

**Microwave Integrated Circuits**  
**Professor Jayanta Mukherjee**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Bombay**

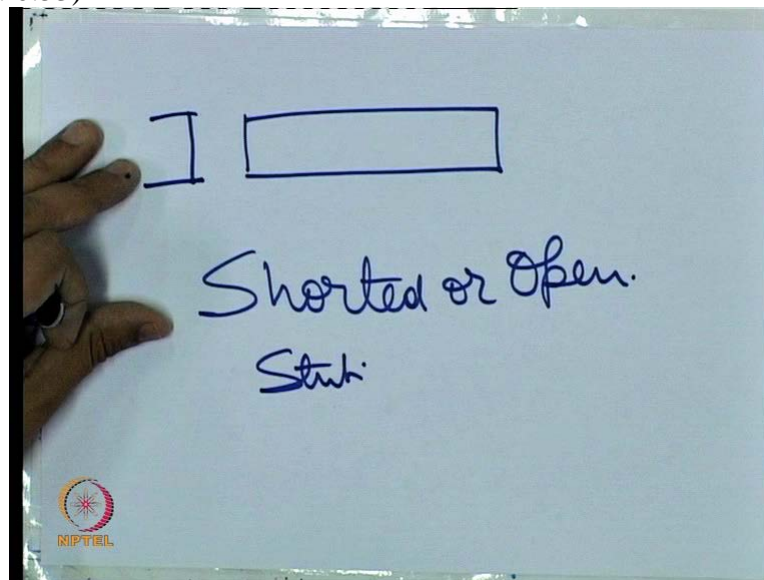
**Module 5**

**Lecture No 20**

**Narrow band filters**

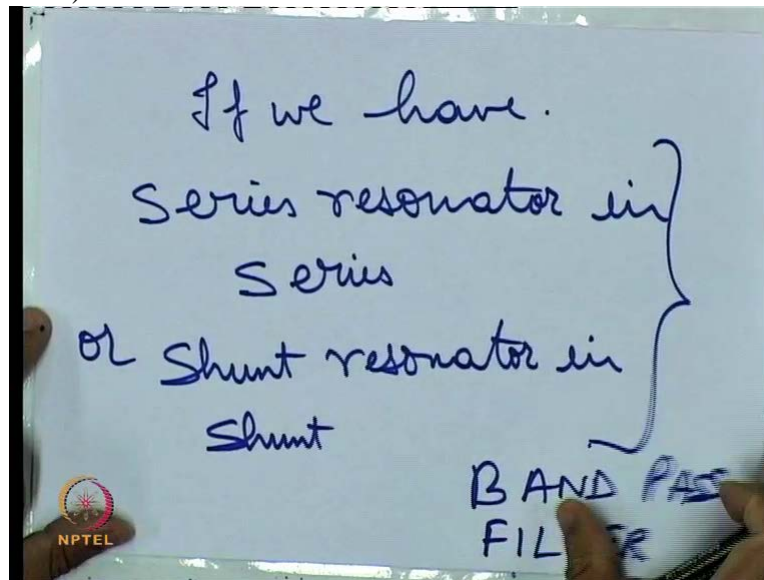
Hello, welcome to another module of this course, microwave integrated circuits. We are in week 5. In the previous module, I had covered the basics of resonators and I had shown that how using simple microwave components like a shorted stub or an open stub, we can build both shunt and series resonator. In this module, we shall be using those resonators to build the actual narrowband filter. So let us review what we did in the previous class in the previous module.

(Refer Slide Time: 0:53)



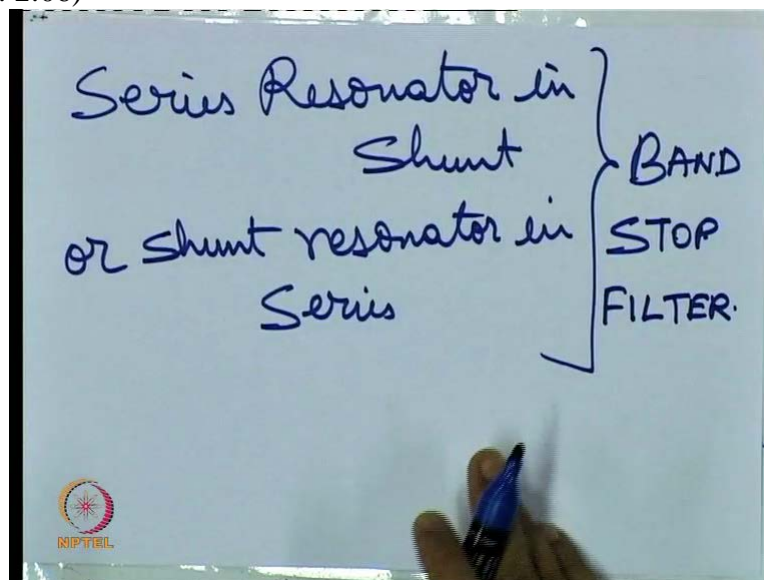
We had a shorted or a shunt stub and we saw that we could improve the performance by adding a gap like this.

(Refer Slide Time: 1:25)



So the 1<sup>st</sup> principle of a narrowband filter is that if we have a series resonator in series or shunt resonator in shunt, this gives rise to a band pass filter.

