Microwave Integrated Circuits
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Module 7
Lecture No 29
Noise

Hello, welcome to another module of this course, in NPTEL mocks program, microwave integrated circuits. In the previous modules in this week we had covered the design design aspect using gain circle. In those modules, we covered how to achieve a particular gain whether it is a transducer power gain or available power gain or operating power gain. So in this module we will be covering another important aspect of amplifier design which is the noise aspect. In the 1st part, we will be covering what is noise, how to characterise noise and inthe 2nd part, we will be discussing about noise circle and how to achieve a particular noise figure.

And then we will also see what is in noise figure. What does the term, noise figure mean and how to achieve if you have given noise figure specifications and how we can achieve that using noise figure circuits.

So, 1st question is, what is noise?

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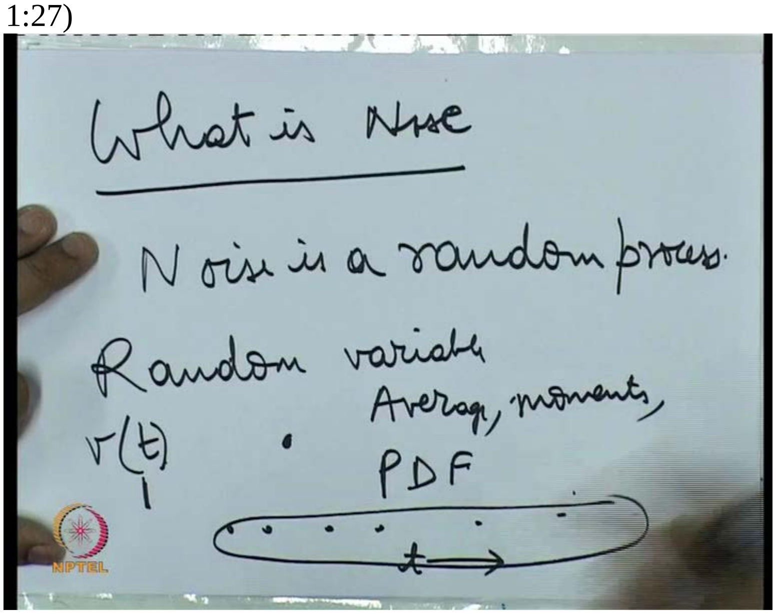

Noise if you have any idea about statistical statistics, noise is known as a noise is a random process. What do we mean by random process? 1st, let us see what is a random variable. A random variable is a quantity which does not have any fixed value unlike deterministic quantities say a voltage. It has, at a particular instant it will have a definite value. But say, noise voltage if you say, it does not have any particular value. Instead, what we can characterise it is with some average or moments or what we call probability density function. Now, an ensemble of such random variables over trying if time is also now available, then such a process is called a random process. And noise is one such random process.

Now how to as I said, you cannot have a deterministic value. For example I say that the voltage, if you measure the voltage of a battery you might find it to be 2 volts then you say that the voltage of the battery is 2 volts. But on the other hand let's say noise voltage, at any instant it may not be the same value. It keeps on varying. The value of the noise voltage keeps on varying. But then, the average value if you find the average of the noise voltage, then that remains constant.

So that is how we characterise noise. Even though we cannot exactly specify what the real voltage will be at a particular instant of time, we can say what the average will be over time. So in that sense, there are 2 types of average actually.

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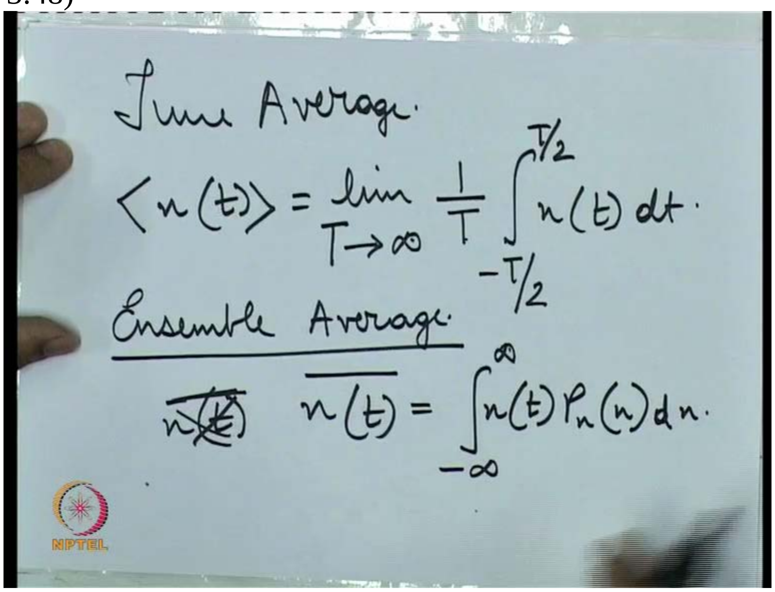

One is what is known as the time average. That is represented by this symbol and it is defined as like this. So it is like the average of noise voltage or any other random process with time. Another kind of average that we can define is what is known as the in ensemble average.

