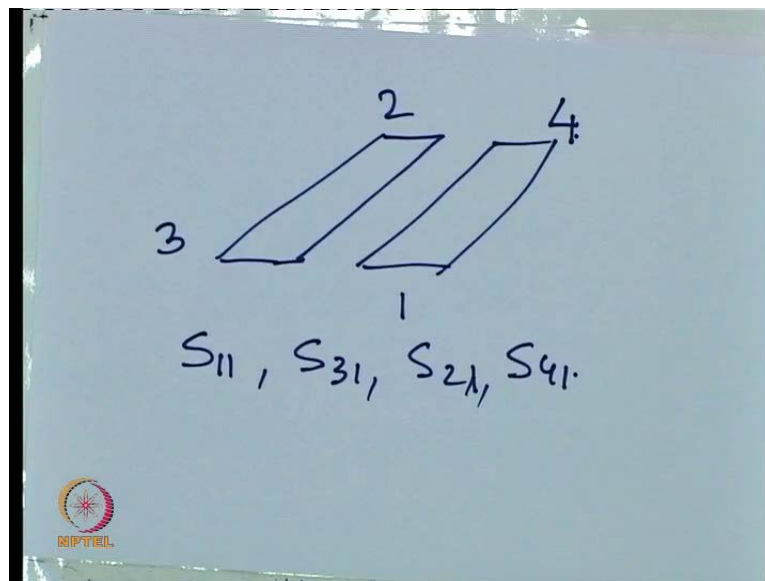
(Refer Slide Time: 4:33)



For the odd mode, when one line has is charged at a positive volt and the other one is that a negative voltage, then the, so in this case if we redraw this cross-sectional diagram, in the odd mode means one is at a distance, each are at a equal and opposite voltages. So, in that case the capacitance that exist between the lines C_{12} , that, we draw a line through it, that can be considered as a combination of 2 series capacitances $2C_{12}$, $2C_{12}$.

And since this is positive and this is negative and the voltages are equal and opposite, has this line must represent the short and therefore the Ceffective as you know of go C_{11} is same as C_{22} , Identical, these 2 lines are identical, then C_{11} will be equal to C_{22} . Now, C effective will be simply equal to $C_{11} + 2C_{22}$. So, in that case for the odd mode Z_{0o} will be given by. Now this is lesser than Z_0 even as we had seen, so this is something we have to understand for the coupled lines, there is the concept of odd mode and even mode impedances, characteristic impedances. And we saw that the even mode characteristic impedance is greater than the odd mode characteristic impedance.

(Refer Slide Time: 7:05)



Now, with this knowledge, let us proceed to the coupled line coupler, so the coupled line coupler as I said is again very simple in construction. And so, this is the input port, say this is the isolated port, now S_{11} , S_{31} , S_{21} and S_{41} are just like we had seen in the case for branch line coupler.

(Refer Slide Time: 7:42)

IIT Bombay
Directional Couplers

Even mode Odd mode

- Given a_1 at port 1 the reflected waves of the 4 port network are given by :

$$b_1 = S'_{11e} \frac{a_1}{2} + S'_{11o} \frac{a_1}{2} \Rightarrow S_{11} = \frac{1}{2} S'_{11e} + \frac{1}{2} S'_{11o}$$

