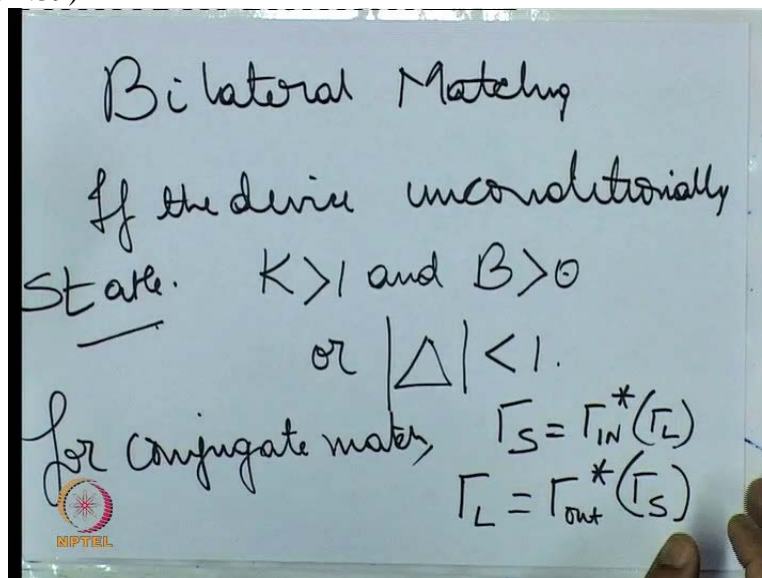


Microwave Integrated Circuits
Professor Jayanta Mukherjee
Department of Electrical Engineering
Indian Institute of Technology Bombay
Module 7
Lecture No 28
Gain Circles (contd.)

Hello, welcome to another module of this course, microwave integrated circuits. In the previous module, we are in week 7 now. In the 1st module of week 7, we had covered about design using the gain circle for the unilateral phase using the transducer gain as our specification for the gain. And then we had seen what we had also discussed about the unilateral figure of merit to show how good an approximation, the unilateral approximation was. I had also mentioned in the previous module that next we will be covering the bilateral case. That is the case when we do not make the unilateral approximation. So let us see how, what we can actually do.

Now one thing I would like to state for the bilateral phase is that unlike the unilateral case there is not much design flexibility. I mean what we can achieve mathematically is to specify the maximum gain that we can get unlike the unilateral approximation case where you could specify a particular gain and then draw a load side and source side gain circle. In the bilateral case, we cannot do that.

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Now if the device unconditionally stable that is $A > 1$ and $B > 0$ or $\Delta < 1$ then for conjugate match, we have to have and Γ_L should be equal to now we

know what is the equation for gamma in and the equation for gamma out and if we solve these 3 equations simultaneously then the solution that we will get is given like this.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the maximum source reflection coefficient is given as $\Gamma_{MS} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$, with the condition $S_{12} \neq 0$ written to the right. Below this, the maximum load reflection coefficient is given as $\Gamma_{ML} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$. Underneath these, the word "where:" is written, followed by the definitions of B_1 and B_2 : $B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$ and $B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$. At the bottom, the definitions of C_1 and C_2 are given as $C_1 = \Delta S_{22}^*$ and $C_2 = S_{22} - \Delta S_{11}^*$. A small logo for "RIPTIL" is visible in the bottom left corner of the whiteboard image.

Where this B_1 , B_2 and C_1 , C_2 are given by...

Now if we substitute these values into the gain equation then the value of gain we call the maximum gain for the non-unilateral case that is when we do not make that is when S_{12} is not equal to 0. I'm not repeating that equation but if we substitute these values of Γ_{MS} and Γ_{ML} in that equation, what we get is actually the maximum possible gain, maximum possible value of G_T for the bilateral case. And that value of G_T , that maximum value of G_T that we get is given by...

