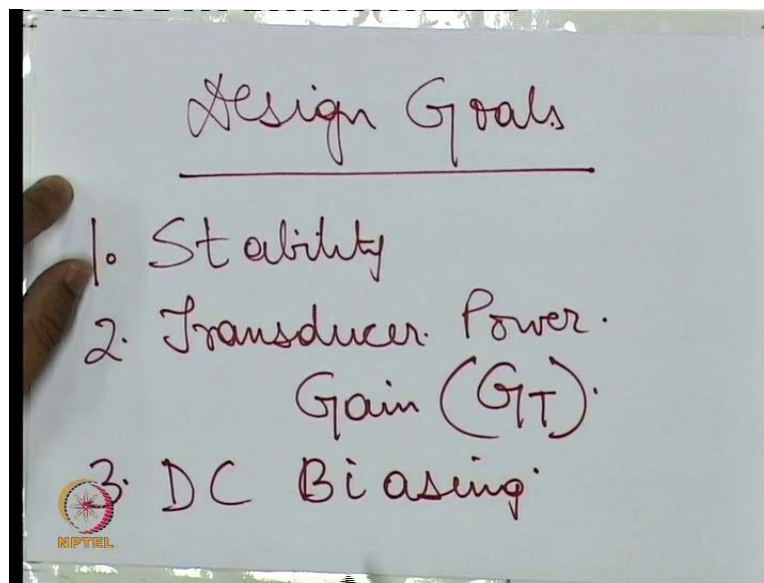


**Microwave Integrated Circuits**  
**Prof. Jayanta Mukherjee**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Bombay**  
**Mod 06, Lec 25**  
**Amplifier gain stability**

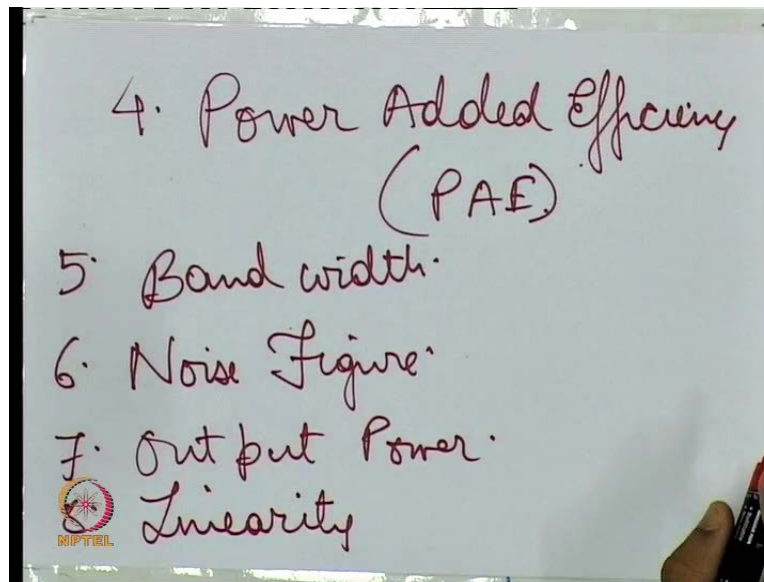
Welcome to another module of this course 'Microwave Integrated circuits'. As you must be must have noted for the past few modules we have been discussing about active circuit design. In the previous modules we were discussing about impedance matching as related to an amplifier especially the input and Output impedance matching. Now, what is the goal of this matching that is the first thing that has to be established? There are various goals while we design an active circuit like an amplifier. Let us list the goals.

[Refer Slide Time: 01:05]



So one first there are various goals that you need to first of all the circuit should be stable. So stability, then something called transducer power gain we shall see in a moment in the next few slides may be, what is this transducer power gain and this is represented by the symbol  $G$ . Then DC biasing that is another important because all active devices need to have a proper DC for the supply for them to work unlike passive devices where they simply work on the input power active, device needs a separate DC wires.

