Microwave Integrated Circuits.
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Lecture -22.
Filter synthesis, Kuroda's identity.

Hello, welcome to another module of this course microwave integrated circuits. In the previous 2 modules we had covered the various techniques for filter synthesis. In 2 modules that we had introduced, he had introduced you to the concept of narrowband filters using the resonators. And in the last module we had discussed about image filters. Now both the narrowband and image filters as we had seen, they rely on certain set structures, for example in the narrowband filter case, there were resonators.

And for the image filters, there was the concept of unit cells. The synthesis techniques were very limited, for example in a narrow band filter case, we could only change some gap, introduce a gap, put it in shunt or in… Or put a transmission line element in Cascade and something like this. And for image filters we see this that our design is limited to just designing the unit cells. And basically they were just 2 types of unit cells that we discussed, one was the constant K and the other was M derived.

But really there is no way of designing the response. So, in this module we shall be talking about designing particular frequency response. How to design a filter that will provide us a particular desired frequency response?

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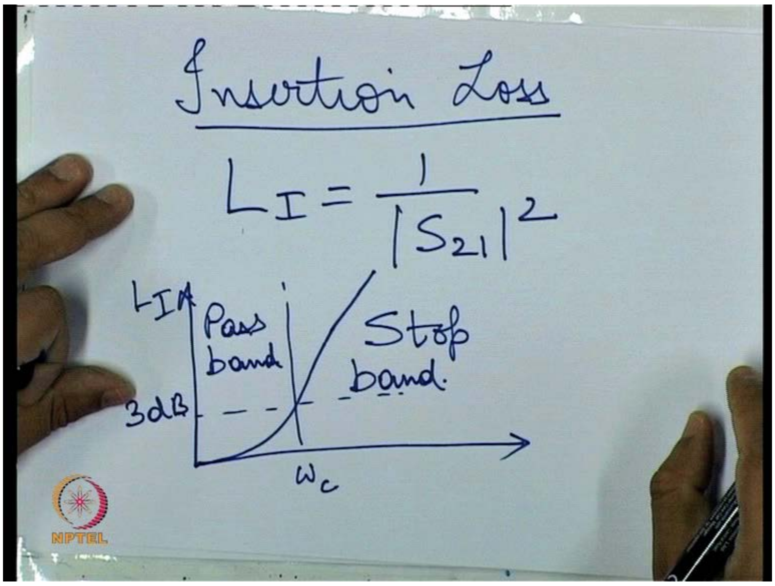

So, let us see how to do that. Recall I had introduced you to the concept of insertion loss and oa few modules back. So, LI, this insertion loss is given by this relationship and for say a low pass filter, LI should be high, should be low in the passband and high in the stop band.

So, if we make a plot between LI and omegaC and suppose OmegaC is our cut-off frequency, then it means that the insertion loss should increase to a maximum of 3 DB for frequencies less than omega C and beyond omega C, it would increase. So, this is our passband and beyond omega C, we have our stop band. Now the technique for this synthesis is based on this insertion loss. Suppose we are given a certain insertion loss, what circuit can be realised, can be designed so that a particular insertion loss characteristic is…

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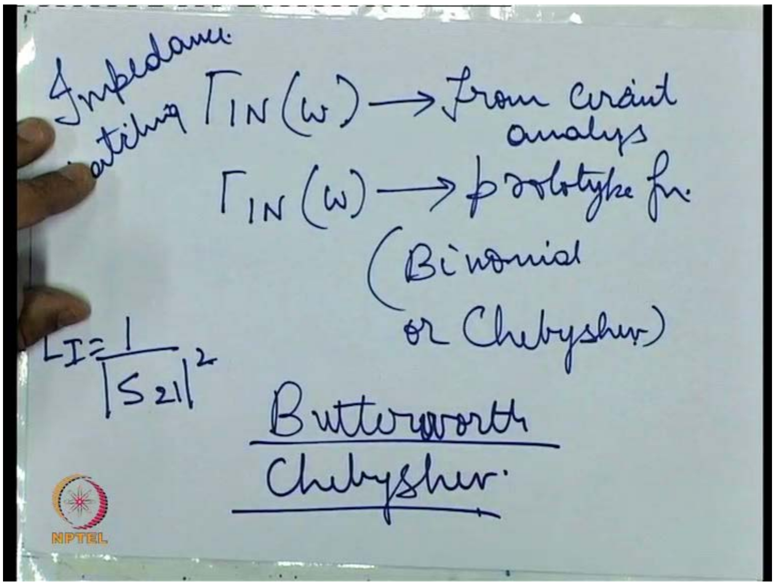