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The whiteboard contains the following handwritten text and equations:

$$G_{Tmax} = \left| \frac{S_{21}}{S_{12}} \right| (K - \sqrt{K^2 - 1})$$

G_P G_A

$$G_{Tmax} = G_{Pmax} = G_{Amax}$$

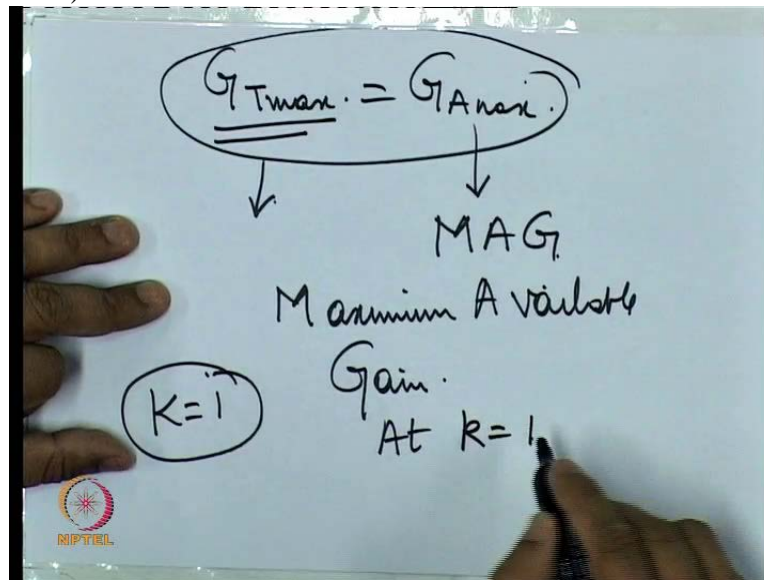
$\rightarrow G_{Tmax} = \left| \frac{S_{21}}{S_{12}} \right| \frac{K + \sqrt{K^2 - 1}}{K - \sqrt{K^2 - 1}}$

$= \left| \frac{z_{21}}{z_{12}} \right| = \left| \frac{y_{21}}{y_{12}} \right|$

A small logo for NIPTEL is visible in the bottom left corner of the whiteboard image.

Now we can show, we have already talked about GP that is the power gain or the operating power gain and GA is the available power gain. Now we can show that GT Max will be equal to GP Max which is equal to GA Max. Now this can also be simplified like this. Now just dividing the numerator and denominator by $K + \sqrt{K^2 - 1}$, what we will get is this. This S_{21} upon S_{12} is equal to any of the values. That is this is also equal to Y_{21} upon Y_{12} . You can verify that from the conversion formulas for between Z and S parameters. Now, we saw that this is the case for the unconditionally stable case.

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Now for the unconditionally stable case, this GT Max that we just found out which is equal to GA Max. Now this is actually a figure of merit. This is actually a specification given in the datasheets of RF devices that suppose if you buy from the market, then the GA Max will be a specification that is given with the device. This is also known by the term MAG. That is, maximum available gain. Now we see that K equal to 1 defines the boundary between unconditionally stable and potentially unstable. So we see that at K equal to 1, where this K is the stability factor that we had studied about previously. And that is also related to this GT.

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$$G_{Tmax} = \left| \frac{S_{21}}{S_{12}} \right| (K - \sqrt{K^2 - 1})$$

$$G_P \quad G_A$$

$$G_{Tmax} = G_{Pmax} = G_{Amax}$$

$$G_{Tmax} = \left| \frac{S_{21}}{S_{12}} \right| (K + \sqrt{K^2 - 1})$$

$$= \left| \frac{z_{21}}{z_{12}} \right|$$

So if we just come back to the previous slide, now this GT Max is the related to K. K comes into this equation and this value of K is the same value. This K is the same value as the K that we discussed while discussing stability.

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$$\underline{G_{Tmax} = G_{Amax}}$$

MAG
Maximum Available
Gain.