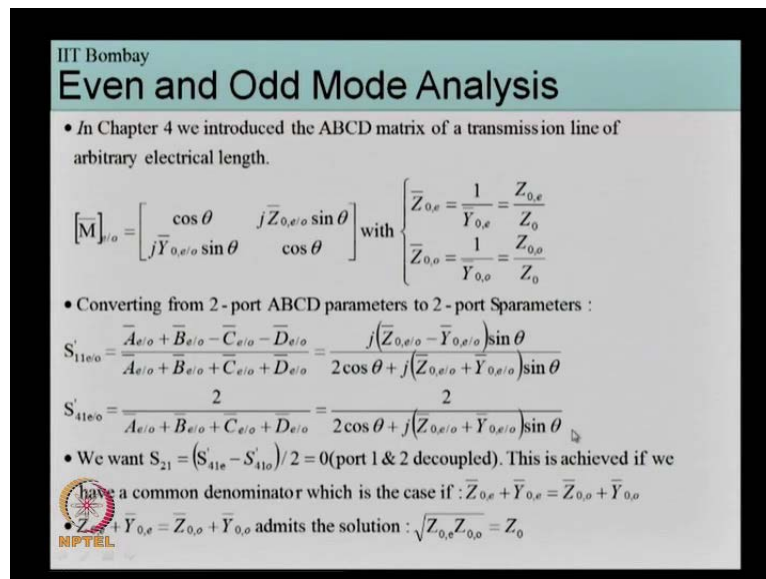
$$b_3 = S'_{11e} \frac{a_1}{2} - S'_{11o} \frac{a_1}{2} \Rightarrow S_{31} = \frac{1}{2} S'_{11e} - \frac{1}{2} S'_{11o}$$

$$b_4 = S'_{41e} \frac{a_1}{2} + S'_{41o} \frac{a_1}{2} \Rightarrow S_{41} = \frac{1}{2} S'_{41e} + \frac{1}{2} S'_{41o}$$

$$b_2 = S'_{41e} \frac{a_1}{2} - S'_{41o} \frac{a_1}{2} \Rightarrow S_{21} = \frac{1}{2} S'_{41e} - \frac{1}{2} S'_{41o}$$

Here also if you go to the slides on monitor, if we could go to the yah, here again the equations for these S parameters can be written in terms of the S parameters of the even and odd mode like this. Here of course 1 difference is that S_{31} is now equal to this instead of S_{21} as we had seen, which was say for the branch line hybrid S_{21} was equal to S_{11E} upon 2 - S_{11O} upon 2, in this case, this is S_{31} .

(Refer Slide Time: 8:14)



IIT Bombay
Even and Odd Mode Analysis

- In Chapter 4 we introduced the ABCD matrix of a transmission line of arbitrary electrical length.

$$[M]_{l/o} = \begin{bmatrix} \cos \theta & j\bar{Z}_{0,e/o} \sin \theta \\ j\bar{Y}_{0,e/o} \sin \theta & \cos \theta \end{bmatrix} \text{ with } \begin{cases} \bar{Z}_{0,e} = \frac{1}{Y_{0,e}} = \frac{Z_{0,e}}{Z_0} \\ \bar{Z}_{0,o} = \frac{1}{Y_{0,o}} = \frac{Z_{0,o}}{Z_0} \end{cases}$$

- Converting from 2-port ABCD parameters to 2-port S parameters :

$$S'_{11e/o} = \frac{\bar{A}_{e/o} + \bar{B}_{e/o} - \bar{C}_{e/o} - \bar{D}_{e/o}}{\bar{A}_{e/o} + \bar{B}_{e/o} + \bar{C}_{e/o} + \bar{D}_{e/o}} = \frac{j(\bar{Z}_{0,e/o} - \bar{Y}_{0,e/o}) \sin \theta}{2 \cos \theta + j(\bar{Z}_{0,e/o} + \bar{Y}_{0,e/o}) \sin \theta}$$

$$S'_{41e/o} = \frac{2}{\bar{A}_{e/o} + \bar{B}_{e/o} + \bar{C}_{e/o} + \bar{D}_{e/o}} = \frac{2}{2 \cos \theta + j(\bar{Z}_{0,e/o} + \bar{Y}_{0,e/o}) \sin \theta}$$

- We want $S_{21} = (S'_{41e} - S'_{41o})/2 = 0$ (port 1 & 2 decoupled). This is achieved if we have a common denominator which is the case if : $\bar{Z}_{0,e} + \bar{Y}_{0,e} = \bar{Z}_{0,o} + \bar{Y}_{0,o}$
- $\bar{Z}_{0,e} + \bar{Y}_{0,e} = \bar{Z}_{0,o} + \bar{Y}_{0,o}$ admits the solution : $\sqrt{Z_{0,e} Z_{0,o}} = Z_0$

Now this is, this coupled line coupler consists just of 2 pieces of transmission line, so for the even or odd modes cascades or ABCD matrix is very simple, it is same as that of the transmission line of length of electrical length theta. Except that instead of Z0 being same for the even and odd modes, we will have 2 different values of Z0, one for the even mode and one for the odd mode. And then using these using this ABCD matrix we can write down the S11 and S41, we can find out the S11 and S41 parameters using the conversion equations.

