Microwave Integrated Circuits

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Module 1

Lecture No 2

Reflection Coefficient, VSWR, Smith Chart.

Hello welcome to another module of this course, microwave integrated circuits. In the previous module, we saw the properties of microwave circuits, the wave nature and the distributed and then we also found the solution of transmission lines equation is for dealing with distributed circuit elements. In this lecture let us continue with the discussion.So the 1^{st} so one more concept that is associated with microwave circuits is the concept of impedance.

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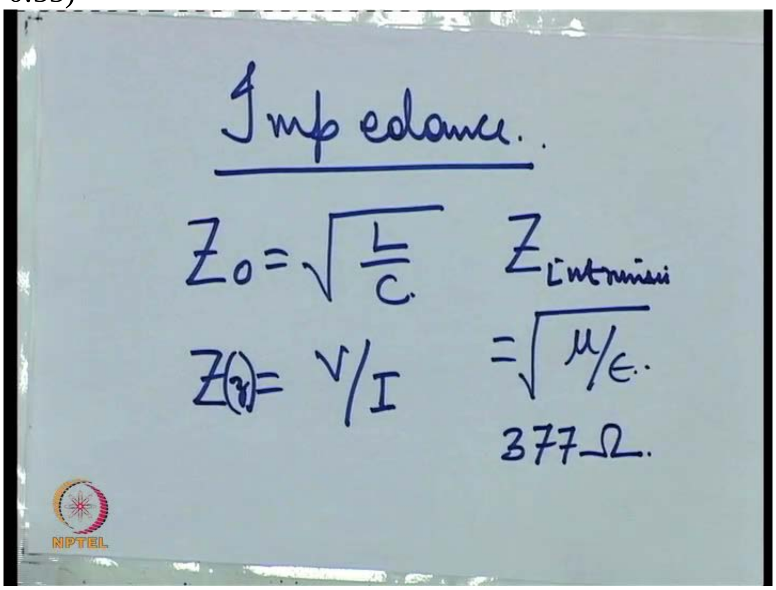

The impedance can be defined in various ways. One is that characteristic impedance associated with distributed circuits that which we found out to be L and C. Then Z also is defined as V and I as we saw in the case of distributed circuits. We can have V and I solutions so then we can also have Z. Z of course is a function of z the small z the distance from the source. Then there is also another kind of impedance that is defined which is called Z intrinsic which is related to the properties of the material. For example for air this ratio comes out to be 377 ohms. So this has nothing to do, this definition of impedance has nothing to do with voltages and currents. Still this holds.

In fact for microwave engineering as we saw for single conductor waveguide. You know that in a single conductor waveguide we cannot have a unique definition of voltage and current. But we can have a definition of impedance. Either from this relationship that is we have an electromagnetic wave passing through some media. Then you can find out Z intrinsic from this relation.

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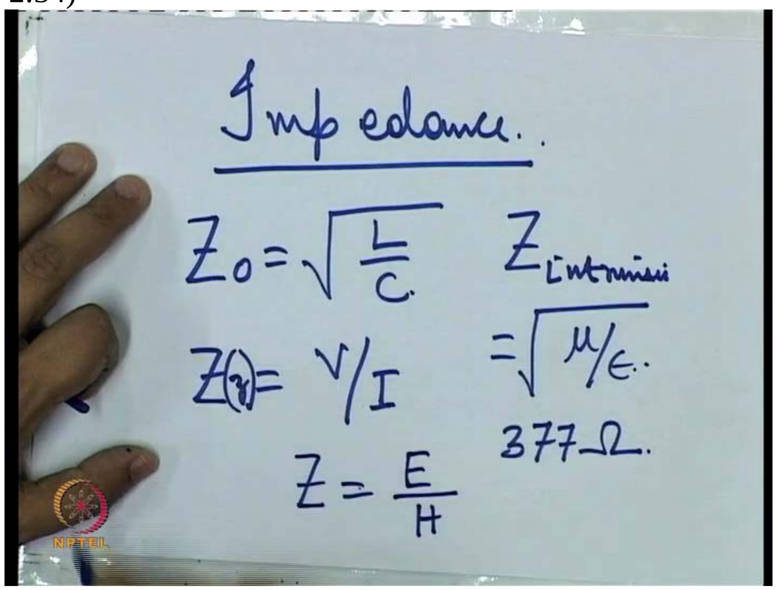

Or Z is also yet another definition of Z is ratio of electric to magnetic field. So these are the various definitions of impedance.

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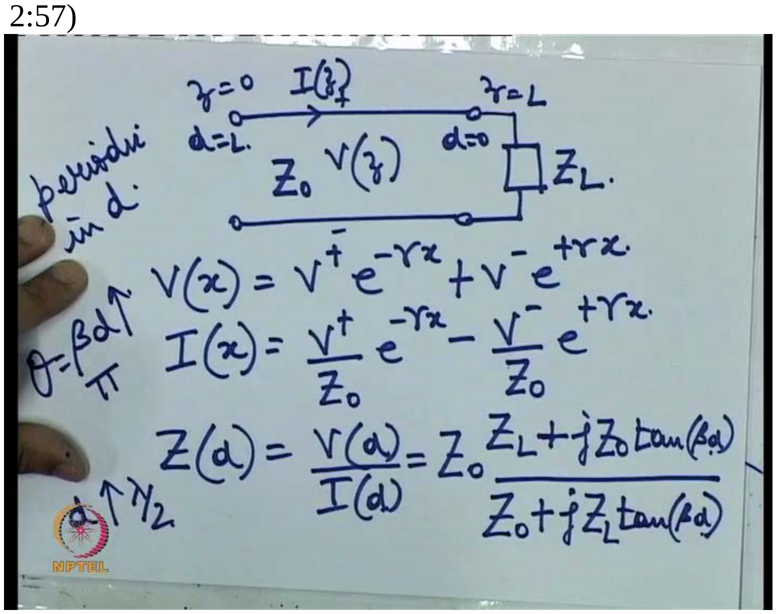

Now let us see let us come back to our definition come back to our transmission lines where we had found out the solution for Vx or Z. This is Z equal to 0 this is Z equal to L. Where V at a particular Z and I at a particular Z. Zo is the characteristic impedance.