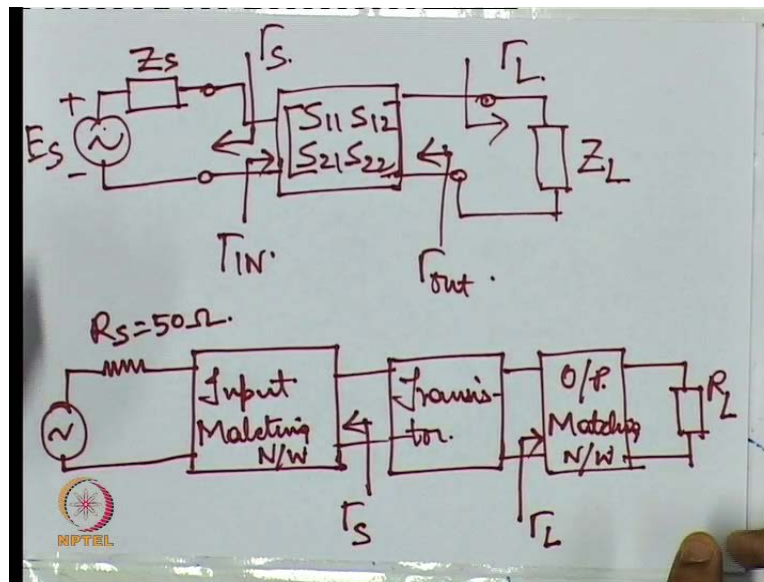
[Refer Slide Time: 02:07]



Then there are other design goals as well like Power Added Efficiency or PAE. Then we have band width. Then we have noise or as we call Noise figure. We shall all we shall take up these aspects one by one when we discuss the amplifier design. Then output power we may because that is also an important figure of merit, how much power not just the efficiency or the band width or how much DC power it takes but also how much output power it gives and finally the linearity.

This is another very important aspect of the amplifier design how linear it is that is how close the input output relationship of the amplifier are towards are to a linear relationship that is output should be proportional to the input. So let us let us take up these things one by one but before going there we would like to define some of the, so let us let us see...

[Refer Slide Time: 03:30]



So we have our amplifier with a certain X parameter matrix then we have a load  $Z_L$  and a source  $E_S$  and as I said earlier the reflection co-efficient of the load and of the source and the load are given by  $\gamma_S$  and  $\gamma_L$  respectively. And the input reflection coefficient and the output reflection coefficient of the amplifier are given  $\gamma_{in}$  and  $\gamma_{out}$  and then I had also mentioned this earlier that for proper input matching we need to place something called input matching network between the source and the amplifier input of the (ampli) of the transistor or the active device that you are working.

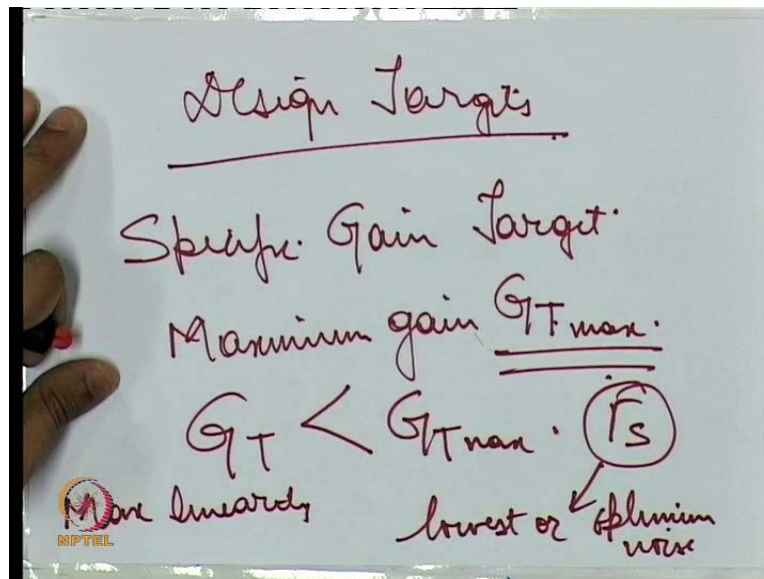
So let's call that transistor and then we also need an output matching network which will be between the output load and the output of the transistor. So this is my  $R_L$  so this is more or less the design you know this input matching and output matching and what the purpose of this input and output matching networks is to obtain an appropriate  $\gamma_S$  and  $\gamma_L$  that will satisfy the conditions the various you know input matching and output power noise and all those criteria that I mentioned earlier earlier in the lecture today.

So how to select question is how to what should we select because there are so many design codes so you know one value of  $\gamma_S$  might give you impedance matching one another value might give you output power added efficiency. How do you select optimum  $\gamma_S$  and  $\gamma_L$  and the and mind you when I say how do you select since you know one value of

gamma S or one value of gamma L may not be able to satisfy all the design parameters it may be that if I have a single goal say stability.

Then there may be more than one values of gamma S or gamma L that can satisfy the stability criteria if I have more than one values of gamma S and gamma L for satisfying the stability criteria then that allow gives us some allowance in choosing the gamma S and gamma L to satisfy the other criteria like the noise or say the gain...

