Microwave Integrated Circuits.
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Lecture -24.
Microstrip Matching (contd.), Mason's Rule, Power Gain Equations.

Hello, welcome to another module of this course microwave integrated circuits. In the previous module we had saw how a basic microstrip, how basic matching lumped elements can be done and how using short lengths of high impedance or low impedance microstrip lines they can be used to substitute for inductances and capacitances. But I also mentioned that that is not enough because it may not always be possible to have high impedance or low impedance lines of the desired characteristic impedance. It might not be possible to implement them in a circuit and also the lengths might be too small then that what is feasible.

When we look from a purely transmission line point of view, the matching of circuits is similar to the matching using lumped elements but a little different principles are involved.

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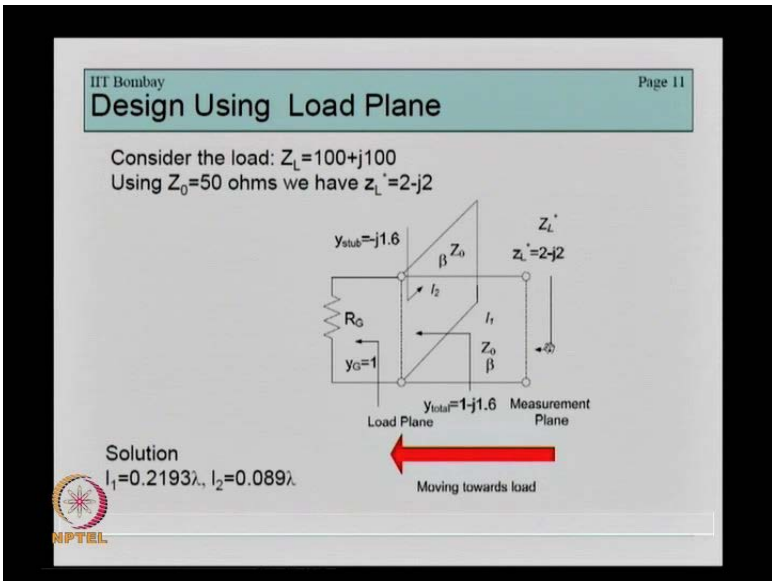

So, if we can go to, if we can go to our slides on the monitor. Suppose we have a load. I think the best way to show this is by example. Suppose we have this load ZL, given by the ZL equal to 2 - J2 and we have to match it to a generator RG where the RG is given by 50 ohms. So, then the normalised value of RG will be equal to 1.

Now how can we do this. If we have already seen using lumped elements in the previous class where we connected either an inductor in series, inductor in shunt or capacitor in series, capacitor in short. But in this example, let us suppose that we want an implementation like this, that 1st we connect the length of a transmission line and then followed up by a shorted stub as shown. Shorted transmission line stub.

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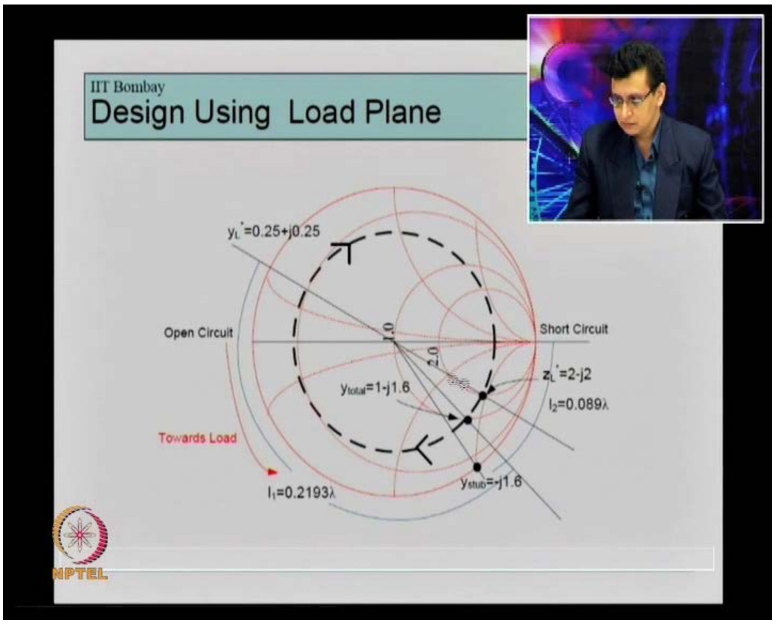

Now 1st let us see how to do this. If we go, this is our Smith chart implementation of the circuit. This is the value of ZL, I beg your pardon, ZL was equal to yah ZL yah, ZL is equal to 2 - J2. This is the value of ZL and we have to bring it to the centre that is 50 ohms.

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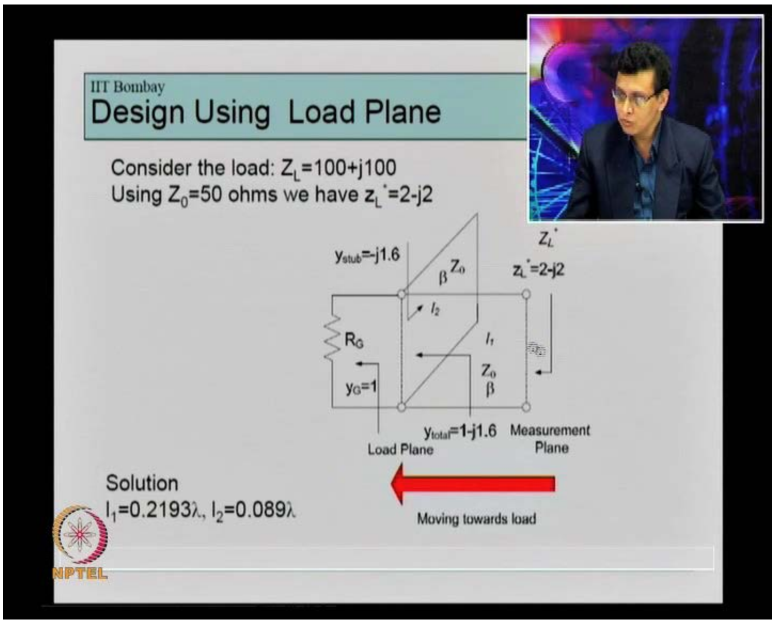

The 1st thing we note that as I had mentioned that when we go along a transmission line, if this is supposed the implementation that we want, that is 1st length of the transmission line followed by a shorted stub. When we move along a transmission section of a transmission line, the magnitude of the reflection coefficient along the transmission line remains constant.