Here, instead of at a particular instant of time what is the average value that the random process takes? So the representation of ensemble average by this symbol. Similarly these are the time average and the ensemble average. If we consider a power content in any signal whether it is noise or any other random process, then 2 other definitions that we can use is what is known as mean square power.

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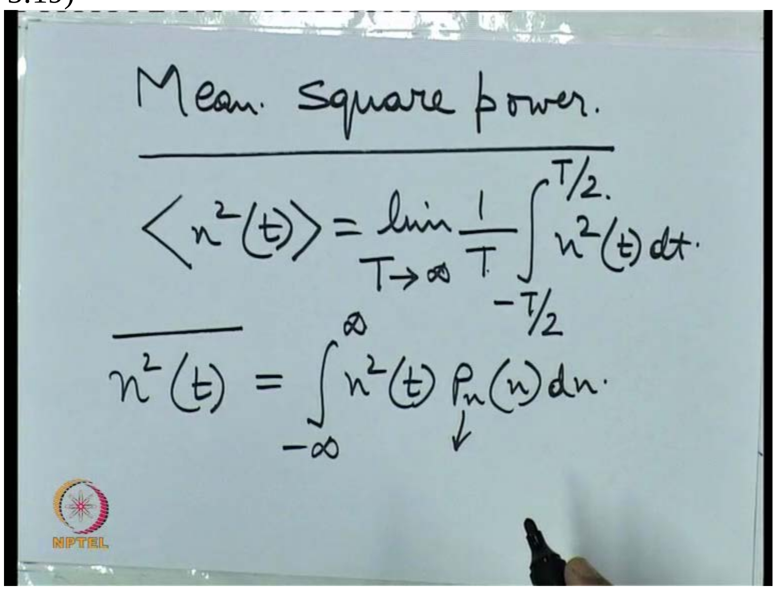

Now the mean square power can be defined in terms of its variation with time just like the time average that we defined. Or in terms of the ensemble that is now here this term, this PN what for the ensemble everything that I was discussing in the previous slide.

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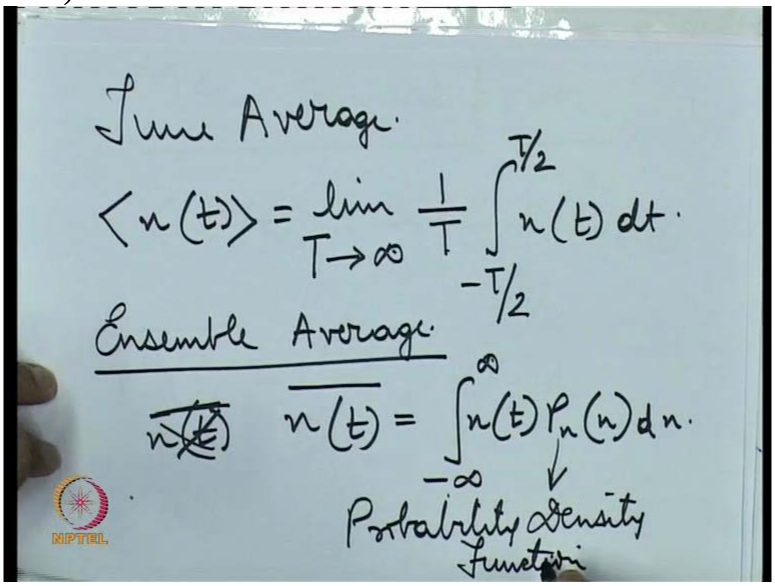

This slide I was discussing about an ensemble average. I wrote this equation. This PN represents the probability density function. This is like, what is the probability of finding a particular value of N among the various possible values that N can take? Probability density function.

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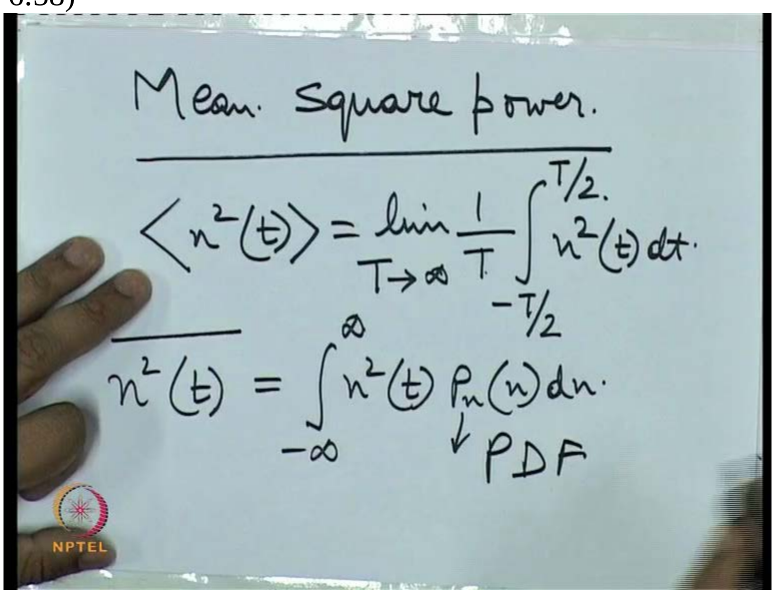

Here also, in the case of this mean square power, the ensemble average, this PN represents the probability density function or simply PDF.

