

Microwave Integrated Circuits
Professor Jayanta Mukherjee
Department of Electrical Engineering
Indian Institute of Technology Bombay
Lecture No 33
Linearity

Hello, welcome to another module of this course Microwave Integrated Circuits. We are now in week eight and in the past few weeks we have been covered covering the various aspects of amplifier design, especially active circuit design like gain, then the frequency response then noise, matching and other things like that.

So here we will take yet another concept that is frequently associated with Linear Circuits or amplifiers and mixers which is the Linearity, the measure of Linearity. So in this module the first thing what is Linearity, what do you mean by Linearity?

So if you consider a memory less system.

(Refer Slide Time: 01:18)

Memory less System

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x(t)^2 + \alpha_3 x(t)^3$$

$$\text{For } x(t) = A_{in} \cos \omega t$$

$$y(t) = \alpha_0 + \frac{\alpha_2 A_{in}^2}{2} + \left(\alpha_1 A_{in} + \frac{3\alpha_3 A_{in}^3}{4} \right) \cos \omega t$$

Now in a memory less system, the output and input relationship can be written like this. So if my input $x(t)$ is like this of this form $A \cos \Omega t$, then the $y(t)$ that I get will be like something like this, where $y(t)$ follows this relationship that I have given earlier. So $A \cos \Omega t + \dots$ + just if I continue here.

So this is this whole thing multiplied by $\cos \Omega t$ and then below that we have

(Refer Slide Time: 03:08)

$$+ \frac{\alpha_2 A_{in}^2}{2} \cos 2\omega t + \frac{\alpha_3 A_{in}^3}{4} \cos 3\omega t$$

$$A_{out} = \alpha_1 A_{in} + \frac{3\alpha_3 A_{in}^3}{4}$$

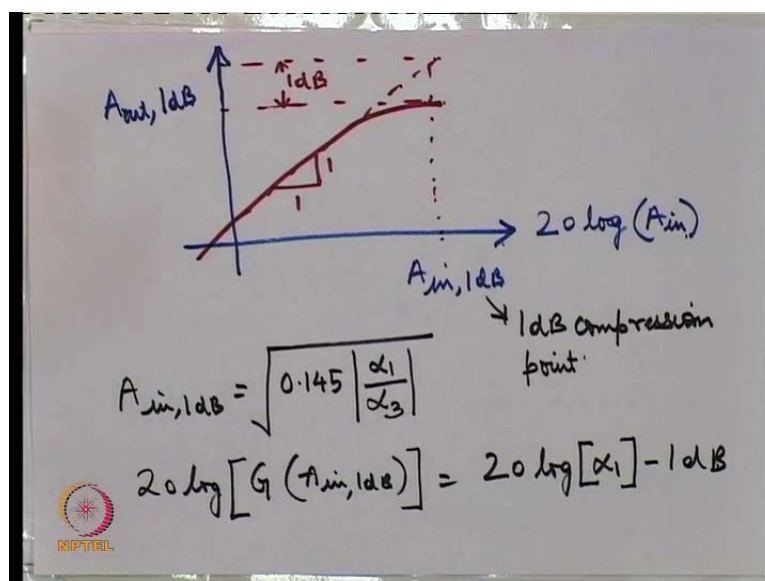
$$Gain(A_{in}) = \left| \alpha_1 + \frac{3\alpha_3 A_{in}^2}{4} \right|$$

$\alpha_3 \Rightarrow$ usually negative

So you see I just consider the fundamental component, then that fundamental component will have amplitude $\alpha_1 A_{in} + 3\alpha_3 A_{in}^3 / 4$. So now so the output at the fundamental frequency will have amplitude given like this. And again at the fundamental frequency for particular amplitude A_{in} will be equal to.

Now the thing with this α_3 is that it is usually negative, if it is usually negative then you see with increase in A_{in} , this value will become, this component will become more and more negative and it will subtract from this α_1 . And as a result what happens is this gain, it will have follow-up curve like this with increasing amplitude. The gain if I use the red pen.

(Refer Slide Time: 05:08)



So, you see this gain would follow a curve like this and because that Alpha 3 is negative, so as the input amplitude increases, the gain will gradually reduce from what it should have been by 1 dB for a certain value of the input power which is known as the 1 dB compression point. So, this is that value of the input power for which the value of the gain will be 1 dB lesser than what it should have been.

And in fact, in terms of those Alpha 1 and Alpha 3, this 1 dB compression point is given by... So at the 1 dB, we will have $20 \log g A$ in 1dB that is this power. At this power level we will have the gain which should have been, had it been a perfectly linear system then this would have been the gain, 1 dB less than this.

