

**Microwave Integrated Circuits**  
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**Lecture Number 27**  
**Gain Circles**

Hello! Welcome to another module of this course “Microwave Integrated Circuits”. In the previous modules for the past few weeks we had covered the topic of stability and as I had said stability is the very important aspect of amplifier design, and in this module we will be covering about amplifier, about amplifier gain how to design an amplifier with a given gain. So already we have discussed about the various types of gain that we talk about, among them the transducer gain is the most effective; it is the most practical definition of gain.

But then problem with transducer gain is that we cannot design  $\Gamma_S$  and  $\Gamma_L$  independently, hence to get rid of that problem we have some other definitions of gain like operating power of gain and available power gain, and then I had also mentioned for the unilateral case that is when  $S_{12}$  is equal to 0, the definition of the transducer gain can be somewhat simplify. So let us start the discussion today on how to design for the unilateral case where we have  $S_{12}$  is equal to 0.

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Unilateral Case

$$G_{TU} = G_S G_0 G_L$$

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii} \Gamma_i|^2} \geq 0$$

$|S_{ii}| < 1$

$$\begin{bmatrix} i & S & L \\ ii & 11 & 22 \end{bmatrix} \quad 0 < G_i < G_{i_{max}}$$

Now, this  $G_S$  or  $G_L$ , I can write by the general term like this. Now for an unconditionally case stable case we have seen that  $S_{ii}$  modulus should be lesser than 1, this is of course considering that this case where the stability circle is encompassing the center of the switch

now for this case we have SII lesser than 0 and our GI is between 0 and some value GI max.  
 Now if we define a term small GI which is defined as follows...

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$$g_i = \frac{G_i}{G_{i\max}} \quad G_{i\max} = \frac{1}{1 - |S_{ii}|^2}$$

$$G_i = g_i G_{i\max} \quad 0 \ \& \ S_{ii}^*$$

$$0 < g_i \leq 1 \quad G_i = 1, \Gamma_i = 0$$

$\Gamma_i \rightarrow$  which give a const  $G_i$

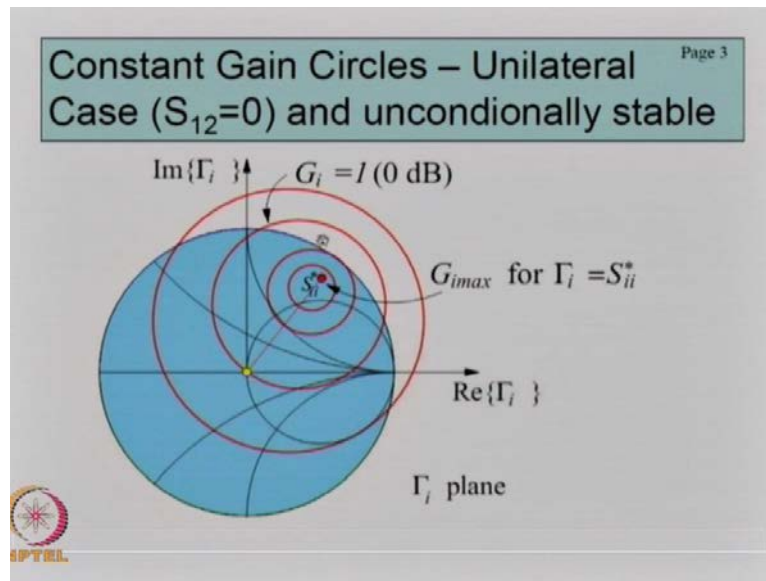
$$|\Gamma_i - C_i| = R_i, \quad C_i = \frac{g_i S_{ii}^*}{1 - |S_{ii}|^2 (1 - g_i)}$$

$$R_i = \frac{\sqrt{1 - g_i} (1 - |S_{ii}|^2)}{1 - |S_{ii}|^2 (1 - g_i)}$$

Say capital GI is the gain that you want to achieve and GI max is the value of the maximum gain, maximum value of GI where GI is either GL or GS. Then we know that GI max is given by this equation, and GI will verify this equation. This intern verifies that GI, small gi is between 0 and 1. Now the locus of those values of gamma I which give constant GI now that is a circle and the equation for that locus is given by this where CI, RI is given is like this.

