Microwave Integrated Circuits.
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Lecture -04.
Smith Chart.

Hello, welcome to week 4 of this course microwave integrated circuits. In the previous module we had covered the concept of reflection coefficient and VSWR. In this module we are going to cover other important aspects of microwave engineering, this is known as the Smith chart. As we have discussed in the previous module that as we move away from the load, we traverse a clockwise rotation along a constant various circle. So, the concept of Smith chart arises from this interesting phenomenon that we see in transmission lines.

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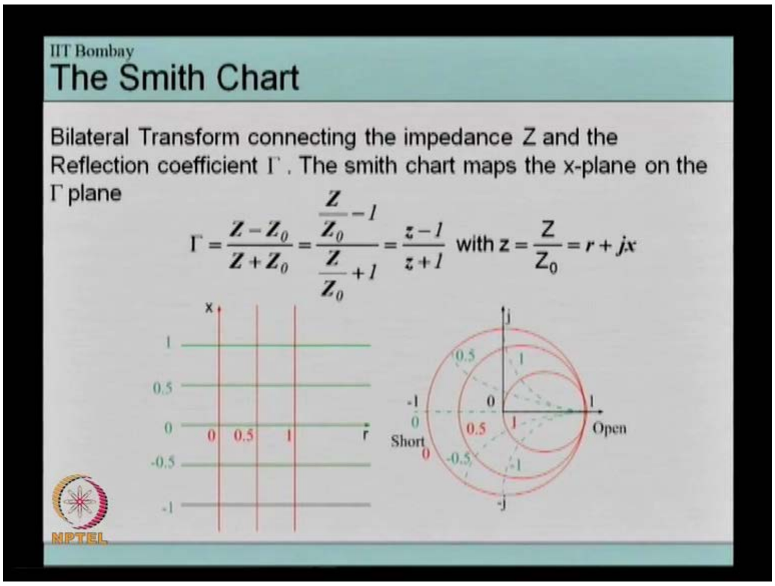

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The Smith Chart

Bilateral Transform connecting the impedance Z and the Reflection coefficient Γ. The smith chart maps the x-plane on the Γ plane

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1} = \frac{z - 1}{z + 1} \text{ with } z = \frac{Z}{Z_0} = r + jx$$

But as we move away or towards the load, the magnitude of the reflection coefficient remains constant. And the phasor keeps on changing.

Now the Smith chart is what we call the bilinear transformation of the impedance from the Z plane to the Gamma plane. As you can see this formula, even though I am saying it is a bilinear star formation, this formula is analogous to the formula for the reflection coefficient. So, a Smith chart is nothing but a conversion of Z impedances into reflection coefficients. Usually even though Gamma is given… The transformation is given like this, it is more convenient to express this transformation like this.

Now Z upon Z0, I call this the normalised impedance and represented by z. So, then Gamma is equal to [z-1] upon [z+1] and if I plot these constant resistance lines represented by these are red lines and these green lines are representing what I am calling constant reactance lines. If for these lines if I do this transformation, then these are red and green lines are transformed to these circles, these red and these green circles. These green dotted lines are actually portions of circles, now for Z for say x is equal to 1, I get a circle like this. For x is equal to 0.5, I get a circle like this. So, this circle gets converted to this circle, I beg your pardon, this straight line gets converted to this circle.

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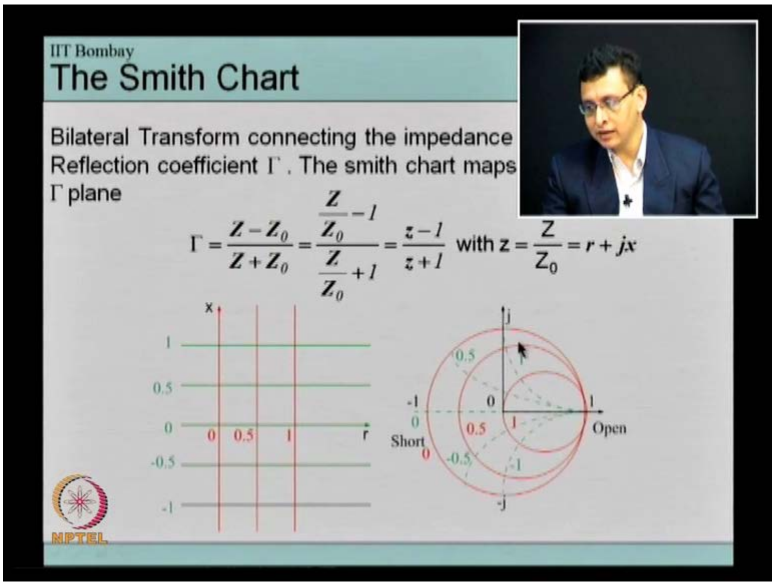