Now we want S21 equal to 0, so to achieve that, 1st thing is that you know if S41E - S41O should be equal to 0, then the denominators for the even and odd mode of S41 should be the same. And that is the case if this condition is satisfied. Just by looking at this equation also we can find that term. If the denominators have to be equal, then Z0O + Y0O, that is this expression should be equal to Z0E + Y0E. Now on solving this equation we get this value, so the relationship between Z0E and Z0O is given by this where Z0 is the matching impedance or the impedance to which the coupler is connected at all the ports.

(Refer Slide Time: 10:02)

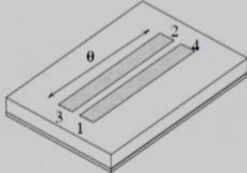
IIT Bombay
Proof of Isolation

- Let us prove that $\bar{Z}_{0,e} + \bar{Y}_{0,e} = \bar{Z}_{0,o} + \bar{Y}_{0,o}$ admits the solution $\sqrt{Z_{0,e}Z_{0,o}} = Z_0$

Substituting $\frac{Z_{0,e}}{Z_0} = \frac{Z_0}{Z_{0,o}}$ in $\frac{Z_{0,e}}{Z_0} + \frac{Z_0}{Z_{0,e}} = \frac{Z_{0,o}}{Z_0} + \frac{Z_0}{Z_{0,o}}$

we get: $\frac{Z_0}{Z_{0,o}} + \frac{Z_{0,o}}{Z_0} = \frac{Z_{0,o}}{Z_0} + \frac{Z_0}{Z_{0,o}}$

- Therefore we have $S_{21} = (S'_{11e} - S'_{11o})/2 = 0$ (port 1 & 2 decoupled) for ALL θ
 This coupler provides broadband isolation between port 1 and 2.



NPTTEL

We could skip this slide.

(Refer Slide Time: 10:12)

IIT Bombay
Broadband Matching

- We have obtained the 2 port even/odd S' parameters :

$$S'_{11e,o} = \frac{j(\bar{Z}_{0e/o} - \bar{Y}_{0e/o})\sin\theta}{2\cos\theta + j(\bar{Z}_{0e/o} + \bar{Y}_{0e/o})\sin\theta}$$

- We want port 1 to be matched: $S_{11} = (S'_{11e} + S'_{11o})/2 = 0$. Since we have the same common denominator for $\sqrt{Z_{0,e}Z_{0,o}} = Z_0$ this is achieved if :

$$\bar{Z}_{0,e} - \bar{Y}_{0,e} + \bar{Z}_{0,o} - \bar{Y}_{0,o} = 0$$

Substituting $\frac{Z_{0,e}}{Z_0} = \frac{Z_0}{Z_{0,o}}$ in $\frac{Z_{0,e}}{Z_0} - \frac{Z_0}{Z_{0,e}} + \frac{Z_{0,o}}{Z_0} - \frac{Z_0}{Z_{0,o}} = 0$

we get: $\frac{Z_0}{Z_{0,o}} - \frac{Z_{0,o}}{Z_0} + \frac{Z_{0,o}}{Z_0} - \frac{Z_0}{Z_{0,o}} = 0$

- Therefore we have $S_{11} = (S'_{11e} + S'_{11o})/2 = 0$ and port 1 is matched for all θ .
 This coupler provides broadband matching.

NPTTEL

Other thing that we note is that condition if the denominators are equal, then the conditions of matching, that is $S_{11} = (S'_{11e} + S'_{11o})/2 = 0$ is automatically satisfied. So, therefore for the coupled line coupler, we obtained broad band matching, that is they are always matched.