Now the center of all the circles for the various values of GI are located between 0 and SII star so if we draw a line between SII conjugate and the center then all the centers will be lying on that line now other observation be may we see is that GI is equal to 1 for gamma I equal to 0 see in other words GI is equal to 0 (())(5:54) there is no gain for gamma I equal to 0. So if now go to the slides on the monitor we see that if we draw these circles then it will the circles will look something like this actually.

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Now, what is these red circles are the these red circles are the gain circles of the locus of the gamma I for a particular value of capital GI and this blue circle is the Smith chart. Other thing to note as I had said the line joining the origin of the smith chart to this SII conjugate the center of all these red circles will lie along this line. Now what we had so far discussed is for the unconditionally stable.

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Potentially Unstable Case.

$$G_{TU} = G_S G_0 G_L \quad \underline{\underline{G_{i\max.}}}$$

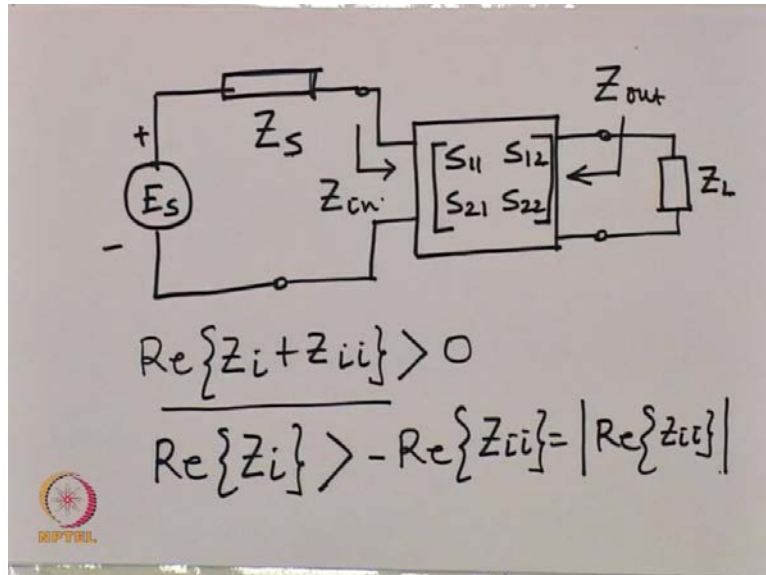
$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2} \rightarrow \infty$$

$$1 - S_{ii}\Gamma_i = 0 \quad G_i \rightarrow \infty$$

Now, what happens when we have a potentially instabilities in the case in this case the GTU will continue to be written by this formula, and this can go all the way till infinity, that is the difference between potentially unstable and a unconditionally stable is that there is nothing called GI max actually, for the unconditionally stable case we have a certain concept of GI

max for that we don't have, now we don't have such a thing for the potentially unstable case. Now this happens when  $1 - \text{ASSI } \gamma I$  this becomes equal to 0 then GI will tend to infinity.

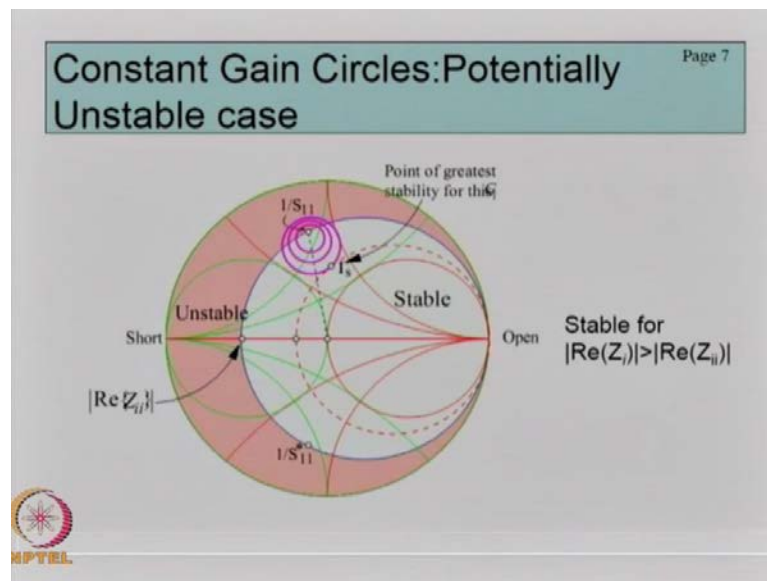
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The other thing about stability is that it requires... If I say again if we draw our basic 2 port diagram, now stability for stability as we have already seen this is the condition and if we write it in a different way then we can this thing can be written as... Now the question is that if you see that since no writing in this way now  $Z_S$  or  $Z_L$  both will be will have positive real parts so we can say that real of  $Z_I$  will always be positive.