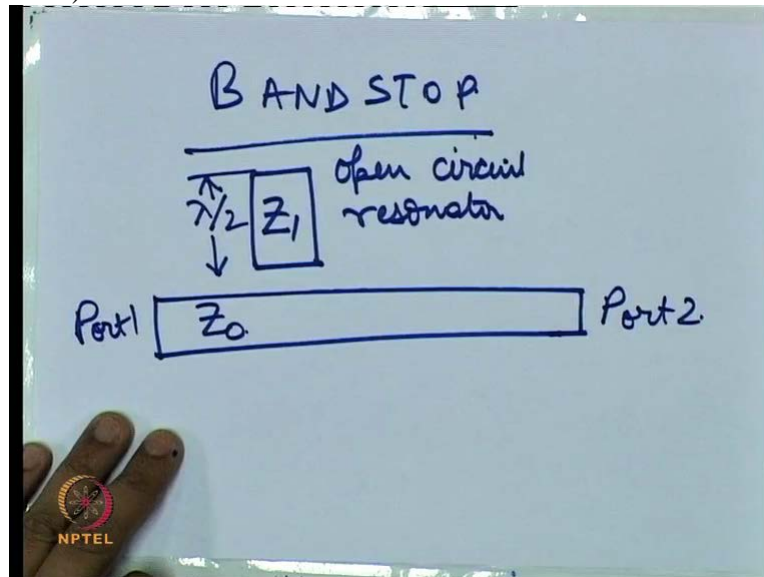
(Refer Slide Time: 2:06)



And if we have the converse, that is so whenever we want to design a band pass or band stop filter we have to keep this in mind. Now the resonator that we had discussed in the previous module using an open stub coupled with a gap, that is an example of a series inductor. Well, that can act as a series resonator or a shunt resonator depending on the frequency. Let us say we are

using it in a frequency range where it acts as a series resonator. So in that case, how do we make it a, use it as a filter?

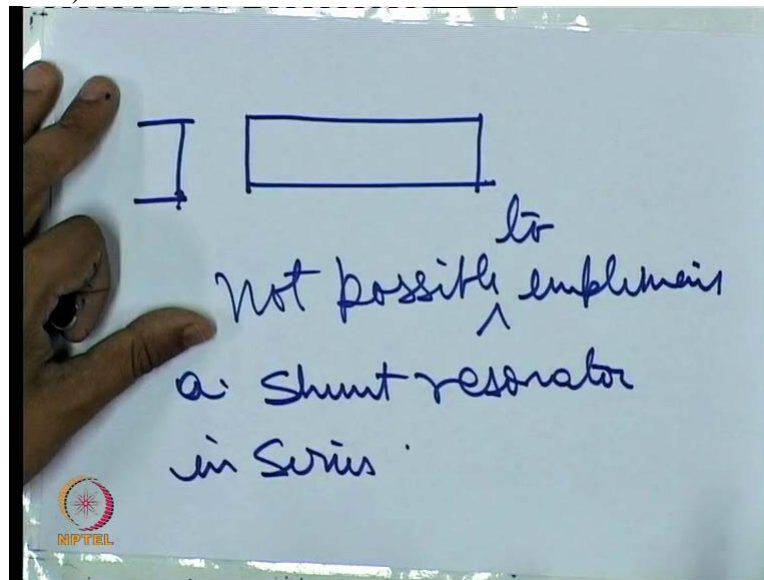
(Refer Slide Time: 3:16)



So Let us see how we can make a band stop filter using that resonator. If we have a simple cross motion line like this with a filter with a gap coupled open circuit resonator, the characteristic impedance of that open circuit resonator is  $Z_1$  then this is a series resonator in shunt. Hence this whole concentration will act as a band stop filter. Now, even better version of this is to have 2 band stop is to have 2 resonators. Now if we could have a series resonator, see in a band stop filters we need a shunt resonator in series or a series resonator in shunt. So we saw how to implement a series resonator in shunt.

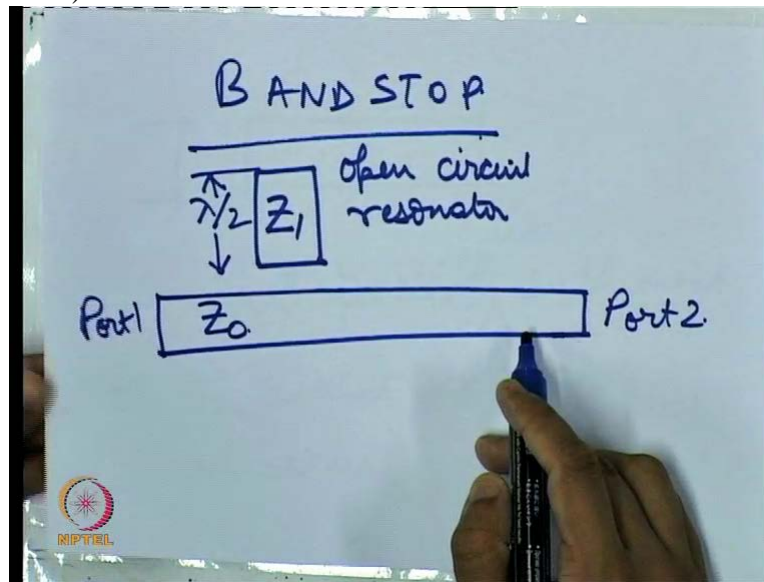
Now if we want to do the other way that is we want to implement a shunt resonator in series, then that becomes a problem because shunt resonators using same open stub is difficult to realise. Why is it difficult to realise?