So this is one way of characterizing the linearity of a system. This 1 dB compression point is a very important figure of merit and you will see various circuits are characterized by 1 dB compression point. However, the limitation of this 1 dB compression point is that this is figure of merit at a particular frequency only. We are not considering the effect when we say multiple frequencies present.

In other words, if we have multiple frequencies present, will that also have an effect on gain reduction like we had at a single frequency? Let us see that. So suppose we have a system where we are introducing 2 frequencies, closely spaced frequencies. So this is our amplifier and say we have 2 frequencies at ω_1 and ω_2 and say this has some kind of, if this is a non-linear system.

(Refer Slide Time: 08:46)

Diagram illustrating a non-linear system (amplifier) receiving two input frequencies, ω_1 and ω_2 . The output shows the original frequencies ω_1 and ω_2 , along with intermodulation products $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$.

Input signal: $x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$

Output signal: $y(t) = (\alpha_1 A_1 + \dots) \cos \omega_1 t + (\alpha_1 A_2 + \dots) \cos \omega_2 t + \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 - \omega_2)t + \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 - \omega_1)t$

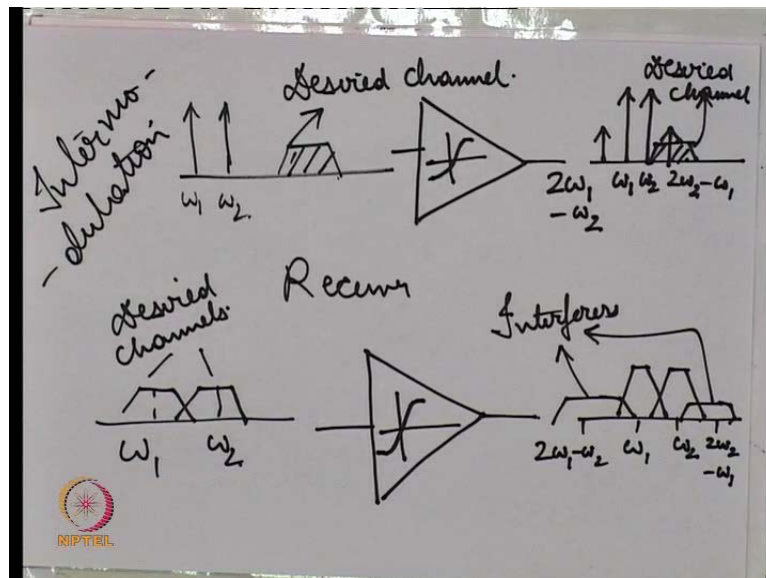
The input-output relationship is of some non-linear way. Now let us see, if we suppose our $X(t)$, since now we have 2 tones or 2 frequencies at the input, we can write our input $X(t)$ like this. So then now this $Y(t)$, if we use that same expression for Y that we had used earlier that is $Y(t)$ and $X(t)$ are related by this relationship, this equation.

So then with now two tones present, the output $Y(t)$ will be given like... Okay, will be something like this. Now, what you see here is now that we have 2 additional components frequency components, one is at a frequency $2\omega_1 - \omega_2$ and other at $2\omega_2 - \omega_1$. So at the output what we will have is, we will continue having our original tones at ω_1 and ω_2 with amplitude given like this and this.

And 2 other components at $\omega_1 - \omega_2$ and $\omega_2 - \omega_1$ with amplitude given by these values, okay. Sorry, that will be $A_1^2 A_2$, this will be $A_2^2 A_1$, this is $A_1^2 A_2$, okay this is note correction. So the output will now look like this, so we will continue having our original frequencies at ω_1 and ω_2 . In addition we will also have $\omega_1 - \omega_2$ and yet another frequencies at $2\omega_2 - \omega_1$.

So this is the problem here. Now what happens is that the problem with this setup is somewhat like this actually.

(Refer Slide Time: 11:56)



Suppose, originally we had our 2 tones and then this is supposed these are two interferers at ω_1 and ω_2 and say if this is our desired channels then say this is our desired channel of frequencies that we want to amplify.

Now after passing through this amplifier, we will get outputs like this. But then now this interferer will now occupy our desired channels, so this is our desired channel, this is our desired channel and then this is the problem. You know that, this is the problem that is seen in the receiver, that we have a desired channel but because of the interferers, there are now additional components in the desired channel and thereby it corrupts the desired channel.

