Microwave Integrated Circuits.
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Lecture -03.
Reflection coefficient, VSWR.

Welcome back to module 3 of the 1st week. In the previous module we had covered transmission lines and the equations governing them. In this module we will be covering what is known as reflection coefficient and VSWR.

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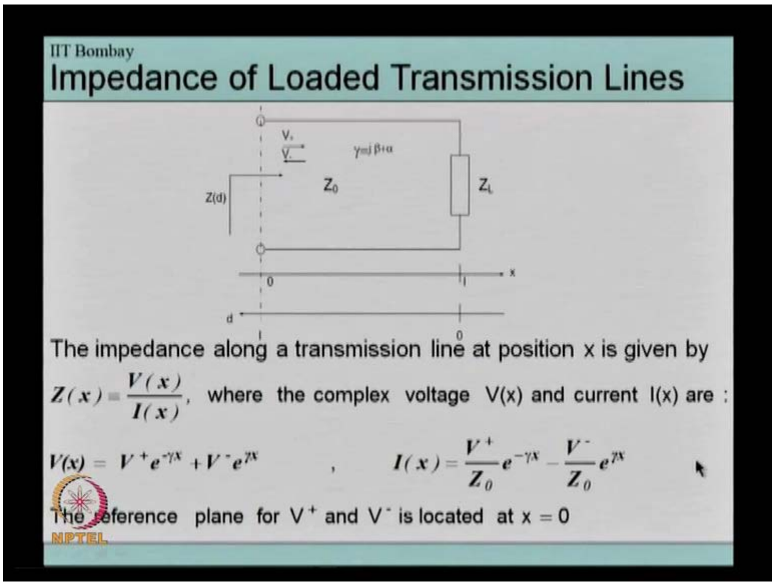

So, while solving the transmission lines equation, we have come across 2 questions, one for the voltage and one for the current. The voltage equation was given like this and the current equation was given like this. Here once again Gamma represents the propagation constant and Z0 is what is known as the characteristic impedance. So the impedance then should be equal to the ratio of Vx upon Ix, this is from the definition.

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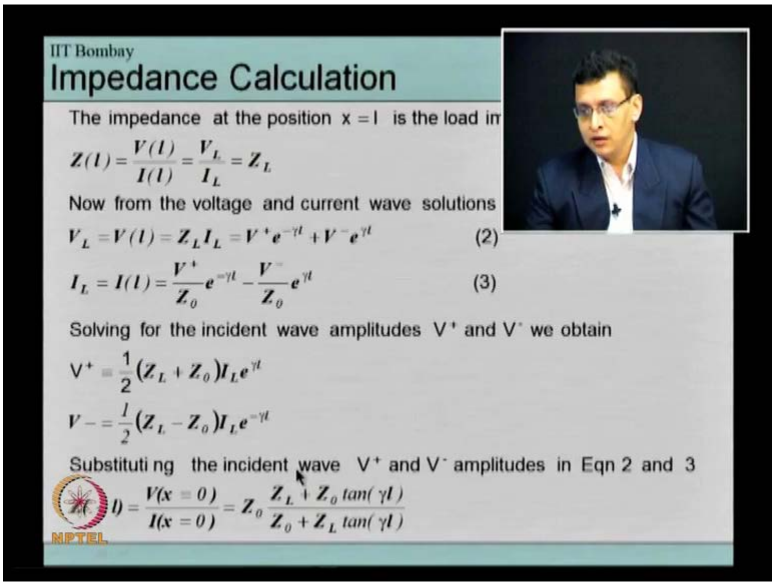

Now often what happens is that if we are calculating from the source reference plane, then this X that we have in this equation represents the distance from the source.

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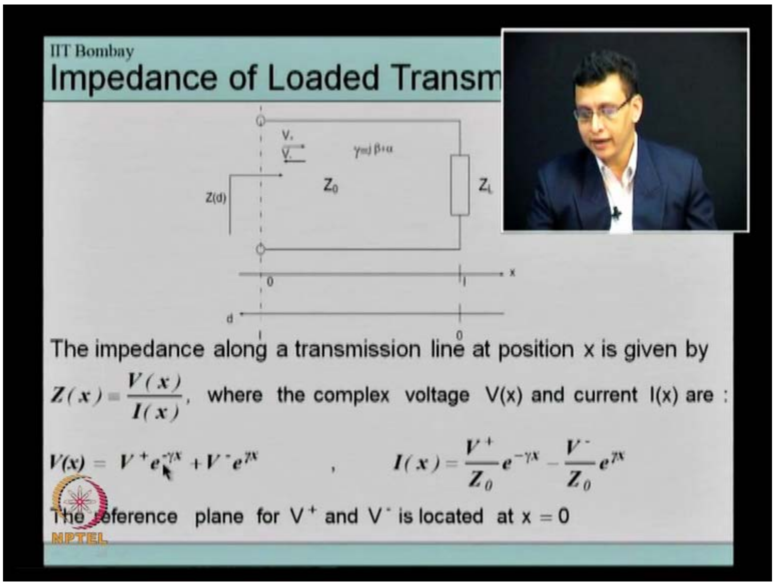

But that poses a challenge, the challenge being that what is the absolute quantity at the source that we know? If we have a source connected, then you might tell me that source itself is a reference. But there suppose we just take a transmission line with a load ZL connected to one end and the other end, we call it the source end, then you see we do not have any reference in the absence of any source that is really connected, we do not have any reference.