(Refer Slide Time: 10:48)

IIT Bombay
Coupled and Through Gains

- We have obtained the 2-port even/odd S' parameters :

$$S'_{11e/o} = \frac{j(Z_{0,e/o} - Y_{0,e/o}) \sin \theta}{2 \cos \theta + j(Z_{0,e/o} + Y_{0,e/o}) \sin \theta} \quad S'_{41e/o} = \frac{2}{2 \cos \theta + j(Z_{0,e/o} + Y_{0,e/o}) \sin \theta}$$
- The coupled and through gains are then given by :

$$S_{31} = \frac{1}{2} S'_{11e} - \frac{1}{2} S'_{11o} \quad \text{and} \quad S_{41} = \frac{1}{2} S'_{41e} + \frac{1}{2} S'_{41o}$$
- Using $Z_0 = \sqrt{Z_{0,e} Z_{0,o}}$ this can be simplified to

$$S_{31} = \frac{jk_c \sin \theta}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta} \quad \text{and} \quad S_{41} = \frac{\sqrt{1-k_c^2}}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta} \quad \text{with} \quad k_c = \frac{Z_{0,e} - Z_{0,o}}{Z_{0,e} + Z_{0,o}}$$

$$Z_{0,e} = Z_0 \sqrt{\frac{1+k_c}{1-k_c}} \quad \text{and} \quad Z_{0,o} = Z_0 \sqrt{\frac{1-k_c}{1+k_c}}$$

NPTEL

Now the S 31 parameters and S 41 parameters can be found out using this equation which we have already discussed. And then from the value of Z0 given by this equation, so, S 31 is found out from S11 E - S 11 O, S11 E and S 11 EO and S41EO, we have already found out, plug-in these values into these equations, we get this as the expression for S 31 and this as the expression for S 41.

Now, here S 31 contains as you can see both these S 31 and S 41 contains at KC, KC is defined by this equation, it is like a measure of the mismatch between Z0 E that is the characteristic impedance of the even mode and Z0 O which is the characteristic impedance of the odd mode and on solving for Z0 E and Z0 O in terms of KC and using this equation, we can get the following equation for Z0 E and Z0 O in terms of KC and Z0. So, we have analysed the coupled line couplers and found out these equations, now when you want to design this coupler, you have to have however go in the opposite direction.

You will be given a certain KC and you will be asked to find out Z0 E and Z0 O and then from Z0 E and Z0 O, we can find out the C effective and from C effective we can find out the interline capacitance and the capacitance between the line and the ground plane and from there we can completely design our system.

(Refer Slide Time: 12: 52)

IIT Bombay
**Frequency Dependence of Coupling
 And Through Gains**

- $S_{31} = \frac{jk_c \sin \theta}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta}$ and $S_{41} = \frac{\sqrt{1-k_c^2}}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta}$ with $k_c = \frac{Z_{0,e} - Z_{0,o}}{Z_{0,e} + Z_{0,o}}$

maximum coupling coefficient at $\theta = \beta l = \pi/2$ ($l = \lambda/4$)

NPTTEL

If we further go back to the slides once again, the S_{31} and S_{41} parameters, if we plot, if we plot the squares, then the sum of the magnitude squares of the S_{31} and S_{41} will always be constant. However S_{21} represented by Square represented by this Red Line and S_{41} square represented by this dotted line will vary.

This is of course logical, this shows that the total power leaving from S port 3 and port 4 is equal to the total power entering port 1. So, we have discussed a number of couplers in this course so we 1st started with magic tee which is an example of a 180° coupler then we discussed about branch line coupler and the the coupled line coupler which are examples of the 90° couplers.

