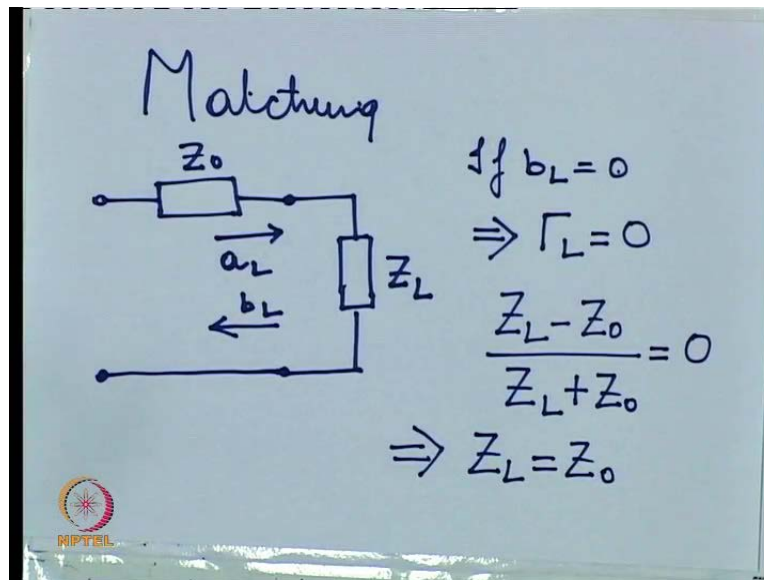


**Microwave Integrated Circuits**  
**Prof. Jayanta Mukherjee**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Bombay**  
**Broadband Impedance Matching**  
**Mod 02, Lec 07**

Hello! welcome to another module of this course 'Microwave Integrated Circuit'. We are here, we are starting with week 2. In the previous module we had covered microwave components. And in this module we are going to cover an important application of microwave engineering, which is matching. Now we have already discussed what is matching in the previous classes. Let me just refresh what we mean by matching.

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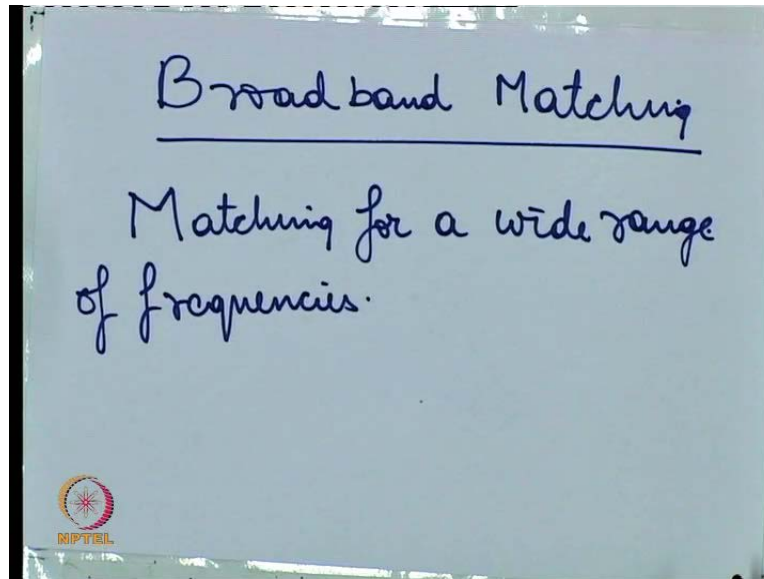


So matching we saw; was if you have a load connected and if this load is supplied by a transmission line or any other network which has a characteristic impedance  $Z_0$  say; and there is an incident wave  $a_L$  and a reflected wave  $b_L$  then; our load  $Z_L$  is said to be well matched with respect to the transmission line having a characteristic impedance  $Z_0$ , if  $b_L$  is equal to 0. This is also; this also can be written as  $\Gamma_L$  is equal to 0. Now  $\Gamma_L$  we know is equal to  $Z_L$  minus  $Z_0$  upon  $Z_L$  plus  $Z_0$ , so then this should also be equal to 0, and this being 0 implies  $Z_L$  should be equal to  $Z_0$ . Now this is the basic matching problem.

However; when, see the transmission line or whichever device which we connect to the end of the transmission, to the end of the load, if these conditions are satisfied then it will be matched.

But then often it happens that this is true only for a particular frequency, not for all frequencies. So in that cases if we want to say match this load for a wide range of frequencies, not just for a single frequency; then what do we do? So that is what we call broadband matching.