The other scenario could be you know suppose at the especially at the transmitter side, transmitter side say we have 2 desired channels with centre frequencies at ω_1 and ω_2 that we want to transmit. What happens in the output is that in addition to the desired channels, so the desired channels will now be amplified, we also have now interferers. These are the interferers.

In fact, so significant is this distortion produced by this effect this intermodulation, this effect is known as intermodulation. Not harmonic modulation, intermodulation. It is interference between caused by 2 different frequency tones.

So significant is this distortion that there is actually just like that 1 dB compression point there is actually a measurement quantity, a quantity used to characterize this distortion and it is called the in order intermodulation product or I_{p3} .

(Refer Slide Time: 15:33)

Third order Intermodulation Point: IP_3

$$|\alpha_1| A_{in, IP_3} = |A_{out, IP_3(2\omega_2 - \omega_1)}|$$

$$= \frac{3}{4} |\alpha_3| A_{in, IP_3}^3$$

$$A_{in, IP_3} = \sqrt{\frac{4}{3} \frac{\alpha_1}{\alpha_3}}$$

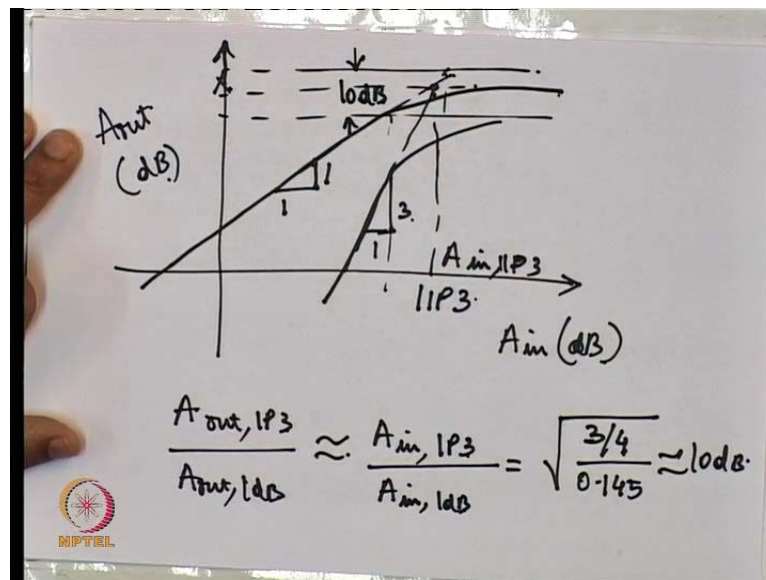
NIPTEIL

You know we can solve that A_{out} that particular value of input for which, so I_{p3} is defined as that value of input power for which the fundamental amplitude of the fundamental signal is equal to the amplitude of the intermodulation products, so we have α_1 is

equal to okay. And this in turn as you know the amplitude of intermodulation products is equal to... So from here we get this A in I p 3.

So this is the slope of the intermodulation products which is 3 is to 1, where as the slope of the fundamental is 1 is to 1.

(Refer Slide Time: 17:21)



And the point where the two meet is called the that input power at that particular is called I i p, it is the A in I I p it is I p.

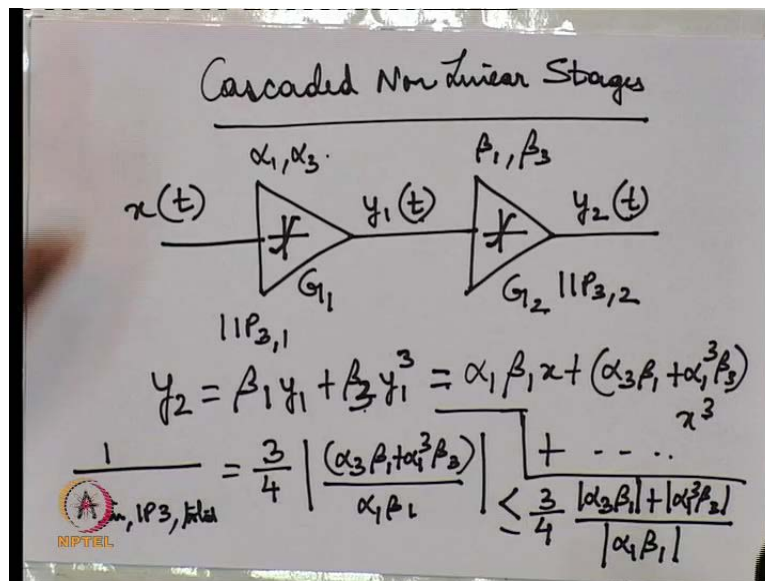
So this is the definition of the intermodulation products, so here you have A out in dB, this is A in along the X axis and this is the slope that the third order modulation product follows and this is the slope that the fundamental tones follows.