Now solving this equation you can obtain the solution for Zd given by Vd/Id where d is the distance from the load end that is here d is equal to 0 and here d is equal to L. And this comes out to be equal to Zo(ZL+jZo).

We see that this definition of Zd means Zd is periodic in d i.e. when every time d increases by lambda/2 we get the same value of Zd. Or when theeta given by beta d increases by pi we also get the same value of Zd. In fact from this we can derive the expressions of input impedance i.e. Zd for some special types of transmission lines.

So we have obtained solution for the input reflection coefficient of input impedance. I beg your pardon. We have not come across the concept of reflection coefficient yet. We have found that the input impedance of transmission line loaded with an impedance Z. Now let us see what happens if we consider some special cases. So the special cases are:

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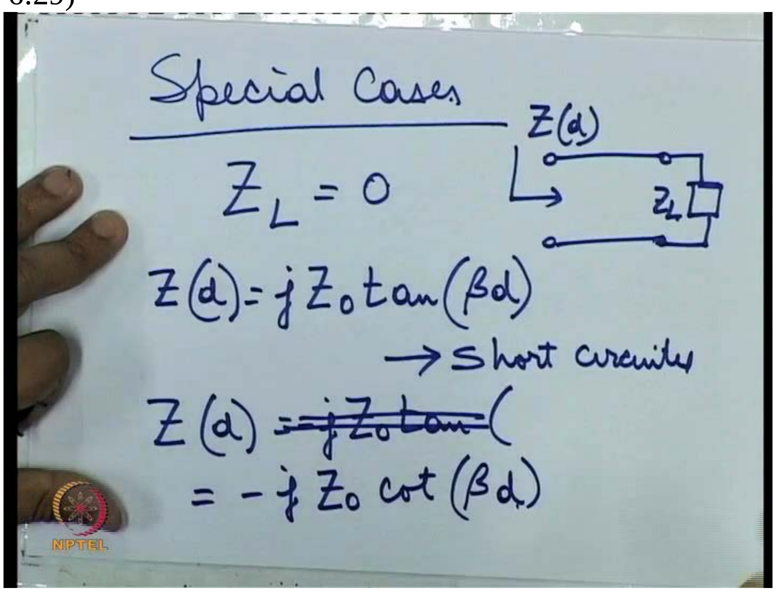

1st is when ZL is equal to 0. So when ZL equal to 0. So this is our transmission line once again, ZL the load. Then, Zd this is for a short-circuited line. So short-circuited line will have this kind of an input impedance. For an open circuited line this will be sorry this will be -jZo quat of beta d.

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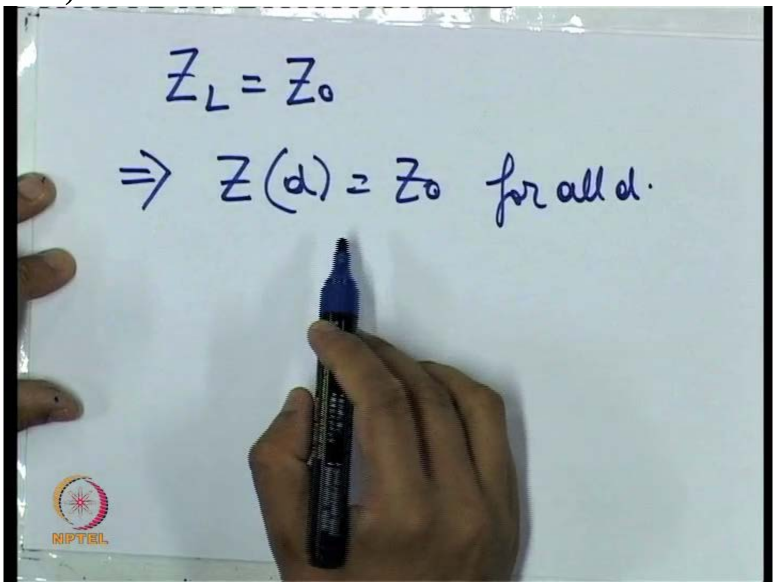

And for the case when ZL is equal to Zo, Zd will be simply equal to Zo for all d. Means as if the transmission line is never present. If we plot a curve the value of Zd for the shorted transmission line, it appears to be something like this.

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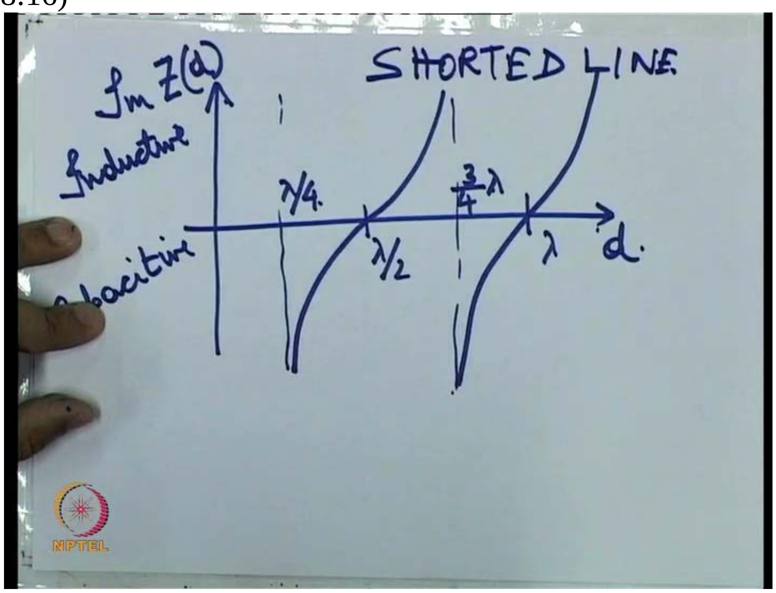

So we can see a shorted transmission line can take on both positive values as well as negative values for different values of d. For example, when d is equal to 0, it has a 0. So it is like a short. When d is between lambda/2 and 3lambda/4, it is positive. Zd is positive for a shorted line. So this is for a shorted line. Zd is positive. So it acts as an inductor. When d is between lambda/4 and lambda/2, Zd is negative and hence the shorted transmission line acts as capacitor. A special type of transmission line which is very commonly used is what is called as a quarter wave Transformer.