[Refer Slide Time: 07:20]

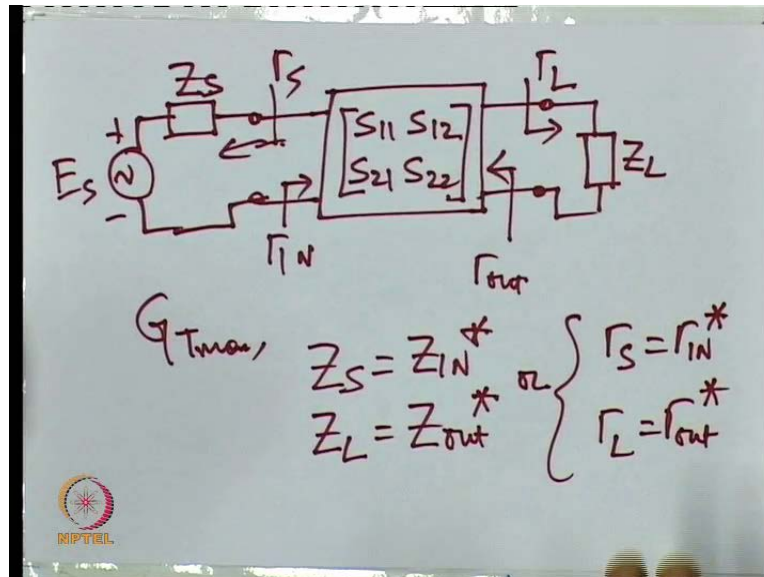


So say the design targets that we have specific gain target say maximum gain  $G_T$  there might be a maximum gain for  $G_T$  possibly again let if you are not clear what is  $G_T$  is for now just take it that this is the most correct definition of power gain in an amplifier the most useful value of power gain even though it is the most useful it may not be the easiest to compute or easiest to design as we shall see later.

So there might be a maximum gain  $G_{Tmax}$  and all other gain will be lesser than  $G_{Tmax}$  other possible is that say a particular gamma S which gives lowest noise or optimum noise as we call it or say it might give the maximum linearity. So let us first concentrate we'll see how we can achieve all these targets this noise stability and gain but first let us suppose we are concentrating on the gain aspect so if I want to achieve the maximum gain then what should it be?

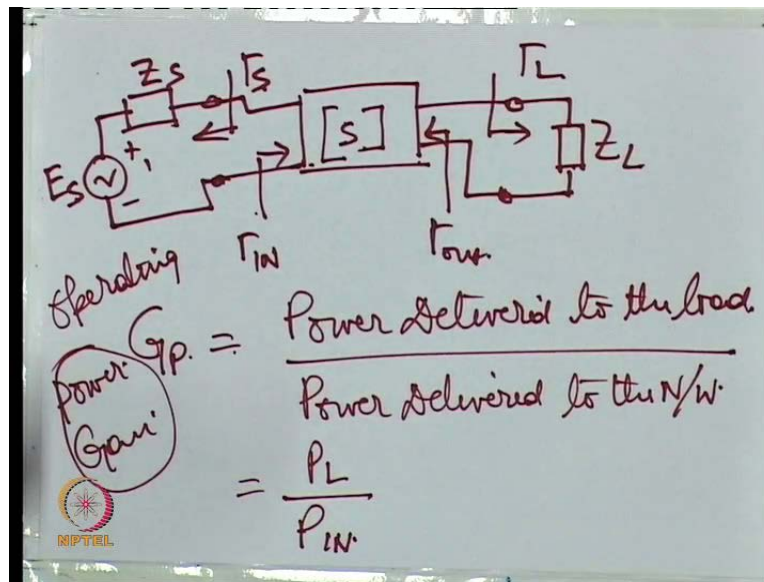
Now the preliminary answer can be given by common knowledge which is that gamma S should be the conjugate of gamma IN and gamma L should be the conjugate of gamma OUT.

[Refer Slide Time: 09:30]



In other words suppose we have an amplifier like this so preliminary answer for maximum transducer power gain  $G_{Tmax}$  will be  $Z_S$  should be equal to  $Z_{IN}$  star and  $Z_L$  should be equal to  $Z_{OUT}$  star or gamma S should be equal to gamma IN star and gamma L should be equal to gamma OUT star that is the preliminary answer based on this not based on this as I said I had said earlier that I would give you a number of definitions about these power gains so let us now one one one by one see the various definitions.