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The Smith Chart

Bilateral Transform connecting the impedance Reflection coefficient Γ. The smith chart maps Γ plane

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1} = \frac{z-1}{z+1} \text{ with } z = \frac{Z}{Z_0} = r + jx$$

This constant reactance straight line is converted to this constant reactance circle. This constant resistance straight-line is converted to this constant resistance circle.

So, it is a kind of conversion from straight lines to circles. But another advantage is that the entire rights half of the Z plane, that is those values of Z for which r greater than 0 is now enclosed within this circle having radius 1. So, in other words, all passive impedances are now concentrated within this circle and I can visualize all these impedances in a small space. Outside this biggest circle, by biggest circle I mean the circle having radius 1, the real part of the impedances will be negative and that corresponds to active devices.

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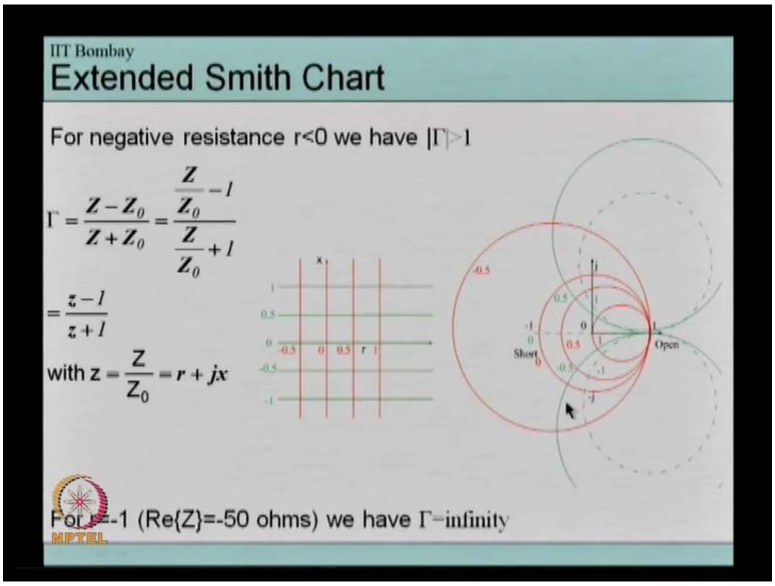

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Extended Smith Chart

For negative resistance r<0 we have $|\Gamma|>1$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1}$$

$$= \frac{z-1}{z+1}$$

with $z = \frac{Z}{Z_0} = r + jx$

For r=-1 (Re{Z}=-50 ohms) we have Γ=infinity

So, this is a more elaborate view of the transformation, as I said, these green lines which are confined within the circle having radius 1, they are actually the portions of bigger circles. And as you can see, impedance having constant resistance of -0.5 will be this large circle which is outside the unit circle.