(Refer Slide Time: 4:58)



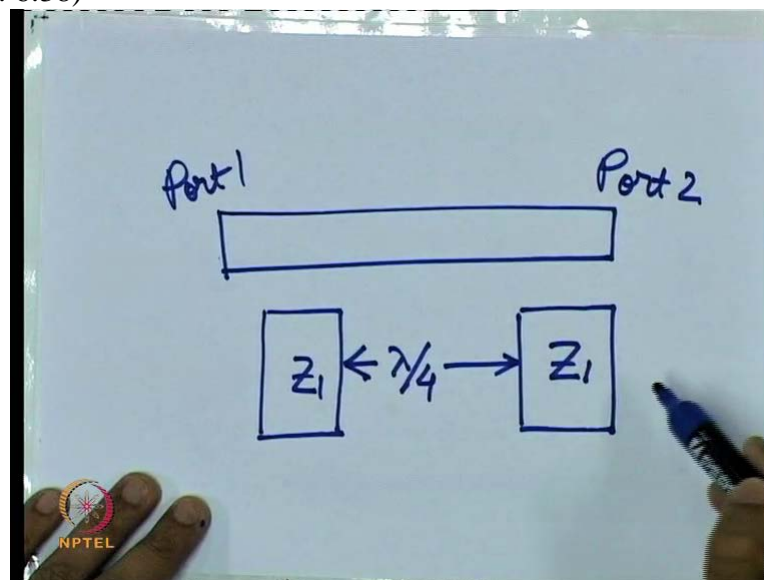
See, an open stub resonator will always have one end either open, it has to have open. Since it is an open stub resonator it has to have this open and the other end it can be either directly connected or gap coupled. While this can be easily realised in shunt the because in shunt, this end will hang from the circuit but in series, it is necessary that something at the other end is connected which is not possible because this is an open shunt open stub. It has to be open. So using an open stub gap coupled resonator, it is not possible to implement a shunt resonator in series. It can act as a shunt resonator because it can for certain frequencies give high impedance. But it will not be able to but since it cannot be connected in series, hence it cannot be used as a resonator in series. So that is one fundamental limitation.

(Refer Slide Time: 6:15)



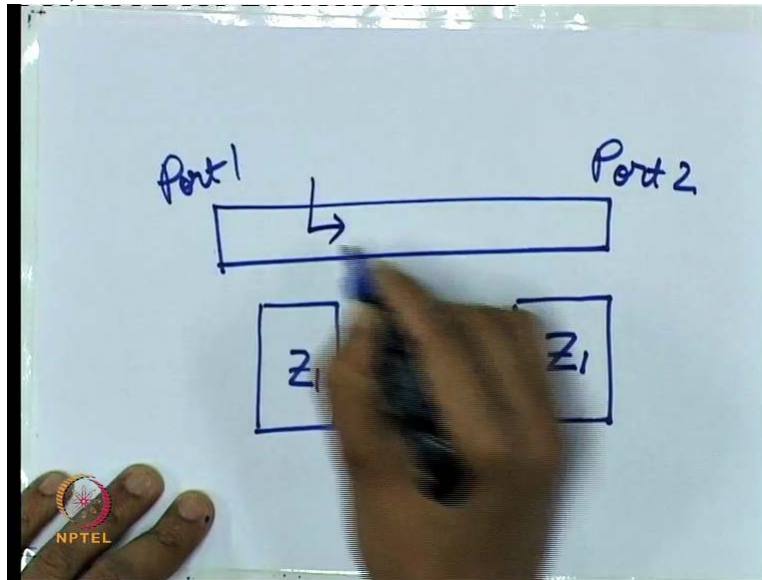
Then coming back to this circuit that we discussed a few minutes ago, if we have to use another resonator, we essentially need a circuit, a hunter resonator in series.

(Refer Slide Time: 6:38)



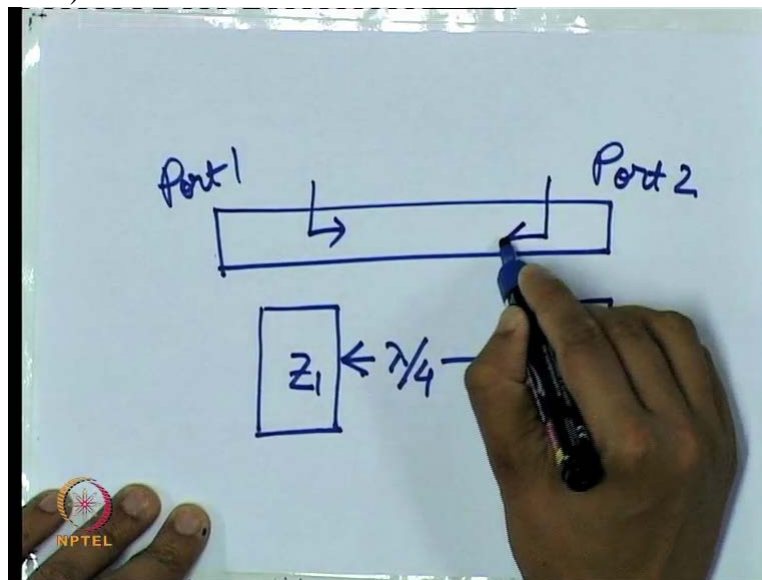
Other way of doing it is, we have 2 open stub resonators gap coupled resonators in shunt like this with a lambda by 4 distance between them. Since lambda by 4 implies a quarter wave Transformer, what this does is, because of the presence of a quarter wave Transformer, the property of a quarter wave Transformer that it can invert the impedance that is connected to the other end.

(Refer Slide Time: 7:22)



From this point, even though this is a series resonator in shunt, it actually appears as a shunt resonator in series from this point.