And not only positive but greater than the negative of the negative real part of  $Z_{11}$  or  $Z_{22}$  so since the negative of the negative real part will be positive that's why we can write this equation write this so then what is that you know writing in a different way will also we can say that the real parts since the real part of  $Z_S$  or  $Z_L$  should be greater than the modulus of  $Z_{11}$  or  $Z_{22}$  the locus of  $\gamma_s$  and  $\gamma_l$  can be expresses in this form.

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If we go back to the slides on the monitor you see the requirement for stability is that the real part of  $Z_i$  should be greater than the modulus of real part of  $Z_i$  this intern translates into this circle so this is our stability circle for  $Z_i$  now you might ask here that how is this stability circle different from the stability circles we have already covered so far the difference is for those stability circles say you are considering  $Z_1$  then the locus will be for  $Z_L$  whereas in this case the locus for  $Z_S$ .

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$$g_i = G_i (1 - |S_{ii}|^2)$$

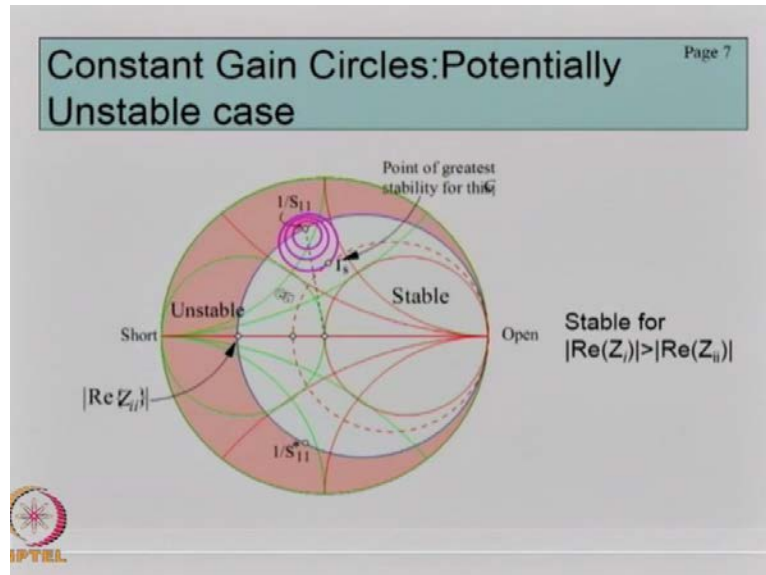
$$C_i \quad R_i = \frac{\sqrt{1 - g_i} (1 - |S_{ii}|^2)}{1 - |S_{ii}|^2 (1 - g_i)}$$

$$= \frac{g_i S_{ii}^*}{1 - |S_{ii}|^2 (1 - g_i)}$$

That's why you know it is not we cannot apply those same theory that we discussed for the previous case what about the gain circles that was about the stability circles but what about the gain circles so for the gain circles our definition of small  $G_i$  remains the same which is