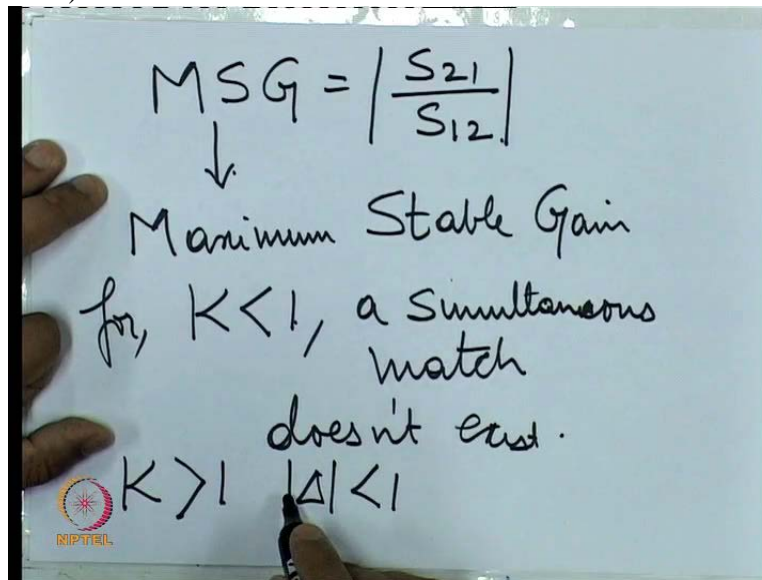
At $K=1$ $K=1$

$$G_{Tmax} = \left| \frac{S_{21}}{S_{12}} \right| \text{MSG}$$

$K=1$

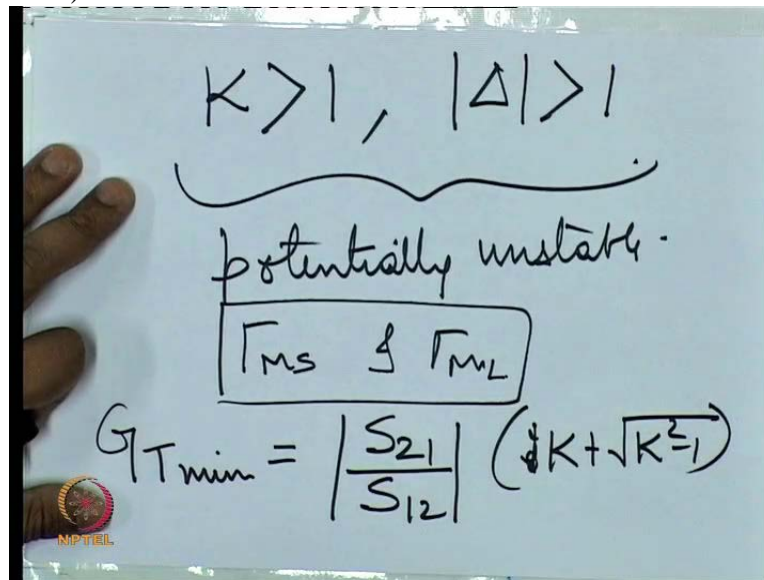
Now at K equal to 1, this GT Max becomes 1. GT Max becomes simply equal to S21 upon S12. That's it. So at the stability boundary, this is the value of GT. And in fact, this is also a figure of merit and it is known by the term MSG which means MSG which is equal to S21 upon S12 and this is known as the maximum stable gain.

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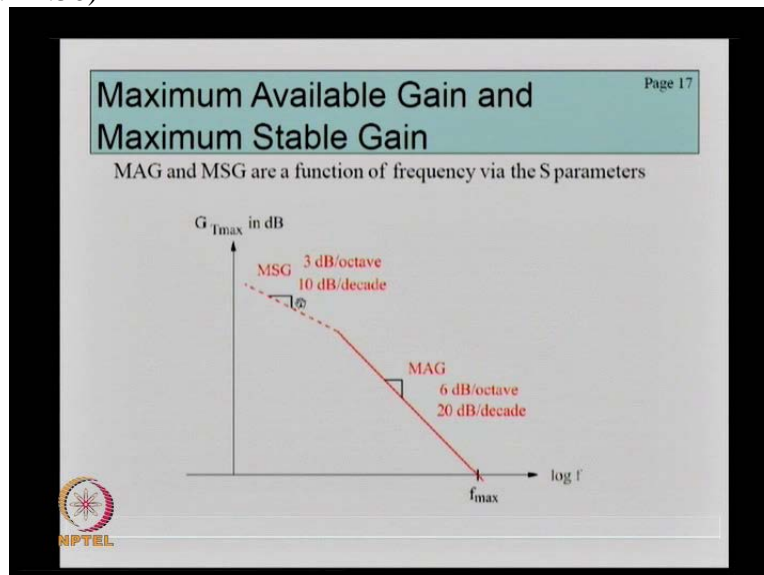
For K lesser than 1, there is for the potentially unstable case, a simultaneous match does not exist. So at K equal to 1, we have a match as possible and the gain and that value is given by this. For K lesser than 1 that is now we are having a potentially unstable region, a simultaneous match does not exist. It is not possible to find a matching network which can satisfy the 2 equations that I described before. However, for the K greater than 1, now for unconditional stability we saw that we need K greater than 1 and $|\Delta| < 1$. If both these 2 conditions are not satisfied, then it is potentially unstable.