The procedure is very similar to the procedure that I had described while designing impedance matching networks. In the impedance matching networks, we 1st had found out a relationship for, gamma in omega and then we had equated it to a prototype gamma in omega. So, this was our prototype function. So, either it was a binomial or Chebyshev and this was from circuit analysis. So, this was the case for impedance matching now for this filter synthesis also, the principle will be the same. We will be given a certain S21 or certain insertion loss transfer function and we will be equating it to a prototype case.

Now we can do it both for binomial and Chebyshev but I will just show you for the, for the I beg your pardon, we will be equating this to 2 prototype functions one is known as the Butterworth prototype and the other is the Chebyshev prototype.

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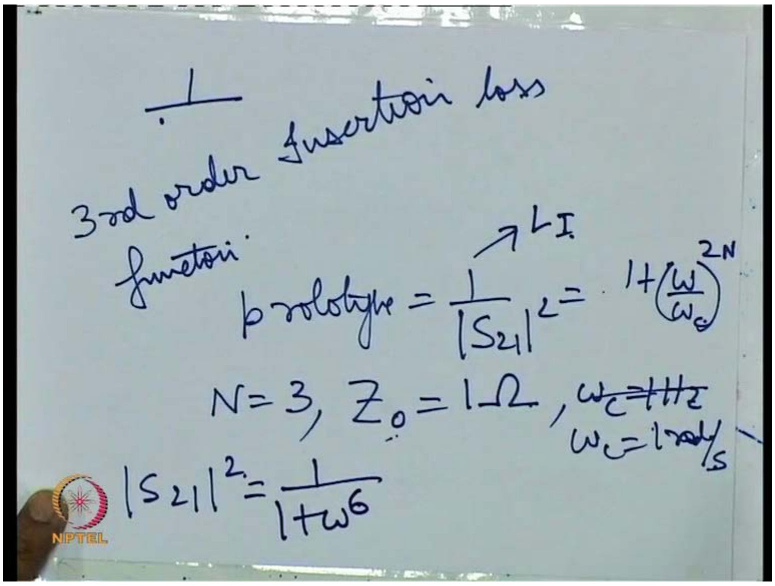

So, let us instead of discussing the theory behind it, let us currently go to the example. Now let us suppose that we are given a 2nd order insertion loss function as shown here. So, suppose our 1 upon, suppose we are given a 3rd order, I beg your pardon, a 3rd order insertion loss function.

Now the Butterworth 3rd order insertion loss function prototype is given as this one. Now, this can be equated as Suppose say for our case we have N equal to 3 and say have input and output impedance match required is 1 ohm and say our cut-off frequency is 1 Hz. Now, this is not the usual case, usually you will have Z0 as 50 ohms and omega C at some high-frequency or various frequencies according to the requirement. But just for this case, 1st we will be deriving our prototypes for these idealised values and then we shall be finding the transformation or how to transform the circuit so that we get whatever impedance matching we want and whatever cut-off frequency that we want.

So, with that, there are some scaling techniques which shall cover later. But for now we shall be considering our Z0 to be 1 ohms, omega C to be, omega should be 1 radians per second and this is our 3rd order Butterworth prototype. So, I said that the Butterworth autotype is given by this function. So then $S21^2$ I should emphasise that this is the insertion loss LI. Now $S21^2$ for this particular values of Z0 and omega C will become 1 + omega raised to 6 because N equal to 3, so this is our $S21^2$. And then from here, once we know $S21^2$ in terms of omega, we can convert it into Laplace domain that is S domain.