(Refer Slide Time: 14:09)

IIT Bombay
Coupled and Through Gains

- We have obtained the 2-port even/odd S' parameters :

$$S'_{11e/o} = \frac{j(\bar{Z}_{0,e/o} - \bar{Y}_{0,e/o}) \sin \theta}{2 \cos \theta + j(\bar{Z}_{0,e/o} + \bar{Y}_{0,e/o}) \sin \theta} \quad S'_{41e/o} = \frac{2}{2 \cos \theta + j(\bar{Z}_{0,e/o} + \bar{Y}_{0,e/o}) \sin \theta}$$

- The coupled and through gains are then given by :

$$S_{31} = \frac{1}{2} S'_{11e} - \frac{1}{2} S'_{11o} \quad \text{and} \quad S_{41} = \frac{1}{2} S'_{41e} + \frac{1}{2} S'_{41o}$$

- Using $Z_0 = \sqrt{Z_{0,e} Z_{0,o}}$ this can be simplified to

$$S_{31} = \frac{jk_c \sin \theta}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta} \quad \text{and} \quad S_{41} = \frac{\sqrt{1-k_c^2}}{\sqrt{1-k_c^2} \cos \theta + j \sin \theta} \quad \text{with} \quad k_c = \frac{Z_{0,e} - Z_{0,o}}{Z_{0,e} + Z_{0,o}}$$

$$Z_{0,e} = Z_0 \sqrt{\frac{1+k_c}{1-k_c}} \quad \text{and} \quad Z_{0,o} = Z_0 \sqrt{\frac{1-k_c}{1+k_c}}$$

NPTTEL

And just to show you one thing that it is indeed a 90° couplers, if we can go once to the slides on the monitor, you see that S_{31} and S_{41} are shifted by 90° due to the presence of this J term. Hence it is indeed a 90° coupler.

So, so in summary we saw that couplers represent 4 port microwave devices and there are various kinds of couplers, 90° couplers are one which provide phase shift of 90° between the coupled and through ports, 180° couplers are ones which provide 180° phase shift between coupled and through ports. Now, you might ask me what is the application of a coupler because if a coupler is just providing power division between the coupled and through ports, then it looks very similar to a power divider. So, let us see some applications of a coupler.

(Refer Slide Time: 15:09)

IIT Bombay
Network Analyzer Test Set

- 20 dB couplers are typically used in a network analyzer to separate the incident and reflected waves at port 1 and 2.

The diagram illustrates a network analyzer test set. It starts with an RF source connected to a circulator. The circulator directs the signal to a 20 dB coupler. This coupler has three ports: one for the incident wave (a_1), one for the reflected wave (b_1), and one for the wave going to the Device Under Test (DUT). The DUT is connected to a second 20 dB coupler, which also has three ports: one for the incident wave (a_2), one for the reflected wave (b_2), and one for the wave going to a load. The diagram shows the incident and reflected waves at both ports of the DUT.

- Since a low coupling (20 dB) is used most of the power is sent to DUT.
- DUT = device under test

NPTEL

If we can go to the slide on the monitor, one major application of a coupler is network analyser. As you know network analyser are devices for finding out, which directly measure the S parameters of the circuit or the device under test or DUT as we call it. Now, since the network analyser actually deals with the reflected and incident wave and we saw that the reflected and incident wave are not so easy to separate, then the couplers come handy in separating the incident and reflected waves.

Now, here we have RF source supplying a certain incident wave to this coupler, now a part of this input signal will go to the coupled port, no part of the input signal will come back to the will go to the isolator. Here of course note one thing that this port is isolated with respect to this port only, it however acts as the coupled port with respect to this port. For this port, this

port is the isolated port. Now, after the incident wave travels to the through port and it reaches the DUT, that is the device under test and that it undergoes some reflection which is B1 which is revisited by the normalised voltage B1.

Now this B1 will again see this port as the input port, no part of this input signal here will go to this port and all the signal will pass through this coupled port. So, for this port, this is the coupled port. So, this way we have completely separated A1 and B1 and then we can find out the ratio of A1 and B1. So, this is 1 important application of a coupler.