Now this deviation from the straight line is due to the compression, again compression. However, if you ignore the gain compression, then there would have met at this point, at a certain point and that point is the I P 3 value. In fact, there is a relationship between I p 3 and output intercept point and the output 1 dB compression point and that is nearly equal to the input third order intercept point and the input 1 dB compression point.

And that is equal to 3 upon 4 point upon point 1 4 5 and that is nearly equal to 10 dB, okay. So what you have is that this say if this is my I p 3 point and say this is my 1 dB compression point, then there will be 10 dB difference between the two on the output axis.

If you want to find out the output of a cascaded, then suppose we have number of stages cascaded, so we have stages like this.

(Refer Slide Time: 19:54)



So these are the non-linear coefficients of the non-linear polynomial. So you have this Y 2 is equal to Beta 1 times Y 1 + Beta 3 times Y 1 cube. This Y 1 and if I further write the relationship between Y 1 and X t, the relationship comes out like this.

And then the total input in the you know the total I p 3, if suppose A in is the total I p 3, and this will be less or equal to 3 by 4 Alpha 3 Beta 1 + Alpha 1 cube Beta 3 upon Alpha 1 Beta 1. And this relationship can simply be written as...

(Refer Slide Time: 22:13)

$$\frac{3}{4} \frac{|\alpha_3 \beta_1| + |\alpha_1^3 \beta_3|}{|\alpha_1 \beta_1|}$$

$$= \frac{1}{2} \frac{1}{A_{in, IP3,1}} + \frac{\alpha_1^2}{A_{in, IP3,2}}$$

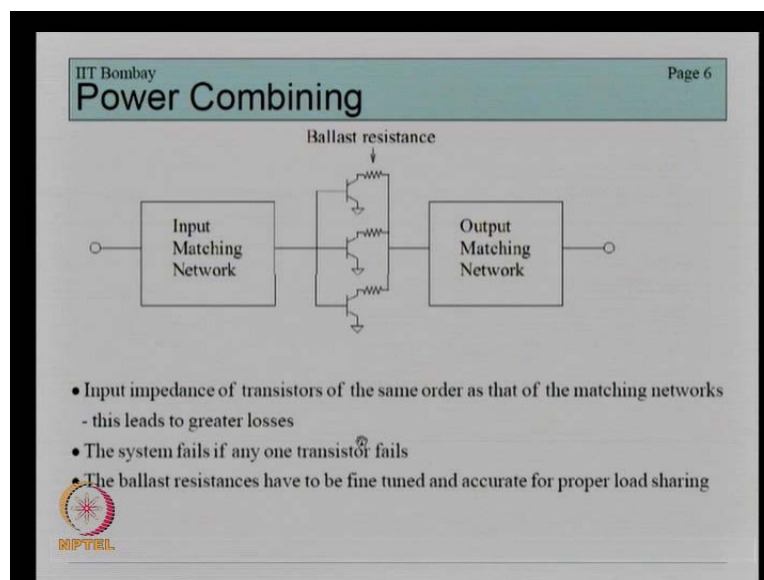
$$\frac{1}{P_{IP3, total}} = \frac{1}{P_{IP3,1}} + \frac{G_1}{P_{IP3,2}}$$

So I have 3 upon 4 Alpha 3 Beta 1 + Alpha 1 cube Beta 3 upon Alpha Beta 1. This is equal to 1 over the input intercept point of the first stage like this.

So, from here we can say that one over the input intercept point total is equal to 1 over input intercept point the first stage multiplied by the gain of the first stage over I P 3 of the second stage. So from here that we see that the input, that I p 3, the linearity of the end-stage in the in the chain of cascaded stages matters the most, so this is opposite to the effect of the noise figure, in case of noise figure we saw that the noise figure of the 1st stage in the chain matters the most.

In the case of I p 3, the linearity of the last stage matters the most. So far, we have been analyzing some of the parameters used to characterize the linearity of a circuit, but let us now see some of the methods in which how we can improve the linearity. So one of the ways you know if we go to the slides on the monitor is a method called “Power Combining”.

(Refer Slide Time: 24:02)



Now power combining seems as you see that the linearity will decrease with increase in input power, so if we could separate, if we could have individual amplifiers which which will individually amplify the signal, and then at the output we combine the various powers and that would increase the improve the linearity of the circuit.