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For a particular case when Delta so for K greater than 1 and mod Delta greater than 1 this is also potentially unstable. For this particular case also, it is possible to find the value of gamma MS and gamma ML and these values are same that we obtained for the unconditionally stable case. Since this is now potentially unstable, instead of GT Max we will have a GTmin that is the minimum value of the transducer power gain. And that value is given by. Instead of the minus, it is now plus here.

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And if we go to the slides on the monitor for a moment, this is the typical GT vs frequency curve for a standard RF device. We see that for high frequencies it is unconditionally stable and that's why, this MAG exists and for lower frequencies, this MSG is the specification at the stability boundary that is given. This is often a graph that is supplied with RF devices. And we see that at lower frequencies the device is less stable as compared to higher frequencies and for a particular value of F called F Max, the the gain, the value of GT or GT Max becomes one. That is beyond F max it no longer gives any usable gain.

And this F Max, MSG and MAG are the parameters that are supplied in the datasheets of many RF devices. So we have discussed how to design using the transducer power gain for the unilateral case and also how to find out the maximum available gain or the maximum stable gain for the bilateral case depending on whether we are in the unconditionally stable or in the potentially unstable regions of the RF device.

But then we see that if we have the bilateral case, then we cannot design. We cannot set a particular gain for the, a particular value for the transducer gain. So to get around this problem, we use the other 2 gain definitions that we have used. That is the operating power gain and the available power gain for designing RF circuits. So let us see how to design using these 2 power gains.

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Operating Power Gain Circles $G_p(\Gamma_L)$

$$|\Gamma_L - C_p| = r_p$$

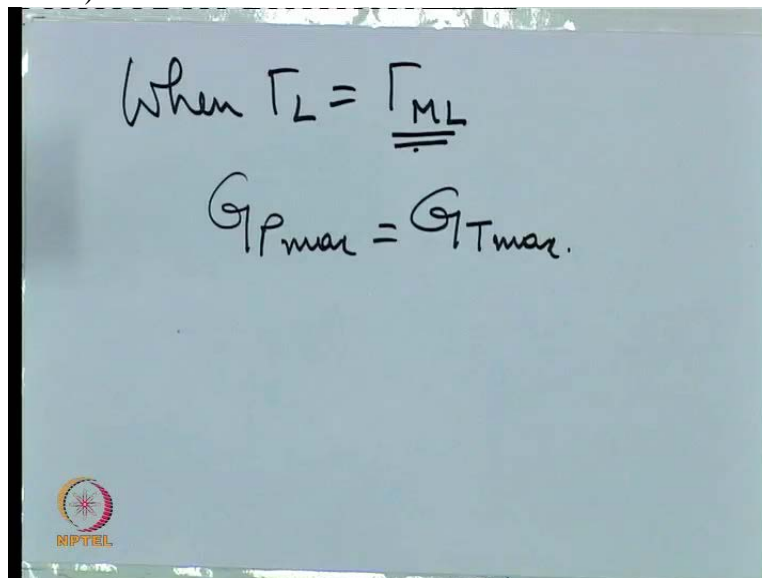
$$C_p = \frac{g_p C_2^*}{1 + g_p (|S_{22}|^2 - |\Delta|^2)}$$

$$r_p = \frac{(1 - 2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2)^{1/2}}{|1 + g_p (|S_{22}|^2 - |\Delta|^2)|}$$

Now if you fix a particular value of the operating power gain, then the locus of gamma L we call that in power gain, operating power gain GP is a function of gamma L, the load reflection coefficient then the locus of gamma L for a constant value of GP can be shown, the derivation is there in the book by Gonzales. It can be shown that the locus is like this where this CP is given by...