[Refer Slide Time: 10:50]

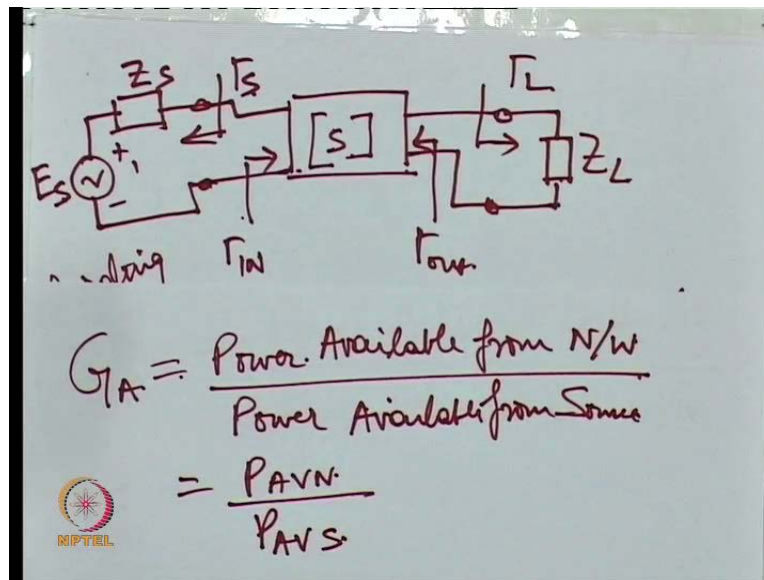


First so again let me just draw my amplifier once again the first type of power definition is GP this is quite simply known as power gain or alternatively as operating power gain. I can call this as operating power gain or simply power gain GP is equal to power delivered to the load upon power delivered to the network. So what it means is that this numerator here, I can write it like this also that is  $P_L$  upon  $P_{IN}$  so  $P_L$  is the total power reaching the load ok that is the incident power minus the reflected power.

The net power that is reaching the load and  $P_{IN}$  is the net power that is reaching the network it means that at the input of this network or at the input of this amplifier. I have an incident power and a reflected power that incident power minus the reflected power is the net power that is going inside the network and that is represented by  $P_{IN}$  so this is the definition of GP.

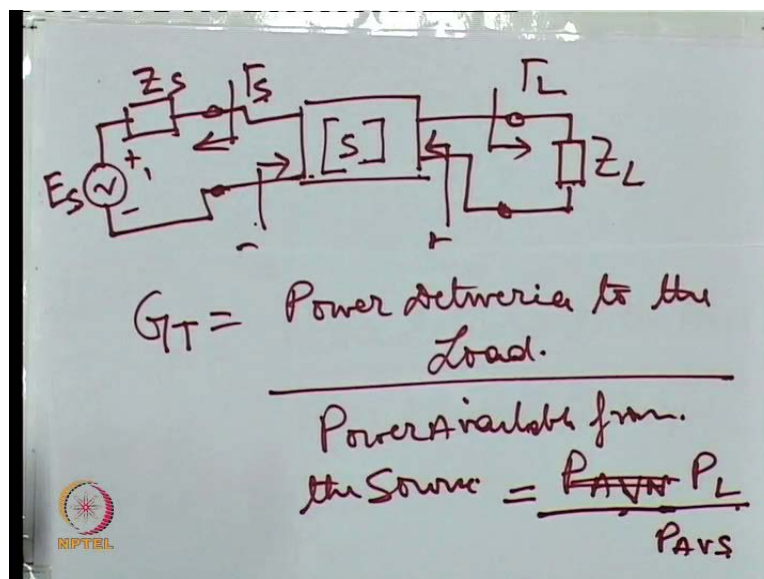


[Refer Slide Time: 13:20]



Then I have another definition which is called  $G_A$  which is so this is my once again this is my network in the top  $G_A$  is given as power available from the network upon power available from source so this is equal to  $P_{AVN}$  upon  $P_{AVS}$ . So  $P_{AVN}$  is the power available from the network it basically means the maximum power that can be supplied by the network to the load. And  $P_{AVS}$  is the power available from the source it basically means the maximum power that can be obtained from the source.

[Refer Slide Time: 14:10]



And finally the GT which I have been mentioning more than once previously is power delivered to the load upon power available from the source. So this is sometimes this is can be written as PAVN, sorry this is PL upon PAVS. And this is the gain actually that matters the most because see gain when we when we mention the term gain it basically means what is the total power delivered to the load which is PL upon the total power that can be supplied to the source that is what we care for.

If we have a source and load then we measure how much power has actually gone to the load and how much power was originally available from the source and that's why this is the most practical and most useful definition of power gain. But then the problem as we shall see when we derive the expression for this GT it contains some terms which make the design of a network with pure GT difficult and therefore we will actually design with GT in mind but with some simplifications we shall see this later.

Now these power gain equations can be obtained from the signal flow graph because these are nothing that for example PL is nothing but the incident power minus the reflected power (( ))(16:06) so these can be analyzed in the same way that we analyzed in the previous module using signal flow graphs. And I I, you can do that as an exercise or you can refer to the book by Gonzalez which is mentioned in this the syllabus for this course the book by Microwave Transistor Amplifiers by Guillermo Gonzalez (( ))(16:33) check the latest edition.