So this is one method where you have an input matching network and then three individual transistors each of which produce their amplification and then finally at the output of the transistors, the powers are combined through an output matching network. so but the problem with this particular circuit is that, if any of the transistors fail, then the whole system fails and also these resistances, the output resistances have to be fine tuned.

So that you know there is proper balance between the individual circuits. So yet another way of achieving this power combining is, using a power divider or repeatedly using power dividers we repeatedly divide the powers and then combine, then amplify the individual signals and then combine it using combine.

(Refer Slide Time: 25:32)

IIT Bombay Page 7

Power Combining

40 W Amplifier

- A 40 W amplifier can be realized with four 10 W amplifiers ($M = 2, N = 4$)
- In practice some power is lost in the power combiner/dividers
- Efficiency $\eta = \eta_{\text{amplifier}} \times \eta_{\text{combiner}}$ with $\eta_{\text{combiner}} = 1^M$ and $N = 2^M$

NPTEL

One other way in, what happens during combining is that the overall linearity of the transistor or I p 3 of the transistor is not improved, but just that the curve is shifted away.

(Refer Slide Time: 25:42)

IIT Bombay Page 9

Power Combining

- Power saturation is the same
- Gain degradation possible
- Less intermodulation products

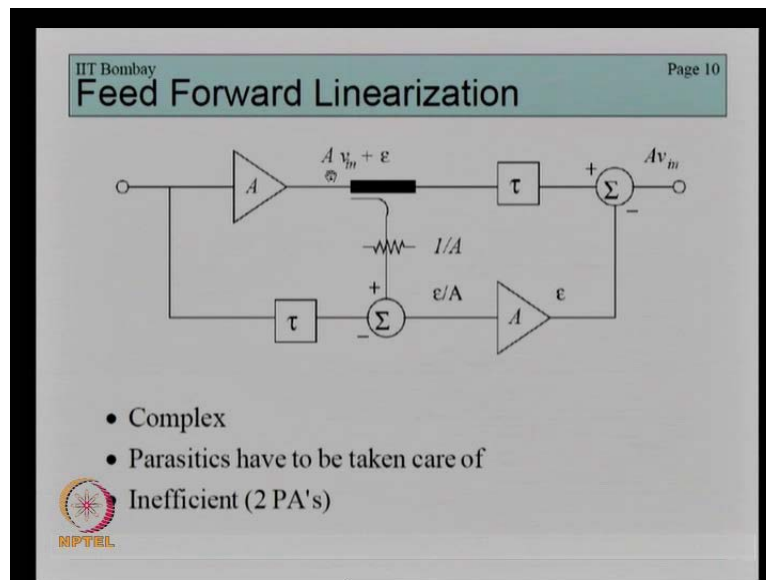
NPTEL

So if initially this blue curve was representing the transistors input output characteristics, now due to this power combining the entire characteristic shifts rightwards and so we get that the

input power at which the saturation or gain reduction starts happening is now shifted to a higher value and that is why we are getting the higher value of gain.

Then yet another method of achieving this linearity is what is known as Feed Forward Linearisation.

(Refer Slide Time: 26:35)



So here since you know the output of this amplifier can be thought of as the combination of the correctly amplified input signal + some error.

Then if we now have a system where we can use this error, where we can you know remove this error, so to do that what we do is this output signal is now subtracted from its own version using a delay, so as to account for this delay of this amplifier. Then if that you know the correctly the linearly amplified part is subtracted, then all we will have remaining is the error part.

And if that is also you know while combining in this summer, if we pass it through an attenuator having an attenuation equal to the amplification of this amplifier, then what we ultimately get here is the error signal divided by the amplification factor A. Now if you again pass this through an identical amplifier assuming this as very little noise or which produces very little nonlinearity because the input signal is at very low level.

So, it is much [fur] further away from its nonlinear region. So what we get at the output is the absolute, that is the error factor again and now if we again subtract it from this signal after

proper delays, then we can get back the correctly amplified version of the input with no nonlinearities present.

Ideally no non-linearities should be present, but in practice some non-linearities always present. The problem with this is the correct tuning of the devices, that is we need two identical stages and then this delays have to be tuned, this amplifier has to be tuned and also we need to summers which have to be accurately working.

Also, since we need to amplifiers for this one, it is also inefficient, it improves the linearity, but it reduces the efficiency. So, so with this we come to end and to this module, in this module we have discussed about the various parameters used to characterise the linearity of a circuit and some of the techniques used to improve the linearity. Thank you.