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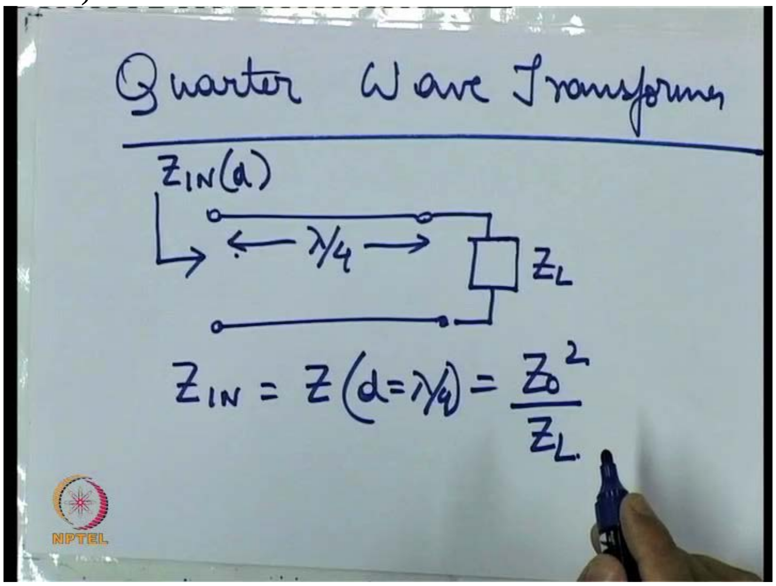

For quarter wave Transformer is a transmission line which is quarter wavelength long i.e. lambda/for long. It is also terminated by a load ZL. Now, in place of Zd I can also write this as Zind. So Zin will be equal to Zd equal to lambda/4 and this comes out to using the equation that I just gave if we put in the equation that I had previously stated that if this equation

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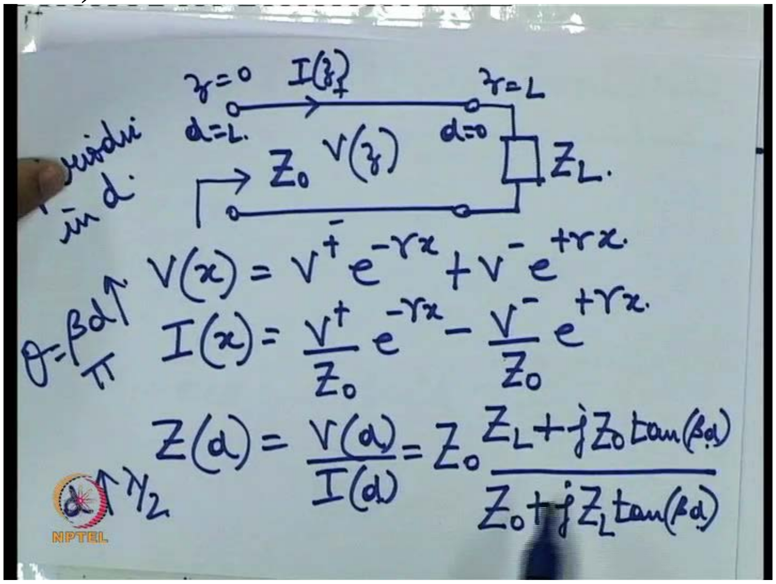

this equation if we substitute in place of d as lambda/4 and beta as 2pi/lambda then we get this particular relationship.

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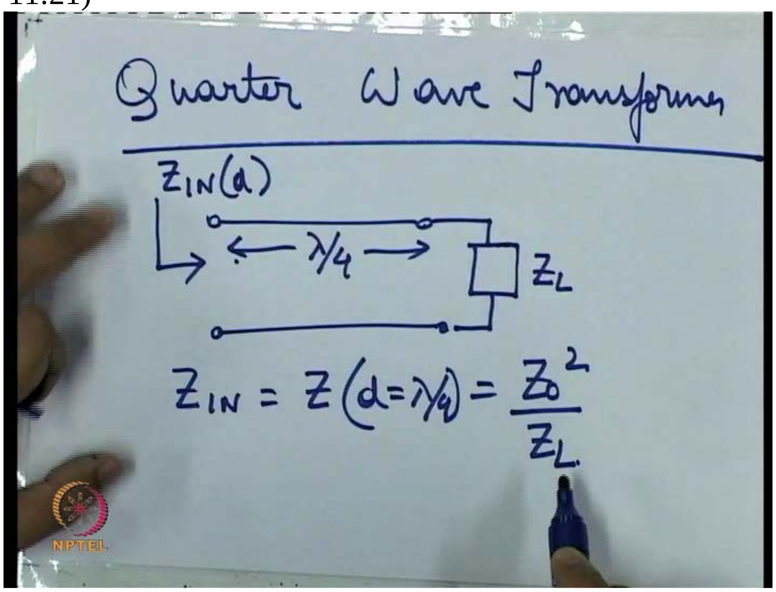

So what we see is that the input impedance is proportional to the inverse of the load impedance. If the load impedance ZL shorted, input impedance will appear to be open. If the load is open, the input will appear to be shorted.