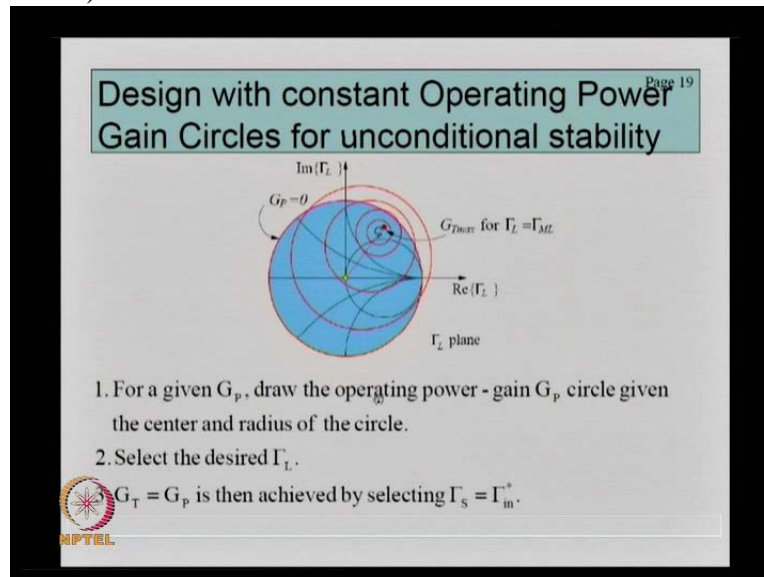
This is the value of the CP the centre of the circle and RP, the radius of the circle. The Centre of this output, this recall is an output stability circle because it is a function of gamma L. The centre of the output stability circle, the centre of the constant GP circle and the point gamma L are equal to 0 are collinear.

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$$\text{When } \Gamma_L = \underline{\underline{\Gamma_{ML}}}$$
$$G_{Pmax} = G_{Tmax}.$$

The other thing is, when gamma L is equal to gamma ML, this is the solution obtained for the bilateral case of the transducer power gain then GP Max is equal to GT Max.

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So if we go to the slides on the monitor, these operating power circles are like this. This blue circle is the, is our Smith chart and this red circle are the operating power circles. As we see that all the centres of all these circles are collinear and that is because of this.

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Operating Power Gain Circles $G_p(\Gamma_L)$

$g_p = \frac{G_p}{|S_{21}|^2}$

$|\Gamma_L - C_p| = r_p$

$C_p = g_p C_2$ $C_2 = S_{22} - \Delta S_{11}^*$

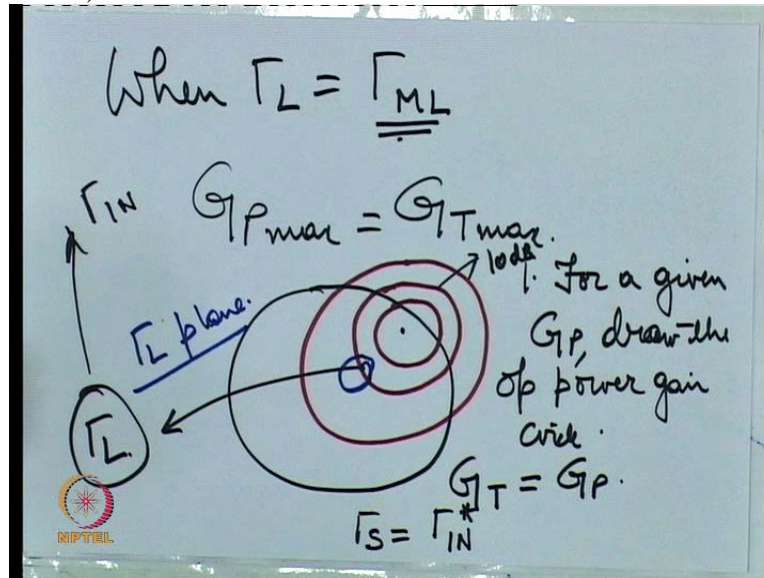
$r_p = \frac{(1 - 2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2)^{1/2}}{1 + g_p(|S_{22}|^2 - |\Delta|^2)}$

NPTEL

If we come back to the slides on the, if we come back to the slides, written slides, the CP is given by this expression and the value of C2 is given as S22-Delta S11 star. This is a vector quantity. Others are scalar quantities and therefore all the centres of all the operating gain circles will lie on a straight line. This small GP, I did not mention what is the definition. This small GP is

defined as the actual power gain divided by S_{21} square. So this is the normalised, so this small GP is the normalised power gain or the normalised operating power gain. So, how to design using the operating power circle?