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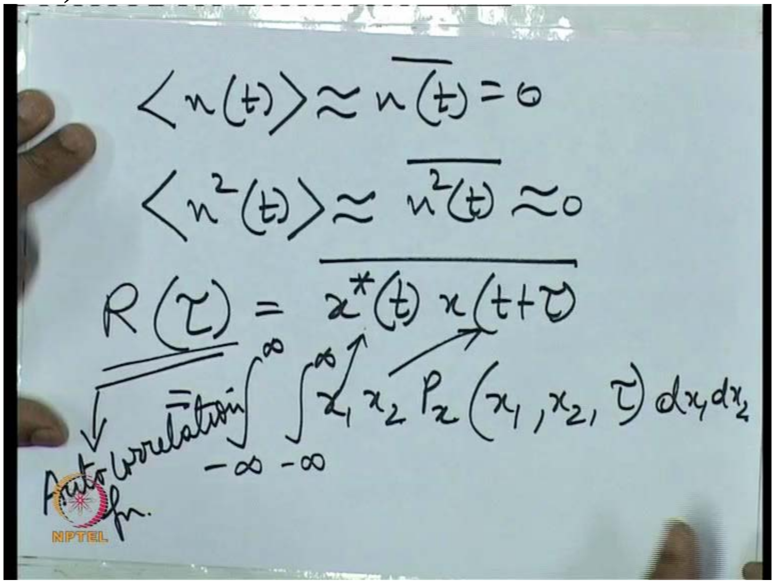

Now for our purpose you know when we define noise, the average value that is the ensemble average or the time average both are equal to 0. And so is the case with the mean square power. Now the time average for a stationary random process another quantity that is frequently discussed is what is called as the autocorrelation function which is given like this and this is describing at the moment why this autocorrelation function is so important.

This is the definition of autocorrelation function which is the same function X which is a kind of relationship between t and x at T+ Tao that is at a time spacing. X1 represents X of T. X2 represents XT of Tao and the importance is that if we take the Fourier transform of this function it is the autocorrelation function. Then we get what is known as the spectral density.

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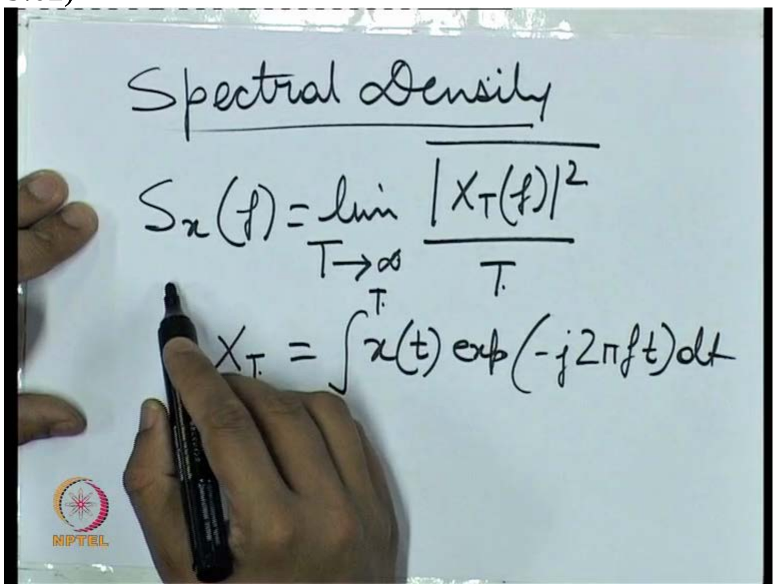

…where this XT is given as…as I said X of T is a measurable quantity known as the spectral density. This is something you can actually measure using an instrument called as spectrum analyser and…