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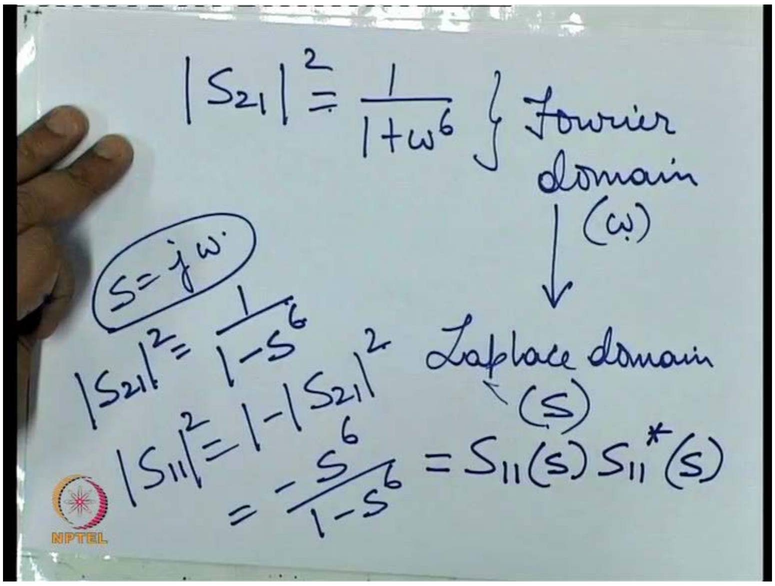

So, to do that we have our $S21^2$ as 1 upon omega 6. Now the usual or the most common way to convert from the…, So this is in the Fourier domain, if I have to convert it to the Laplace domain that is in, so this is an omega, Fourier domain but Laplace domain is in terms of S. So, usual transformations that are used is S equal to J omega. And I am substituting this value here, what I get is… Now we should know that this is not always possible, we cannot always have conversion from the for a domain to the Laplace domain using this transformation.

For example there are some functions like the unit step functions which does not have Fourier transform but has a Laplace transform. But anyway, we are given that our function, our particular transfer function or our particular insertion loss function has both the Fourier transform as well as the Laplace transform. So, once we do this, $S21^2$ comes out to like this and if we assume that a circuit is lost less, then we know how to find out $S11^2$, it will be 1 - S21 whole square and this will be equal to - S raised to 6 upon 1 - S raised to 6. Now this S 11 square, modular square, S 11 modular square can also be written as S11 S Times S11 conjugate of S.

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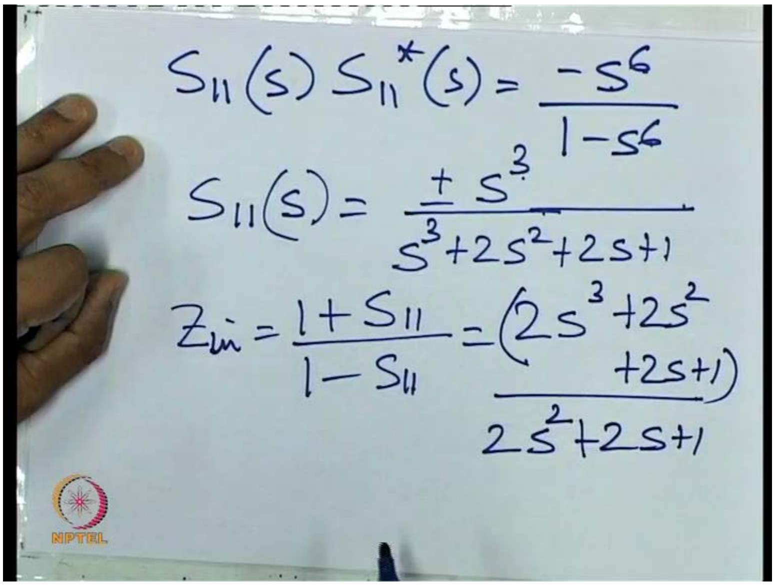

So, with this in mind, we can find out what are the, we can find out the value for this S 11 transfer function. Now to find out S 11, let me just write it once again. S11 conjugate S, so this is what we had found out. If we find out the roots of this denominator, then there are 6 roots and if we select only those roots that provide the stable, that is those roots that lie on the left half of the S plane. Then after taking only those roots, we find out S11 S to be equal to… Here I can have the numerator to be positive as well as negative.

And we shall see that we have 2 different realisations for positive values of numerator and negative values of numerator. So, then from here, I can find out Zin as equal to and this comes out to be.