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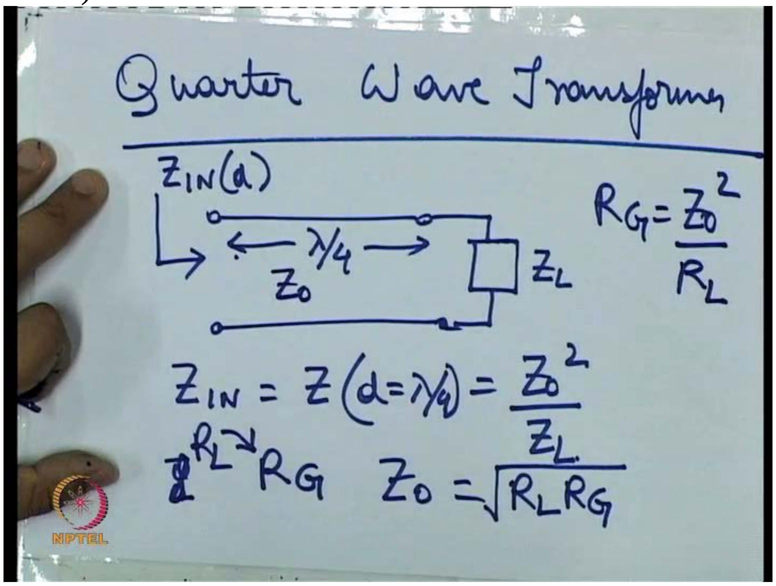

Conversely, if you want to convert any impedance say you want to convert resistance RG sorry resistance RL connected at the load to some value RG then you choose a transmission line which a characteristic impedance square root of RL RG and using this, you will get RG is equal to Zo square upon RL. So we see, by connecting our RL this is converted to RG when we have this transmission line with a Zo given by this value. This is one of the that is why it is called a Transformer.A transfer from one value of load impedance to another value of source impedance. Now coming to yet another important concept that is frequently associate it with transmission lines that is called as the reflection coefficient.

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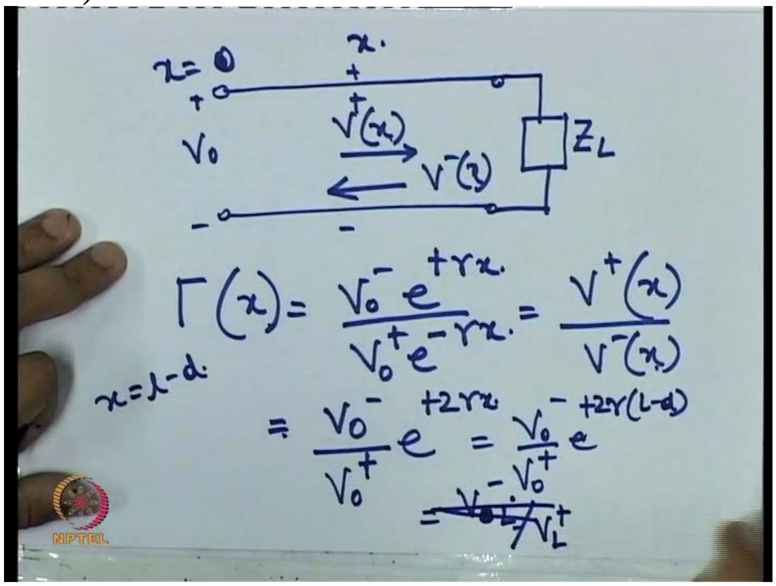

So once again if we have our transmission line with a load ZL connected like this and suppose the voltage as I said there are 2 concepts, one is V+ and V-then the reflection coefficient at a particular length x is given by V- or E+ gama into upon (V+-gama x).

Now this actually, the reflection coefficient is given as simply V+x upon V-x. We know we can write V+x in terms of the voltage at the source, Vo like this. So from here we get this is nothing but Vo-the reflected wave at the source upon Vo+2gama x. If we do a coordinate transformation where we write X is equal to L-d then this becomes so then now if you take out this E raise to +2gamaL then this Vo-upon Vo+ is converted to VL- upon VL+. Let me write it on a different sheet.

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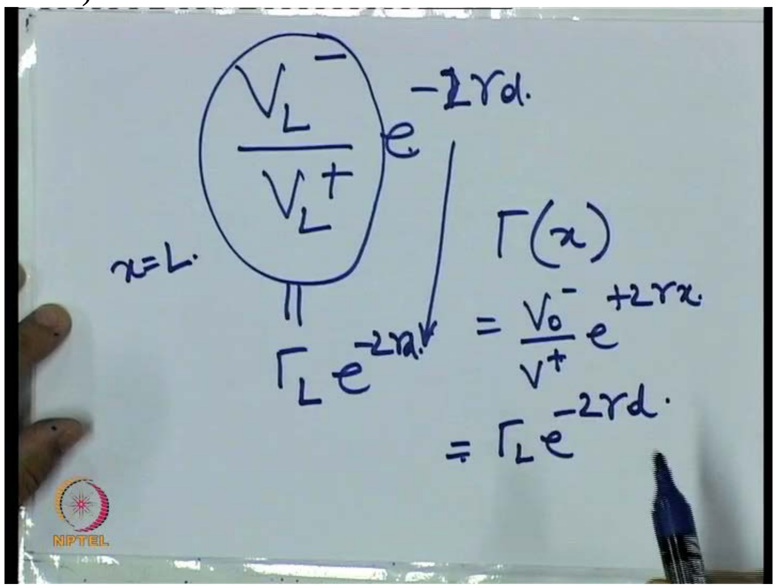

This is converted to VL –VL+E raised to -2gama d. This is nothing but the ratio of the reflected wave to the incident wave at the load end. That is at X equal to L and this is nothing but the reflection coefficient at the load end and this is simply 2 gamma d. Gamma at a particular point X in terms of the source voltages, reflected and incident voltages is given by this and in terms of the load parameters, it is given by this. So we have both key, you can express gamma at any point in terms of both the source parameters as well as the load parameters.

