Microwave Integrated Circuits
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Mod 02, Lec 08
Multi-section transformer

Welcome back to another module of this course, Microwave Integrated Circuits. In the previous module we had talked about the broadband impedance matching & how quarter wave transformers are used for obtaining a certain bandwidth of impedance matching, but then we also saw that quarter wave transformers cannot provide a very high bandwidth, they have their own limitations of bandwidth & one of the ways that I suggested in the previous module is to increase the bandwidth could be to have multiple sections of transmission lines cascaded, featured. So let us see how to do that.

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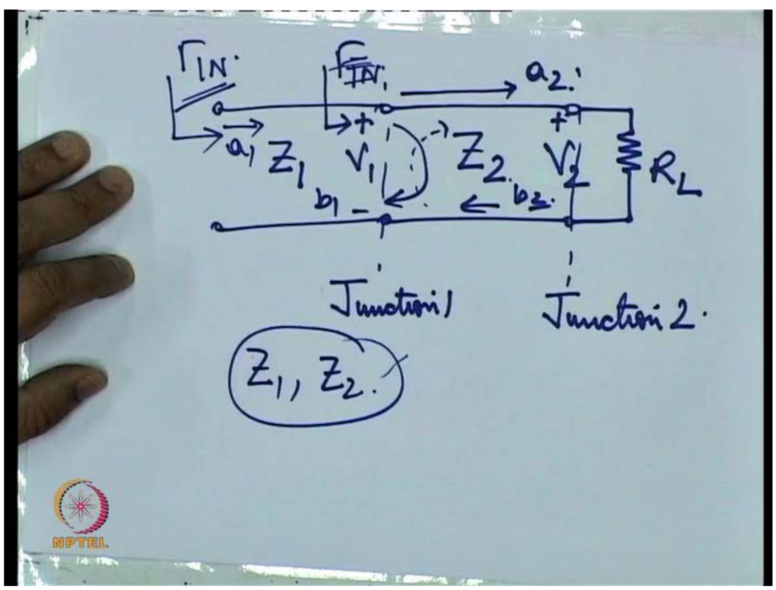

The 1^{st} problem if we have with multi sections transformers is, if we can draw this multi sections. Suppose we have 2 sections of transmission line, one having characteristic impedance z1 the other having characteristic impedance z2, connected with each other in cascade & suppose v1. Ok so this is junction 1, junction 2. Now if we… And suppose say gamma in is the input reflection coefficient before the first section. No sorry it’s here, before the first section, this is where the gamma in is. Now the first problem is that, if suppose there is an incident wave a1 entering the first section then it will be reflected from this junction.

Some of the incident wave will be reflected from this junction & say produce a wave b1 & some of them will continue with second junction say a2 & after reflection will be b2. Now had it been till this point only we would not have any problem. Problem arises, if we can go back to the circuit once again, the problem arises that this reflected wave b2 will also, there’s a possibility that this b2 will also be reflected back again. And here lies the problem. Now this reflected wave will be further reflected wave, and so on that will be an endless process. And it becomes really complicated as we shall see.

Now instead of that if we put, if there was some kind of possibility by which say every incident wave undergoes only one level of reflection. That is it’s reflected either at this junction or at this junction. There is no multiple refection happening, then our expression for this input reflection coefficient becomes much easier. See the problem here is to find out this expression for input reflection coefficient. If we could have a simple expression for gamma in then it would be easier to design sections.

Of course if you want a very rigorous analysis then you have to have proper well defined closed form solutions for gamma in. but if we can, if there are certain conditions, where we can neglect the second level reflections, then perhaps, as we shall see, we can obtain a simple expression for gamma in. Now the condition which allows this to do so is the case when z1 & z2 are very close to each other. The closer they are, the lesser will this second level of reflection.

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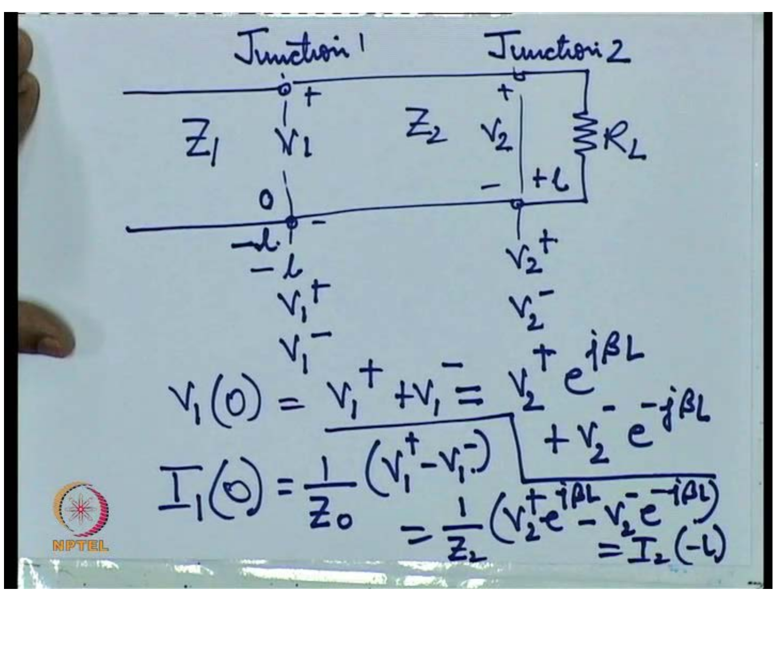