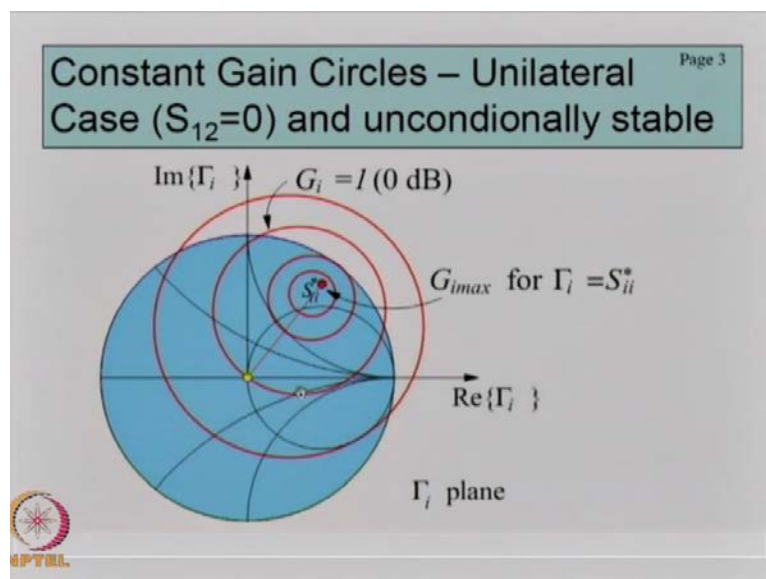
small GI is equal to, so that the definition remains the same however not that we don't have any concept of GI max here there is no concept of GI max but the definition remains the same and the definition of the CI and RI also remain the same. That is CI is equal to GI times SII conjugate upon 1 - SII square and RI is equal to that is the these definitions remain the same.

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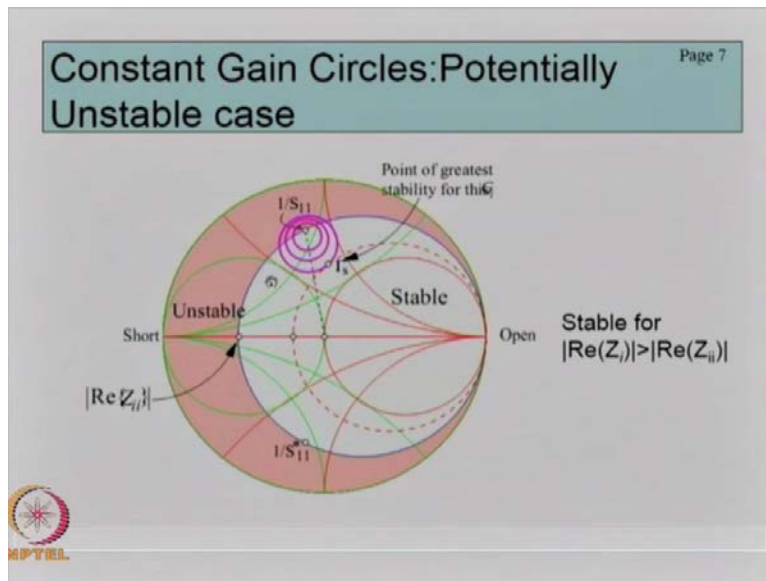
And based on this if you again go back to the diagram on the slide then gain circles that we have drawn are like this.

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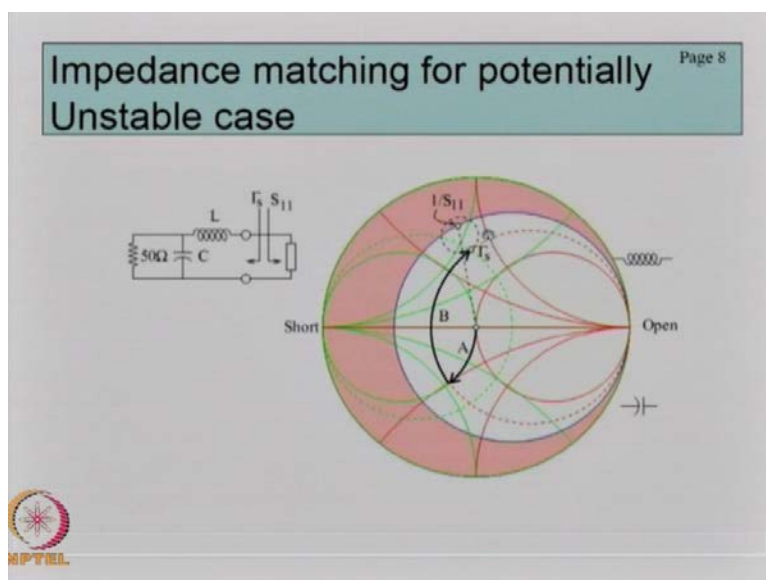
Now if I just compared with the previous one we see that the SII the center or you know we had obtained a GI max for gamma is equal SII start.