On the other hand the load that is connected to this end is a standard reference, by standard reference, I mean this value of load remains constant and does not change even if we have some variables at the source end, variables meaning if we have different types of sources. So, we can see that then the load at the load end, the presence of this ZL as a reference point may accept more standard reference as compared to the source.

So, then we should, we might as well start thinking of distances from the load end instead of the source end. So, that is why what we did was we had another reference plane D which moves in the opposite direction as compared to X and then by doing a coordinate transformation from X to D, what we get is this equation.

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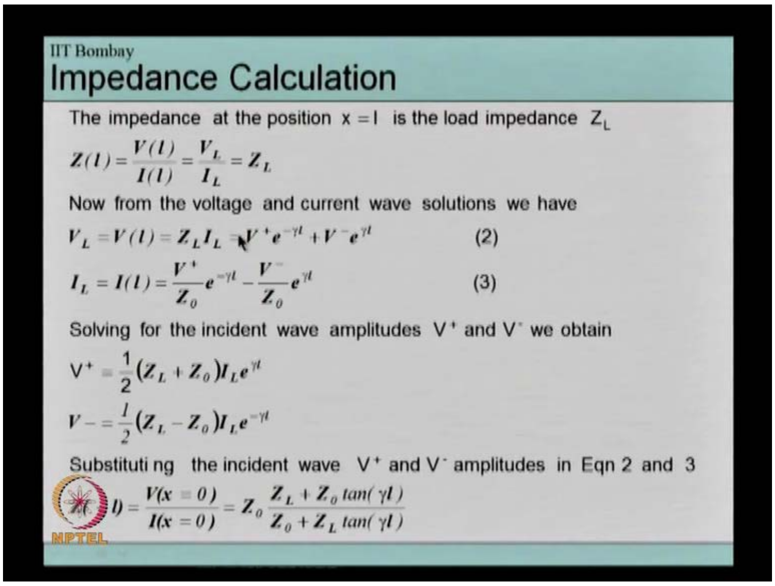

So, these equations you know are what we obtained when we do the coordinate transformation from X to D. And if the variable distance along D, we call that as L, then we get these equations and then on further solving these equations, we get the values of V+ and V- as this and then finally on solving for the impedance, we get this equation.

So, the input impedance for transmission line with load ZL connected is given by this equation.

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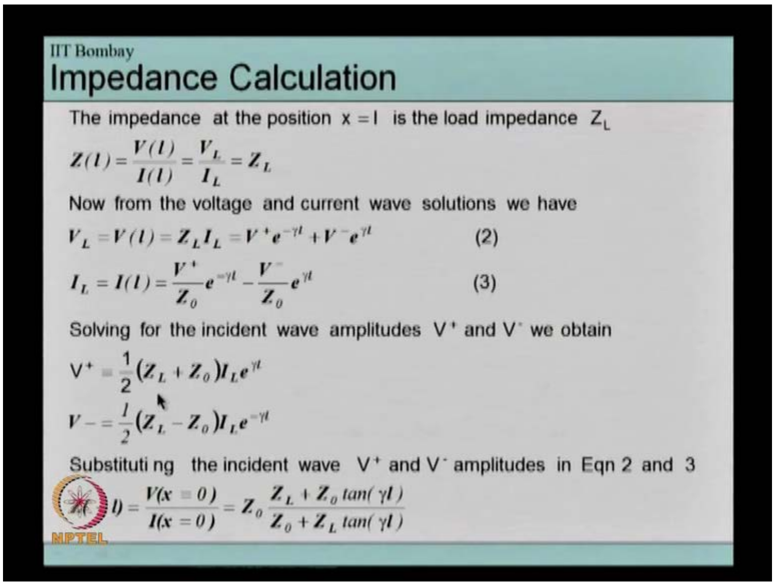

Repeat. Repeat. Now, if we continue using X as the variable, then we get the following relations for VL and IL, that is for voltage and current at the load end. The ZL, the impedance at the load end is equal to VL upon IL and that is equal to ZL. And from these equations if we solve for V+ and V-, we get these 2 equations. And then for finding out Z of -L, that is the impedance at the source, we get this equation.

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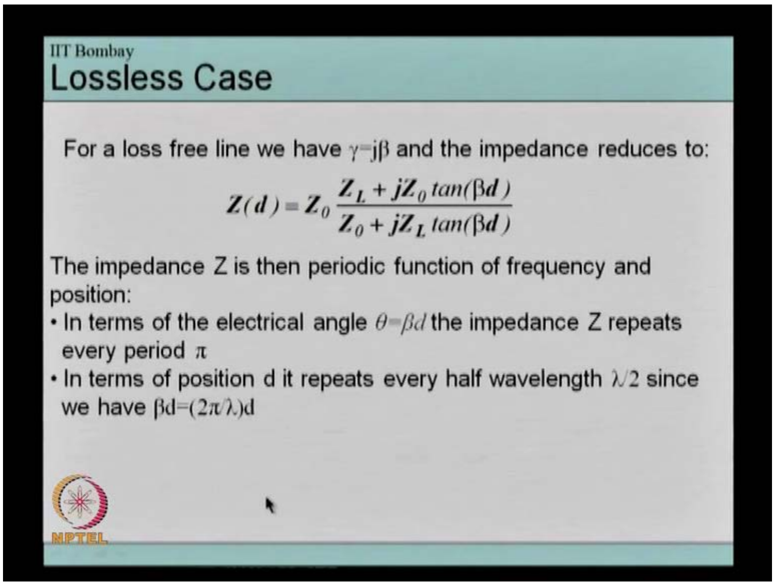

Now after doing the coordinate transformation from X to D, we get this equation and we see that from this equation what we can see is that the value of impedance for the transmission line does not keep increasing or decreasing monotonically with increasing D, it actually repeats because the Tan function actually repeats itself. In terms of electrical angle, every after a period Pie, this value of ZD will repeat itself. Or in terms of position, after every Lambda upon 2 lengths, ZD will repeat itself. So, it is periodic, both in electrical as well as wavelength and as well as frequency as we shall see later.