And there you will find the derivations for all these gain terms I am just giving you the final result if you could go to the slides on the monitor.



[Refer Slide Time: 17:01]

IIT Bombay Page 7

### Power Gain Equations

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2} = G_T(S, \Gamma_s, \Gamma_L)$$

$$G_P = \frac{1}{1 - |\Gamma_m|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = G_P(S, \Gamma_L)$$

$$G_A = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{in}|^2} = G_A(S, \Gamma_s)$$

Block diagram showing a Transistor with an Input matching network and an Output matching network. The input matching network is connected to a source with resistance  $R_s = 50\Omega$  and reflection coefficient  $\Gamma_s$ . The output matching network is connected to a load with resistance  $R_L = 50\Omega$  and reflection coefficient  $\Gamma_L$ . The reflection coefficient at the input of the transistor is  $\Gamma_m$  and at the output is  $\Gamma_{out}$ .

$$\Gamma_m = S_{11} + \frac{S_{12}\Gamma_s S_{21}}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{21}\Gamma_s S_{12}}{1 - S_{11}\Gamma_s}$$

NPTEL

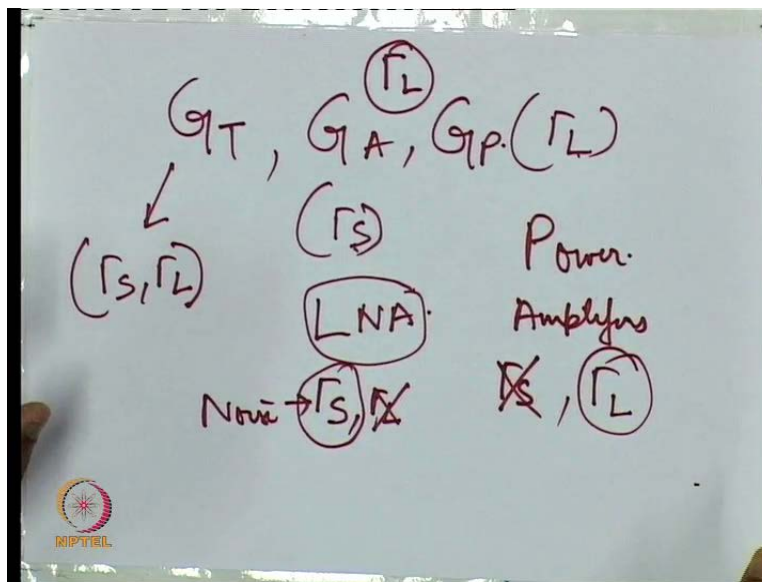
So this GT is given by this expression GP is given by this expression and GA or the available power gain is given by this expression Now you might ask me what is the problem with GT see GT if you notice carefully it has it is a function of this gamma OUT this term But then gamma OUT itself is a function of gamma S isn't it here for example here I have written the expression for gamma OUT as we can see gamma OUT is a function of gamma S. So suppose we design we have a given suppose we have been given some value of GT that we have to achieve then we design our gamma L and then we design our gamma S fine.

Now once I design gamma S I fix my gamma OUT if I fix my gamma OUT then notice here I also have to satisfy the condition that gamma OUT should be equal to the conjugate of gamma L for maximum power transfer that is not possible I if I fix gamma S then I am fixing gamma OUT and then I have to fix gamma L based on the value of gamma OUT I can't independently fix gamma S and gamma L. That is the main problem of this GT expression.

Now there is an alternative expression for GT where I have instead of gamma OUT I have gamma IN and instead of gamma L I have gamma S and instead of gamma S I have gamma L, an analogous expression in terms of gamma IN there also if we fix gamma IN then we are fixing (gam) we are fixing RL or this impedance we are fixing and once we fix gamma IN we have to for matching we have to have gamma S equal to conjugate of gamma IN.

On the other hand the expression for GP, you see that GP is purely a function of gamma IN and gamma L. Now gamma IN itself is a function of gamma L therefore this GP is entirely a function of gamma L. And since it is purely a function of gamma L we have no restrictions on gamma S or RS. Similarly is the case for GA where just the converse of this GP takes place that is this is an expression in terms of gamma OUT gamma OUT depends on gamma S, since only gamma S and gamma OUT is present in the expression for GA this is GA is purely a function of gamma S and therefore we have no restrictions on RL. So we can independently fix RL and ZS.