(Refer Slide Time: 17:16)

IIT Bombay

IQ Modulators

- In digital communication we need to modulate the RF carrier in both amplitude and phase. This is usually done using in-phase I(t) and Q(t) and quadrature baseband signals :

$$b_{RF}(t) = I(t)\cos\omega t - Q(t)\sin\omega t$$

- The following circuit can be used :

- A quadrature hybrid is used to generate a 90 degree phase shift for the L.O. Both in-phase and in-quadrature components are then summed using a power combiner.

NPTEL

One other important application of a coupler is in IQ modulators. Here we use the 90° couplers.

So, assume that this is an LO or local oscillator source inputting, connected to the input port of a coupler, at the coupled port, there will be a 90° phase shift with respect to the through port and this in turn will be multiplied with this I and Q signals which are baseband signals and then when they are passed through what we call a combiner, a combiner is nothing but power divider in reverse, we will get the combined signal, which is the frequency modulated signal. If we want to do the demodulation, this is the modulation function, if we want to do the demodulation function, then also couplers come in handy.

(Refer Slide Time: 18:09)

IIT Bombay
IQ Demodulators

- Given a modulated RF signal $b_{RF}(t) = I(t)\cos\omega t - Q(t)\sin\omega t$ we can recover the $I(t)$ and $Q(t)$ baseband signals using the following demodulator:

NPTEL

Here we have that modulated signal passing through a multiplier, the other end of the multiplier is connected to an LO signal which produces 2 signals which are 90° phase shifted and this in turn when multiplied lead to and pass through a lowpass filter produced the I and Q signals back. So, this way we recovered the I and Q signals from the RF input or the modulated input.

(Refer Slide Time: 18:45)

IIT Bombay
Balanced Amplifiers

- With 2 identical 90° hybrid couplers we can realize a balanced amplifier.

Identical hybrid couplers

The input impedance is always matched even if the amplifiers are not.

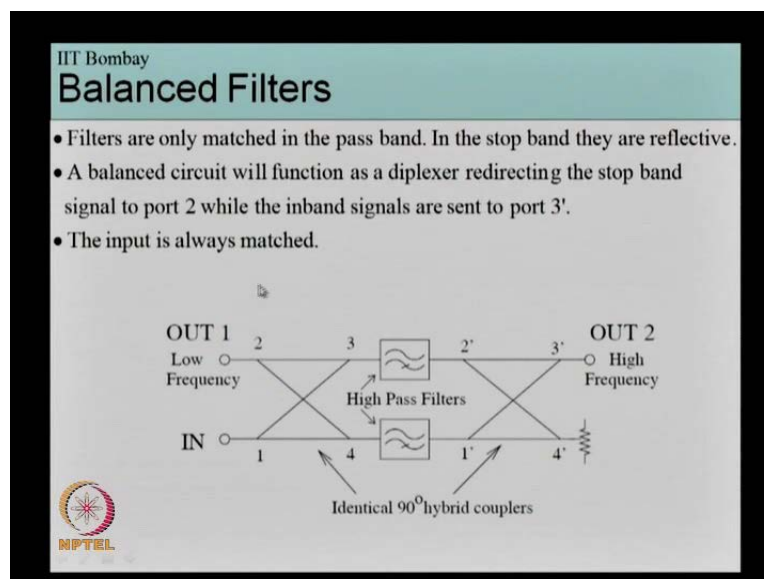
NPTEL

One other important application of a coupler, especially the 90° coupler is in what we call balanced amplifiers. A balanced amplifier is an amplifier where the S_{11} parameter is 0 over a wide frequency range. The way these balanced amplifiers achieve that is not that it reduces the reflected signal from an amplifier, but it kind of causes the reflected signal to be diverted

to the isolated port. So, this will be clear once I explain this. You need to have 2 identical amplifiers, these 2 identical amplifiers are connected to a coupler as shown. Now any input signal that reaches port 1 it will be divided at port 3 and port 4, here at port 3 and port 4, they will undergo reflection and these reflected waves will again be power divided at port 3 and port 4.