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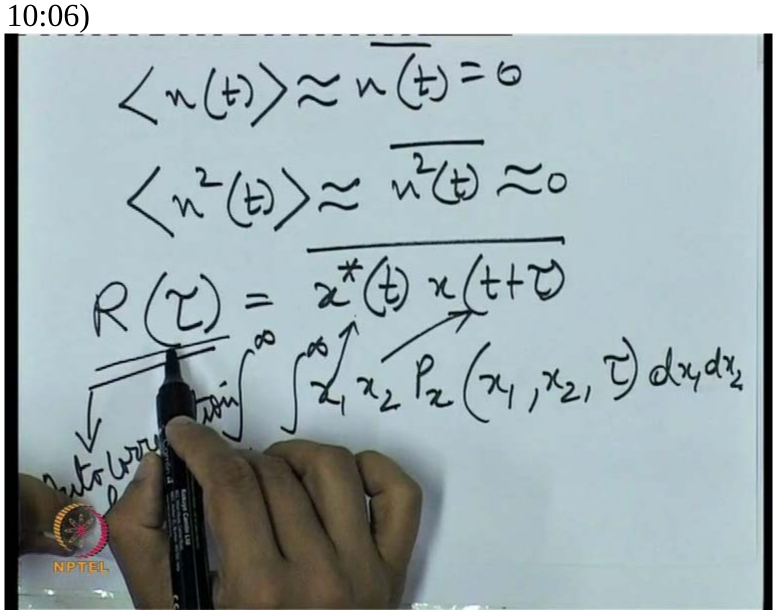

…this is Fourier transform of this RT that is just defined. Now, ideally what we see when we take any device or if we measure the noise power, then there are various kinds of noise power. One of them, various kinds of noise power that we see depends on what kind of spectral density they have, what kind of spectral distribution they have.

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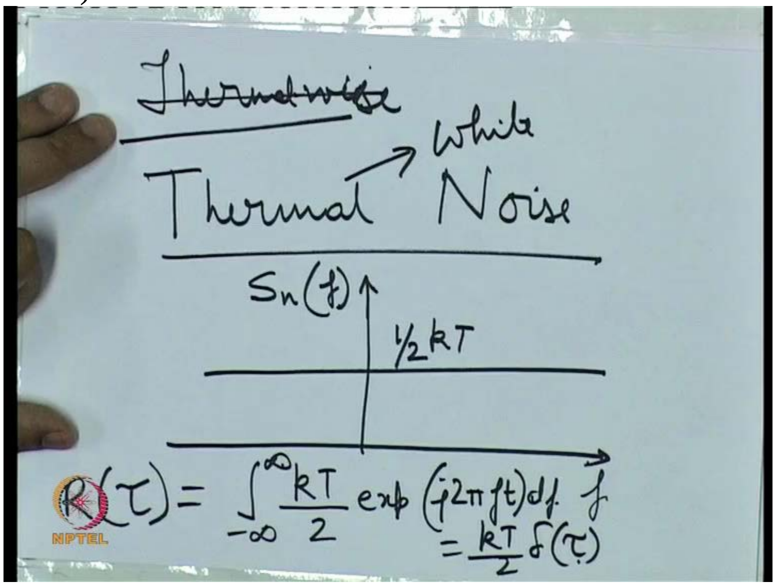

One of them is what is known as a thermal noise. Thermal noise is a distribution like this. Autocorrelation function for this thermal noise is given by this equation.

Thermal noise is also known by the term known as white noise. Now, it can be shown that for a resistor, a simple resistor that we have if it exhibits, if we consider a noise power that is present inside a resistor then the noise power can be given by this.

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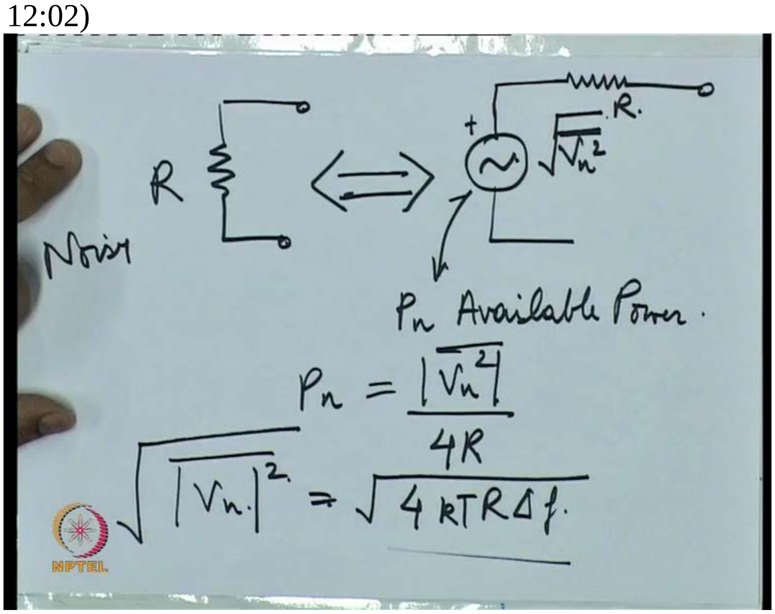

And this available power value is given by PN is equal to VN square by 4R from which this noise voltage value can be found out to be 4KTR delta X. So this is the equivalent noise voltage that is always present when you have a resistance. Now that the revision can be seen, the detailed derivation can be seen in the book by Gonzales.