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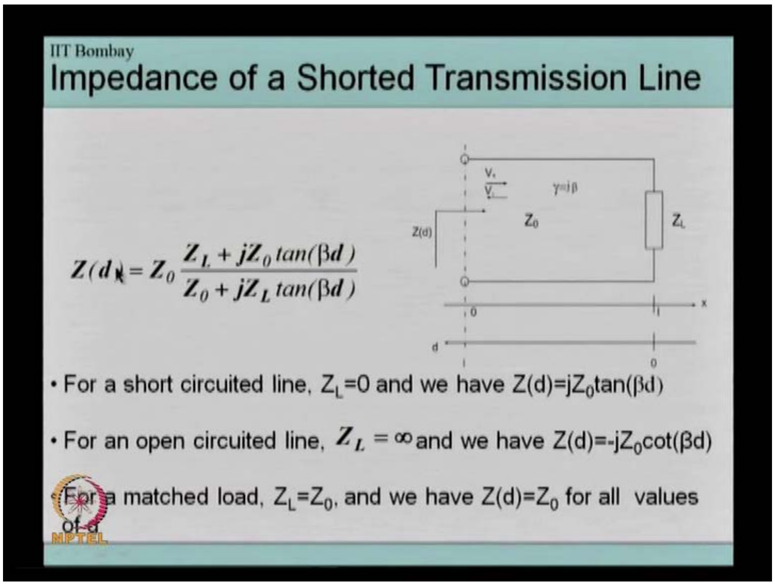

Now the impedance of this transmission line with the load connected at one end, we saw from the previous slide that the input impedance ZD here is given by this equation. Now based on this equation, we can derive some interesting cases. For example, if suppose ZL is 0, then we have the input impedance given by this equation, and if we have ZL is equal to Infinity, that is if this end is open circuited, then ZD is given by this equation and if we have ZL is equal to Z0, then we have ZD is equal to Z0.

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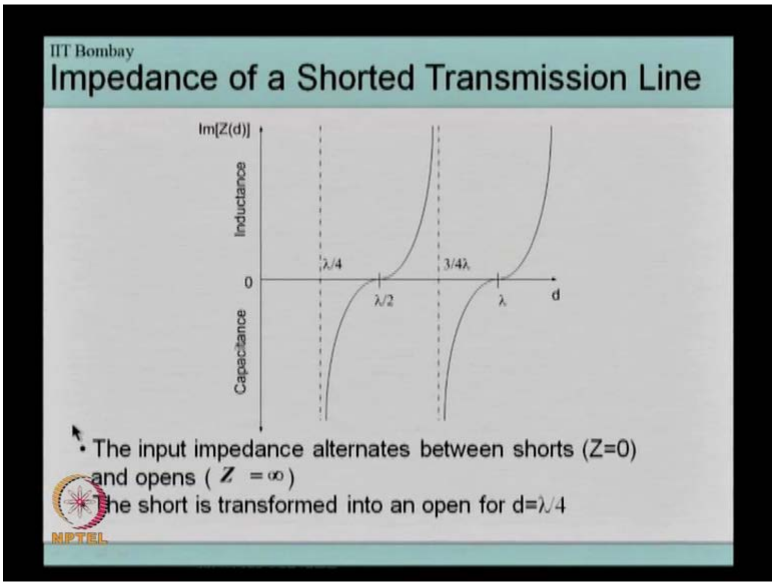

Now the question arises is this whether for the shorted line or for the open line using the formula that we saw in the previous slide, whether the transmission line acts as, shorted or open circuited transmission lines act as inductors or capacitors.

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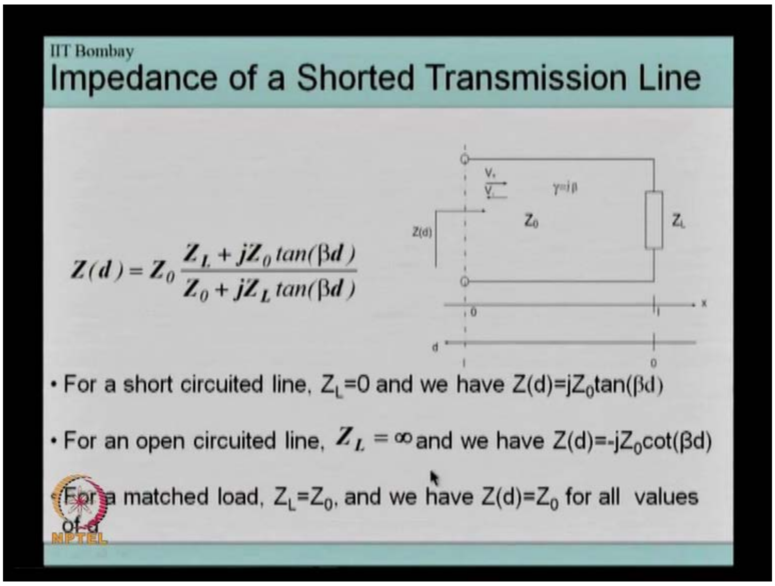

The answer is not so straightforward because as you can see from these equations, ZD is a tan function for a short-circuited line and for an open circuited line, ZD is a cot function. Now tan and cot, both can acquire positive and negative values, therefore the values of ZD for both the short-circuited lines and open circuited lines can be negative or positive depending upon the values of Beta D.