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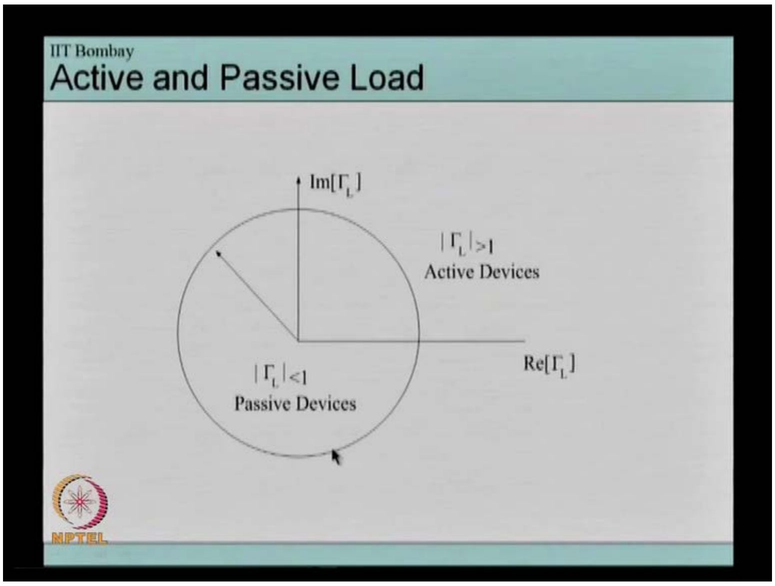

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Active and Passive Load

So, this is once again summarising what I was saying. All passive devices having Gamma L (i.e. reflection coefficient) less than or equal to1 will be confined within this unit circle whereas all active devices having reflection coefficient greater than 1 will be outside the unit circle.

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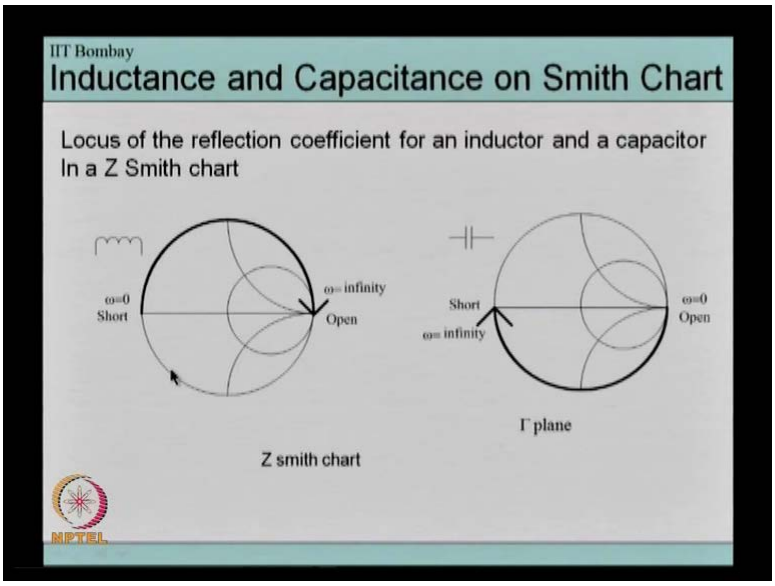

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Inductance and Capacitance on Smith Chart

Locus of the reflection coefficient for an inductor and a capacitor In a Z Smith chart

Now let us see the implications of this. Suppose we have a lump element inductor, then at Frequency Omega is equal to 0, the reactance will be 0. As the frequency increases, we will be moving along this constant resistance circle. This constant resistance circle has a radius of unity and we shall be moving along the circumference of this constant resistance circle till become to this point where Omega is infinity. So, in other words, the left-hand portion of this Smith chart corresponds to a short and the right-hand portion corresponds to an open.

Similarly if we consider capacitance, then for Omega is equal to 0, it will be infinite impedance reactance and for omega is equal to Infinity, reactance will be 0. So, we are moving along this constant resistance circle, this unit circle, along the lower half. So, for inductance we move from… As the frequency goes from 0 to Infinity, we move from the left to the right along the top half portion and for the capacitance we move from the right to the left along the bottom half. Again just like for inductance, this portion for the capacitance, this part is the short and this part is the open.