So, if the magnitude of the reflection coefficient remains constant, then this also we had discussed earlier that we are moving along a circle having constant radius.

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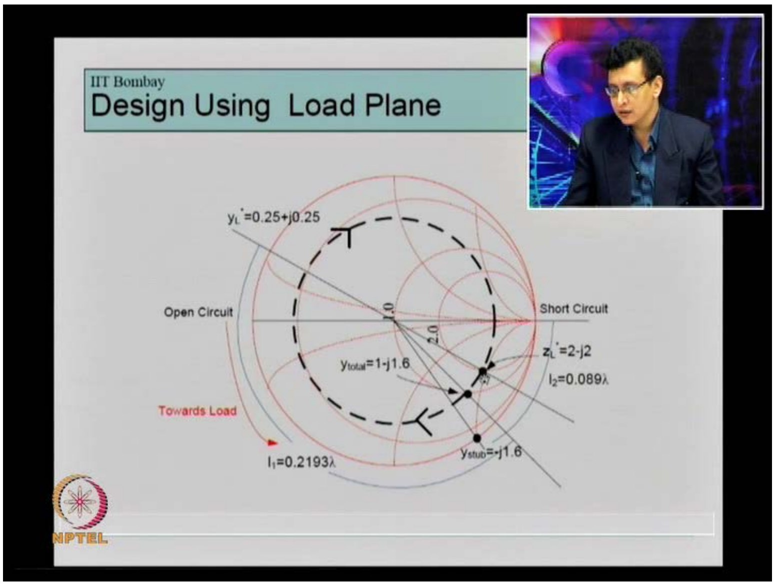

So, if we move along a circle having a constant radius in the clockwise direction, in the clockwise direction because we are moving away from the load, then we are, we can say that we are moving along, we are going away from the load along a transmission line.

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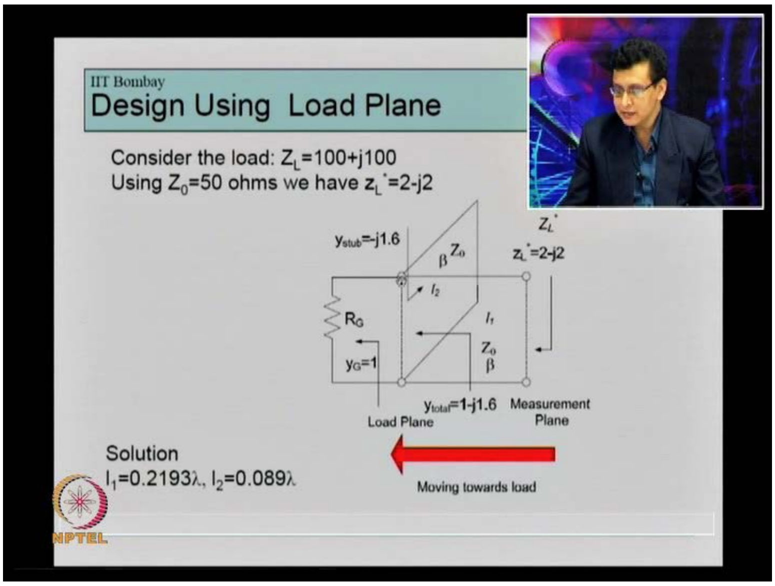

And then at the other end, at this end, whatever we keep moving till the point that we reach the constant resistance circle corresponding to R equal to 1

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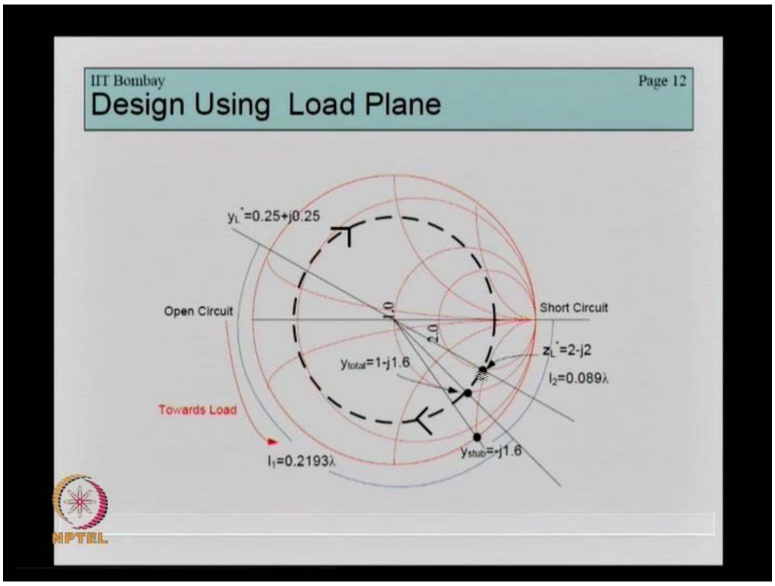

that is this circle and then whatever admittance that we have, we cancel it by connecting a shorted stub in shunt. Now this I have also mentioned that shorted stubs can always be connected in shunt, this we had seen while discussing filters.

But then we have to understand something that if we are connecting a shorted stub in shunt, then would not be more convenient if we are doing, if we are considering the Y Smith chart because in the Y Smith chart, we use the admittance values and when we are cancelling the admittance value by connecting a shorted stub in shunt, we are cancelling the admittance. So, that is why we will be considering the admittance values of this impedance or in other words, we will be plotting this impedance in the Y Smith chart.

Now in the Y Smith chart, if you want to go from the Z Smith chart to the Y Smith chart, we know that we can, all we have to do is we have to take a 180° turn from the point that it corresponds to the Z Smith chart and if you take the 180° turn, you will reach its the its position, the position the corresponding, the position corresponding to the Y Smith chart. So, that is what we have done here.