Now it can be shown that on division the signal reaching port 2 from this amplifier and the signal reaching port 2 from this amplifier will add constructively at port 2, whereas the single reaching port 1 from the this amplifier and the signal reaching port 1 from this amplifier will add destructively at this point. So, if they are adding destructively, it means that no reflected wave appears at port 1 and all reflected waves appear at port 2. So, this way, at port 1, there is no net reflected wave coming and hence S_{11} appears to be 0. Now, here I must mention this appears to be, there is not that the reflected wave does not exist. It is simply that were diverting it to port 2.

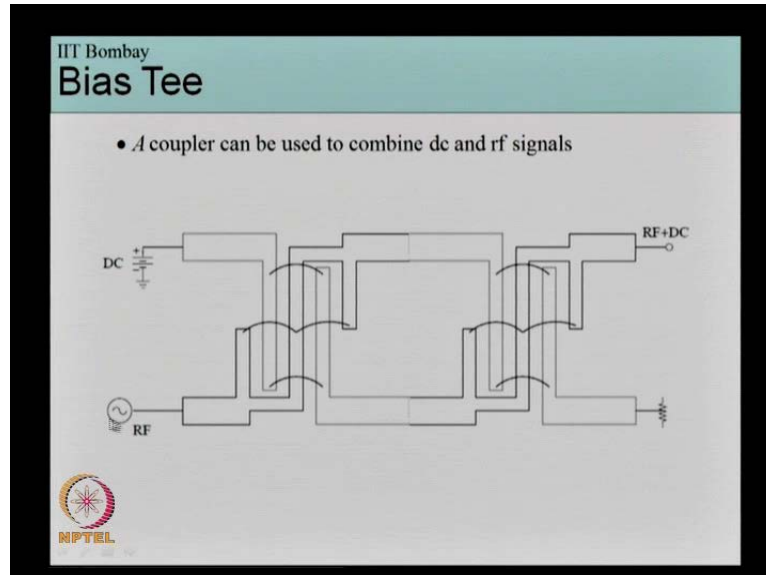
(Refer Slide Time: 20:54)



A similar circuit is what is called a balanced filter. Now filter by definition is not matched at all frequencies, for example a high pass filter will reflect back lowpass signals and only allow high frequency signals to pass through it. And the principal for balanced filter is exactly the same as that for a balanced amplifier, that is at port 2, the reflected wave add constructively add at port 1, the reflected wave add destructively as a result, even though our filter is fundamentally not matched for the low-frequency signal, at the input port, we will not see any reflected signals.

Hence the input port remains matched over a wide frequency range. Now this is at the input side of the filters and at the output side of the filters also, we can apply a similar logic, so the output port will also be matched over a wide frequency range.

(Refer Slide Time: 22:08)



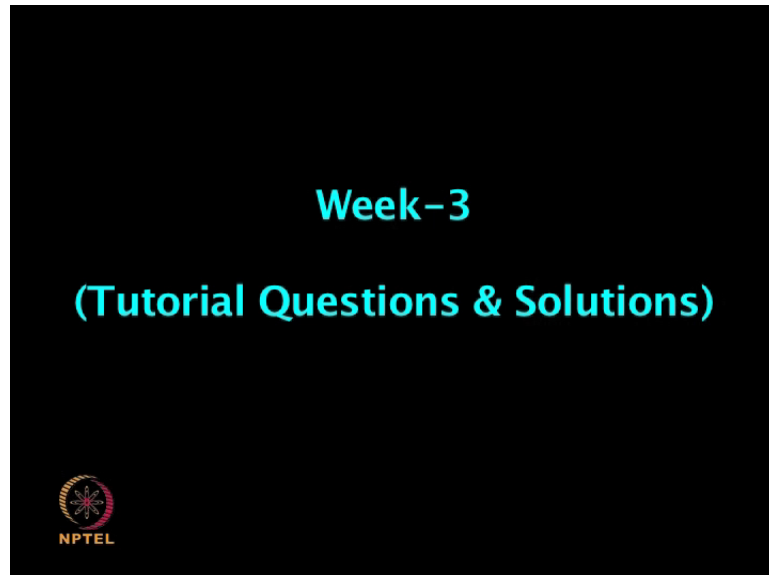
One other important application of a coupler is what is known as the biased tee. Now, a biased tee is one which combines DC signal with RF signal and this kind of combined RF and or high-frequency and DC signal is necessary for various microwave devices like amplifiers or Mixers.