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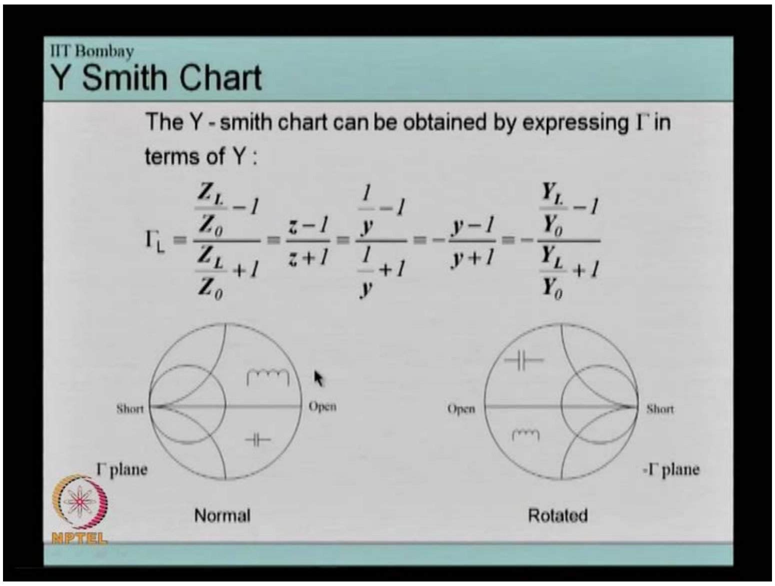

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Y Smith Chart

The Y - smith chart can be obtained by expressing Γ in terms of Y :

$$\Gamma_L = \frac{\frac{Z_L}{Z_0}-1}{\frac{Z_L}{Z_0}+1} = \frac{z-1}{z+1} = \frac{\frac{1}{y}-1}{\frac{1}{y}+1} = -\frac{y-1}{y+1} = -\frac{\frac{Y_L}{Y_0}-1}{\frac{Y_L}{Y_0}+1}$$

Normal

Rotated

Excuse me, now analogous to the Z Smith chart… The previous Smith charts that we considered were based on the bilinear transformation of Z and we can have a similar Smith chart based on the bilinear transformation of Y. Now as we can see that the Y Smith chart and Z Smith chart, the same point on the 2 Smith charts or the same impedance of the 2 Smith charts will be rotated by 180° by virtue of the presence of this negative sign.

So, if suppose this is our normal Smith chart, then we saw that for a normal Smith chart, the far end was short and then this near end was open. In this Y Smith chart, this far end will be open and near end will be short. And you can also rotate the Y Smith chart so that this part becomes open and this part becomes short.

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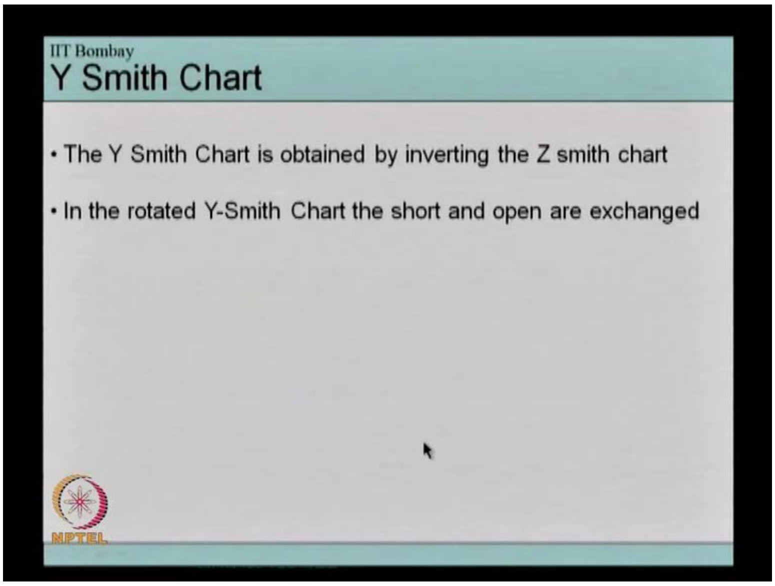

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Y Smith Chart

- The Y Smith Chart is obtained by inverting the Z smith chart
- In the rotated Y-Smith Chart the short and open are exchanged

So, this is the summary of what we observed. Y Smith chart is obtained by inverting the Z Smith chart. In the rotated Y Smith chart, the short and open positions are exchanged.