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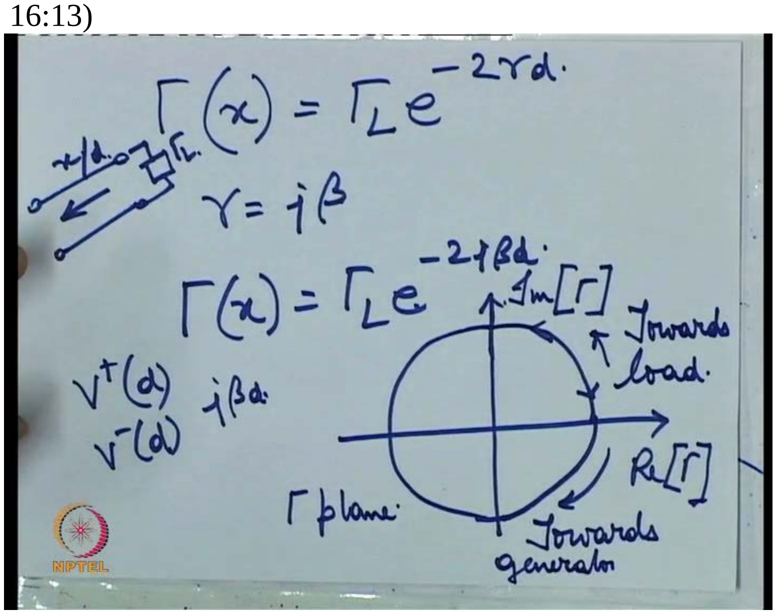

Now one interesting thing that we observed if I write the gamma X value now assuming this is a non-dispersive media gamma is equal to jbeta then we can write gamma X is equal to gamma L raised to -2jbeta d. Now what we see is that we had our transmission line at a particular point X or d. So d is away from the load. This is our gamma L and this is we are going away from the load. Now as we go away from the load, the phase of V+d or V-d changes by jbeta d only. Plus or minus jbeta d. But the phase of this reflection coefficient changes twice. So if we can draw this, as d increases or we are going away from the load, our phase angle keeps on decreasing and phase angle decreasing means we are moving in a clockwise direction.

The other thing that we note is that the magnitude of the reflection coefficient as we go away from the load remains constant and if we are going towards the load, then we move in an anticlockwise direction. If we are moving in a clockwise direction if we are moving towards the load, if we are going away from the load, we are moving in a clockwise direction. If we are moving towards the load, we are going in an anticlockwise direction. And the phase transit is twice the phase transit of as of that what we encountered while going along the transmission line.So this is one important concept associated with transmission lines. The reflection coefficient is changes twice faced change for the voltage for the incident and reflected waves.

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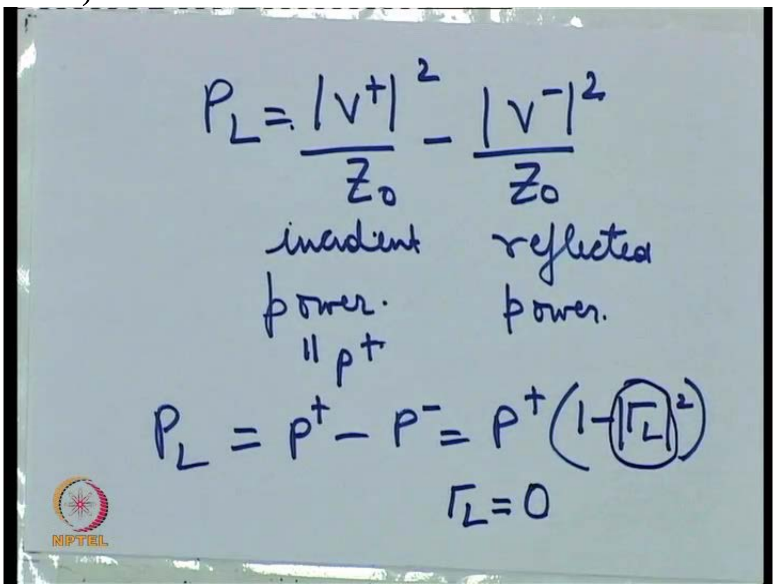

Now we had already come across this formula for the total real power that is delivered to the load and that was discussed in the previous module like this. Where say, V+ and V- are the voltages are the incident and reflected voltages at the load. Now, if we represent this incident power so this I can call incident power and this I can call the reflected power.

If I call denote it by the term P+ then PL can be written as P+-P-which in turn can be written as P+ if I take this V+ square upon Zo common 1 - gamma L whole square. Because V- upon V+ is nothing but gamma L. And so, we see that for total power transmission that if we want the entire incident power to go to the load gamma L should be equal to 0. Because for any other values of this gamma L some component of power will be reflected back and only when gamma L is equal to 0 we have complete transmission of the incident power to the load.

No see one thing about this gamma L is that this is a constant quantity. Gamma L does not change. It only depends on the load. Whereas the other components are not fixed. For example, what is the input reflection coefficient or what is the reflection coefficient at any other point along the transmission line, gamma L is a quantity that depends purely on the load property. So can we then derive a formula of gamma L in terms of the load impedance?

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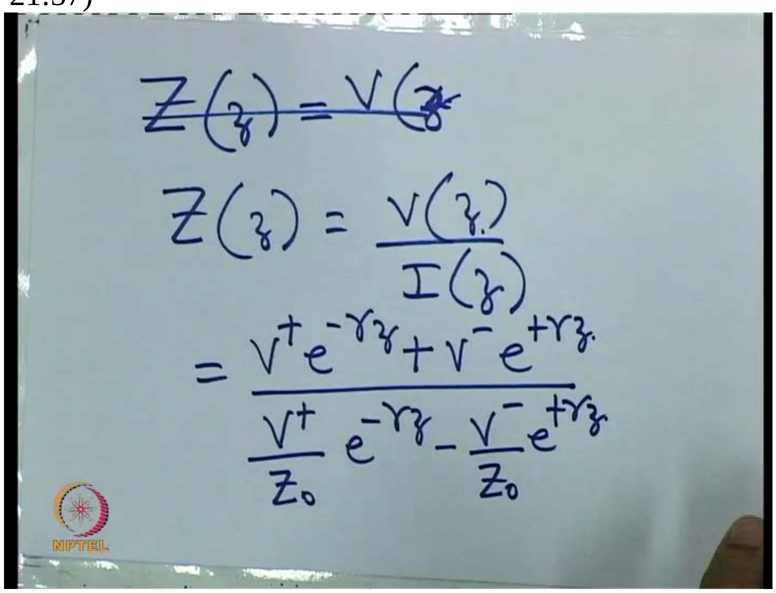

The impedance at any point Z or X's whatever. We can take X also in place of Z is given Z of small is equal to V of Z upon I of Z. If I write the voltage and current relationship at Z, that is…now this can be simplified to this relationship.