So if we can go back to our figure, let me draw a fresh figure. So we have one transmission line with characteristic impedance z1, & another with characteristic impedance z2. V2+ is the incident wave at junction 2 & v2 - is the reflected wave at junction 2. V1+ is the incident wave at junction 1, v1- is the reflected wave at junction 1.

Now if we consider this point to be 0, then for reference system where a coordinate origin is at this point, then this junction 2 will be + L & for origin system when this junction 2 is the origin point for that system this junction1 will be –L. Ok so then v1 of 0 in this coordinate system, where junction 1 is the origin, that will be equal to (v1+) + (v1-) & that in terms of v2+ will be given by. And i10, which we saw, is the difference; this can be given as, like this. So then I hope this is visible.

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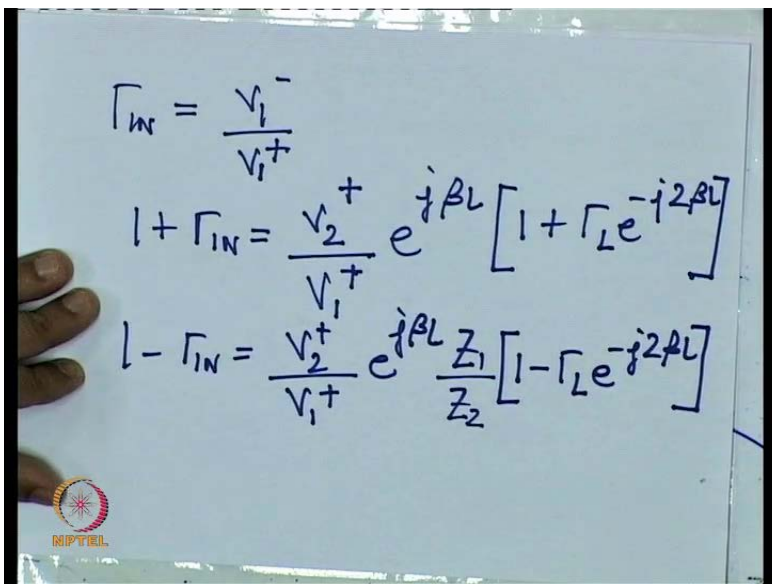

So i1 of 0 is i2 of –L with appropriate coordinate transformations & then gamma in, which is defined as d1- upon d1+, that with this definition, we can write 1+ gamma in equal to v2+ upon v1+. So this is the 2 expressions that we can find for 1+ gamma in & 1-gamma in.

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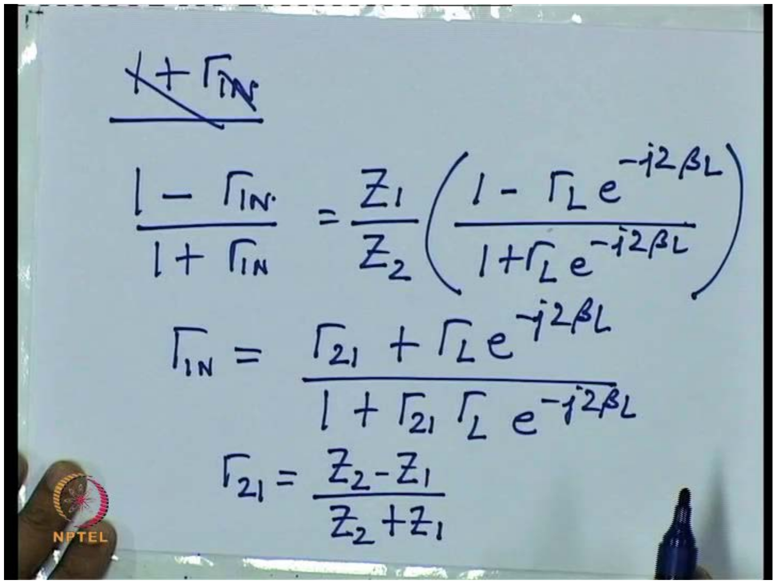

And if we then take the ratio of these expressions that comes out to 1- gamma in upon 1+gamma in equal to z1 upon z2 upon…. When solving for gamma in gives this expression where gamma 21 equal to this expression. Now this is of course the complete expression for gamma in with no assumptions with what the values of z2 & z1 are. For any values of z1 & z2 you will get the same expression. And of course this expression is only when there are 2 sections, if we suppose have a third section, then we have to again to the same process, we have to repeat the process for the third section, and the more the number of sections, this expression will become more & more complicated.