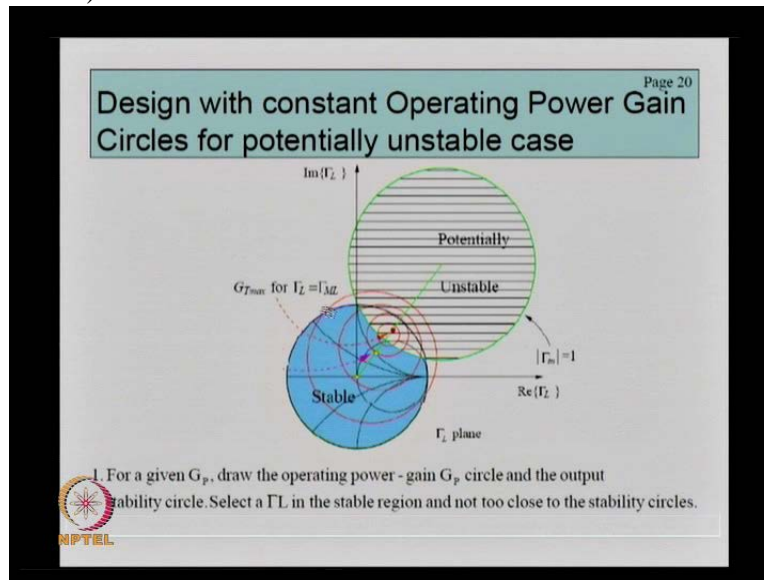
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So for a given, suppose this is our smart chart and say the power gain circles that we obtained are like this. So the steps to design are 1st, for a given GP, draw the operating power gain circle. Now, since this is the gamma L plane, select the desired value of gamma L. So suppose this is the gain circle that you are interested in, select a point, gamma L then that is the value of gamma L that you choose for your design.

And if you want to make the transducer power gain equal to the operating power gain that is for making G_T equal to G_P , so you choose the value of gamma L here. This is the value of gamma L and say this is the gain circle, this is the say you need a GP of 10dB and say this is the 10dB gain circle, then you choose a point on the circle that gives you the value of gamma L. From here you get gamma L. Once you get gamma L, you can calculate gamma in and then gamma S will be equal to gamma in conjugate. This will also ensure that G_T is equal to G_P . Now for potentially unstable case, Let us see how to design for potentially unstable case.

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For the potentially unstable case, if we go back to the slides on the monitor, say this is your unit circle and this is your stability circle so this will be the output stability circle. So you choose same as the previous case you choose a particular gain circle, select a value of gamma L, ensure that the gamma L that you choose lies in the stable region and then once you choose gamma L, you can find out gamma in and from gamma in, you can find out gamma S. Now, one important factor that you have to take into account when you do this is that you have to draw the input stability circles as well. Because if you choose as I said, you have to choose gamma S is equal to gamma in conjugate.

But then, this gamma S itself should lie in the stable region of the input stability circle. So that, you can do by drawing the input stability circle and then see whether the gamma is that you have obtained using this equation indeed lies in the stable region. If it lies in the stable region, then that is fine. Otherwise, you need to choose a different value of gamma L and repeat this whole process again.

Now note one thing, this operating power gain circle based design can be used for certain types of amplifiers like power amplifiers where the load where the input impedance is fixed but the load impedance can be varied. Then, there are other amplifiers like the low noise amplifier where the output is fixed but the input can be varied. So for such type of amplifiers, as I already

mentioned before, you might want to use available gain circles for design. So let us see what are the available power gain circles.