But just I want to impress on you that resistor is not just a simple resistance as we have known from the circuit theory. There is also a source kind of associated with it. The source does not have any direction. That is, it is difficult to say which is the positive and negative. We can not say it like that. All we can say is that there is a power associated with the source and that power value is given by this equation from which this equivalent noise voltage is given by this equation.

Now, that was the basic noise definition. But what about a 2 port microwave device? How do we characterise the noise associated with our 2 port the microwave device or in other words the noise of an amplifier?

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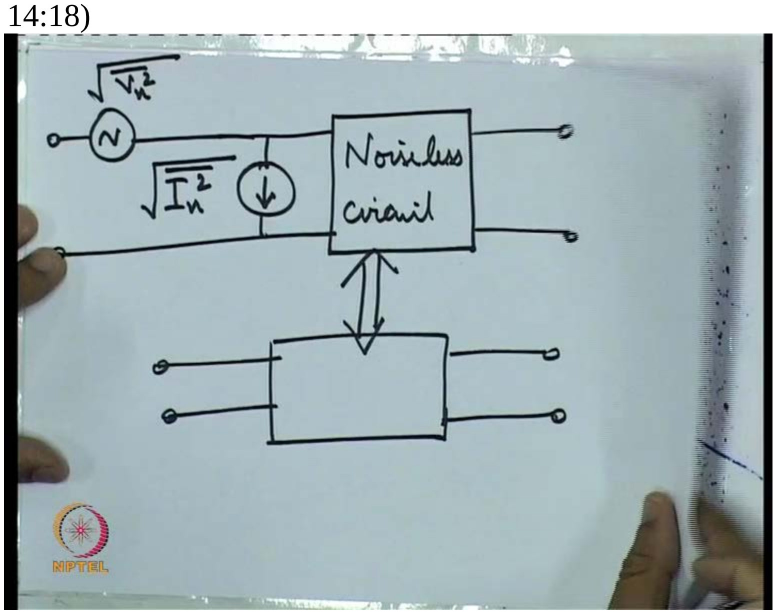

Now for 2 port network like this so for 2 port circuits, this is a noisy 2 port circuit various noise elements present. From that if we have to go to a noise less circuit, then we make a transformation like this. And how you know the derivation of the values of this VN and IN is given in appendix L of the book by Gonzales. Please go through it.

So for example, if we, you can take up various noisy devices like say a MOSFET.

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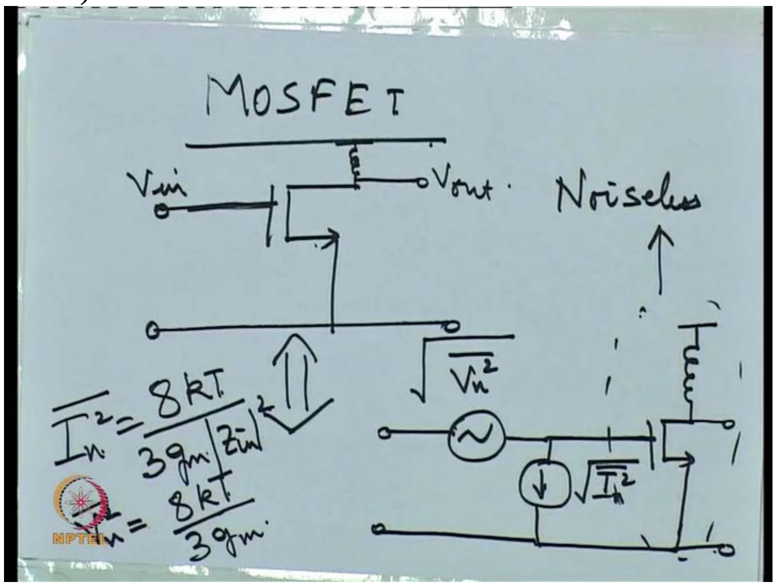

Let us see a simple MOSFET which is an active device and has some noise associated with it. In fact, a simple MOSFET has a circuit like this. Now the equivalent noise circuit for this MOSFET can be given like this. This is noise less and value of this IN square is given by the value of this VN square is given by. And related term that is often associated with 2 port networks is what is known as a noise figure. Now let us see what does the term, noise figure mean?