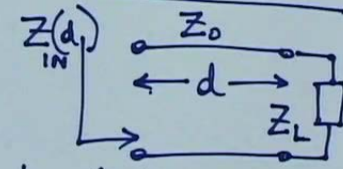
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It means matching for a wide range of frequencies. Now the first thing to understand this broadband matching; the first element that we need to consider is the quarter wave transforming, that we had discussed in the previous class. So once again, let's go back to the quarter wave transformer for a moment.

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Quarter wave Transformers


$$Z(d)_{IN} = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

$d = \lambda/4$     $\beta d = \frac{2\pi}{\lambda} \times \lambda/4 = \pi/2$

Recall how we had derived expression for a quarter wave transformer. The impedance, the input impedance of a transmission line of length D, having characteristic impedance  $Z_0$  and say we have a load  $Z_L$  connected as shown, then the input impedance  $Z_D$ ,  $Z_{in D}$  is given by this expression.

Now when this expression; when we have a length D, given by say lambda by 4 or in other words the electrical angle beta D is equal to 2 pi upon lambda multiplied by the distance lambda by 4 that makes a total electrical angle of 5/2, so either, so the quarter wave length can be defined either ways. If the length D is equal to lambda upon 4, or the electrical angle of the length of transmission line is pi by 2, then what we observe is, when we substitute into the previous expression that I just wrote down.

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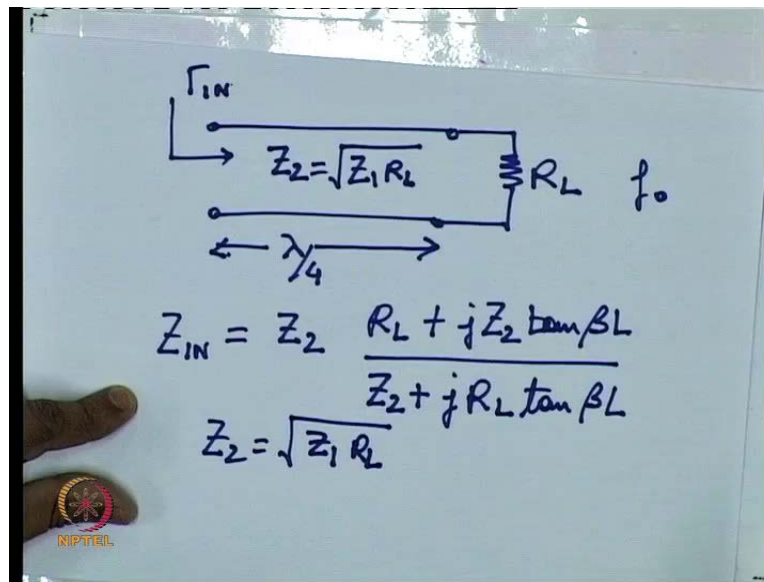
$$Z_{IN} = Z_{IN} (d = \lambda/4)$$
$$= \frac{Z_0^2}{Z_L}$$
$$Z_0 = \sqrt{Z_G Z_L}$$

Then what we get is  $Z_{in}$  is equal to  $Z_{in}$  in  $D$  is equal to  $\lambda/4$  which is equal to  $Z_0^2$  upon  $Z_L$  and from this I said that this is the expression for, it is an inversion expression. What I mean is if  $Z_L$  is the short then  $Z_{in}$  will appear to be as an open or if  $Z_L$  is an open then  $Z_{in}$  will appear to be a short.

And then we also discussed that suppose we want to convert, say we, say we have two values  $Z_G$  and  $Z_L$  and if you want to convert  $Z_L$  to  $Z_G$ ; then we choose a quarter wave length, quarter wave transformer having characteristic impedance  $Z_0$  given by  $Z_G$  upon  $Z_L$ . Here of course both  $Z_L$  and  $Z_G$  have to be real. So this is the basic expression for a quarter wave transformer.

And now that we know what is the basic expression for a quarter wave transformer; let us see the frequency response of a quarter wave transformer. Because this quarter wave transformer will provide the transformation only at a particular frequency, the frequency for which it is a quarter wave transformer and not for other frequencies. Now the reason we see the frequency response of the quarter wave transformer is because, we want to see what happens at say a small offset from this frequency for which the transmission line is a quarter wave is of quarter wave.