(Refer Slide Time: 7:32)



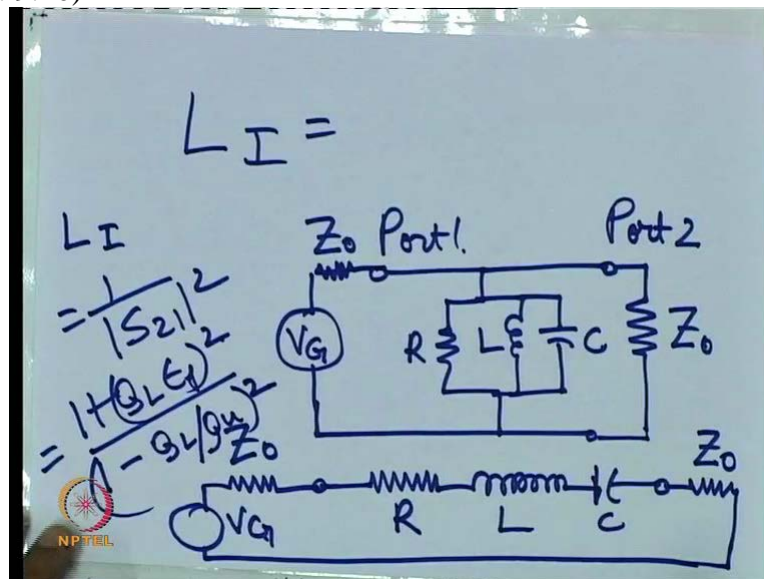
Similarly, from this point, this resonator even though a series resonator in shunt, will appear as a shunt resonator in series. So this is the way to implement a band stop filter using a series resonator. Note one thing that series resonators are the ones that are preferred because as I said shunt resonators have to be connected in series. For a band stop filter, we need shunt resonators to be connected in series which is not possible. Series connection of resonators using open stubs



or shorted stubs is not possible. Open stubs or shorted stubs can always be easily connected in shunt but not in series. Hence we prefer implementations where all our resonators are in shunt. Whether for the band pass or the band stop. So this was something about our band pass filter. I beg your pardon, this was about band stop filter.

Now coming to band pass filters. Again as I said, band pass filter essentially needs a series resonator in series or a shunt resonator in shunt. Since again, I said series resonator in series is not possible to implement using microwave components like open stub or a shorted stub, we will go for the shunt implementation.

(Refer Slide Time: 9:10)

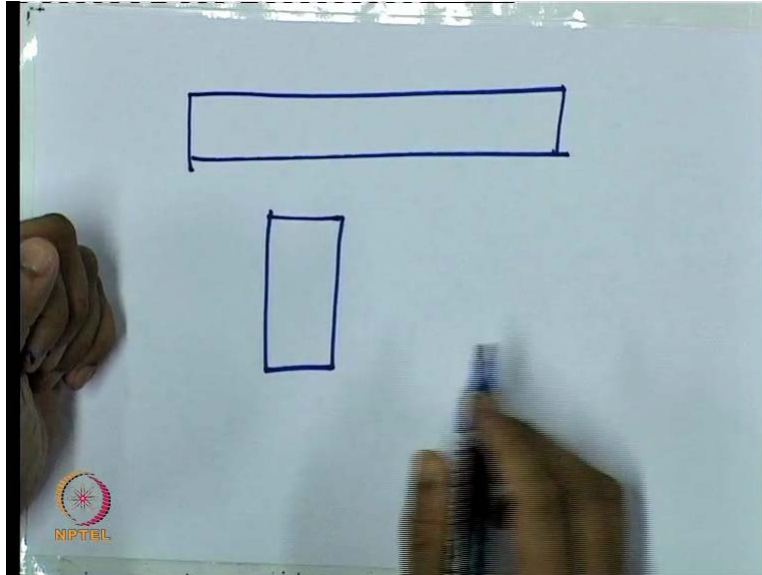


Before we go, we have to note that the insertion loss of a band pass filter, suppose this is our resonator, a shunt resonator in shunt. So this is a shunt resonator in shunt connected to a generator and a load. Generator internal impedance is  $Z_0$ . So this is matched to  $Z_0$  both at the input and output. Or say, if we consider a series resonator in series like this, so this is the band pass case of course because we have either a series resonator in series or a shunt resonator in shunt.

Now it can be shown that the insertion loss,  $LI$  equal to  $1$  upon  $S_{21}$  square is given by this formula. The derivation is again shown in the accompanying our accompanying slides. So I said

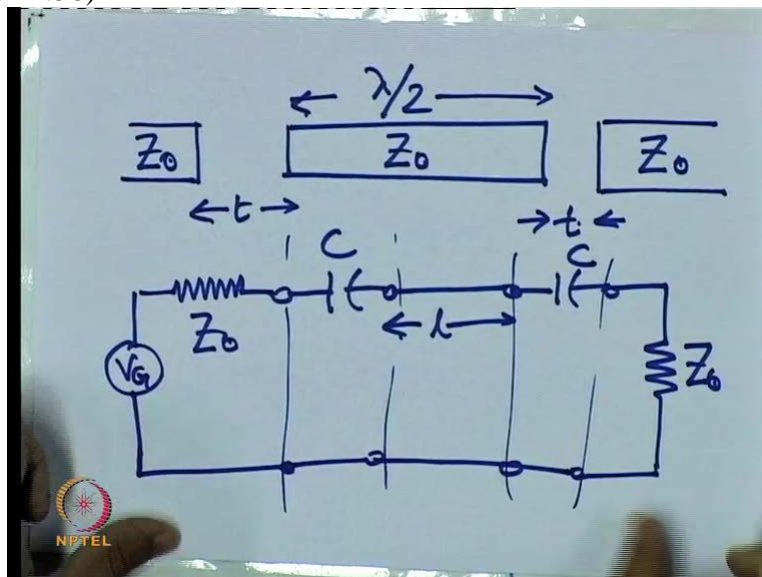
that both for the band pass and band stop filter, we would prefer shunt implementation of the resonator.