We have started with ZL equal to 2 - J2 and after taking 180° turn, we reached this point YL equal to 0.5+ J0 .25. So, we are at this point and we have to go to this point, that is the origin and that is same whether we are in the Y Smith chart or whether in the Z Smith chart. So, what we do is we travel along this constant resistance circle from where this dotted line, this black dotted line, till we reach the constant resistance circle, R equal to 1, it intersects at this point. So, this is the length that is required, from this point to this point, this is the length that is required.

Then at this point, our admittance, our conductance value is matched to the origin value about the susceptance still needs to be cancelled and to cancel that, we connect a shorted stub and what should be the length of that shorted stub. The length of the shorted stub corresponds to this value, Ystub equal to - J1 .6. Because this is 1+ J1 .6, we have to cancel that + J1 .6, so we have the stub admittance should be - J1 .6. And since it is a shorted stub, the length of the stub that we choose will be from this point that is the short point on the Y Smith chart to the point corresponding to Y - J1 .6. So, this length is 0.089 and the length from here to here is 0.2193.So, this is how we implement that.

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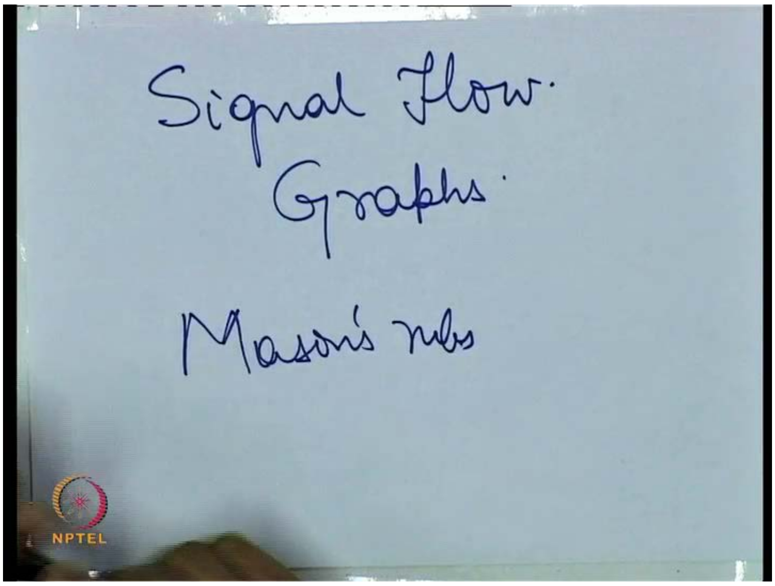

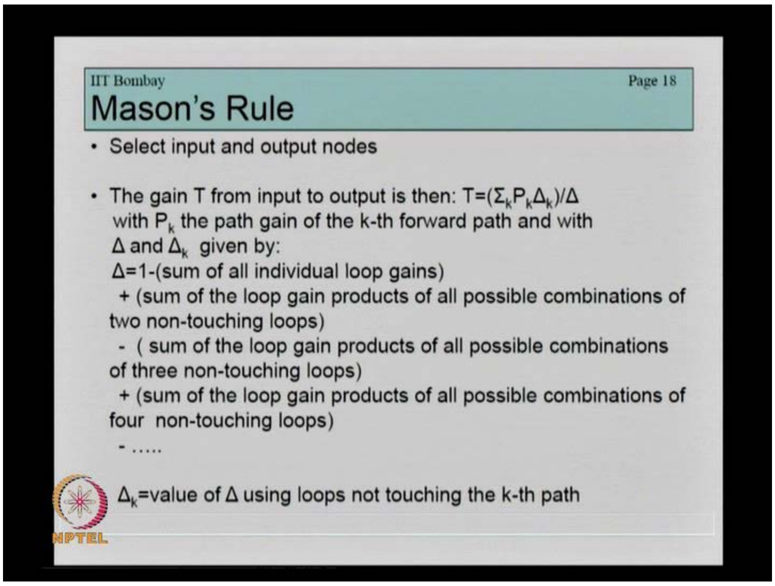

The signal flow graphs are as we have already discussed about signal flow graphs in the previous modules. At the time of discussing I had mentioned that there are some rules, I had also discussed the ways of simplifying the signal flow graphs and in addition to the methods that we had discussed then, there is also something called Mason's rules. So, Mason's rules are somewhat of an easier way of achieving implementation of signal flow graphs and if we go to the slides on the on the monitor then the gain that is…

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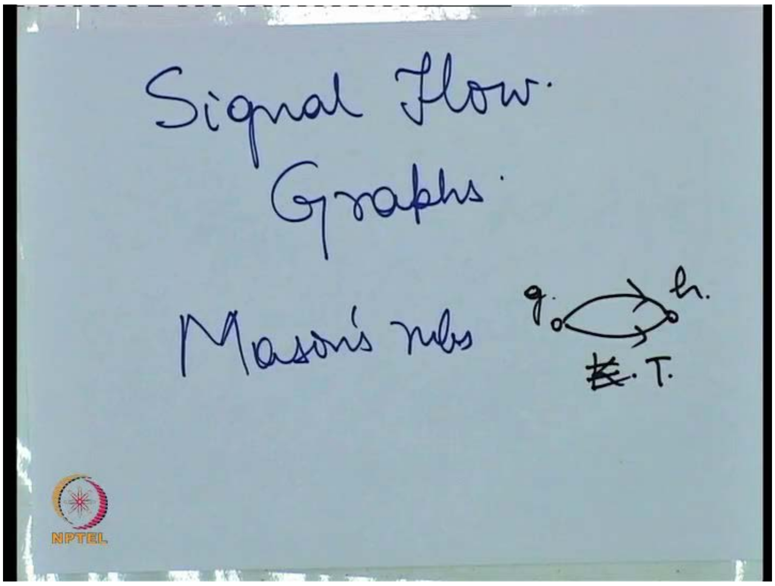

Suppose we have 2 nodes, if we can come back to the slides on the on the writing slides.