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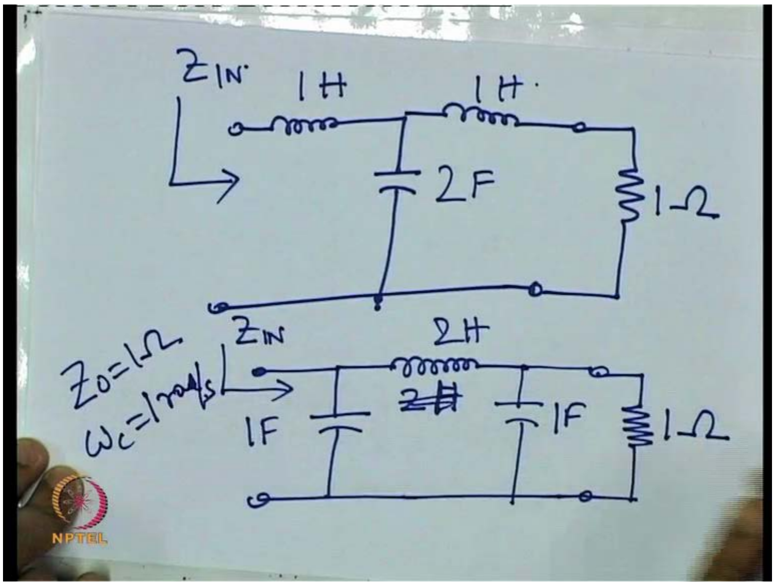

Now if I were to realise this Zin, then the kind of circuit that I get is like this. This is my Zin, had I taken the negative value in the numerator, then the realisation would be… In fact it can be shown that for any Nth order transfer function or any Nth order Butterworth prototype transfer function that we take, the values of these inductors and capacitors, whether for the T implementation or the pie implementation, provided we take Z0 as 1 ohms and omega C1 radians per second.

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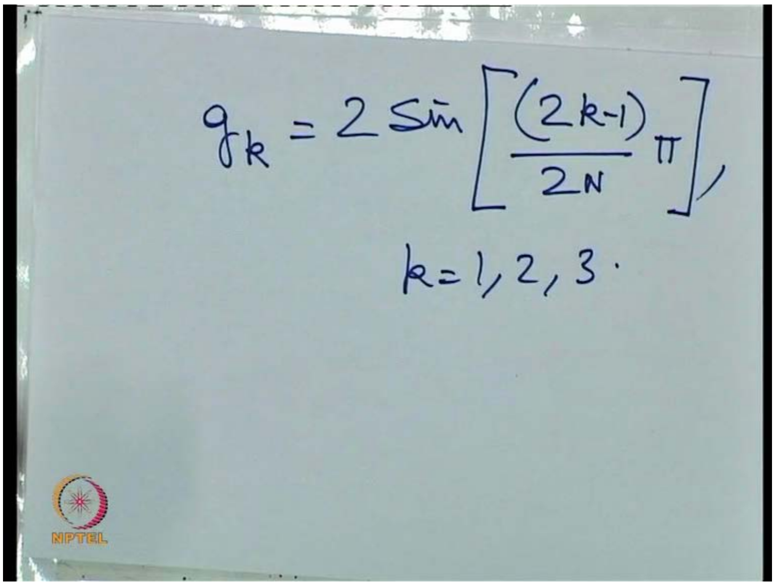

For any Nth order transfer function, the values of these inductors and capacitors will be given by this general formula GK. Where GK is like this one. For example if I take this prototype, I can call this as G1, this as G2, this as G3. So, it is like this. Now one example of this Butterworth prototype is that it automatically gives you at the output impedance as we saw, for example whether for the T equivalent or pie equivalent, we saw that the output impedance is 1 ohms.