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The question here is what is this SII start for this potentially unstable case. Now we note one thing like in that SII start, since there is see first of all there is no concept of GI max okay so there will not be a point for which the radius is 0 that will not happen. However if we have to locate the line along which all the circles will have their centers then we can instead of plotting S 1 1 conjugate we simply plot 1 upon S 1 1 because 1 upon S11 has the same phaser as S 1 1 conjugate.

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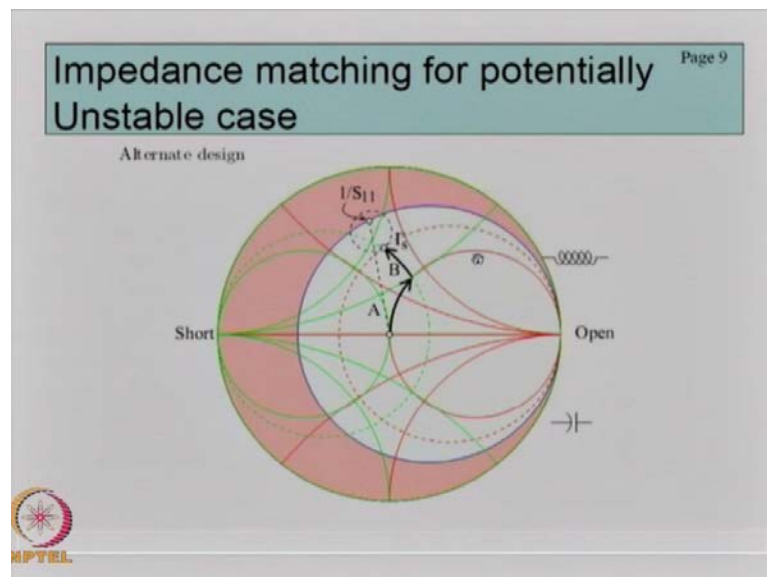


Now that was that was about the gain circles but about the impedance matching let's go back to our slides on the monitor suppose this dotted line this small circle that we see is the gain circle for some value for capital GI and if we go from then if we have to provide a matching

network so first of all note that this is the value of gamma S that we have to achieve and for matching we need you know we need a gamma S to be matched to our characteristic impedance which say for our case is 50 ohms.

So as we know for matching what we do we find a path from the center to that particular point, and why take this particular point by the way, we should have taken any of the points along this dotted circle and all the points of this dotted circle which lie in this white region will be stable. Why do we choose this point? We choose this point because this is the most stable point this had the maximum, this creates the most I can say the most or the highest or give the highest value of the real part of  $Z_S$  plus  $Z_{11}$  that why we choose this point.

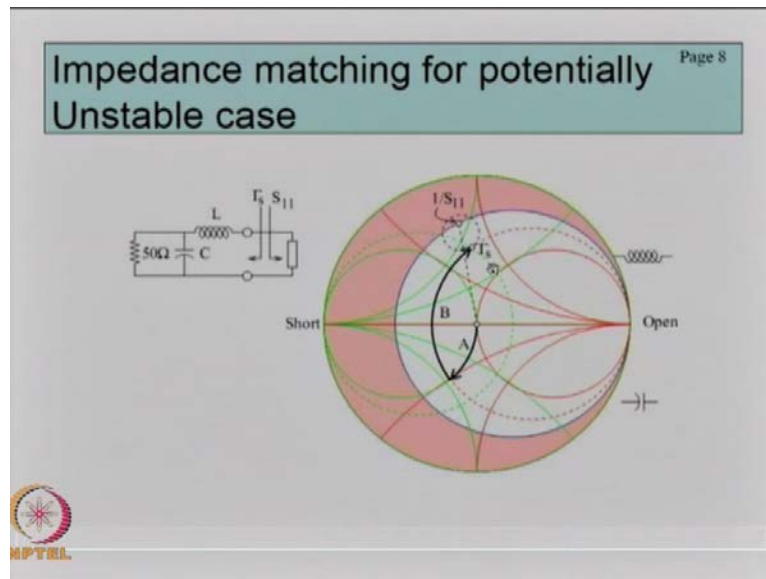
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Alternatively we have could have chosen this path as well from the center to the this point gamma s which is the most stable point, and in this case going along a constant resistance circle upwards mean inductive in series, and then going upwards again along a constant conductance circle also means an inductance but in shunt.