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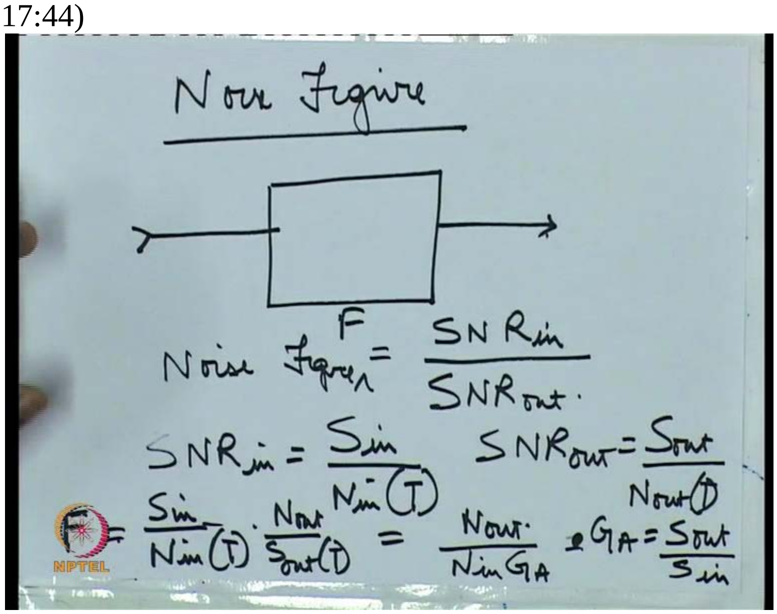

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Suppose we have a 2 port network. That is say an input signal and an output signal. The noise figure is defined as SNR in upon SNR out. No it is not SNR out upon SNR in. It is SNR in upon SNR out unlike other terms. Unlike gain term where you always have output in the numerator and input terminal denominator. That is not the case here. Now, SNR in is given by Sin upon Nin T. Here, Sin is the signal power of the input i.e. the part of the input without the noise and Nin is the noise power at the input.

That is only the noise power and SNRout is equal to Sout upon Nout. Now this T that is there just to signify that noise is associated with heat or the temperature and it is a function of temperature. So the SNR without changing anything in this circuit if you change the temperature, Nin and Nout will change and therefore the SNR values will also change.

So Fin FT this noise figure F, F is given as SNRin upon SNRout which I can write as Sin upon Nin as a function of T multiplied by Nout over SNRout as a function of T. This is equal to Nout over Nin removing the T term upon GA. So GA is nothing, we are zooming that this is the actual power delivered to the load upon the power available from the network. That is why this Sout upon Sin is equal to GA. and GA is equal to Sout over Sin.

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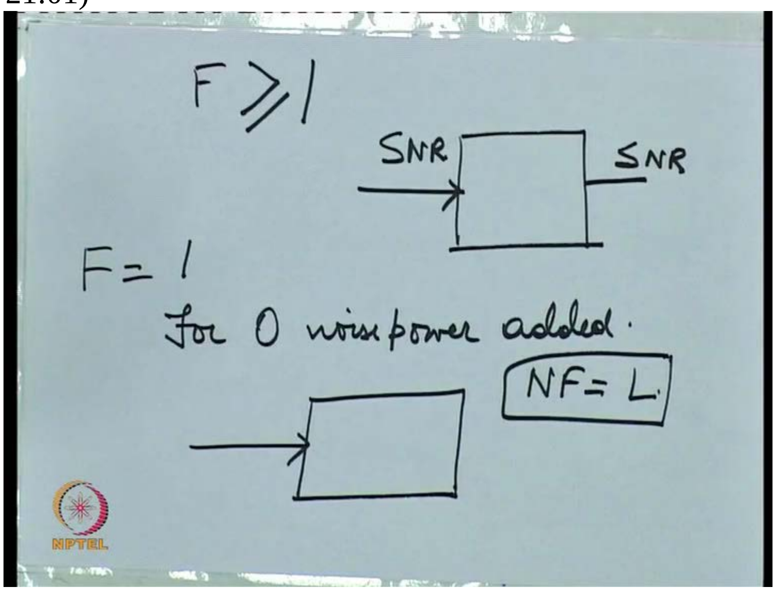

Now some of the properties of noise figure are that this F is always greater than or equal to 1. It cannot be less than one because in any 2 port network, the SNR of the input will always be greater or at best equal to the SNR of the output. They will be always sent out noise contributed by the network. Hence the signal quality will either remain the same or will be a little bit degraded than the output. Hence, SNRin can be never less than SNRout. Therefore F will always be greater than or equal to 1. F is equal to 1 for 0 noise power added that is when a network is perfectly noiseless, contributes absolutely 0 noise. Only then F will be equal to 0.

So, one of the properties of this noise figure for a passive device we saw that what is the definition of SNR for an active device. For a passive device, this noise figure. You can show that this noise figure is simply equal to the attenuation. That is, L is the attenuation. So attenuation is the reverse of gain. If instead of gain, we get attenuation then we can show that the noise figure will be equal to the attenuation.