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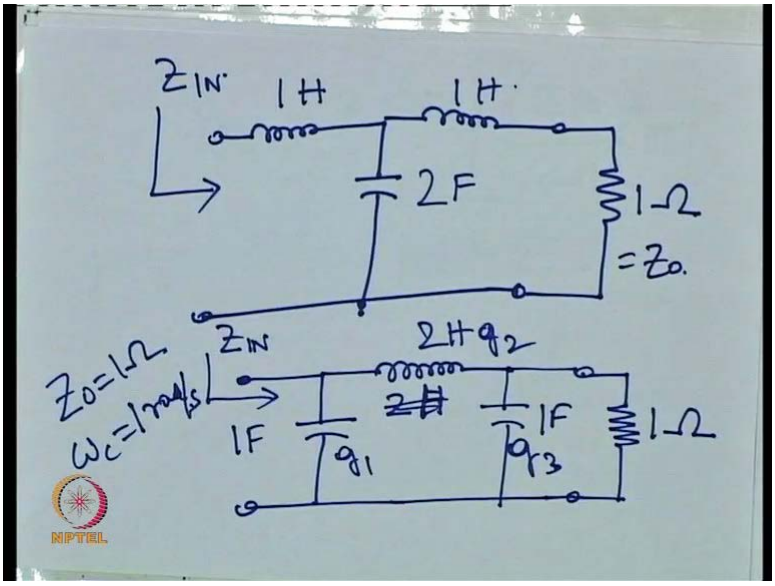

So, that is also equal to Z0. So, in other words, Butterworth prototype functions are auto matched.

But if we, you know, we can have a similar implementation for this Chebyshev function as well. Chebyshev prototype function as well. And there we will observe that at the output, we do not get 1 ohms impedance. So, the outputs are not matched and some kind of matching structures have to be present at the output to achieve this. At the beginning, I have said that we can do a transformation from lowpass to high pass prototypes. So, if we go to the slides on the monitor, these are some of the ways we can do it actually.

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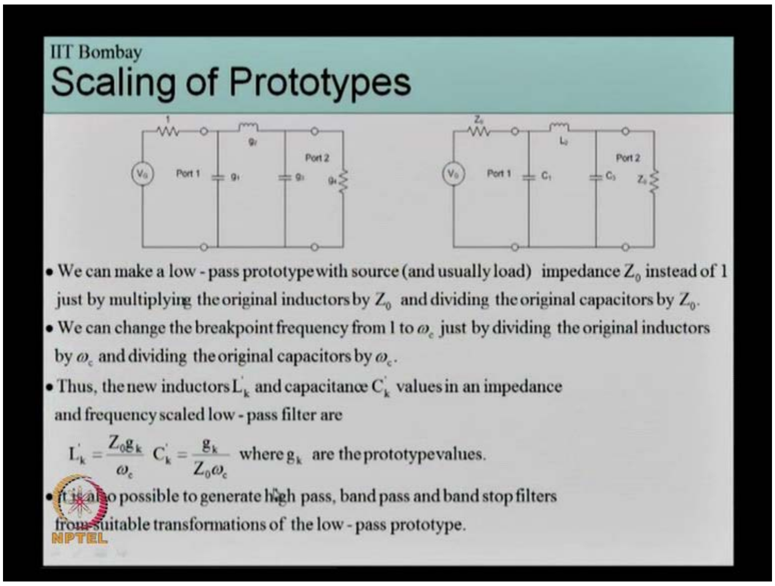

This is a lowpass prototype where the pie or… In this case we have pie equivalent. Now suppose we want to achieve, this was a prototype with 1 ohms value of Z0 and 1 radians per second value of omega C. Now how to go from here to the circuit where the cut-off frequency is no longer 1 radians per second and the input and output matching impedances are no longer equal to 50ohms. No longer equal to 1 ohms, I beg your pardon. So, the transformation, for example G1, G2, G3 are the prototypes with Z0 equal to 1 ohms and omega C equals to 1 radians per second. Then say for any arbitrary value of that 0 and omega C, the values to which these GK should be scaled is given by this equation.

For example, if GK was the prototype inductance, then it should be scaled up to a value of LKdash given by this equation. And similarly if capacitance, if GK was a capacitance value, then it should also be scaled according to this value. Now similar to this lowpass to lowpass transformation, we can use these prototypes to also achieve lowpass to high pass transformation.