Suppose this is a point say G and this is the point say H. And after simplification the gain between G and H is some value K or say T, the gain between G and G to H is T, then and said there are a lot of paths between this node G to H, many complex paths maybe. Then the gain T can be given by a formula this as shown here.

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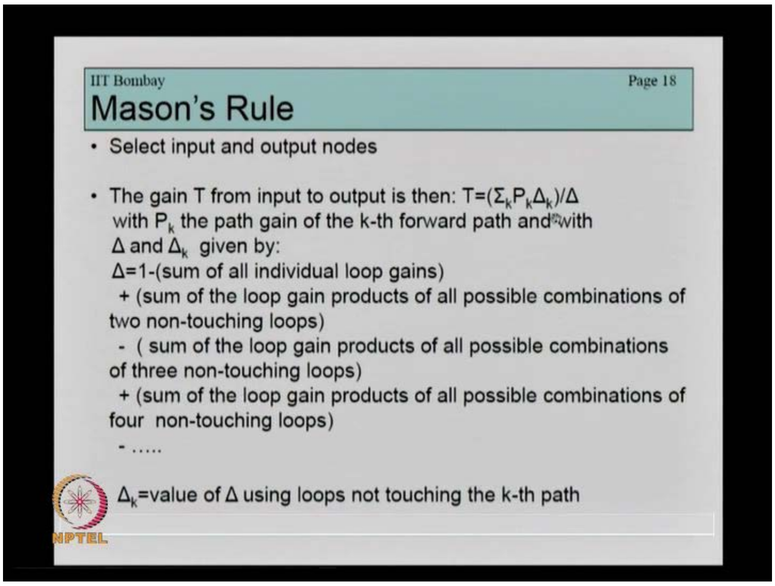

This T will be equal to Sigma K PK Delta K upon delta. Now I will just give the definition of this PK, Delta K and delta and we will take an example to understand what these individual terms mean. PK is what is known as the path again of the Kth path between the nodes, delta is the quantity given by this and delta K is the value of delta using loops not touching the Kth parth.

So, it might not be very clear but let us see an example.

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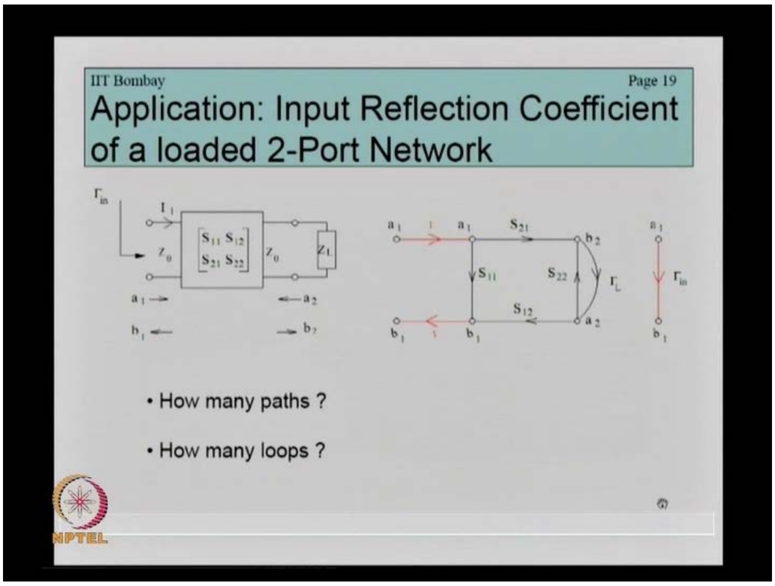

Suppose we have a 2 port network as shown here, S parameters are shown. Then it is corresponding signal flow graphs diagram as we have already seen earlier is given like this and so from this signal flow graphs we have to 1st determine how many paths are there. Suppose we consider the path between, the gain between A1 and B1, then how many paths are there between A1 and B1 and how many loops are there in this signal flow graph. So, 1st we note that this is the path, starting from A1, going like this, this is one path.

Another path exists between A1 and B1 like this. It starts from A1, goes straight and go along gamma L and come back along S12 and then reaching B1. So, these are the 2 paths that we have and in the loop is defined as a, as a path which starts and ends at the same point. For example, this path start from A2, goes all the way to B2 and comes back to A2. So, this is a loop.

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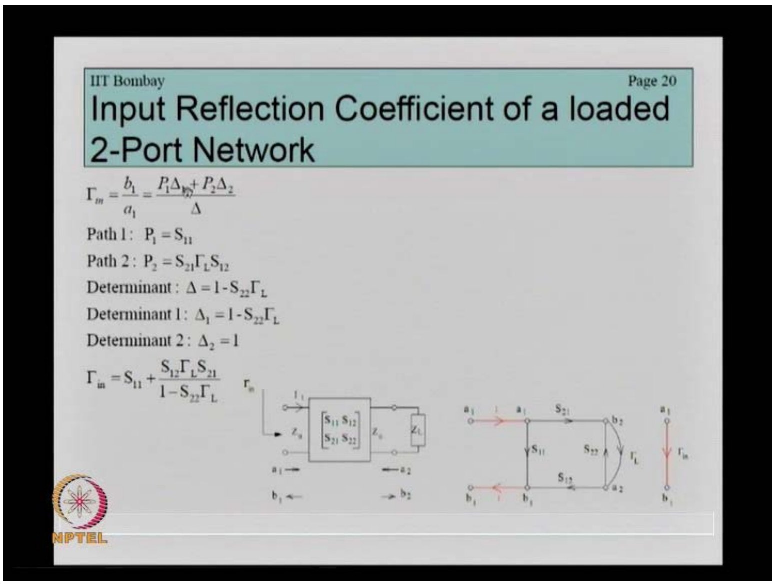

Now, so between A1 and B1, there are 2 paths, hence applying that Mason's rules formula that I just showed earlier, gamma in equal to, (that is the gain between B1 and A1) should be given by this formula.

That is Path 1 multiplied by path 1 gain multiplied by delta 1 + path to gain multiplied by delta 2 upon delta.