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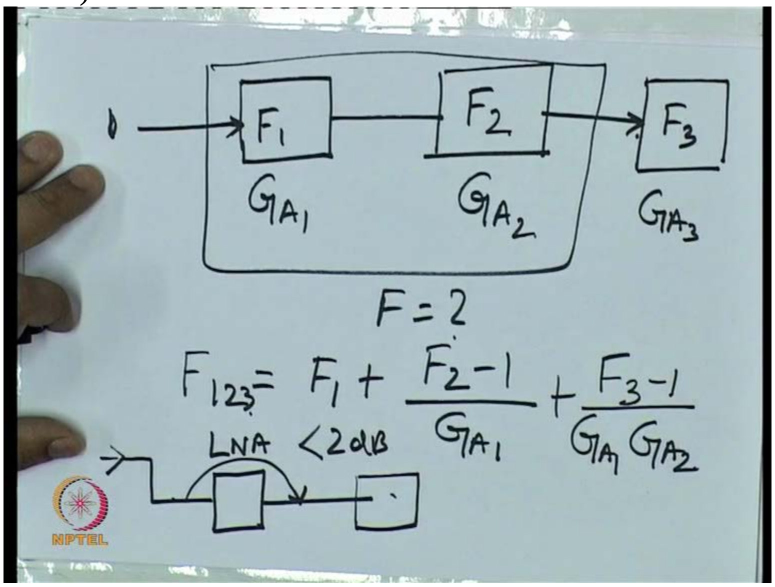

Now one other property of noise figure if we have a cascade of networks, say 2 networks, this has a noise figure value of F1 and available gain value GA1. This has available gain value GA2 and noise figure F2. Then, what will be the overall noise figure for the system? It can be shown that overall noise figure F12 will be equal to F1+F2-1 upon GA1. Suppose there was another stage with a noise figure S3 and GA3 then the value of the total noise figure that is F123 would be F3 -1 upon GA1 multiplied by GA2.

So what we see here is that when we have a cascade of elements connected, the highest noise contribution to the overall noise figure is from the 1st stage. Other subsequent stages are scaled down by the value of the available gain of the QCD stage. That is why, when we have a receiver connected to an antenna, the 1st stage is often what is known as the low noise amplifier. The purpose of the low noise amplifier is to boost this small power signal received from the antenna without contributing too much noise. That is why LNA is supposed to have a very good noise figure. In fact, it should be less than 2 dB.

And the purpose of LNA is not to provide power gain so much but to ensure that the signal is amplified from a similar value received, low-power value received from the antenna to a value which can be used by subsequent stages. And this LNA because the reason this LNA has to have a low noise figure is because of this equation. That is, it does not really matter what the subsequent stages what is the noise contributed by the subsequent stages as long as the noise contributed by the 1st stage that is the LNA is less. The overall noise figure of this chain of elements will also be low if the 1st element has a noise figure, low noise figure.

And special case, this is the case when we have the gain stage connected to an attenuator

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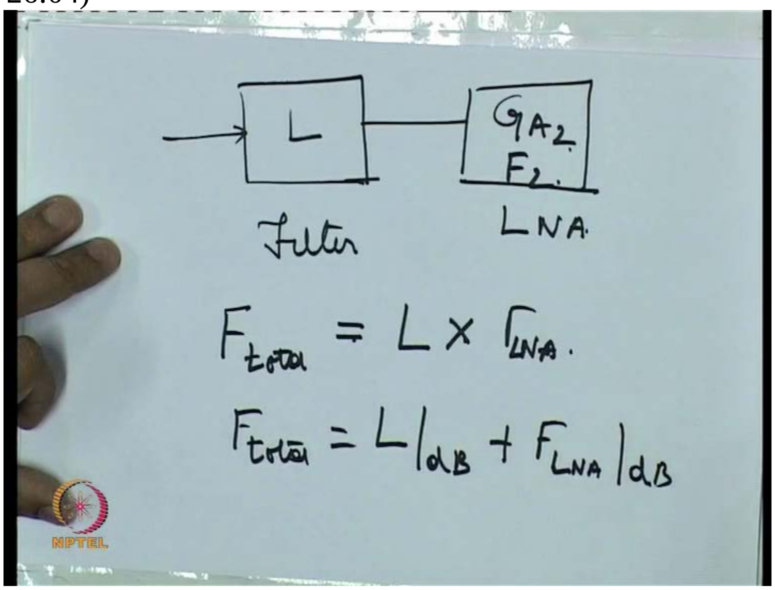

. So suppose, we have an attenuator connected in cascade with the gain stage. So see a this is an LNA and this is a filter. Now this is a passive stage. It does not have any gain. Instead, it has an attenuation, L. I said, the noise figure of a passive device that is one which attenuates the signal is same as the attenuation. But this LNA will have say again A2 and a noise figure F2. It can be shown that the total gain total noise figure is simply L multiplied by F LNA. Now in terms of dB it is L in dB plus F LNA that is a noise figure of the LNA in dB.

A related concept is what is known as a noise measure.