If you want a rigorous solution then you have to solve it completely. But suppose you have that mismatch between z2 & z1 as small, then we can think of a different solution. The solution is that this quantity, gamma 21 which is dependent on the difference of z2 & z1. If suppose z2 & z1 are not that difference, z2 is very close to z1…

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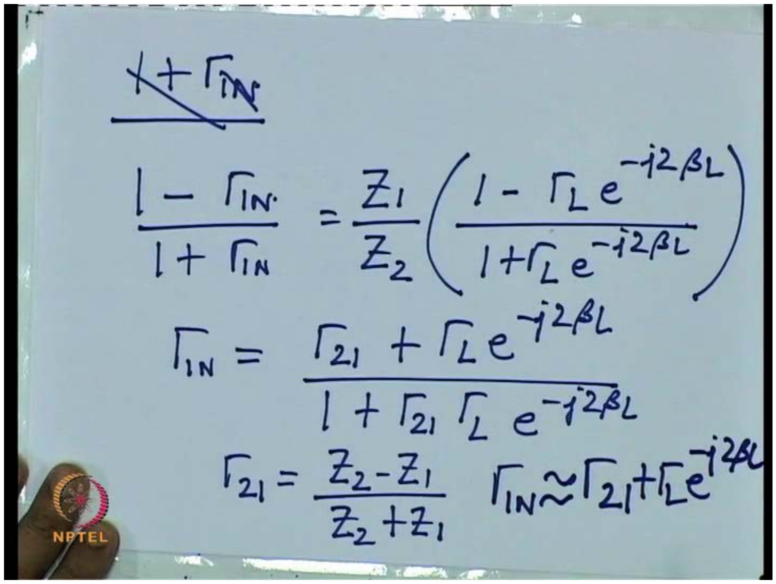

Then gamma in can approximately be written as, like this.

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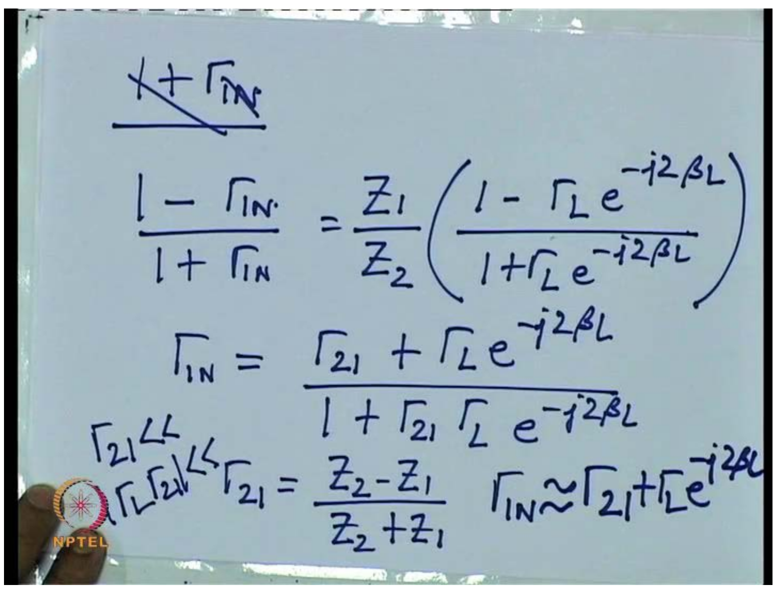

What happens is when z2 is very close to z1 then this gamma21 is small & so this expression, gamma L & gamma21 magnitude, will be even smaller because gamma L is either 1 or lesser than.