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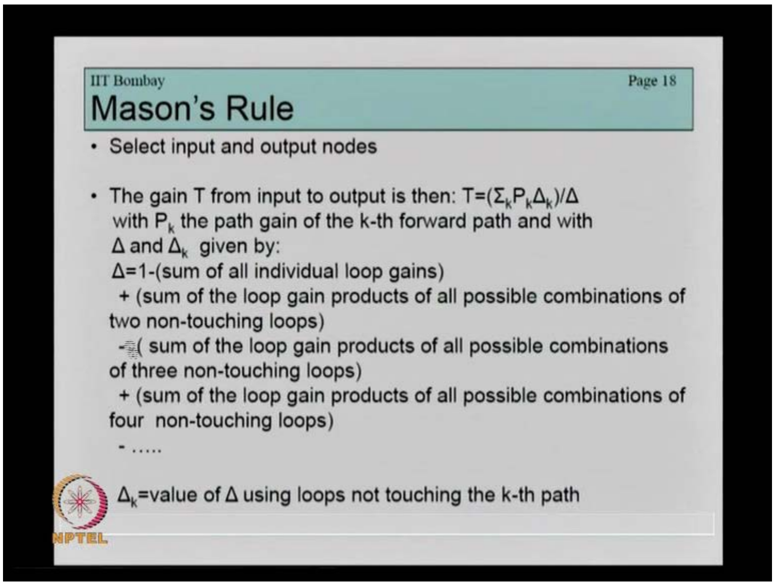

And let us go back to the definition of delta. Delta is 1 - the sum of all individual loops gains. So, if we have one loop, then we will have only one loop gain, if we have 2 loops, then we will have 2 loops games. So, this term within this bracket refers to the sum of all the loops gains, i.e. if there are 2 loops, then the gain of loop 1 + the gain of loop 2 + the gain of look 3. Now this come under this bracket refers to some of the loop gain products of all possible loop, all possible combinations of 2 non-touching loops.

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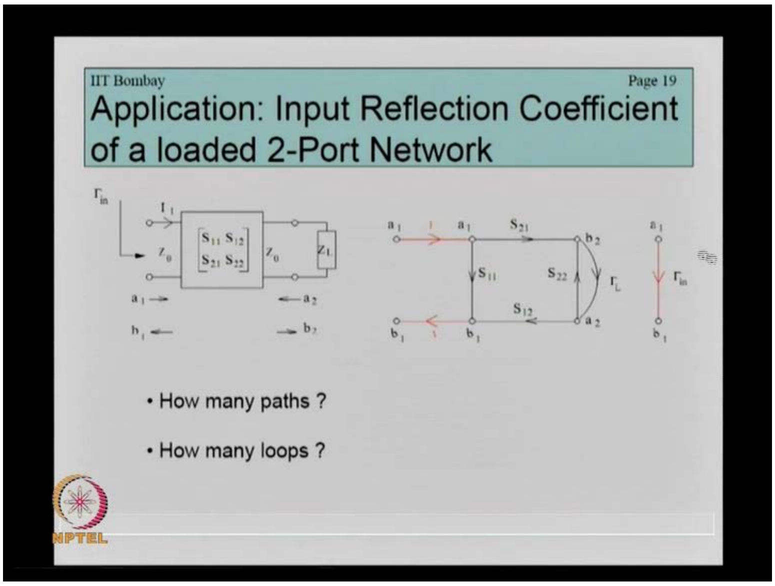

Now imagine this is the signal flow graphs diagram which we were considering earlier. Now consider a case where there is additional another loop. And these 2 loops do not touch each other, so then they become non-teaching loops. And then say there is another loop in some signal flow graphs diagram which also does not the other 2, so we have 3 non-touching loops. So, 3 non-touching loops, if they want to choose any 2 of those 3, we can do that in 3 ways. That is loops 1 and loop 2 together, loops 1 loop 3 together and loop 2 loop 3 together.

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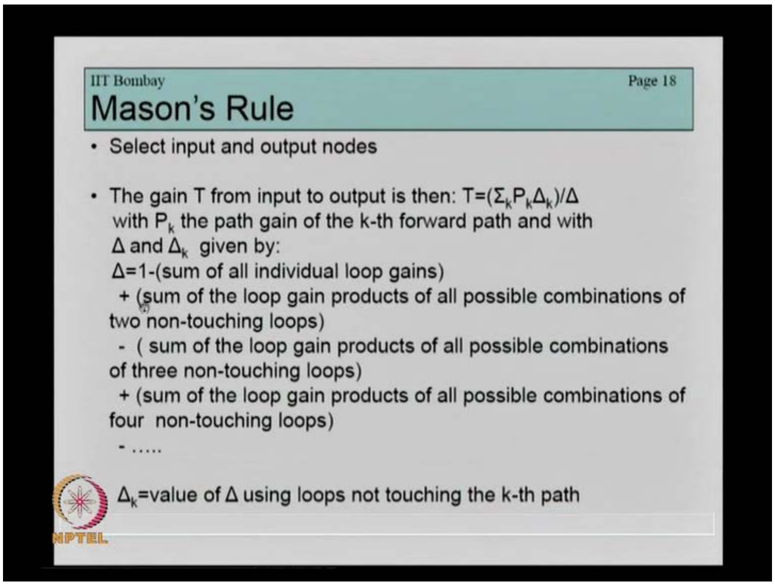

Now this term under this bracket shows the loop gain products of all possible combinations of 2 non-touching loops.

So, if we take to loops at one time, that is say loop 1 and loop 2, if we find out their loop gain products, loops 1 and loop 3, loop 1 and loop 2, loop 2 and loop 3, so we will have 3 product terms, each containing the products of 2 non-touching loops. So, if we have 3 non-touching loops, we will have 3 terms within this bracket. And then this term is an extension of this concept where instead of considering 2 non-touching loops, we will be considering 3 non-touching loops and so on. As long as we have any such combinations, if we say have 5 non-touching loops, then we will have this term as well. If we do not have 5 non-touching loops, then this delta will end here.

And if we do not have more than 2 non-touching loops, then our delta term will end here. And delta K is those… is the value of delta using loops not touching the Kth path. So, if we have any loop that is touching the Kth path, then that loop will not be considered while finding out Delta K. All other loops which do not touch the Kth path will be considered while finding out Delta K. I hope I am clear.