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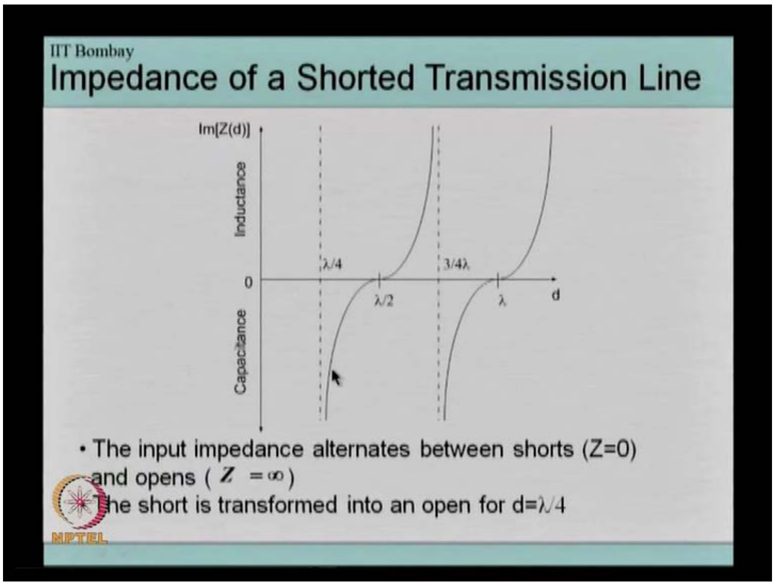

For a shorted transmission line or a short-circuited transmission line, if we plot the imaginary value of ZD and we get a curve like this, which is basically the Tan function. And you can work it out that for open circuited transmission line also, the shape will be the same except that the points of 0 crossing will be different.

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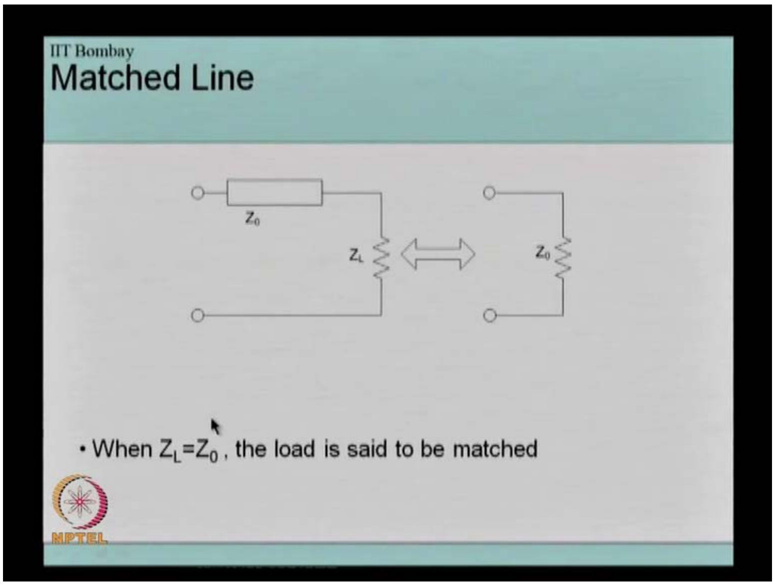

A matched line is a special case of transmission line where a load ZL with a transmission line having characteristic impedance Z0 is connected. Now, if ZL is equal to Z0, then the line is said to be matched. This will have other meanings also that we shall see later, that matching also will mean that when the reflection coefficient of the load end is 0. That means that there is no reflection at the load end, this will come in if you moments.

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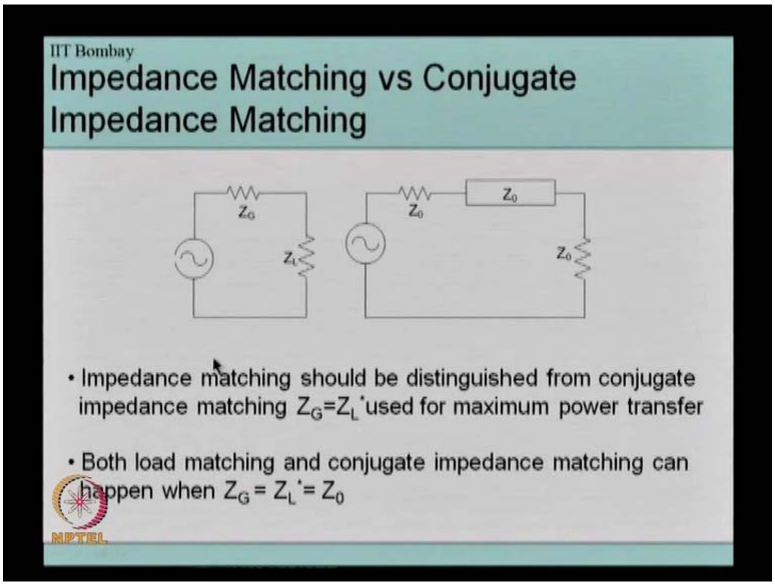

Now matching, note, is different from conjugate matching. Conjugate matching is a concept that is frequently used in lumped element circuits. The conjugate matching is a concept where it relates to the maximum power transfer to the load. If ZG is the source impedance, then maximum power will be transferred to the load when ZG is the conjugate of ZL, whereas matching in the transmission line sense is a case where there is no reflection on the load. So, these are 2 different components, 2 different concepts we have to keep in mind.