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Design Using Available Power Gain Circles

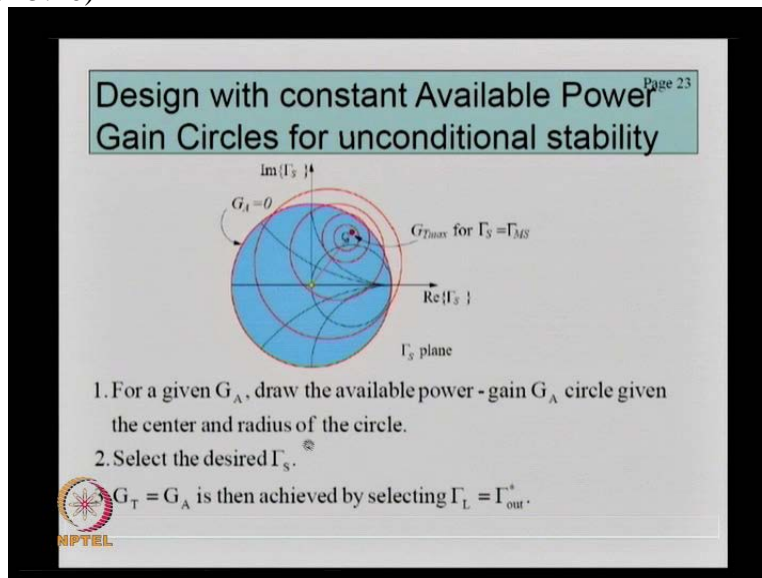
$$|\Gamma_S - C_A| = r_A$$

$$C_A = g_A C_1^*$$

$$r_A = \frac{(1 - 2K |S_{12} S_{21}| g_A + |S_{12} S_{21}|^2 g_A^2)^{1/2}}{1 + g_A (|S_{11}|^2 - |\Delta|^2)}$$

So available power gain circles are very analogous to the operating power gain circle. We chose a different...so they are very analogous to the discussion that we had for operating power circles that is the locus of gamma S for a constant available power gain will be given like this where this...this is the equation for the available power gain.

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And if we go to the slides on the monitor, just like the operating power gain circle, the centre of all the available power gain circles will be collinear. This is of course the Γ_S plane. For the operating power circles, it was a Γ_L plane and so the way to select, so the way to design is suppose this is the 10 dB available power gain line and you need a gain of 10 dB, then choose a point on this circle, this red circle that lies within the Smith chart.

This is of course, we are assuming this is the unconditionally stable case. We chose the value of Γ_S and then find out what is the value of Γ_{out} just like we found out what is the value of Γ_{in} for the operating power gain circle case. Once you find out Γ_{in} , Γ_{out} I beg your pardon, you set Γ_L equal to Γ_{out} conjugate and this is the value of Γ_L . When you do this, the G_T will be equal to G_A . For the potentially unstable case, you draw the available power gain circles.

You also draw the input stability circle, select a point on this available power gain circles that lies in the stable region. Once you select a point, say that a certain value of Γ_S , find out the value of Γ_{out} and then set your Γ_L is equal to Γ_{out} conjugate. Once you do that you now draw the output stability circle where we are in the Γ_L plane and see whether this Γ_L that you just obtained using this equation really lies in the stable region.

If it lies in the stable region then fine. If it does not lie in the stable region, then you need to find another value of Γ_S and repeat this process till the time that Γ_L , was Γ_S and Γ_L are in the stable regions of the input and output stability circles respectively. So, in summary, in the structure, we covered the bilateral cases, in the previous module, we had covered only the unilateral transducer gain case. For bilateral matching, for the bilateral for finding out the if we consider only the transducer gain we saw that bilateral design is mathematically very difficult.

And what we can obtain is the maximum, specify the input and output reflection coefficients for opening the maximum value of G_T . For the available power gain and the operating power gain, we saw that we can do a design even for the bilateral case. Of course for the available power gain circles, the load is always, the load reflection coefficient is always the conjugate of the output

reflection coefficient and for the operating power gain circle, the input, the the source reflection coefficient is the conjugate of the input reflection coefficient.

So, as I said before, the available power gain circle based design is used for those amplifiers where the output impedance is fixed but the input impedance can be varied and the operating power gain circles are used for design of those amplifiers where the input reflection coefficient is fixed but the output reflection coefficient can be varied. In the next lecture, we shall be covering another important aspect of amplifier designs which is noise. So that is what we will be doing in the next lecture, next module. Thank you.