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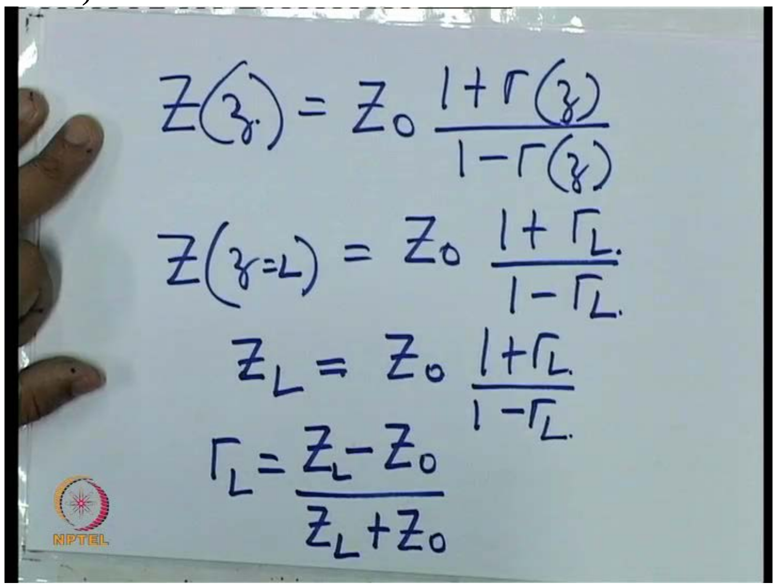

So then the impedance or the input impedance at Z equal to L is nothing but Zo upon 1+ gamma L upon 1-gamma L. So then ZL is nothing but Zo upon 1+ gamma L upon 1 - gamma L . From here, we can write if we want to find gamma L in terms of ZL that will come out to…So this is the value of load reflection coefficient.

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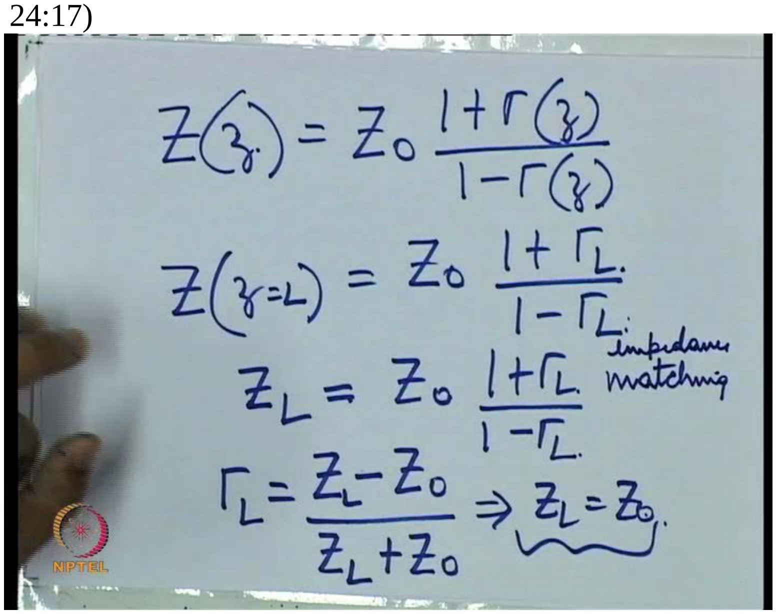

Now for total power transmission, we saw that gamma L should be 0 which also implies ZL should be equal to Zo. And such a condition where the entire incident power is transmitted to the load is called matching, impedance matching. So impedance matching is that condition where the entire incident power is transmitted to the load and that happens when my load impedance is equal to the characteristic impedance. Now this concept of load impedance this reflection coefficient, we saw the relationship that relates the 2, gamma L and ZL. The relationship is nothing but a bilinear transformation from one coordinate to another.

So once again if I write the question for gamma L…

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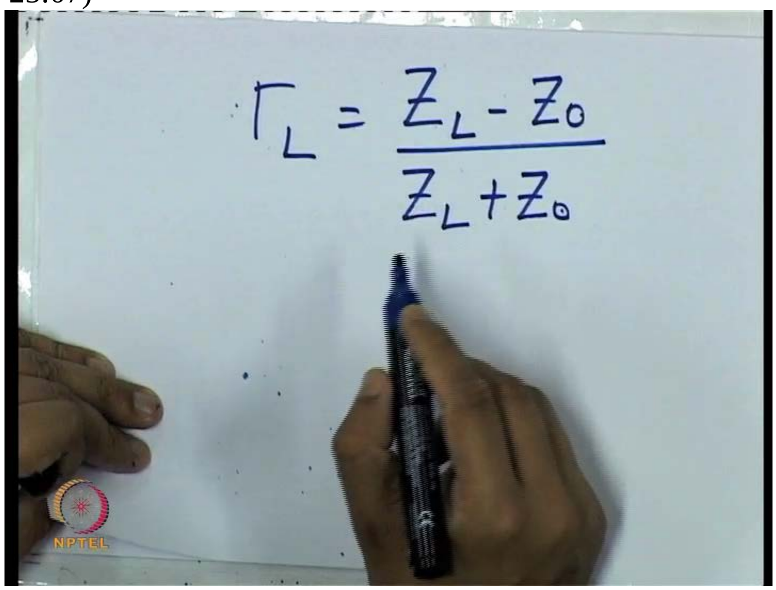

This is a bilinear transformation of Z. A special chart which does this transformation for all impedances is what is known as a Smith chart.