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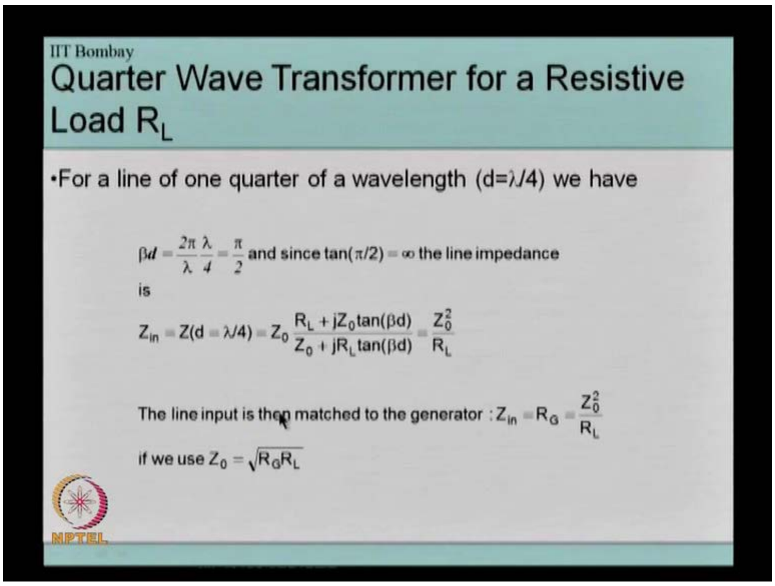

Another interesting type of transmission line that exists is what is known as the quarter wave transmission line. If suppose the total electrical length of the transmission line with loads ZL connected or RL connected at one end is Pie upon 2, this corresponds to a wavelength of Lambda upon 4 or quarter wavelength, then ZIn is given by this interesting relationship ZIn is equal to Z0 square upon RL. Now, if Z0 for the transmission line is kept constant, then we see that ZIn is proportional to the inverse of RL. So, that means a quarter wave Transformer inverts a load to its inverse. What it means that if suppose RL is a shorted stub, that is if RL is 0, then we will see infinite impedance at the source end up or if RL is open circuited, then we will see a short-circuit at the input.

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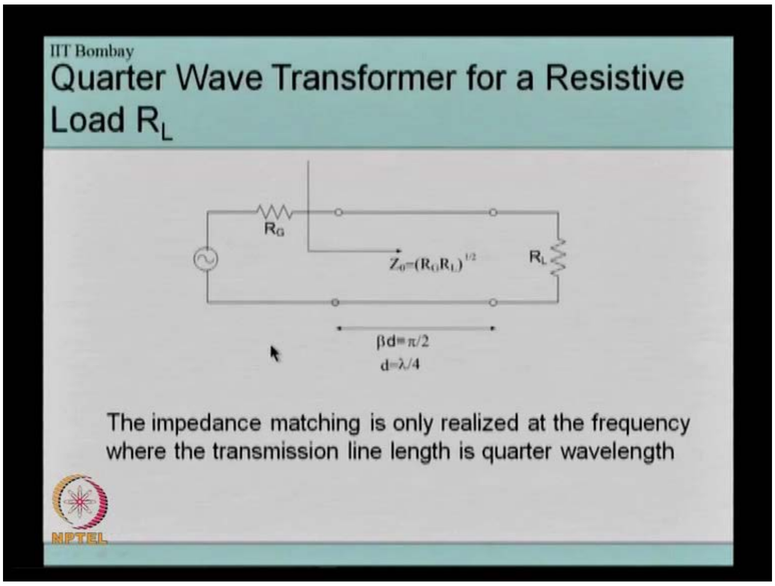

Now a quarter wave transmission line for resistive load is shown here. As discussed, the input impedance that we will see here will be the inverse of RL.

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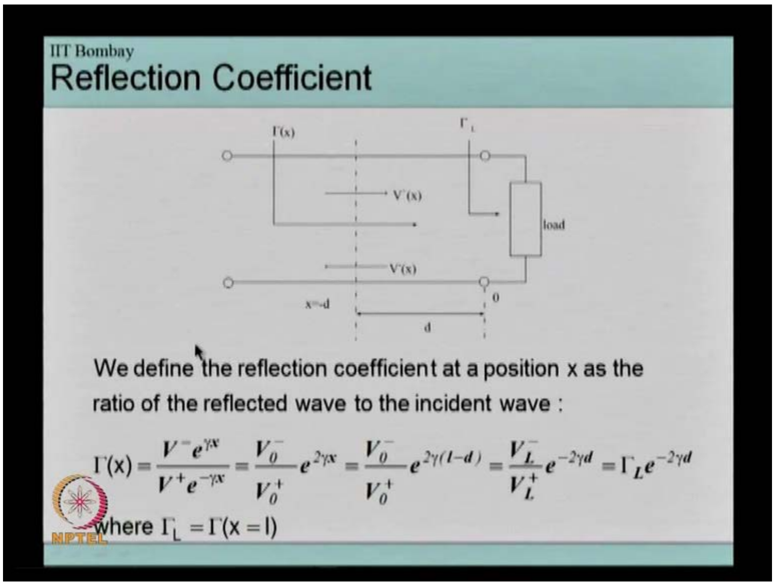