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So if we try to do that, when we go back to our expression, let's suppose we have a transmission line with load  $R_L$  connected and it has, say we have a quarter wave length transformer at a certain frequency  $F_0$ , say. So this is  $\lambda/4$  at frequency  $F_0$  and the characteristic impedance is equal to. Okay, then for small offsets the  $Z_E$  can be written as  $Z_2 R_L$  plus  $jZ_2 \tan$  of  $\beta L$  upon  $Z_2$  plus  $jR_L \tan$  of  $\beta L$  with  $Z_2$  being the square root of  $Z_1$  and  $R_L$ . So  $Z_1$  is the value to which we convert the load  $R_L$  at the frequency for which the transmission line is a quarter wave length.

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$$Z_1 = \frac{Z_2^2}{R_L} \text{ at } f = f_0$$

$$Z_{IN} = Z_2 \frac{R_L}{Z_1} + j \sqrt{\frac{R_L}{Z_1}} \tan\left(\frac{\pi f}{f_0}\right)$$

$$\beta L = \frac{2\pi}{\lambda} \frac{\lambda_0}{4} = \frac{\pi}{2} f/f_0$$

$$Z_{IN} = \frac{Z_2 \frac{R_L}{Z_1} + j \sqrt{\frac{R_L}{Z_1}} \tan\left(\frac{\pi f}{f_0}\right)}{\sqrt{\frac{R_L}{Z_1}} + j \frac{R_L}{Z_1} \tan\left(\frac{\pi f}{f_0}\right)}$$

So now if we substitute; here  $Z_1$  is equal to  $Z_2^2$  upon  $R_L$  or the value of the input impedance at  $F$  equal to  $F_{CL}$ , Now with this expression, if we go back to, no if we go back to the expression for  $Z$  in then it comes, we can we can write it like this; By dividing the numerator and denominator by  $Z_1$  we can write our expression like this. Now here this expression  $I$  by  $2F$  upon  $F_0$  is actually the value of  $\beta L$ .  $\beta$  recall is equal to  $2\pi$  upon  $\lambda$ . But then  $L$  is equal to  $\lambda_0$  upon  $4$ , where  $\lambda_0$  corresponds to the frequency  $F_0$  for which the our transmission line is the quarter, is the quarter wave length length.

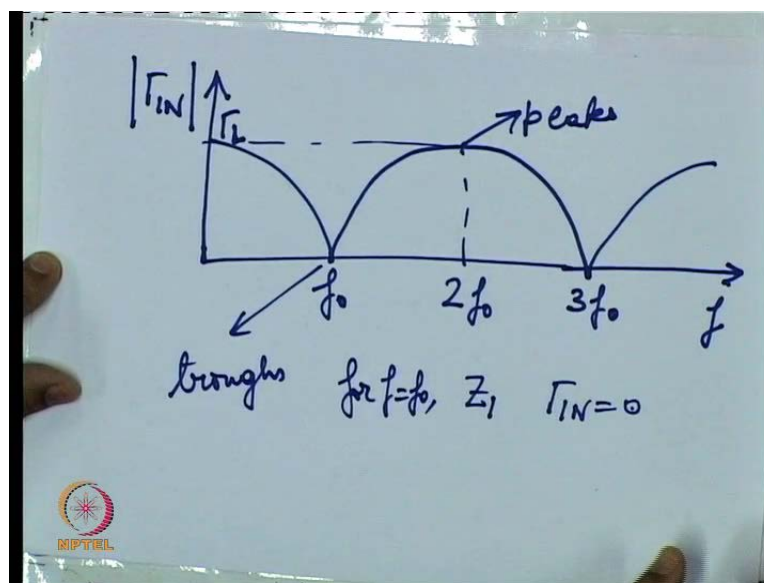
So  $\lambda$  is the wave length of the current frequency that is it may not be equal to  $\lambda_0$ , it may be for a frequency which is slightly shifted away from  $F_0$ . And this is equal to  $\pi$  upon  $2F$  upon  $F_0$ . Then  $Z$  in, sorry I didn't write the whole expression, I will cut this down I'll just do it so that I can write it once again.  $Z_2$  is equal to  $R_L$  upon  $Z_1$  plus  $J$  square root of  $R_L$  upon  $Z_1$ .  $\tan$  of  $5/2 F$  upon  $F_0$ . This whole was, okay.