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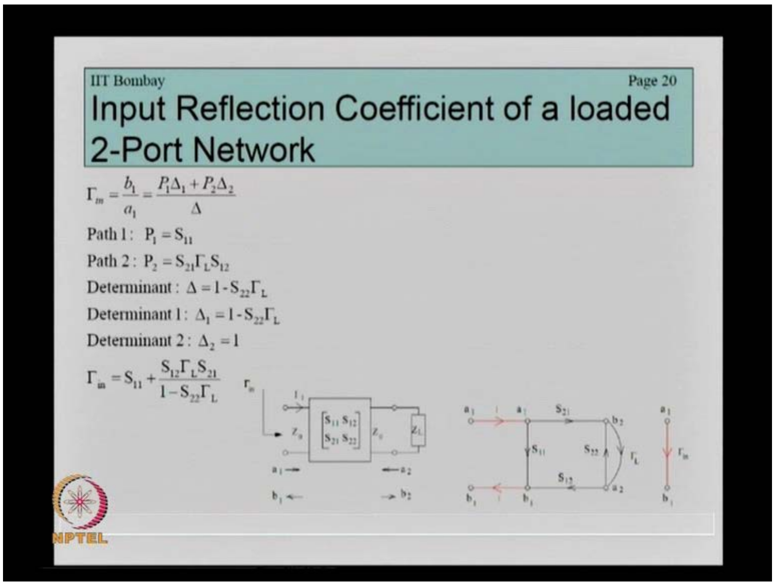

Now then coming back to over 2 port circuit as shown here, the path 1 gain is simply S1, the gain of this path, S1. If we go from A1 to B1 along this path, it is S11, the path gain of path 2, that is starting from A1, going straight, turning and then returning to B1 is S21 multiplied by gamma L multiplied by S12. And then what is delta, how many loops do we have? We have only one loop.

So, then delta will simply be 1 - the low gain product which is S22 gamma L and that is what we have written. Now delta 1, this path does not touch this loop and hence delta one will be same as delta and this path starting from A1 and ending at B1, touches this loop and therefore the only term within delta that will be left using loops that do not touch the path 2, so would not have any loops which do not touch path 2, hence our determinant 2 or delta 2 will simply be 1. So, then now, putting all these values in this equation, if you multiply P1 Times Delta1 + P2 Times Delta 2 and this whole by delta, we get this as the value of gamma L.

I think we had also found out, discussed this formula earlier but then at that time we had done it analytically using equations. Using this Mason’s rule, we can, if we have any given signal flow graphs diagram, we can find out the simplified gain between any 2 nodes without going into those signal flow graph reduction techniques that we have studied earlier. Now let us see some applications of this.

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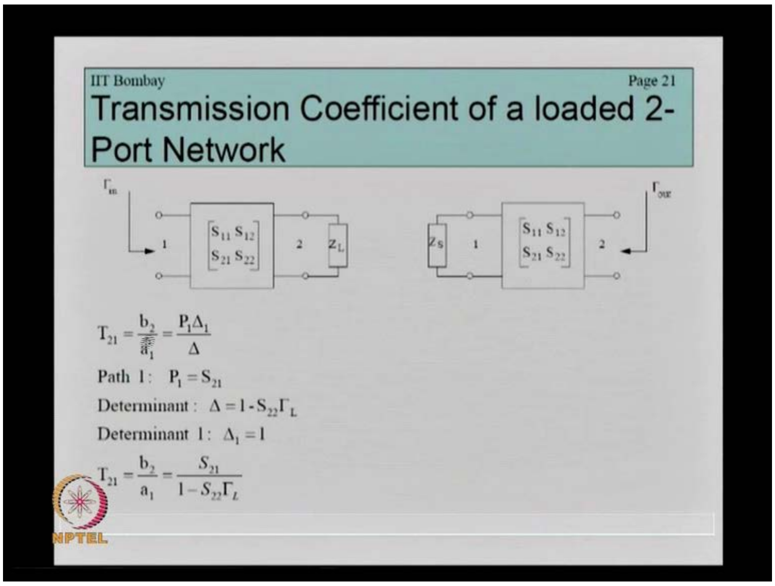

Suppose we want to find out the transmission coefficient, that is T 21 given by B2 upon A1, let us go back to…

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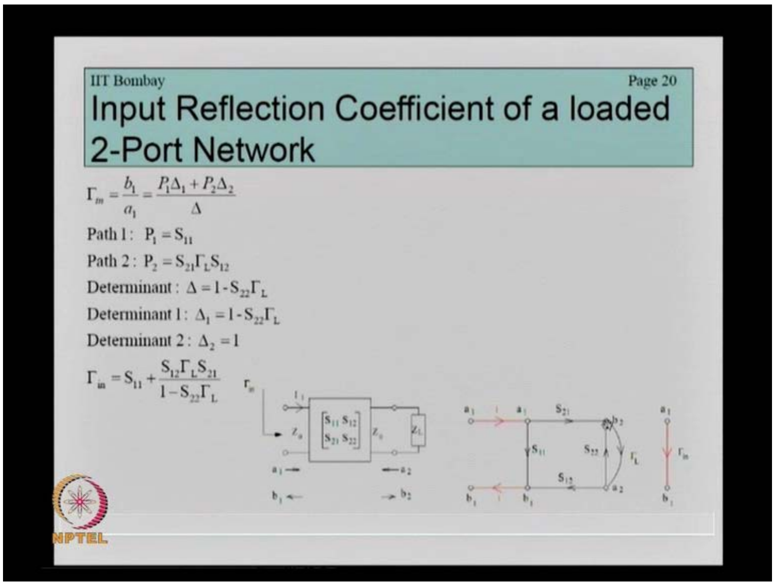

Here we have to find out B2 upon A1. There is only one path between B2 and A1 which is along this path S21 and hence P1 is equal to S 21,