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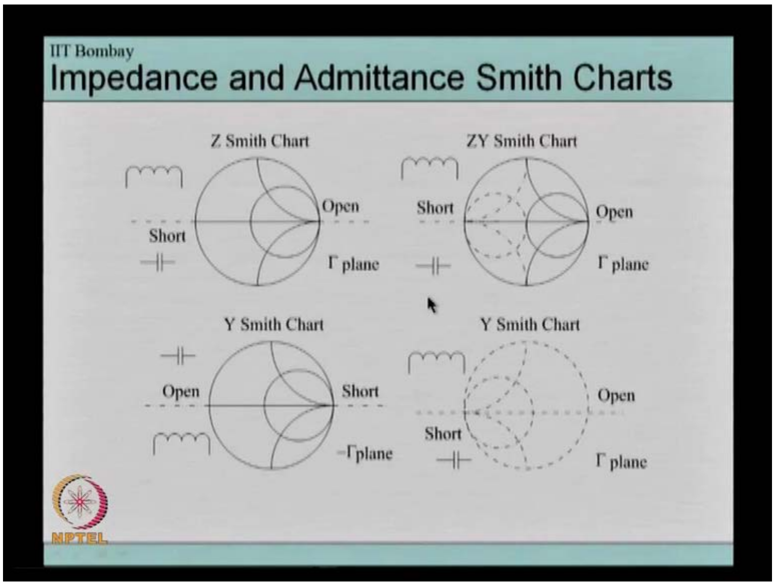

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Impedance and Admittance Smith Charts

So, once again for the Z Smith chart we saw that the far end is short, near end is open and for the Y Smith chart far end is open and near end is short.

Now, one thing we noticed is that if the Z Smith chart is rotated 180° with respect to the Y Smith chart, then we can combine the 2 and form what is called as ZY Smith chart, where a point with respect to the Z Smith chart will also satisfy the position requirement for the Y Smith chart.

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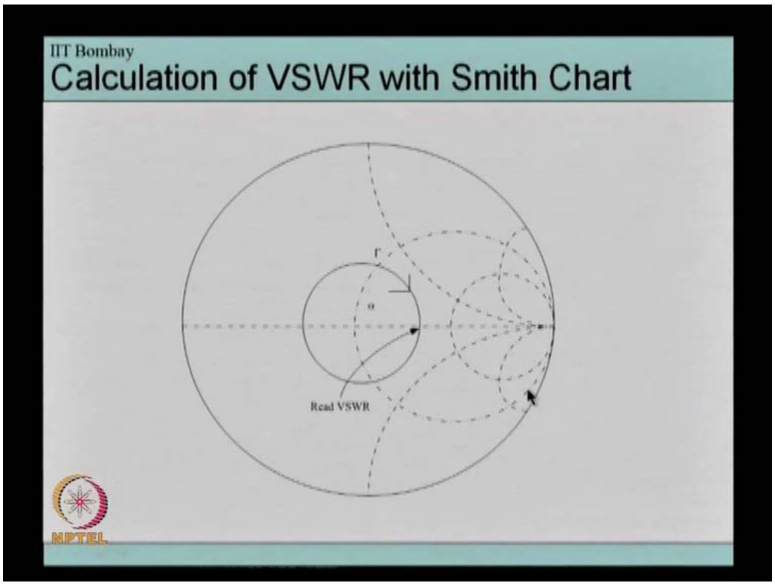

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Calculation of VSWR with Smith Chart

Now, the Smith charts are commercially available, they can be downloaded from the Internet. One interesting application of the Smith chart is to read impedances and impedance transformation. What you have in a real Smith chart is a figure like this with small markings, the various lines have their own reactances, normalised reactance values. And the Central line has many markings corresponding to the normalised resistances. If you want to find out the VSWR of a load which is the reflection coefficient Gamma and which is plotted on the Smith chart like this, then there is a simple way to find out the VSWR directly. What you do is you draw a circle on that point which intersects the x-axis at a certain point. And the reading on the x-axis, reading, not the length or the distance, the actual reading on the Smith chart will be the VSWR for this particular impedance.

That brings us to an end to the module 4 of week 1.