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$$\begin{aligned}\Gamma_{IN} &= \frac{Z_{IN} - 1}{Z_{IN} + 1} \\ &= \frac{\left(\frac{R_L}{Z_1} - 1\right)}{\left(\frac{R_L}{Z_1} + 1\right) + j2\sqrt{\frac{R_L}{Z_1}} \tan\left(\frac{\pi}{2} f/f_0\right)} \\ |\Gamma_{IN}| &= \frac{1}{\sqrt{1 + \frac{4Z_1 R_L}{(R_L - Z_1)^2} \sec^2\left(\frac{\pi}{2} f/f_0\right)}}\end{aligned}$$

So then since I know  $Z_{IN}$  I can find an expression for  $\Gamma_{IN}$ .  $\Gamma_{IN}$ , the input reflection coefficient will be given by  $Z_{IN} - 1$  upon  $Z_{IN} + 1$  and this after some mathematical steps comes out to this expression. And then if I write an expression for the magnitude of this  $\Gamma_{IN}$  it comes out to. So this is my expression for the  $\Gamma_{IN}$  on the magnitude of the input reflection coefficient.

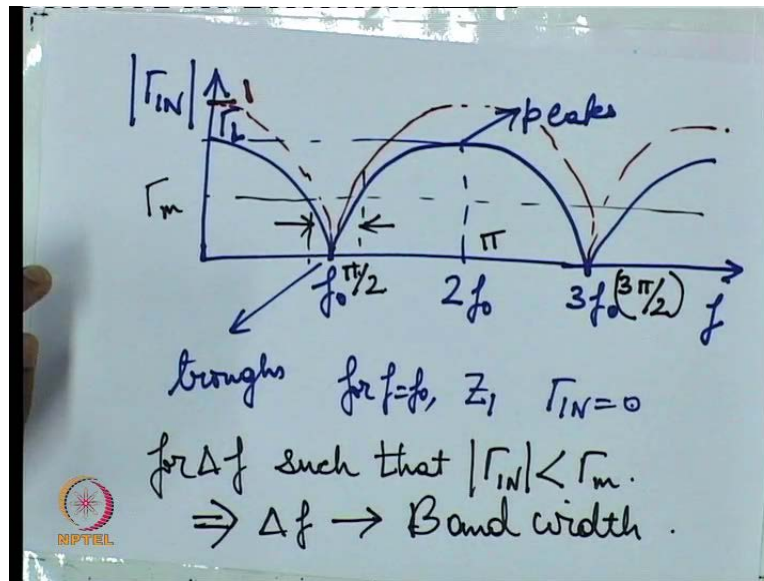
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If I plot this expression, the kind of graph that I get is like this. So there are troughs and then there are peaks. For  $f = f_0$  okay, the input impedance is  $Z_1$ , hence  $\Gamma_{IN}$  is equal to

That is one simple definition. Now this whole expression is repeated here. For every odd multiples of  $F_0$ , it is repeated. and for even multiples you see that gamma in reaches a maximum value and that is given by the value of gamma L. The maximum value that is reached is equal to Gamma L.

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If you change the value of Gamma L, say we keep on changing the value of gamma L, then what happens is, say if we increase the value of gamma L, then our graph changes like this, That is we change our ZL so that gamma L increases. But gamma L will have a maximum value of 1, one which we cannot increase it and uhh, that's all basically. We cannot make any further changes. We can only keep changing our ZL so that our gamma L changes and that way he can control the band width.

So now that I mention the term band width let me define what it is. In the case of impedance matching, the concept of band width is a little different from the normal 3db band width we are familiar with filters. Suppose I choose a certain value gamma N and this is the value as say that any time my value of the magnitude of gamma in is below this gamma M, then that frequency range over which magnitude gamma that is for F such that magnitude of gamma in is lesser than gamma N. I call that range of frequencies or say I say, delta F well that delta F is called a band width. Okay.

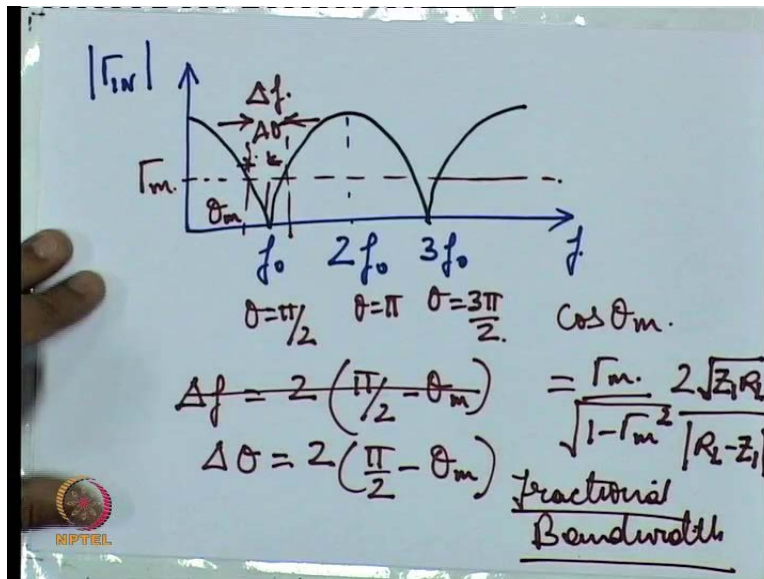


Now this  $2F_0$  also corresponds to the length  $\pi F_0$  corresponds, as we know on a quarter wave is of length  $572$  electrical length and this corresponds to  $3\pi$  upon  $2$ .