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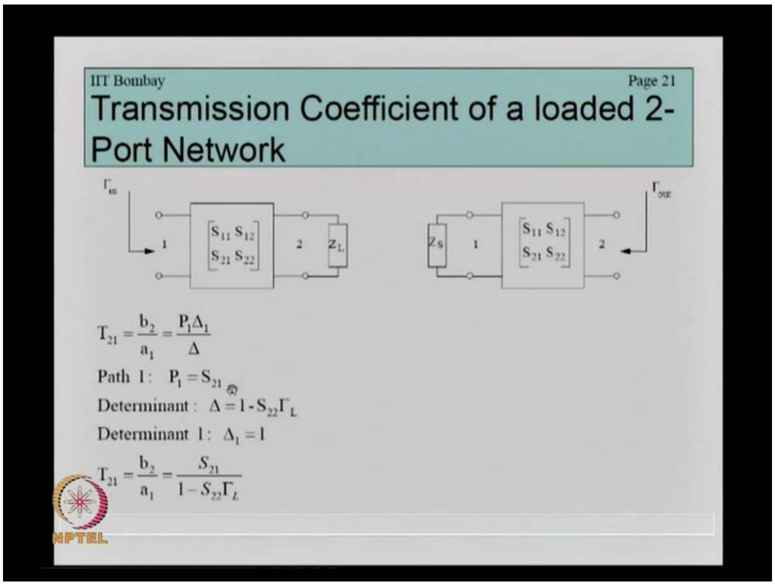

delta remains the same and since this path, P1, it is starting from A1 to B2,

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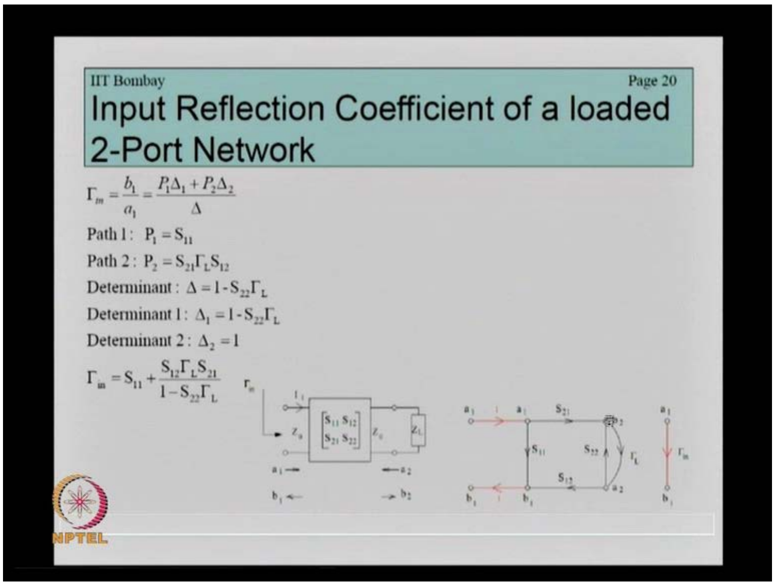

it actually touches this loop and when I say non-touching, it means non-touching at all, including nodes or paths, nothing in this loop including the nodes and paths should touch anything in this inner path in this main path, including the nodes or any sub paths.

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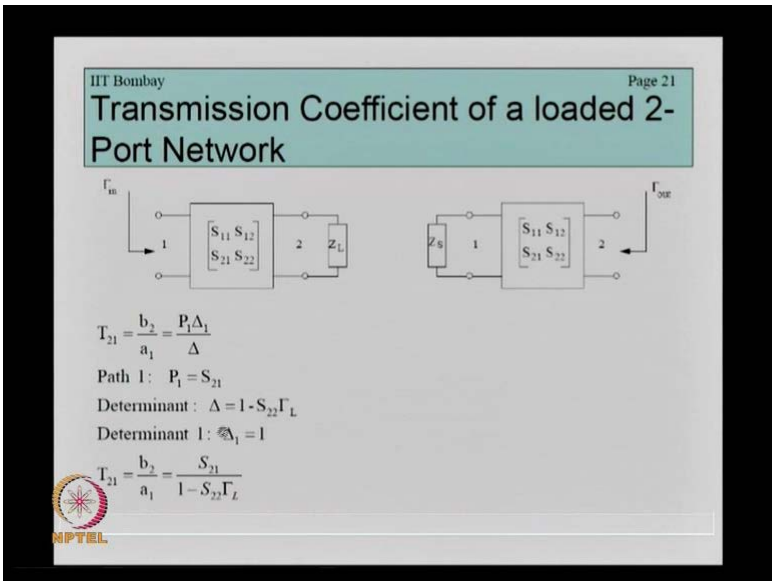

So, then P1 is given by this, delta is given by this and delta 1, that is the loops that do not touch P1, there are no such loops therefore Delta 1 is equal to this. And hence this transmission coefficient is simply P1 delta 1 i.e. S21 multiplied by 1 over delta within 1 - S22 delta.

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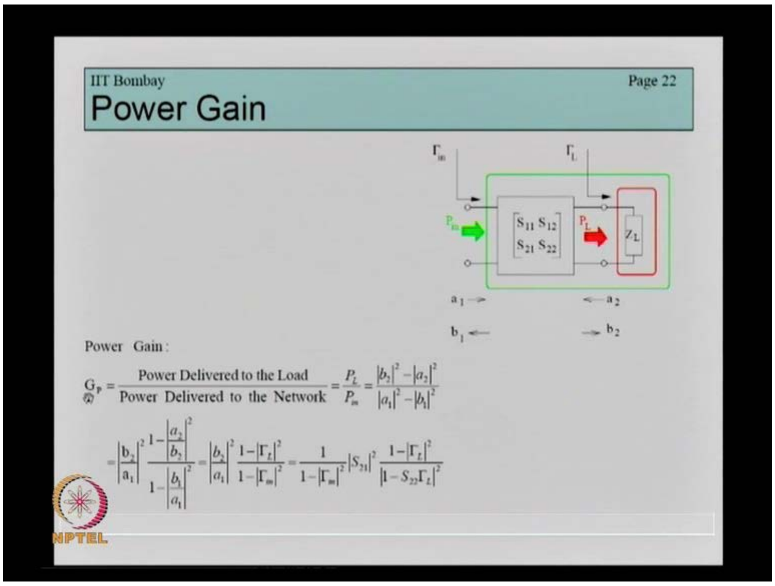

Now, in microwave engineering, we shall of course be studying this in more details. This time, power gain or operating power gain comes across frequently, now what is this power gain? It is like suppose we have 2 port network with a lot connected, then the total power that is actually delivered to the load by actually delivered means actually not… It should not include that component which is reflected from the interface over the power delivered to the network i.e. if we have a source then some part of the power, incident power from the source will be reflected back and some part of the power will enter the network, this network.