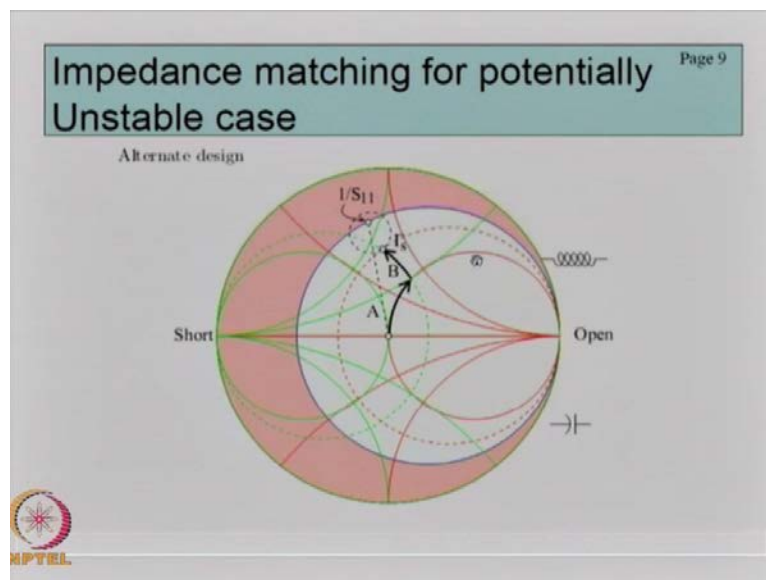


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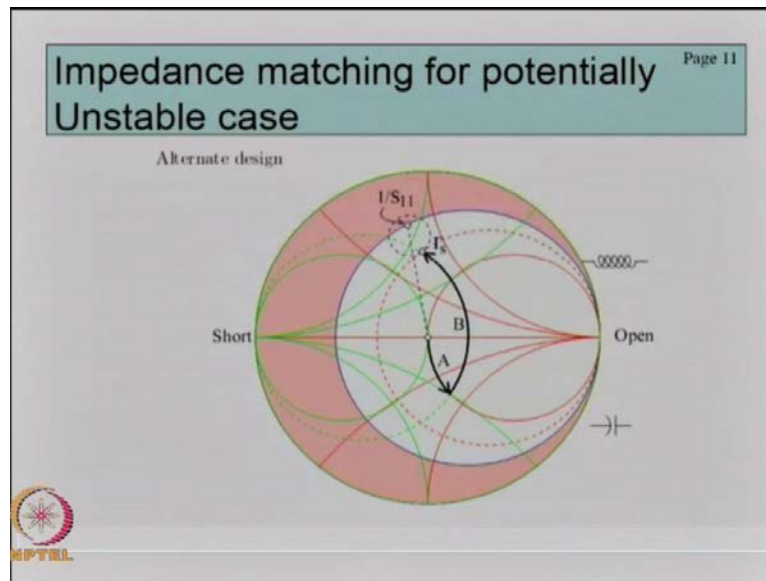
So in this case our matching network for the first previous case where first we went down and went up. So going down along a constant conductance circle means a capacitor in shunt and going up along a constant resistance circle means an inductor in series so that's what we have here. We started from 50 ohms first we have a capacitor in shunt and then an inductor in series.

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In this case also first we will have inductor in series and then an inductor in shunt.

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Alternatively if we go along this path where first we go down along a constant resistance circle, and then go up along a constant conductance circle. Here also we will first have a resistance, I beg your pardon a capacitor in series followed by an inductor in shunt.

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The whiteboard has the title "Unilateral Approximation Figure of Merit". It contains two equations for the Figure of Merit  $G_T$  and  $G_{TU}$ . The NPTEL logo is in the bottom left corner.

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_L|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$\frac{G_T}{G_{TU}} = \frac{|1 - \Gamma_S S_{11}|^2}{|1 - (S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L})\Gamma_S|^2}$$

Now if we come back to our slides on the there is a certain term called Unilateral Figure of Merit or Unilateral Approximation Figure of merit. Now we know that  $G_T$  the real value no without any unilateral approximation is given by this equation. Now  $G_T$  upon  $G_{TU}$  you will be given like this.