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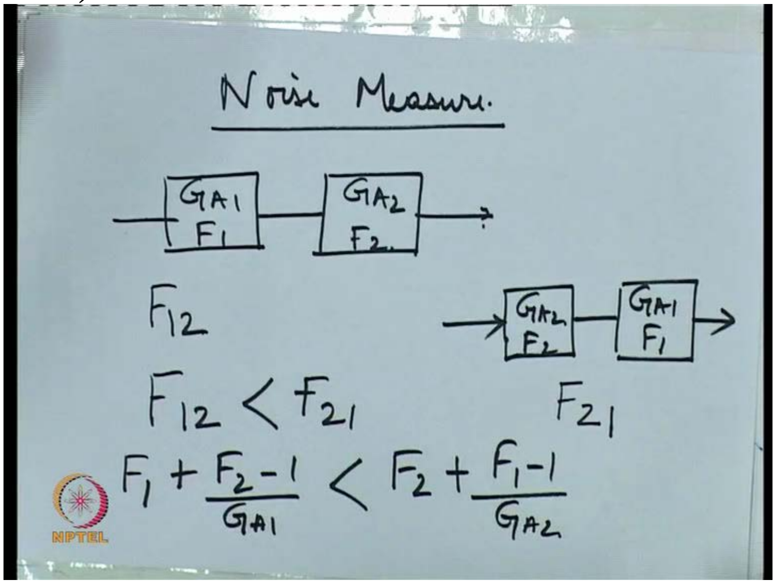

$$F_1 + \frac{F_2 - 1}{G_{A1}} < F_2 + \frac{F_1 - 1}{G_{A2}}$$

Noise measure is a quantity which is introduced because of the problems in determining which stage contributes more. This problem arises specially when we have 2 cascade status. Suppose, we have two one stage, GA1 and F1, GA 2 and F2. Another, GA2 1st, F2 1st and then followed by GA1 and F1. Now which one if I consider the overall noise figure for this case is F12 and overall noise figure for this case is F21, which will be the configuration that gives the lower overall noise figure? That is, if I want to have Let us say F12 lesser then F21 then F12 just from the formula that I gave in the previous slide will be given like this and F21 will be given like this.

Now suppose, the problem with this equation is this has elements of F1 and F2 are mixed on the LHS and RHS.

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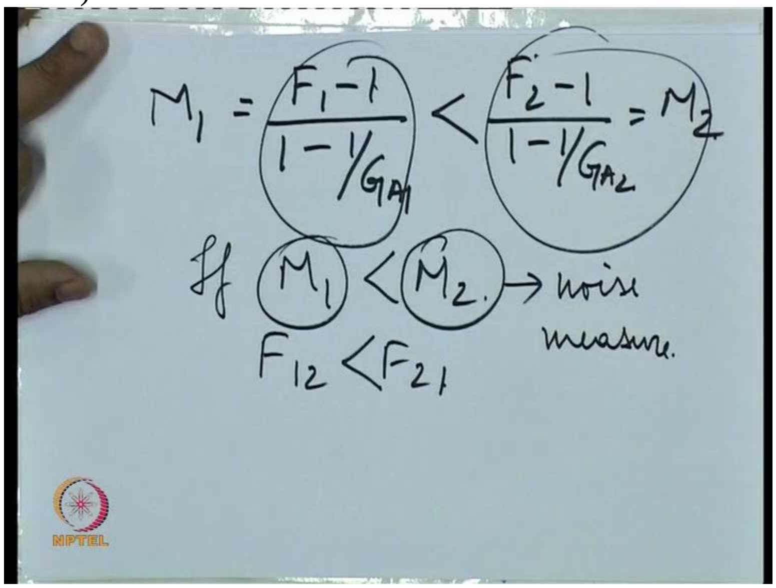

Instead, this equation can be rewritten as, if I rewrite the situation, the same equation like this that M1 equal to F1 -1 upon 1-1 upon GA1 is lesser than F2 -1 upon 1-1 upon GA2. And said this I call M2. If M1 is lesser than M2, both M1 and M2 have terms related to the 1st stage and the 2nd stage respectively. If M1 is lesser than M2, then we can ensure that F12 will be lesser than F2. And these quantities, M1, M2 are called noise measure. So we have noise figure and then noise measure.

So in summary in this module, we covered basics of noise. What is noise? How to characterise noise mathematically and then what is noise figure and how to find out the noise figure of a chain of our stages? Which element contributes most noise figure and then the concept of noise measure. That is when we have 2 elements or various elements in cascade, how to put them in cascade so that the lowest so that the chain will have the lowest noise measure of the various combinations noise figure of the various combinations possible.

And then we found out a new term called noise measure. In the next module, we will discuss about noise figure circuits. We will see that if we set the noise figure value of the circuit at a certain value, if we set if we want to obtain a particular value of noise figure in a 2 port circuit, then the locus of gamma S will be a circle and that circle is known as a constant noise figure circuit and using that, we can design a particular value noise figure that we want to obtain in the same way that we did for the constant gain circuit. So that will be covered in the next module. Thank you.