[Refer Slide Time: 20:30]



So we saw that we have 3 terms  $G_A$   $G_T$   $G_A$  and  $G_P$  we saw that  $G_P$  is purely a function of gamma L  $G_A$  is purely a function of gamma S and  $G_T$  is function of both gamma S and gamma L. In fact  $G_A$  is suitable for designing those amplifiers whose source impedance is already fixed for example low noise amplifiers and  $G_P$  is, I beg your pardon  $G_A$  is useful for designing those amplifiers whose load impedance is fixed but we are free to modify gamma S to obtain appropriate design parameters.

For example in low noise amplifiers noise depends mostly on gamma S rather than gamma L so noise depends mostly on gamma S. Here the gamma L that is the Output impedance of the amplifier is already fixed but to (con) obtain the optimum noise figure we can modify gamma S. On the other hand if we consider power amplifiers for these amplifiers the source is already fixed

so we can't change anything in the source gamma S but gamma L can be designed to obtain the appropriate power gain.

[Refer Slide Time: 22:18]

Unilateral Case.

$S_{12} = 0$

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{41}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

~~$|1 - \Gamma_{IN}\Gamma_S|^2$~~

$$= G_S \times G_0 \times G_L$$

Now let us consider the case as I said even though  $G_T$  is not useful for designing an amplifier we can consider some special cases one of them is what is known as the unilateral case. Unilateral case means  $S_{12}$  is equal to 0 and this assumption is not that par to S because most amplifiers the reverse transmission coefficient is indeed very low it has to be low otherwise power will keep flowing back we don't want that we want power to always flow from input to Output.

We don't want the power to flow back from the Output to the input so this is a somewhat valid somewhat valid assumption though in most amplifiers it is not so low it is not 0 but it is a low value. But then by making this assumption we get some very it makes our life of the designer much easier, how does it make it easier. Because see now the expression for  $G_T$  becomes 1 minus  $S_{22}$  gamma SL whole square.

As you can see it is now there is no gamma OUT or gamma IN term here it is purely a function of gamma S and gamma L and so these terms are can be considered as gain term. If you want in fact we can write it like this this is equal to  $G_S$  multiplied by  $G_0$  multiplied by  $G_L$  so his  $G_S$  is the source gain gain due to the source network  $S_{21}$  is the gain due to the inherent or this S parameters of the network and  $G_L$  is the gain due to the load network.

[Refer Slide Time: 24:30]

When  $\Gamma_S = S_{11}^*$   
 $\Gamma_L = S_{22}^*$   
 $G_T \rightarrow G_{TU_{max}} \rightarrow \text{unilateral case.}$   
 $= \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$   
 $G_{S_{max}} \quad G_0 \quad G_{L_{max}}$

We see that when gamma S is equal to S11 conjugate and gamma L is equal to S22 conjugate then this GT which is its maximum value which I call GTU max. This U represents unilateral so this comes equal to I upon 1 minus S11 whole square, S21 whole square. So this is GS max and G Smax is equal to this value and GLmax is equal to this value.

[Refer Slide Time: 26:01]

Unilateral Case.  
 $S_{12} = 0$   
 $G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$   
 $= G_S \times G_0 \times G_L$

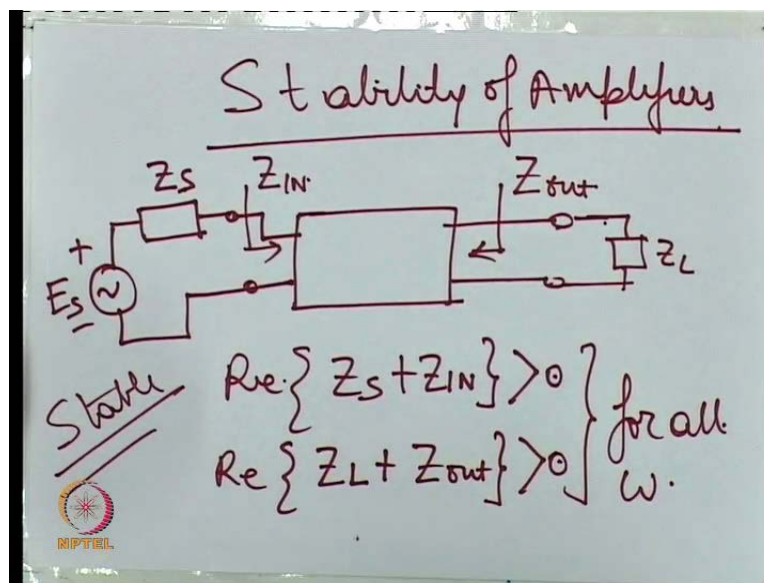
So this is the unilateral design case and as we shall see later when we discuss about gain circles that we the locus of a (lo) of (G) of gamma S or gamma L which gives a constant value of this GS or GL will be circles so this is the equation that we just discussed GTU or unilateral gain and

the locus of gamma S that gives a constant value of this GS is the circles similarly the locus of gamma L which gives the constant value of this GL will also be a circle. We shall discuss this when we are when we discuss about the amplifier gain circles.