As I was discussing in the previous few slides that there is a concept called reflection coefficient. So reflection coefficient is simply the ratio of the negative travelling wave to the positive travelling wave. So, we saw that on solving the transmission line equations, we get 2 components, one is V+ component and the other is V-components. V+ represents the component of the wave travelling in the positive x direction, whereas V- is that component of the wave travelling in the transmission line that is moving in the opposite direction. V+x is also known as the incident wave and V-x as the reflected wave. So, Gamma X or the reflection coefficient is simply the ratio of the reflected wave to the incident wave.

And we see that Gamma X is given like this, on doing the coordinate transformation, we can write Gamma X like this where Gamma L is the reflection coefficient at the load end. So, this reflection coefficient is the value of Gamma X at this point. Now since the load is connected, we will say that his Gamma L is constant, if the load is constant, this Gamma L will also be constant.

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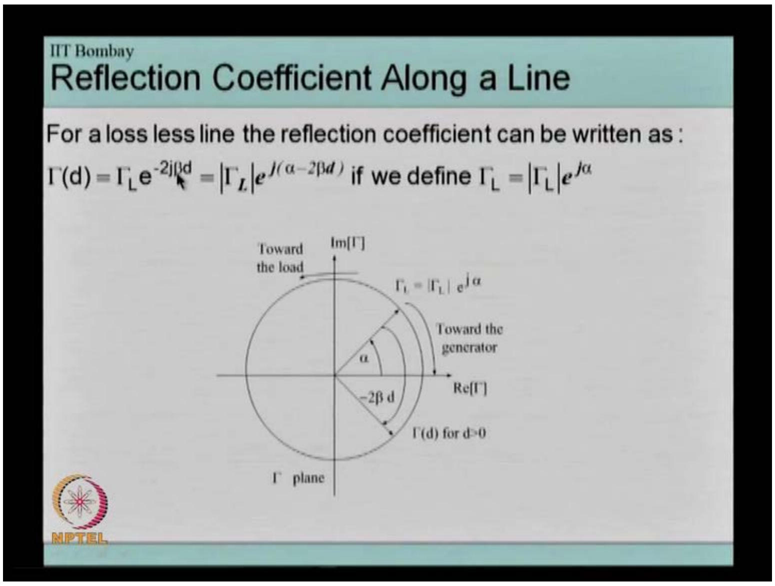

An interesting thing that we note is that if suppose we travel electrical and Beta D in clockwise direction, if we are moving in the clockwise direction, then what is happening is that Gamma D, if I write it like this… If D increases, that is if I am moving away from the load, then my phasor becomes more negative. If it becomes more negative, then I am travelling in clockwise direction. So, everytime if I am moving in a clockwise direction along this phasor diagram or if in terms of transmission line I am moving away from the load, then that translates to a clockwise rotation on this phasor diagram. Now this phasor diagram is simply a plot of the imaginary and real components of Gamma.

We see that whether we are moving away from the load or towards the load, the magnitude of Gamma D remains constant. In other words whether we are moving away from the load or towards the load, the locus that we will be traversing on this Gamma plane is that of a circle. And depending on whether we are moving away from the load or towards the load, we will be traversing in a clockwise direction or anticlockwise direction.

As I said, if I move away from the load, then my D value is increasing, my phasor becomes more and more negative, more and more negative means clockwise traversing. Other interesting thing is that if I traverse an electrical length Beta D along a transmission line, the total phasor change on the Gamma plane is twice of that value. So, if I move Beta D away from the load, I will be moving 2 Beta D angles in clockwise direction.

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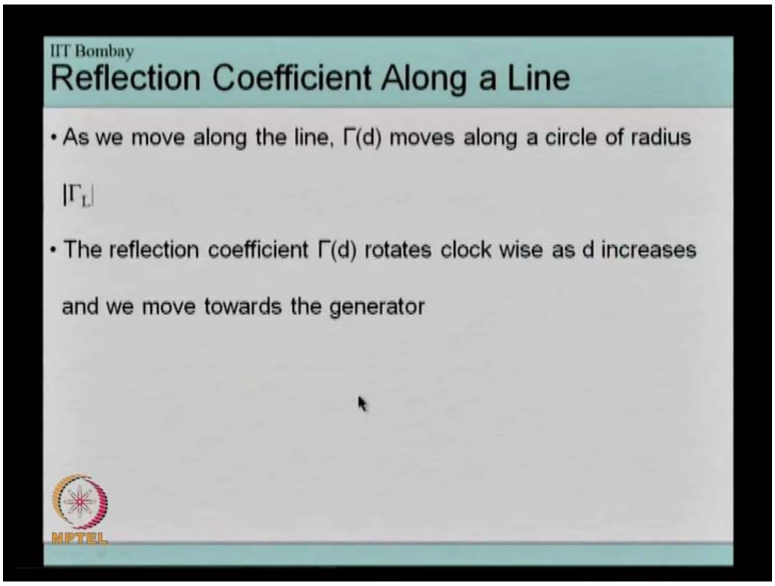

So, these are the conclusions that we can derive. As we move along the line, Gamma D moves along a circle of radius Gamma L magnitude and the reflection coefficient rotates clockwise as D increases and we move towards the generator.