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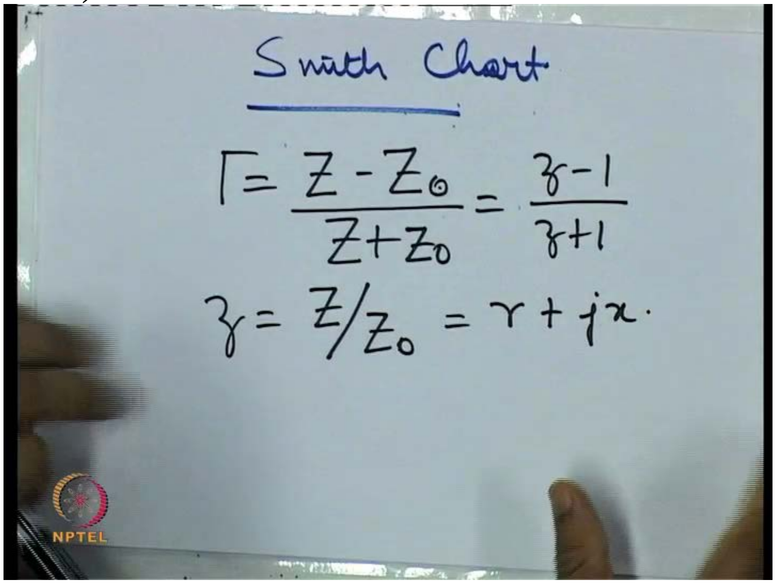

So if we want to draw 1st of all the definition of Smith chart is like this or this can also be written as this small Z where this small Z is equal to the normalised what you call as normalised impedance. This is in terms of the normalised resistance and reactance. Small Z can be written like this.

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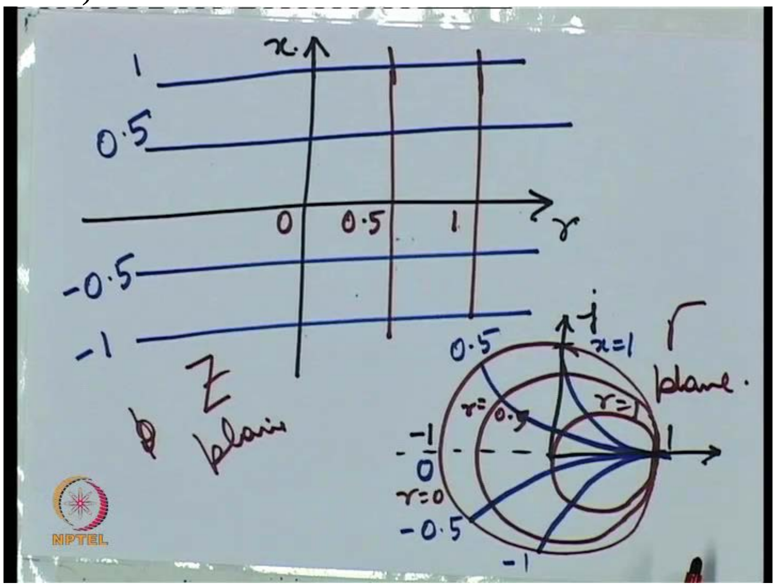

Now, if we plot this transformation, this particular transformation for various values of Z, the curve that we get is something like this. Now this line as you can see, this red and blue line is what is called as a constant resistance and constant a constant reactance line and upon doing this bilinear transformation, they are transformed to these circles. So is the R equal to 0 line, this is the R equal to 0.5 line. Sorry, this is X equal to 0 line, this is X equal to +0.5 line, this is X equal to -0.5 line: this is X equal to-1 line, this is X equal to 1 line. These lines are like the R equal to one line, R equal to 0.5 line. And this is the R equal to 0 line. This is the way conversion is done.

Now 1st thing we have to know is why do we do this conversion? We do this conversion because the entire right half of the Z plain this is our Z plain and this is our gamma plain. The entire right half of our Z plain is now confined within this entire circle. The circle have a radius 1. This helps us to visualise all passive devices because for all passive devices, R is greater than 0 and that is a convenient way of expressing it.

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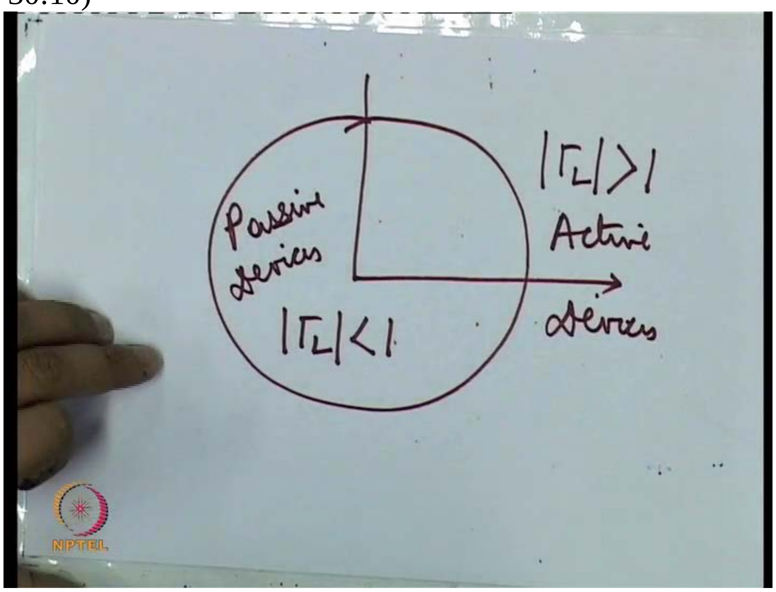

So we can write this as suppose this is a unit circle, the circle have been unit radius and within the circle, we have gamma L modulus lesser than 1 and outside the circle, we have gamma L modulus greater than1 . So this corresponds to all active devices and this corresponds to all passive devices. In the same way that we obtained the curve from Z value, we should also be able to get the curve, Smith chart form the Y values and such a Smith chart is called as Y Smith chart.