For now let us discuss let us start about the stability aspect of an amplifier so what do you mean by stability a circuit see when I say stability it means that if you give an input the output should be bound in other words for bounded input output should be bound if you give an input and then the output amplifies itself to such an extent that it reaches its saturation value then the circuit is not stable. Similarly if you give an input and output becomes so low that it no longer sustains itself the output that is also an unstable circuit.

So stable circuit will be one that when you (con) give a continuous input to that circuit, the output should also be continuous and stable it should not either glow up neither it should dim down. As you know passive if an element is passive that is if there is some real resistance to it if the real part of its impedance is positive, then such a device will always be stable because such a device is a passive device.

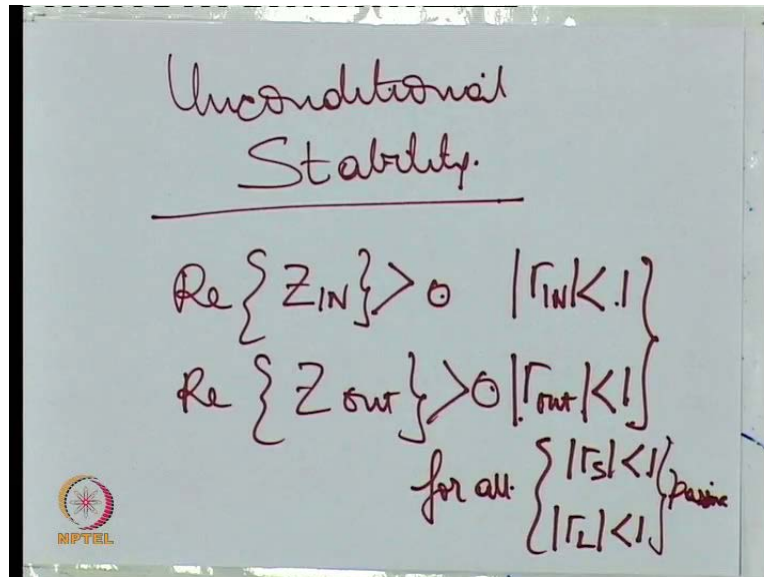
[Refer Slide Time: 28:20]



So then the if we have an say an amplifier with load  $Z_L$  then if we have real part of  $Z_S$  plus  $Z_{IN}$  greater than 0 and real part of  $Z_L$  plus  $Z_{OUT}$  greater than 0 for all frequencies then such a such a (cond) circuit is said to be unconditionally stable or such a.. I beg your pardon such a circuit is said to be stable we shall come to the condition of unconditional stability in a moment.

If these conditions that is you have some  $Z_{IN}$  but then due to  $Z_S$  say  $Z_{IN}$  itself has a negative real part or  $Z_{OUT}$  itself has a negative real part but then on adding  $Z_S$  and  $Z_L$  the real part of the sum of  $Z_S$  and  $Z_{IN}$  becomes 0 becomes greater than 0 then such a circuit is said to be stable How to ensure that a circuit is always stable irrespective of what the value of  $Z_S$  or  $Z_L$  is so such a circuit which is stable irrespective of the value of  $Z_S$  or  $Z_L$  is said to be unconditionally stable.

[Refer Slide Time: 30:15]



So unconditional stability that will be the case when we have the  $Z_{IN}$  itself being passive or  $Z_{OUT}$  itself being passive now this same thing in terms of reflection coefficients is expressed as so for all  $\Gamma_S$  lesser than 1 or  $\Gamma_{OUT}$  modulus  $\Gamma_S$  less or modulus  $\Gamma_L$  lesser than 1 it this basically means for passive values of  $\Gamma_L$  and  $\Gamma_S$ . If  $\Gamma_N$  modulus and  $\Gamma_{OUT}$  modulus is lesser than 1 then the circuit is unconditionally stable unconditionally stable I beg your pardon.



[Refer Slide Time: 31:35]

IIT Bombay Page 14

## Unconditional Stability Criteria

Criteria :

A circuit is unconditionally stable if for all frequencies :


$$K > 1 \quad \text{with} \quad K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{21}||S_{12}|}$$
$$|\Delta| < 1 \quad \text{with} \quad \Delta = S_{11}S_{22} - S_{21}S_{12}$$

Alternate Criteria

A circuit is unconditionally stable if for all frequencies :

$$K > 1$$
$$B_1 > 0 \quad \text{with} \quad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

Proofs are very long. See App B and C in Gonzalez.