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$$\frac{G_T}{G_{TU}} = \frac{1}{\left| 1 - \frac{S_{12} \Gamma_S \Gamma_L S_{21}}{(1 - S_{11} \Gamma_S)(1 - S_{22} \Gamma_L)} \right|^2} \rightarrow X$$

$$= \frac{1}{|1 - X|^2} \quad G_{TU \max.}$$

$$\frac{1}{(1 + |X|^2)} \leq \frac{G_T}{G_{TU}} \leq \frac{1}{(1 - |X|^2)} \quad \Gamma_S = S_{11}^* \quad \Gamma_L = S_{22}^*$$

$$\frac{1}{(1 + U_x)} \leq \frac{G_T}{G_{TU \max.}} \leq \frac{1}{(1 - U_x)^2}$$

And then on further simplification this comes out to be like this and suppose I denote this quantity by the terms  $x$  then this whole thing comes down too. Now depending on what value of  $X$  is whether it is positive or negative, we can say that this  $G_T$  upon  $G_U$ , this equation you know you can work it out you can see that it is really and can verify that it is true the maximum value of this  $G_T$  and  $G_U$  will be when  $x$  is when this is equal to 1 minus modulus of  $x$  square, and the minimum of value of  $G_T$  upon  $G_U$  will happen when this value becomes equal to 1 plus modulus of  $X$  square.

Now  $G_U$  max as we know is obtained for  $\Gamma_S$  equal to  $S_{11}$  conjugate and  $\Gamma_L$  equal to  $S_{22}$  conjugate. Now for that particular case if this is the value of  $\Gamma_S$  and  $\Gamma_L$  we take then this  $G_T$  upon  $G_U$  max will be given by so this will be the range of  $G_T$  upon  $G_U$  max if this is the condition that is maximum gain case if we take. Now here this  $U_x$  is given by... Let me use a different piece of paper...

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$$U_x = \frac{|S_{12}| |S_{11}| |S_{22}| |S_{21}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)}$$

This is the value of U...

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$$\frac{G_T}{G_{TU}} = \frac{1}{\left| 1 - \frac{S_{12} \Gamma_S \Gamma_L S_{21}}{(1 - S_{11} \Gamma_S)(1 - S_{22} \Gamma_L)} \right|^2} \rightarrow X$$


$$= \frac{1}{|1 - X|^2} \quad G_{TU \max.} \quad \Gamma_S = S_{11}^* \quad \Gamma_L = S_{22}^*$$

$$\frac{1}{(1 + |X|^2)} \leq \frac{G_T}{G_{TU}} \leq \frac{1}{(1 - |X|^2)}$$

$$\frac{1}{(1 + U_x)} \leq \frac{G_T}{G_{TU \max.}} \leq \frac{1}{(1 - U_x)}$$

So our this GT upon GTU max should be between this way and whether or not for a particular GT no whether we can make this approximation of not depends on this value of UX if our GT that we obtain does not lie within this range then we can say or this ratio you know if this GT upon GTU max anytime this ratio is not within this given range then we cannot do this unilateral approximation.

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$$U_x = \frac{|S_{12}| |S_{11}| |S_{22}| |S_{21}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$
$$U = 0.03$$
$$\frac{1}{(1+0.03)^2} \leq \frac{G_T}{G_{TU_{max}}} \leq \frac{1}{(1-0.03)^2}$$
$$-0.26 \text{ dB} \leq \frac{G_T}{G_{TU_{max}}} \leq 0.26 \text{ dB}$$


So usually you know the value of  $U$  say usual value of or particular value of  $U$  if it is 0.03 then  $G_T$  upon  $G_{TU_{max}}$  will be so this  $G_T$  upon  $G_{TU_{max}}$  if it is not within this range then we have a problem, we cannot do that unilateral approximation that we have been doing.

So in summary, in this lecture we covered the case when we have the Unilateral Approximation Valid and we saw what happen to the gain circles or what are those gains circles for both the unconditionally stable case and the potentially unstable case. In the next module we shall be covering the case when they when we don't have this unilateral figure of merit and then we shall that our choices are somewhat limited we cannot design it. We cannot do as much flexible design as we have been doing so far that is choosing any particular team. And also how to design using the operating power circles power gain circle out and then available power gain circles.

Thank you!