(Refer Slide Time: 11:19)



So one way of realising is if we have our length tuned properly of this open stub and say the resonant frequency is such that it corresponds to the high impedance state of this gap coupled resonator then that is an implementation of a shunt resonator in shunt. And that way we can realise it. And as I said you know series implementation of this open stub is not possible.

(Refer Slide Time: 11:58)





However, one that might be possible is with not using an open stop but using a simple transmission line element, if we have a structure like this and this is of length  $\lambda/2$ . Let us say at gap of length  $T$  and here also a gap of length  $T$  identical. Now the equivalent circuit diagram of this resonator is like this. Now if we want to find out the ABC parameters of this entire structure then we see that we have 3 circuits. 2 capacitors and a length of transmission line in Cascade. So just like we did for the branch line hybrid, the overall S parameter of this structure can be found from the overall ABCD matrix and to find out the ABCD matrix of this whole structure, we can simply multiply the ABCD matrices of the individual components and find out the overall ABCD the matrix.

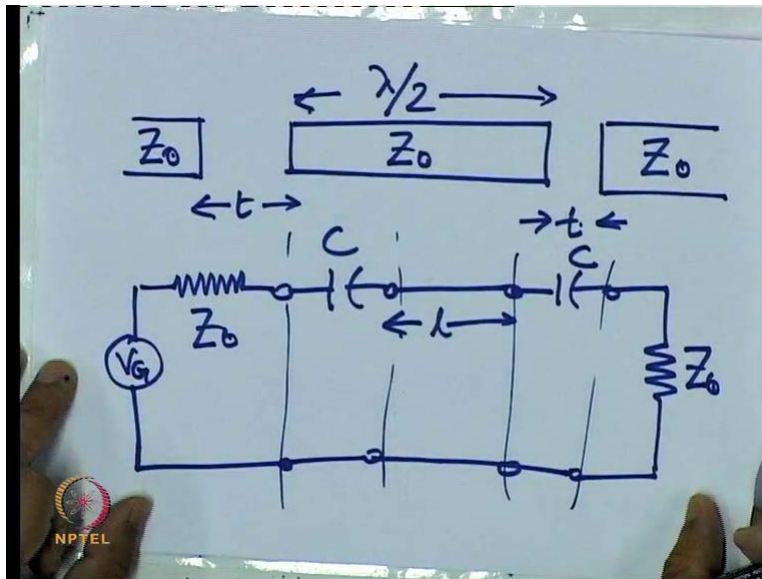
(Refer Slide Time: 13:46)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix} \quad X_c = \frac{1}{\omega C}$$

$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} 1 & -j\bar{X}_c \\ 0 & 1 \end{bmatrix} \quad \bar{X}_c = \frac{X_c}{Z_0}$$

So, for this capacitor, series capacitor, the ABCD matrix is given like this where this  $X_c$  is equal to  $1/(\omega C)$  or if we're using the normalised ABCD parameters then this will be like this.

(Refer Slide Time: 14:46)



So then the overall, the total if we come back to our overall band pass filter where all 3 elements are in Cascade then the overall ABCD matrix can be calculated from the normalised ABCD matrix that is.

(Refer Slide Time: 15:01)

$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi & j \sin \phi \\ j \sin \phi & \cos \phi \end{bmatrix} \times \begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix}$$

And then finally, if we do this multiplication the final value of this matrix that comes is...

(Refer Slide Time: 15:45)

Handwritten equations on a whiteboard:

$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} \cos \phi + \bar{X}_C \sin \phi & j \sin \phi \\ j \sin \phi & \cos \phi \end{bmatrix}$$

$$-I = \frac{1}{|S_{21}|^2} = \left| \frac{\bar{A} + \bar{B} + \bar{C} + \bar{D}}{2} \right|^2$$

The term  $\bar{A} + \bar{B} + \bar{C} + \bar{D}$  is annotated with the expression  $j(1 - \bar{X}_C^2) \sin \phi - 2j \bar{X}_C \cos \phi$ .

Now from this we can of course calculate the value, LI that is the insertion loss which is given by this relationship.

(Refer Slide Time: 16:51)

Handwritten derivation on a whiteboard:

At resonance.

$$L_I = 1 + \bar{X}_C^2 \left[ \frac{1}{2} \bar{X}_C \sin \phi + \cos \phi \right]^2$$

$$L_I = 1$$

$$\frac{1}{2} \bar{X}_C \sin \phi_r + \cos \phi_r = 0$$

$$\Rightarrow -Z_0 \cot \frac{\omega_r L}{v_p} = \frac{1}{2\omega_r C}$$

And then the final value that comes out after doing this mathematical derivation is LI  $1 + X_C$  square half  $X_C$  bar  $\sin \phi + \cos \phi$  whole square. Now the resonant frequency is the frequency at which this LI is equal to 1. So for LI to be equal to 1 within this term within this square brackets has to be 0. And so we should have half at resonance so for this term to be 0 and from this we can get a relation. Now this is an iterative equation. This equation has to be solved

iteratively because there is no close form the value for this omega R. Using a simple if you just write a program in Matlab, you can easily find out using neralical methods the value of omega R.