 NPTEL

If you turn to the notes on the slides on the monitor mathematically if you are given the S parameters of an amplifier or a transistor and you want to know whether it is unconditionally stable then of course you can find OUT Z IN like I said in the previous slide. But in terms of the S parameters there is a specific there is a specific parameter this K and delta.

If you compute this value of K from this equation and the value of delta from this equation then if K is greater than 1 and the modulus of delta is lesser than 1 then the circuit will be unconditionally stable. So (( ))(32:18) given the S parameters of a circuit you simply evaluate K and delta and check whether K is greater than 1 or delta modulus is lesser than 1 then straight away you can say that the circuit is unconditionally stable.

An alternate criteria is this that the K greater than 1 and B1 greater than 0. Here B1 is given by this equation. Now if you want a proof of these equations it is given again in the book by Guillermo Gonzales, I would recommend you refer to that book that is the primary text for this course for these active devices the passive devices it was Microwave Integrated Circuits by Pozar this part the active circuits part is the one by G Gonzalez.



[Refer Slide Time: 33:35]

IIT Bombay Page 15


### Alternate Unconditional Stability Criteria

Stern Criteria :

A circuit is unconditionally stable if for all frequencies :

$$K > 1 \quad \text{with} \quad K = \frac{2\operatorname{Re}\{\gamma_{11}\}\operatorname{Re}\{\gamma_{22}\} - \operatorname{Re}\{\gamma_{12}\gamma_{21}\}}{|\gamma_{12}\gamma_{21}|}$$
$$\operatorname{Re}\{\gamma_{11}\} = \rho_{11} > 0$$
$$\operatorname{Re}\{\gamma_{22}\} = \rho_{22} > 0$$

where  $\gamma_{ij}$  can be either  $z_{ij}$ ,  $y_{ij}$  or  $h_{ij}$



An alternate stability criteria is given like this here  $K$  is defined by this term where this gammas would be anyone of  $Z$  parameters or  $(\ )$ (33:27) parameters or hybrid parameters we have not covered hybrid parameters for now you can take it that  $Z$  and  $Y$  parameters this gamma represents  $Z$  and  $Y$  parameters. Now if the real values here of course we have to ensure that the this  $Z$  say your if gamma represents said then  $Z_{11}$  and  $Z_{22}$  have positive real parts. If you simply calculate the value of  $K$  and see that it is greater than 1 then we can say that the circuit is unconditionally stable.

[Refer Slide Time: 34:15]

IIT Bombay
Page 17

## Property of Stern Stability Constant

New y parameters:

$$y'_{11} = y_{11} + Y_1$$

$$y'_{12} = y_{12}$$

$$y'_{21} = y_{21}$$

$$y'_{22} = y_{22} + Y_2$$

For  $Y_1$  and  $Y_2$  pure imaginary we have:

$$K = \frac{2\operatorname{Re}\{y'_{11}\}\operatorname{Re}\{y'_{22}\} - \operatorname{Re}\{y'_{12}y'_{21}\}}{|y'_{12}y'_{21}|} = \frac{2\operatorname{Re}\{y_{11}\}\operatorname{Re}\{y_{22}\} - \operatorname{Re}\{y_{12}y_{21}\}}{|y_{12}y_{21}|} = K$$

$K$  is invariant under lossless shunt loading

A new condition was given in the year 1992 now this condition states that if you simply find OUT this value of  $\mu$  given by this equation and if the value of  $\mu$  is greater than 1 then the circuit is unconditionally stable  $\mu$  also has a dual instead of this equation you can also write it in terms of 1 minus  $S_{22}$  modulus being on the numerator.

So in this module we covered the basic definitions about the various power gains and also we just introduced we just discussed the beginning of stability. Now mind you one thing that it is not necessary always to for the circuit to be unconditionally stable the circuit has to be stable for the operating frequencies or the operating powers that it is designed for. It is not necessary to make it unconditionally stable that is an over because if you spend if you design for unconditional stability then it might lead to loss or gain or increase noise figure which is undesired.

On the other hand say just at the operating frequency if you can make the circuit stable and stable being the condition that  $Z_S$  plus  $Z_{IN}$  or  $Z_{OUT}$  plus  $Z_{IN}$   $Z_{OUT}$  plus  $Z_L$  is passive. If just at the operating point of the circuit you can ensure that the circuit is stable then that is enough and in fact in the next module we shall be discussing how potentially unstable circuit can be made stable.

Thank you.