The power dissipated on the load, again as I said, power is a very important concept in microwave engineering, we have to calculate at every point what is the power incident or reflected or what is the net power loss.

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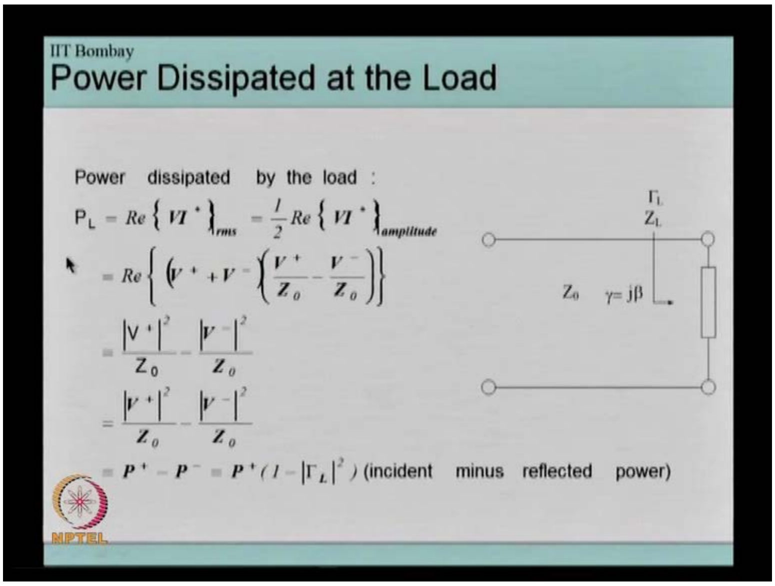

So, if we want to calculate the total power that is transmitted to the load, then we can write it along this equation and then after some mathematical derivations, we will arrive at this point.

Now P+ is a power of the incident wave, Gamma L is the load reflection coefficient and we see that is Gamma L is 0, then PL, that is the power delivered to the load is equal to the incident power. Now the question is… So we see from here if Gamma L is equal to 0, then the entire incident power is delivered to the load.

So, this is why I said that for matched lines, Gamma L should be equal to 0. That is as far as impedance matching in microwave engineering is concerned refers to that condition where all the power that is reaching the load is delivered to it and no component of the incident power is reflected back. And we see that that will happen when this reflection coefficient is 0. The question is what is that condition, when is that load reflection condition is equal to 0 comes?

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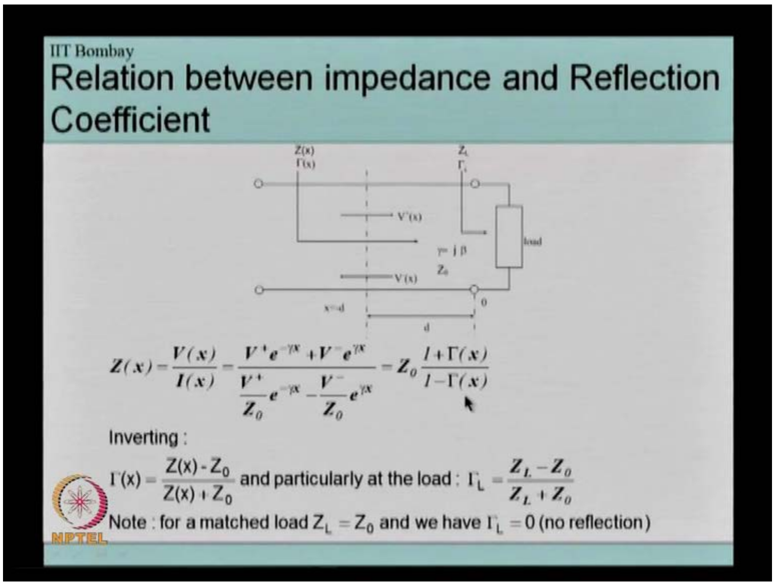

To find that out, what we do is that we find the value of the impedance ZX once again and that is given by this equation from which after some mathematical manipulation, we can get relation like this. So, here we have expressed the impedance ZX which was previously expressed in terms of V+, V-, Gamma and x and Z0 in terms of the reflection coefficient.

Now from this equation, after some mathematical manipulation we can find out an expression for Gamma X in terms of ZX. So, Gamma X at any point is simple equal to the ratio of ZX-Z0 upon ZX+ Z0. So, the Gamma at the load end, that is Gamma L from this equation should be equal to this relationship, where ZL the load impedance and Z0 is the characteristic impedance. For Gamma L to be 0, that is for the matched case that we were discussing, ZL has to be equal to Z0. So, in other words, the condition for matching is that the load connected at the load end should match the characteristic impedance of the transmission line.