Some of the conclusions we can reach is ,okay, one thing that we saw was that, if we keep increasing the value of  $\gamma L$  then the band width actually decreases, Isn't it. Because this becomes narrower the band width decreases with increase in  $\gamma L$ . and the second thing that we see is that the input reflection co-efficient magnitude is a periodic function of frequency, and a perfect match is obtained, wherever we have these troughs, that is odd multiples of  $F_0$ .

Now we have defined what is the bandwidth. we said that whenever the magnitude if the input reflection co-efficient is lesser than certain value  $\gamma_M$  , that we ourselves have say we defined, then that constitutes, those range of frequencies for which the  $\gamma_M$  value,  $\gamma_{in}$  in, pardon, the magnitude of  $\gamma_{in}$  value is lesser than  $\gamma_M$  , is the band width. Band width of the quarter wave transformer.

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So now if we go back once again to our curve, so we have  $F$  this is magnitude  $\gamma_E$ , this is , this is the curve that we just saw and these are the values of  $F_0$ ,  $3 F_0$ , this is  $3F_0$ , and then I define certain value  $\gamma_E$ ,  $\gamma_N$  like this. now. And so suppose this point, where the this value of frequency for which the input the  $\gamma_N$  modulus is equal to  $\gamma_M$ , I call that electrical angle  $\theta_M$ .

So as you know at the point  $F_0$   $\theta$  will correspond to  $\pi/2$  and at  $2F_0$   $\theta$  will be equal to  $\pi$ , at  $3F_0$   $\theta$  will be equal to  $3\pi/2$ , this we just discussed. Now the spoken band width  $\Delta f$  is equal to twice of  $\pi/2$  minus  $\theta_M$ . Okay. Or we can you know say that, I should I should have written this, I should have written this a  $\Delta \theta$ .  $\Delta \theta$  which corresponds to  $\Delta f$ , so I can write here  $\Delta \theta$  also.

$\Delta \theta$  is equal to twice of  $\pi/2$  minus  $\theta_M$ , because this is  $\pi/2$ , this is  $\pi/2$  minus  $\theta$ , twice of that is  $\Delta \theta$ . and this  $\theta_M$  is found out from this relationship;  $\cos \theta_M$  is equal to  $\Gamma_M$  over  $1 - \Gamma_M^2$ , okay. So we kind of find out the value of  $\theta_M$  and from there we can find out the value of, uhh; we can find out the value of  $\Delta \theta$  and we have a term called fractional band width.

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Fractional Bandwidth

$$\frac{\Delta f}{f_0} = \frac{\Delta \theta}{\pi/2} = \frac{2(\frac{\pi}{2} - \theta_m)}{\pi/2}$$

$$= 2 - \frac{4\theta_m}{\pi}$$

So let me define what is fractional band width. Fractional band width is defined as this ratio, which I can also write as equal to  $\Delta \theta$  upon  $\pi/2$  and that is equal to twice of  $\pi/2$  minus  $\theta_M$  upon  $\pi/2$ . Which comes out as  $2 - 4\theta_M$  upon  $\pi$ . Now the way usually these designs happen is that you are given a value of  $\Gamma_N$  from which you find out the value of  $\theta_M$  and from which you can find out the fractional band width. So this is the maximum fractional band width that is possible from a quarter wave transformer.

Now this module was of course on broadband impedance matching, and we see that with a quarter wave transformer you can get only a certain band width. You cannot go on increasing the

band width. You can increase, decrease the band width by increasing  $\gamma L$  or by decreasing  $\gamma L$  you can increase the band width, But there are some limitations.

Now always our requirement may not be same as that what is, what can be provided by quarter wave transformer. so then we have to look to other mark-ups to obtain an even higher band width, such a method is obtained not by one section of transmission line but many sections of transmission lines cascaded with each other. So that is called multi section, that kind of structure is referred to as a multi section impedance matching circuit or a multi section broad band matching circuit, which we shall cover in the next module.

Thank You.