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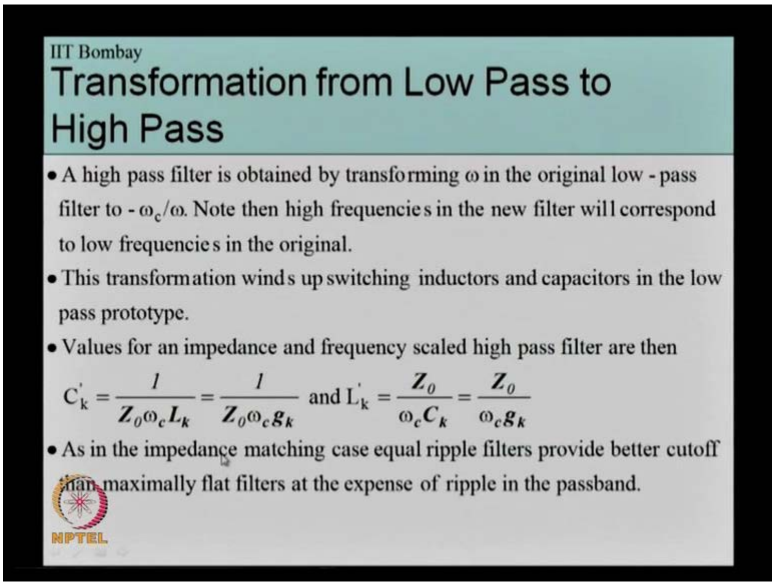

In that case the inductances will be converted to capacitances and capacitances will be converted to inductances as given by this value. So, here GK are the lowpass prototype and CK are the final values for the high pass prototypes with certain Z0 and omega C, they need not be equal to 1 ohms and 1 radians per second.

So, here these formulas directly scale from lowpass prototype to the final high pass filter circuit without going through any high pass prototypes structure.

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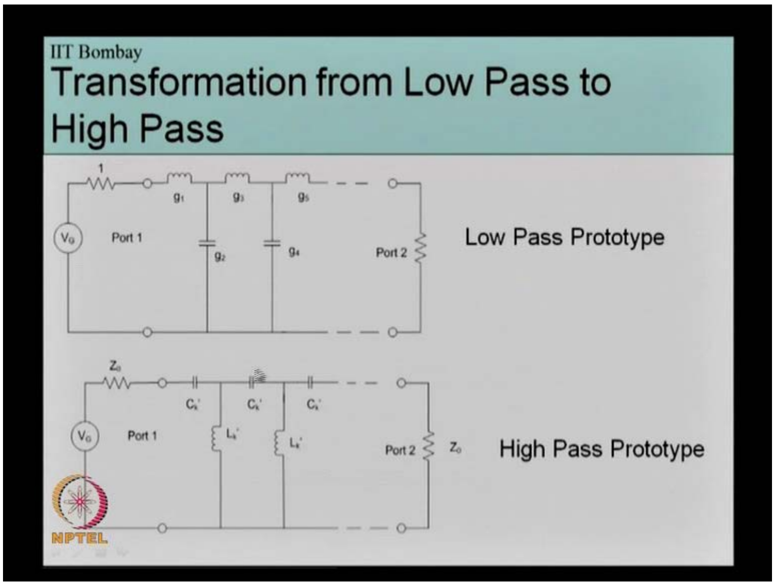

Similarly we can also transform from, so here this is a graph, this shows the transformation, these are the lowpass prototypes and after applying those formulas that are shown in the previous slide, we will get a circuit like this. This is a final high pass circuit.

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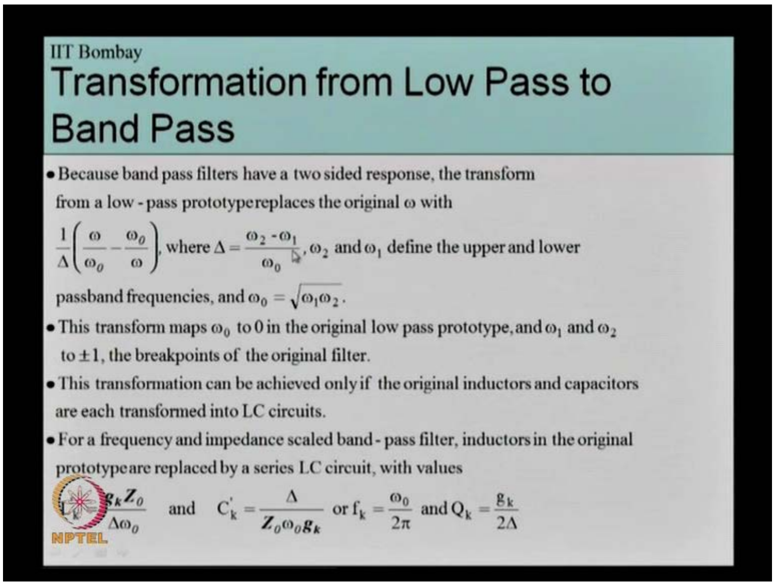