(Refer Slide Time: 18:27)

$$LI \approx 1 + \phi_r^2 \frac{X_{cr}^2}{X_{cr}^2 + 4} \epsilon_r^2$$

$$Q_u = \infty$$

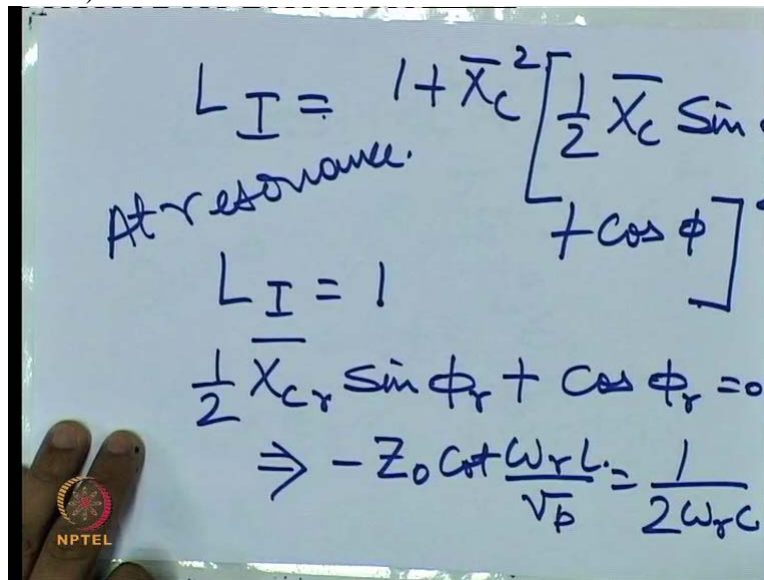
$$= 1 + Q_L^2 \epsilon_r$$

$$Q_L = \frac{\phi_r X_{cr}}{4} \sqrt{\frac{X_{cr}^2 + 4}{X_{cr}^2}}$$

$$\phi_r =$$

To make this calculation a bit easier, this LI can be further simplified I should write it approximately as  $1 + \phi_r^2 X_{cr}^2 / (X_{cr}^2 + 4) \epsilon_r^2$ . And this I just stated the formula for the LI of a band pass filter. Now for a band pass filter that has a lossless resonator in it, that is for which  $Q_u$  is equal to infinity. For that kind of filter since here we are assuming that the resonator we are using, this gap coupled series resonator is lossless, hence LI can be written like this. Now comparing these 2 equations, we can find, we can obtain an equation for  $Q_L$  as  $\phi_r X_{cr} / 4$  upon,  $X_{cr}$  bar this is the value of  $X_C$  that is the capacitance that is the value of reactance of the series capacitance at the resonant frequency. Then  $Q_L$  comes out to be equal to this where  $\phi_r$  comes from this equation that we had earlier derived.

(Refer Slide Time: 20:06)

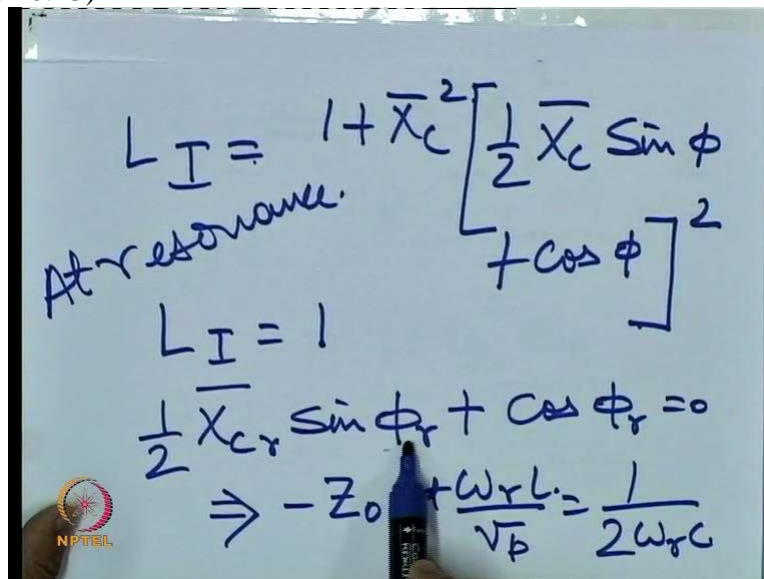


At resonance.

$$L_I = 1 + \bar{X}_c^2 \left[ \frac{1}{2} \bar{X}_c \sin \phi + \cos \phi \right]$$
$$L_I = 1$$
$$\frac{1}{2} \bar{X}_{cr} \sin \phi_r + \cos \phi_r = 0$$
$$\Rightarrow -Z_0 \cot \frac{\omega_r L}{v_p} = \frac{1}{2\omega_r C}$$

Phi R comes from this equation.

(Refer Slide Time: 20:13)



At resonance.

$$L_I = 1 + \bar{X}_c^2 \left[ \frac{1}{2} \bar{X}_c \sin \phi + \cos \phi \right]$$
$$L_I = 1$$
$$\frac{1}{2} \bar{X}_{cr} \sin \phi_r + \cos \phi_r = 0$$
$$\Rightarrow -Z_0 \cot \frac{\omega_r L}{v_p} = \frac{1}{2\omega_r C}$$

I beg your pardon, phi R comes from the solution of this equation this equation involving Sin and Cos.