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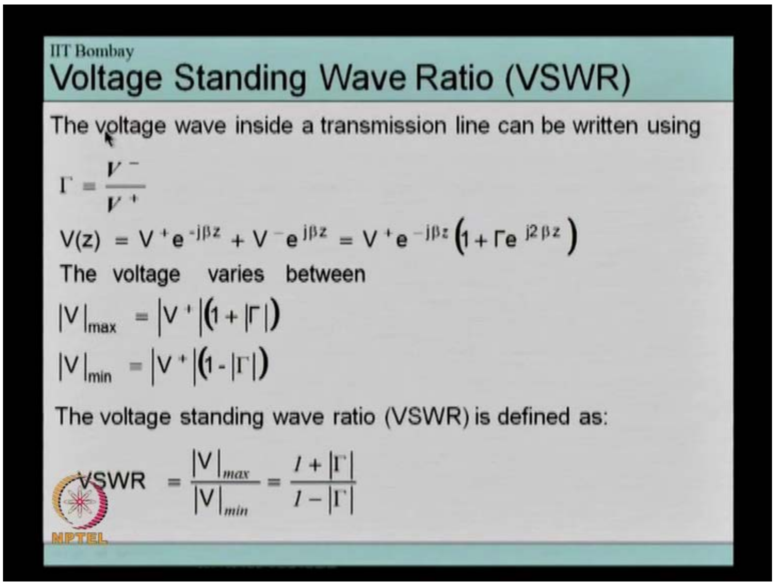

Another interesting concept that is frequently used in microwave engineering is the voltage standing wave ratio. Now this is a ratio that depends only on the magnitude of the waves, of the incident and reflected waves. They have no relationship to the phasor or the phase of the incident or reflected wave. Now the maximum voltage that is achieved at any point on the transmission line will be that, will be given like this when there is constructive interference between the incident and reflected waves and the minimum voltage that will be achieved at any point along the transmission line will be this. And this happens when there is destructive interference along the reflected and incident wave. Note that when Gamma is equal to1, that is when there is perfect reflection of the incident wave, the minimum value will be 0.

Now the ratio of Vmax to Vmin is what is known as voltage standing wave ratio.

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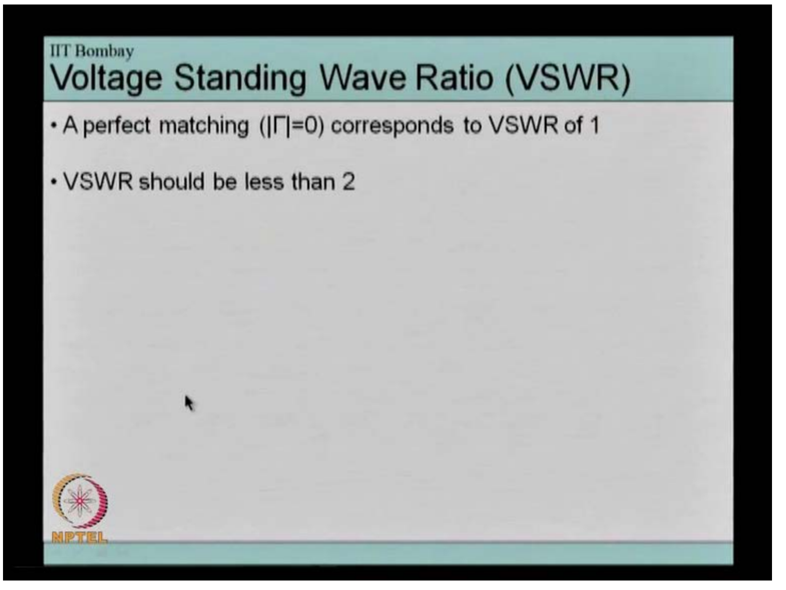

A perfect matching corresponds, that is a perfect matching refers to when the ZL is equal to Z0, then that corresponds to a VSWR of one as seen from this relationship,

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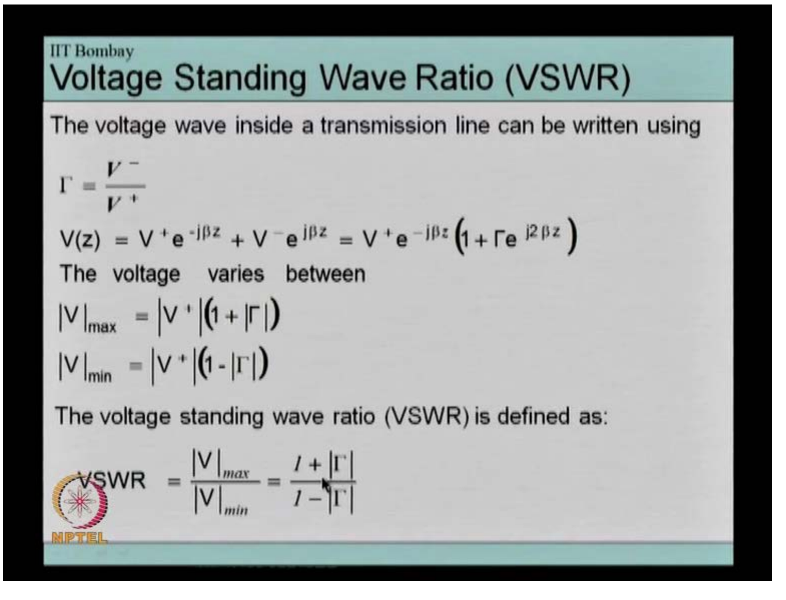

that is when Gamma is equal to 1, so when Gamma is equal to 0, I beg your pardon. We know that when Gamma is equal to 0, there is perfect matching and when that happens, VSWR is equal to 1. Usually for a good, usually in a good design practice, VSWR should be usually lesser than 2.

That brings us to an end of module 3.