So, power delivered to the network refers to that, only that component of people power which actually enters this network and is not reflected. So, that is why GP is given by PL upon Pin and then PL is simply, if suppose B2 is the reflected wave and port 2 at the output port and A2 is the incident wave at Port 2, the output port then we will simply be B2 magnitude square - A2 magnitude square. And P in will simply be A1 magnitude square - B1 magnitude square. And if we simplify this, or if we take B2 common from here and A1 common from here then we can, this equation becomes equal to this equation.

In fact we can do this manually, let us try to establish this relationship.

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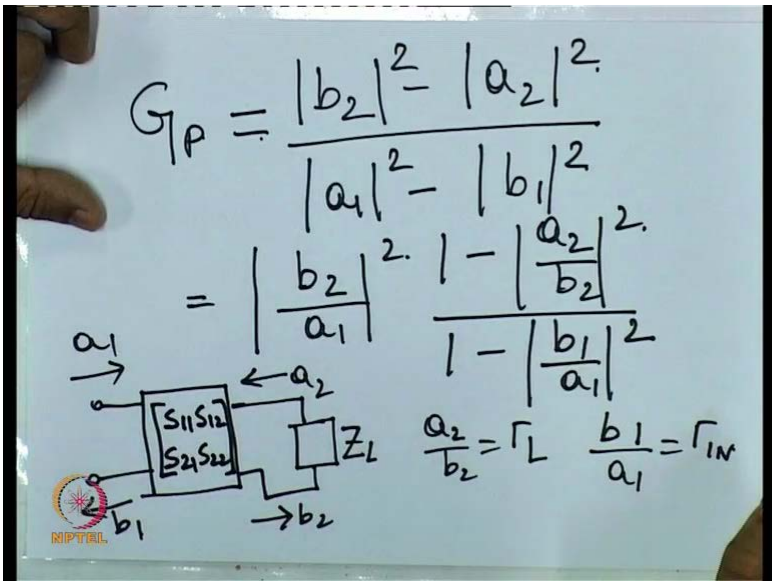

So, we have GP equal to B2 magnitude square - A2 magnitude square upon A1 magnitude square - A1 magnitude square. If I take B2 common here and A1 common here, then this becomes… Now what is this A2 upon B2… we draw a network… This is B2, this is A1 and this is B1. What is this A2 upon B2, A2 upon B2 is nothing but gamma L and B1 upon A1 is nothing but the input reflection coefficient. So, therefore our equation for GP becomes…

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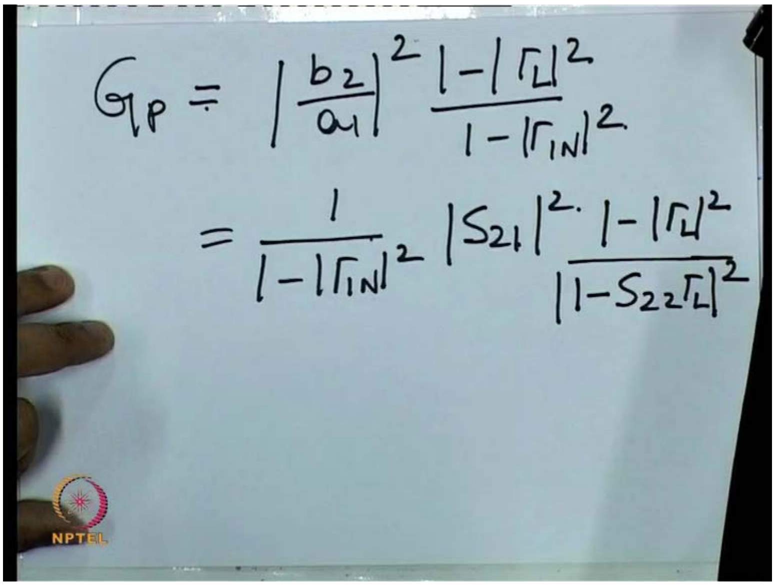

Now we have already just now found out the value of this gamma-in and the transmission coefficient B2 upon A1, so then this simply becomes equal to 1 - gamma-in square S21 square 1 - gamma L magnitude square upon 1 - S 22 gamma L square. So, this is the final value of the GP. So, in summary, signal flow graphs are very, very frequently used as I have mentioned earlier because when we have a large number of ports on any device, it becomes very tedious to actually write all the analytically equations relating incident and reflected waves for all the ports. In fact it becomes easier if we represent the entire microwave device with its various ports using single flow graph.

And then single flow graphs can be analysed using various methods, some of which we have discussion previous modules, the signal flow graph reduction techniques. In this module we have studied somewhat systematic way of analysing the signal flow graph which is called the Mason's rule and I also showed you how we can apply this signal flow graph reduction technique using the Mason's rules to find out the transmission and the reflection, input reflection coefficient of a 2 port network. And then after finding out this transmission coefficient and the input reflection coefficient, we plugged it in the question for the power gain and we derived the expression for the power again.

In the next modules or future models, we will be defining some more power gain terms and we will see that for various situations in a transducer, various definitions of power gain can be used. Some are more practical or more useful but then from the design point of view, other definitions can be useful. And also in this module, we have covered about microstrip matching networks and how to use the Smith chart to opt in the lengths of these microstrip matching networks. Especially I want to emphasise for this microstrip matching networks, please go through the example that I have shown and it is also given in the notes accompanying this module, please go through it to get a better grasp on it.

And to get an even better grasp on it, I believe it will be correct to practice some of these problems, especially those given the books by Gonzales, microwave transistor amplifiers by Gonzales, please see the examples given in the book and also the problems given at the backside of the book. Thank you.