(Refer Slide Time: 20:20)

$$L_I \approx 1 + \phi_r^2 \overline{X_{cr}}^2 \times \frac{\overline{X_{cr}}^2 + 4}{16} \epsilon_f^2$$

$$Q_u = \infty$$

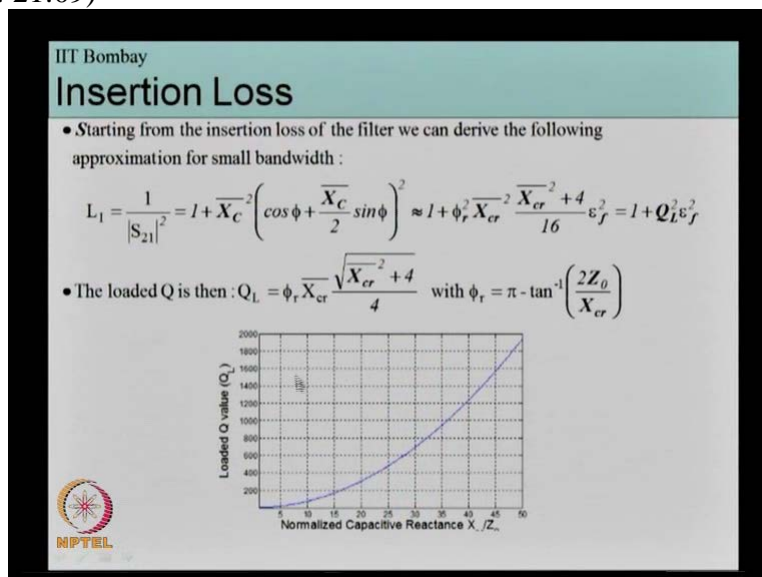
$$= 1 + Q_L^2 \epsilon_r$$

$$Q_L = \phi_r \overline{X_{cr}} \sqrt{\overline{X_{cr}}^2 + 4}$$

$$\phi_r = \pi - \tan^{-1}\left(\frac{4}{2Z_0/\overline{X_{cr}}}\right)$$

So phi R is equal to pi - Tan inverse 2Z0 by XCR. So in summary you know what is initially given is the loaded Q factor that you have to achieve. You are given a certain resonant frequency and the loaded Q factor that you might you have to achieve or you might not even be given the loaded Q factor. You might just be given the two 3dB frequencies. If you know the two 3dB dB frequencies, you can find out the value of QL and then once you know QL, you can use these equations that we just derived to find the values of the to design the resonator.

(Refer Slide Time: 21:09)



See, if you plot this equation, this XCR or the normalised XCR on this plot the X axis is the XCR upon Z0 or the normalised XCR and on the Y axis, we have this QL. So this graph



represents this equation. Now suppose you have been given a certain QL value that you have to achieve, then you can find the value of this XCR upon Z0 from this graph.

(Refer Slide Time: 21:52)

IIT Bombay

## Insertion Loss

- With the ABCD parameters of the filter :
 
$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} \cos\phi + \bar{X}_C & j(1 - \bar{X}_C^2)\sin\phi - 2j\bar{X}_C\cos\phi \\ j\sin\phi & \cos\phi + \bar{X}_C \end{bmatrix}$$
- We calculate its insertion loss :
 
$$L_1 = \frac{1}{|S_{21}|^2} = \left| \frac{\bar{A} + \bar{B} + \bar{C} + \bar{D}}{2} \right|^2 = 1 + \bar{X}_C^2 \left[ \frac{1}{2} \bar{X}_C \sin\phi + \cos\phi \right]^2$$
- At resonance  $\omega = \omega_r$  the bandpass filter is a thru ( $|S_{21}(\omega_r)| = 1$ ) so that
 
$$L_1(\omega_r) = \frac{1}{|S_{21}(\omega_r)|^2} = 1$$
 resulting in (using  $\phi_r = \beta_r l$  and  $\bar{X}_C(\omega_r) = \bar{X}_{cr}$ ) :
 
$$\frac{1}{2} \bar{X}_{cr} \sin\phi_r + \cos\phi_r = 0 \Rightarrow -Z_0 \cot \frac{\omega_r l}{v_p} = \frac{1}{2\omega_r C}$$

This is the resonance condition. For small capacitance values,  $\beta_r l \approx \pi$  and the line is approximately a half wavelength long.

This XCR upon Z0 is by the way from this XCR upon Z0 value that you obtained, you can find out the value of C that is the capacitance you have to connect in this gap. So the remaining things that is left is to find out the value of this Z0 and that Z0 can be found out from this equation.

So once you know XCR upon you know C you can calculate the value of Z0 and the L.

(Refer Slide Time: 22:16)

IIT Bombay

## An Example

Design a bandpass filter to operate at 2 GHz which will have 3 dB cutoff frequencies at 1950 and 2050 MHz. Assume 50 ohm, air-filled lines.

Find the length of the intermediate transmission line.

Sol :- First, since  $Q_L = 2000/100 = 20$ , use the chart to find  $X_{cr}/Z_0 \approx 5.25$

- Since we have 50 ohm lines and desire 2000 MHz as the center frequency, find C from

$$X_{cr} = \frac{1}{\omega_r C} = 50(5.25)$$

$$C = \frac{1}{50(5.25)2\pi(2 \times 10^9)} = 0.303 \text{ pF}$$

- The exact line length can now be found from the resonance condition :

$$\frac{2\pi(2 \times 10^9)}{3 \times 10^8} l = \pi - \tan^{-1}(2/5.25) = 2.776$$

$$l = 0.6663 \text{ m}$$

So here is an example of how we can solve this equation. In this example you have to design a filter at 2 GHz and two 3dB frequencies are at 1950 and 2050 megahertz. And  $Z_0$  is equal to 50 ohms.