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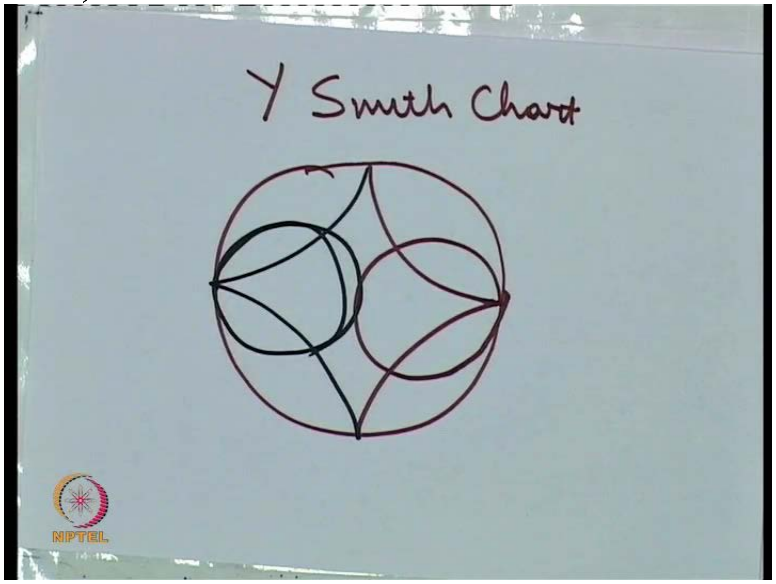

Now suppose this is our Z Smith chart, then the Y Smith chart will be 180° oriented from the Z Smith chart and like this. This will join.

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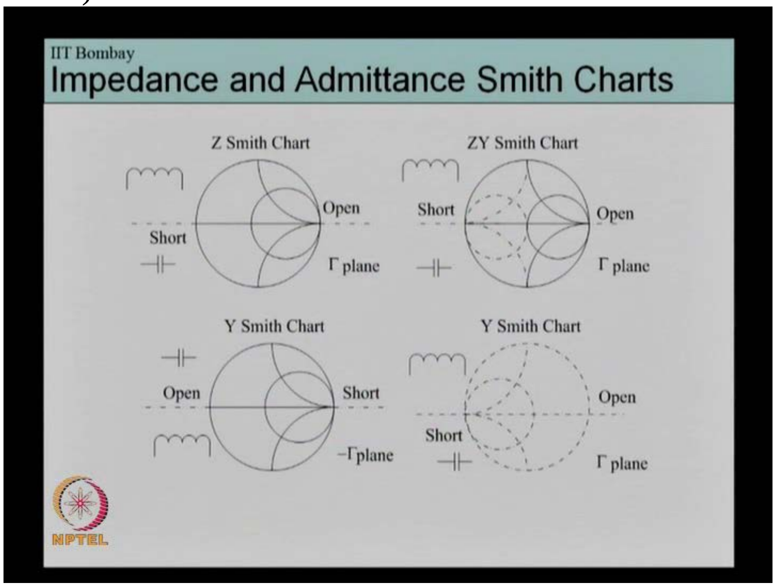

So if we can go back to the slides the monitor for a moment, these are the various configurations of the impedance and admittance Smith charts. Say this is our impedance Smith chart, then the top half we will have inductive i.e for the top half, the reactance X will be positive and for the bottom half, reactance X will be negative.

For the Y Smith chart, it is just the opposite. Top half is capacitive, bottom half is inductive. For the Z Smith chart, the right-hand side is open. Open means infinite impedance. And the left hand side is short or 0 impedance. Now, a combination of the 2 means Z-Y Smith chart as we were discussing as shown here. And a point that is plotted for the Z Smith chart, will have the same sense for the Y Smith chart. That particular point on this Z-Y Smith chart will have the same sense for the Y Smith chart. Now one final concept that I want to introduce is what is called as the voltage standing wave ratio.

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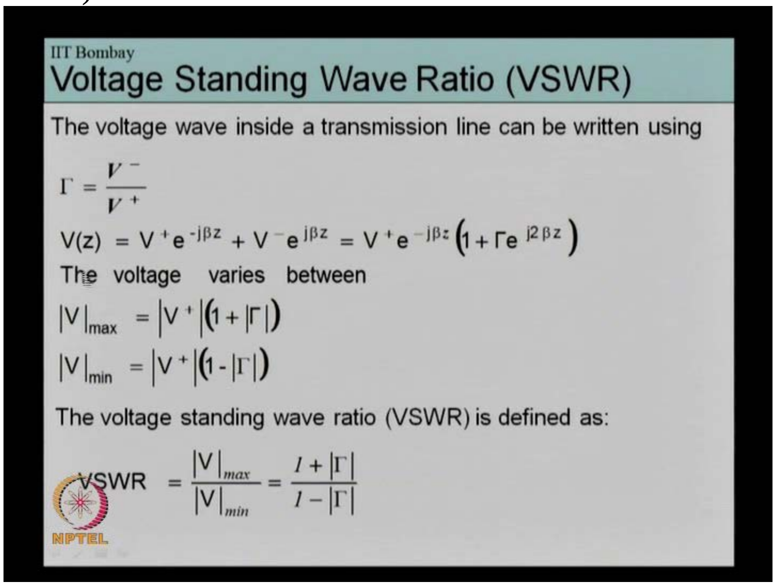

Maybe we can go to the slides on the monitor itself. So we saw that the voltage at any point is on a transmission line is given by this relationship which in turn is also related to the reflection coefficient. The maximum voltage at any point and the minimum voltage at any point, maximum means the maximum that can be achieved over time, minimum also means minimum that can be achieved over time is given by this ratio and by this relationship the ratio of these 2 values is what you called as the voltage standing wave ratio.

Now, VSWR was a very important concept in the olden days when we could not directly measure the reflection coefficient. It is something that can be directly measured with a motor because it is the ratio of the maximum to the minimum voltages. So, usually for good microwave matching, we require VSWR of less than 2. VSWR less than 2 is considered a good matching or as we saw from the concept of impedance matching that when most of the incident power is transmitted to the load. So in terms of reflection coefficient, that translates is to gamma L equal to 0. In terms of VSWR that translates to VSWR lesser than 2.

So in this course, in this module, we covered some of the more standard parameters associated with microwave circuits. In the next module we shall be further covering the more advanced concepts. Thank you