No similar to this transformation from lowpass to high pass, we can also have a transformation from lowpass to bandpass. Now the fractional frequency of, the fractional bandwidth of a bandpass filter is given by this equation where the omega 1 and omega 2 are the 3 dB frequencies. Then as we know for bandpass implementation, we need a series resonator in series or a shunt resonator in shunt. Now, in our low pass prototype, we had seen that we have capacitances in shunt and inductances in series.

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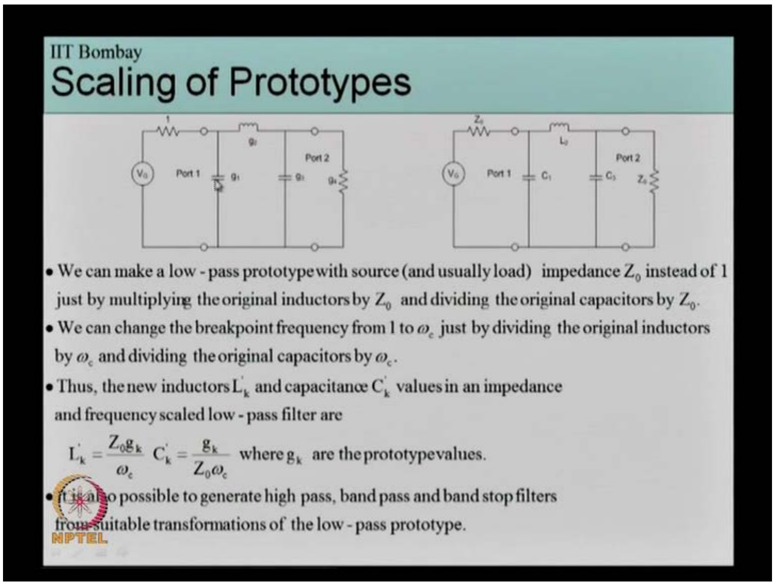

So, to realise a bandpass equivalent, these capacitances should convert to a shunt combination of L and C and these inductances should converted to a series combination of L and C. And the conversion formula is given by this equation.

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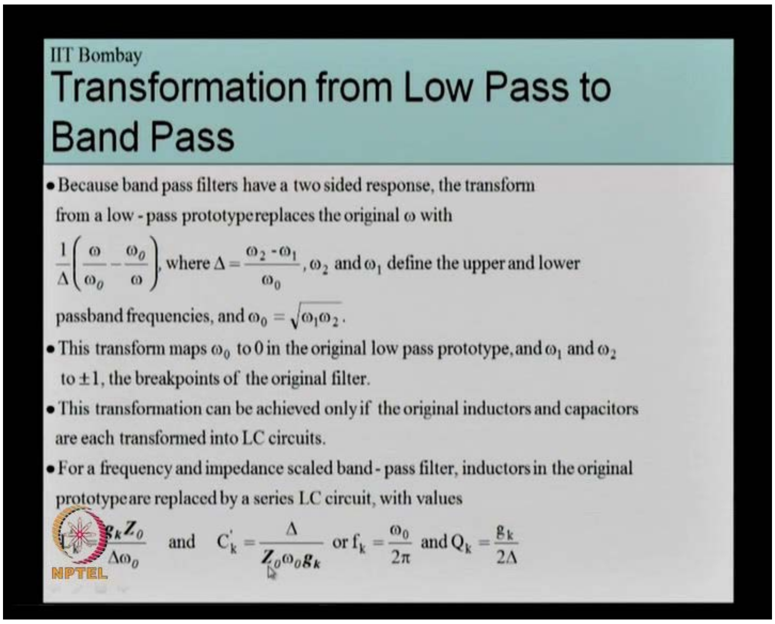

So, GK is our lowpass prototype… If that is an inductor then…