(Refer Slide Time: 22:44)

IIT Bombay

## Insertion Loss

- Starting from the insertion loss of the filter we can derive the following approximation for small bandwidth :

$$L_1 = \frac{1}{|S_{21}|^2} = 1 + \overline{X_C}^2 \left( \cos \phi + \frac{\overline{X_C}}{2} \sin \phi \right)^2 \approx 1 + \phi_r^2 \overline{X_{cr}}^2 \frac{\overline{X_{cr}}^2 + 4}{16} \epsilon_f^2 = 1 + Q_L^2 \epsilon_f^2$$

- The loaded Q is then :  $Q_L = \phi_r \overline{X_{cr}} \frac{\sqrt{\overline{X_{cr}}^2 + 4}}{4}$  with  $\phi_r = \pi - \tan^{-1} \left( \frac{2Z_0}{\overline{X_{cr}}} \right)$

NPTEL

1<sup>st</sup> you find out  $Q_L$  from this centre frequency and the two 3dB frequencies and using this chart you find out that XCR upon  $Z_0$  using this value for this value of  $Q_L$  is 5.25. So you know XCR upon  $Z_0$ .

Then you can find out XCR and C.

(Refer Slide Time: 23:04)

IIT Bombay

## An Example

Design a bandpass filter to operate at 2 GHz which will have 3 dB cutoff frequencies at 1950 and 2050 MHz. Assume 50 ohm, air - filled lines. Find the length of the intermediate transmission line.

Sol :- First, since  $Q_L = 2000/100 = 20$ , us the chart to find  $X_{cr}/Z_0 \approx 5.25$

- Since we have 50 ohm lines and desire 2000 MHz as the center frequency, find C from

$$X_{cr} = \frac{1}{\omega_r C} = 50(5.25)$$

$$C = \frac{1}{50(5.25)2\pi(2 \times 10^9)} = 0.303 \text{ pF}$$

- The exact line length can now be found from the resonance condition :

$$\frac{2\pi(2 \times 10^9)}{3 \times 10^8} l = \pi - \tan^{-1}(2 / 5.25) = 2.776$$

$$l = 0.0663 \text{ m}$$

NPTEL

And the exact line length that is beta RL can be found out from this value. We know XCR upon Z0 so we can plot we can find out phi R and since phi R is equal to beta R upon L. So from here if we know the value of beta R, then we can find out the value of L.

(Refer Slide Time: 23:26)

IIT Bombay  
**Insertion Loss**

- Starting from the insertion loss of the filter we can derive the following approximation for small bandwidth :

$$L_1 = \frac{1}{|S_{21}|^2} = 1 + \overline{X_C}^2 \left( \cos \phi + \frac{\overline{X_C}}{2} \sin \phi \right)^2 \approx 1 + \phi_r^2 \overline{X_{cr}}^2 \frac{\overline{X_{cr}}^2 + 4}{16} \epsilon_f^2 = 1 + Q_L^2 \epsilon_f^2$$

- The loaded Q is then :  $Q_L = \phi_r \overline{X_{cr}} \frac{\sqrt{\overline{X_{cr}}^2 + 4}}{4}$  with  $\phi_r = \pi \cdot \tan^{-1} \left( \frac{2Z_0}{\overline{X_{cr}}} \right)$

NPTEL

So just to summarise once again, 1<sup>st</sup> we find out QL from the given conditions. From that, we can find out XCR using these 2 equations and this plot. This plot is basically a realisation of these 2 equations and once we know XCR upon Z0, we can find out C.

(Refer Slide Time: 24:04)

IIT Bombay  
**Insertion Loss**

- With the ABCD parameters of the filter :

$$\begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} = \begin{bmatrix} \cos \phi + \overline{X_C} & j(1 - \overline{X_C}^2) \sin \phi - 2j\overline{X_C} \cos \phi \\ j \sin \phi & \cos \phi + \overline{X_C} \end{bmatrix}$$

- We calculate its insertion loss :

$$L_1 = \frac{1}{|S_{21}|^2} = \left| \frac{\overline{A} + \overline{B} + \overline{C} + \overline{D}}{2} \right|^2 = 1 + \overline{X_C}^2 \left[ \frac{1}{2} \overline{X_C} \sin \phi + \cos \phi \right]^2$$

- At resonance  $\omega = \omega_r$  the bandpass filter is a thru ( $|S_{21}(\omega_r)| = 1$ ) so that

$$L_1(\omega_r) = \frac{1}{|S_{21}(\omega_r)|^2} = 1 \text{ resulting in (using } \phi_r = \beta_r l \text{ and } \overline{X_C}(\omega_r) = \overline{X_{cr}} \text{):}$$

$$\frac{1}{2} \overline{X_{cr}} \sin \phi_r + \cos \phi_r = 0 \Rightarrow -Z_0 \cot \frac{\omega_r l}{v_p} = \frac{1}{2\omega_r C}$$

- This is the resonance condition. For small capacitance values,  $\beta_r l \approx \pi$  and the line is approximately a half wavelength long.

NPTEL

Once we find out  $C$ , we get one component of our design and then using the value of  $XCR$  upon  $Z_0$  we can find out  $\phi R$  and from  $\phi R$  we can find out  $\beta R$  and the length of this intermediate section.

In summary, this is how, you know in this module we covered 2 specific designs, the 1<sup>st</sup> one being that of a band stop filter using a shunt resonator and the 2<sup>nd</sup> one of a band pass filter using a series resonator. So in the next module, we will see some further implementations further types of filters that are used commonly. So one thing before ending I want to impress upon you is that if you are using open or shorted Stubs as a resonator, just pure open and shorted stubs as a resonator, then you have to connect them in shunt. If on the other hand you are using length of the transmission line which can be connected on both sides as we saw for band pass filter, then you may use a series resonator. But if you are using a purely open or a purely short resonator, that cannot be connected in series. It has to be used in shunt whether you are designing a bandpass or a band stop filter. Thank you.