Now because this is a simple, there is a simple network connection between the input RF and the output and the simple network connection is enforced by these wires which are connected between them, between the arms of the coupler. And since these wires will not allow RF to pass through them, and only DC to pass through them, so any DC signal appearing at this port will be passed onto the output. But then this is the isolated port with respect to this port, so no component of this RF signal will appear here and since this is the coupled port, a component of the input RF signal will appear at this port.

So, both the DC and RF will appear at this point and so we can combine the 2. So, in summary, a coupler is a very widely used microwave device, it is a very useful device. There are some more advanced uses of coupler also which we have not discussed and the good thing about coupler is that it can be matched, be reciprocal and lossless without any mathematical restrictions, it has wide range applications and as we saw that there are various types of couplers, for example 90° coupler, or the 180° coupler, so that is all about 4 port devices and

microwave devices in general. In the next module we shall be discussing about filters. Thank you.

(Refer Slide Time: 24:08)



Problem 1 :

Design a loss-less T-junction Power divider which has 75Ω Source impedance. Find the output characteristic impedances such that input power is divided in 2:1 ratio

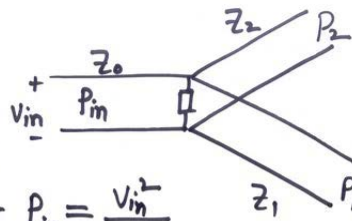
$Z_0 = 75\Omega$
 P_{IN}
 $Z_2 = ?$
 $Z_1 = ?$

NPTEL

Solution

Let

input voltage V_{in} (r.m.s)



$$\text{So input Power } P_{in} = \frac{V_{in}^2}{Z_0}$$

Where $Z_0 = \text{Source Impedance} = 75\Omega$

Now Power at output ports

$$P_2 = \frac{V_{in}^2}{Z_2} \text{ and } P_1 = \frac{V_{in}^2}{Z_1}$$



From specification

$$P_1 = \frac{1}{3} P_{in}$$

$$P_2 = \frac{2}{3} P_{in}$$

$$\text{where } P_{in} = \frac{V_{in}^2}{Z_0}$$

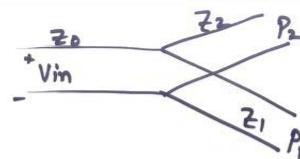
$$\text{So } P_1 = \frac{1}{3} P_{in} = \frac{1}{3} \cdot \frac{V_{in}^2}{Z_0} \quad \text{--- (1)}$$

$$\text{and also } P_1 = \frac{V_{in}^2}{Z_1} \quad \text{--- (2)}$$



Equating ① & ② we get

$$\frac{V_{in}^2}{Z_1} = \frac{1}{3} \frac{V_{in}^2}{Z_0}$$



$$\text{So } Z_1 = 3Z_0 = 3 \times 75\Omega = 225\Omega$$

Similarly for port-2

$$P_2 = \frac{2}{3} P_{in} = \frac{V_{in}^2}{Z_2}$$

$$\text{or } \frac{2}{3} \frac{V_{in}^2}{Z_0} = \frac{V_{in}^2}{Z_2}$$



$$Z_2 = \frac{3}{2} Z_0 = 112.5\Omega$$

Now impedance at

T-junction $Z_{in} = Z_1 \parallel Z_2$

where $Z_1 = 225 \Omega$ and $Z_2 = 112.5 \Omega$

$$\text{So } Z_{in} = \frac{225 \times 112.5}{225 + 112.5} \Omega$$

$$= 75 \Omega$$

Which is matched with Source Impedance,
So there will be no reflection at junction.