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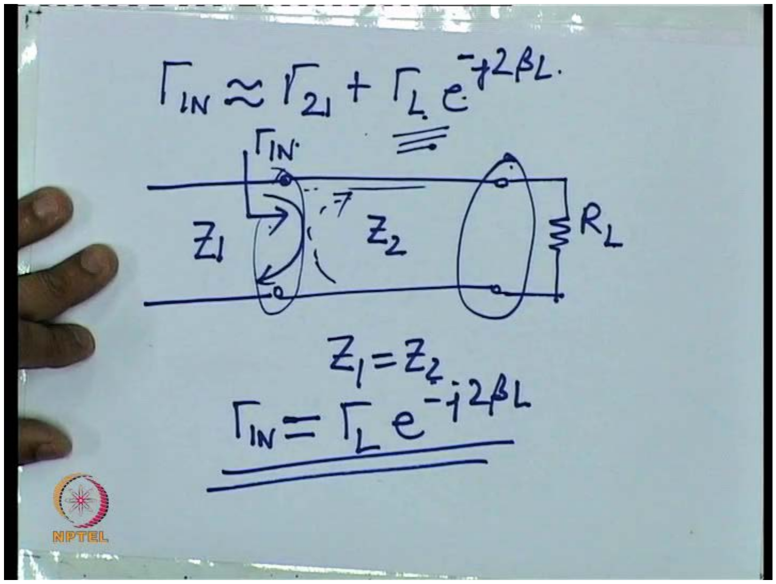

So then in case if this expression holds true then we can simplify this expression with this form. Now what is the significance of this expression, this is of course the simplified sum of two quantities. But if we just want to see the physical significance of this expression then you can go back to our circuit.

Now what it means is that gamma in is the sum of 2 reflections. One is the reflection that has taken place at this junction which is accounted for this term, & other is the mismatch which is happening at this junction. Now this expression is well known. We saw that reflection coefficient is face shifted twice, as twice the amount of face shift provided by a transmission line & so that comes from theory.

And gamma21 is simply the mismatch factor between this z1, this transmission line having characteristic impedance as z1 & this transmission line having characteristic impedance z2. Note that if z1 equal to z2 then gamma in equal to gamma L-j2 beta L which is the expression we know very well. The reason we can do this is because we are only considering a single level reflection which means that one reflection is happening due to the mismatch between z1& z2 & the other reflection at the low level.

So while making the simplification we are not considering the second reflection that might happen i.e. the reflected wave will be re-reflected at junction. So that is the simplification we are making & we are saying that it’s valid because the mismatch between z1 & z2 is very small.

Now that we have this simple expression, this makes it much easier to derive the condition when we have not two sections cascaded like this but then n sections cascaded with each other.

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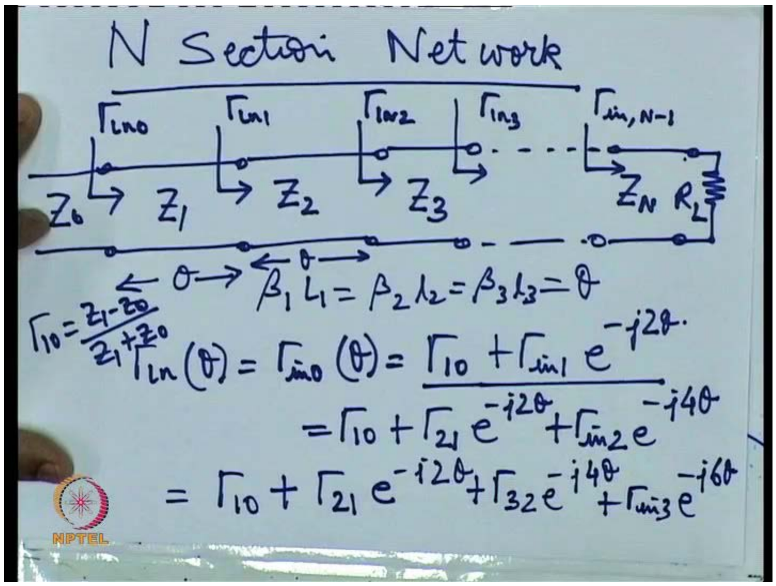

Let us see what happens when n sections are cascaded. So these are n sections of transmission lines cascaded with each other. This is the input section Zn is the last section & Rn is the low. Gamma ins are the reflections coefficients at the junctions of the various sections, now here we assume that all the sections have identical electrical lengths & this is given by beta1L1 is equal to beta2L2 is equal to beta 3L3 equal to some value say theta.

Now if we apply the formula we just discussed in the previous section to each section now, so say we are considering we want to find out the input reflection coefficient. And that input reflection coefficient equal to gamma in 0 theta equal to gamma 10 + gamma in 1-jp theta. So this is the expression, so this is like gamma in1 which is the reflection coefficient at this point multiplied the total by e raised to –j2 theta.

The theta is the electrical length of this section & the electrical length of all the sections. And gamma 1 is the mismatch between this beginning sections z0 & z1. So gamma10 equal to z1-z0 upon z1+z0, so we have this expression, now gamma in1 can itself be further expanded. We have gamma10+gamma21 raised to –j2theta+gamma in2 raised to j4 theta. So this expression I've obtained by expanding gamma in following the same formula & I get this further expression.

Now if I expand this gamma in2 further according to same formula I get this equal to. So I can continue like this for all n sections.

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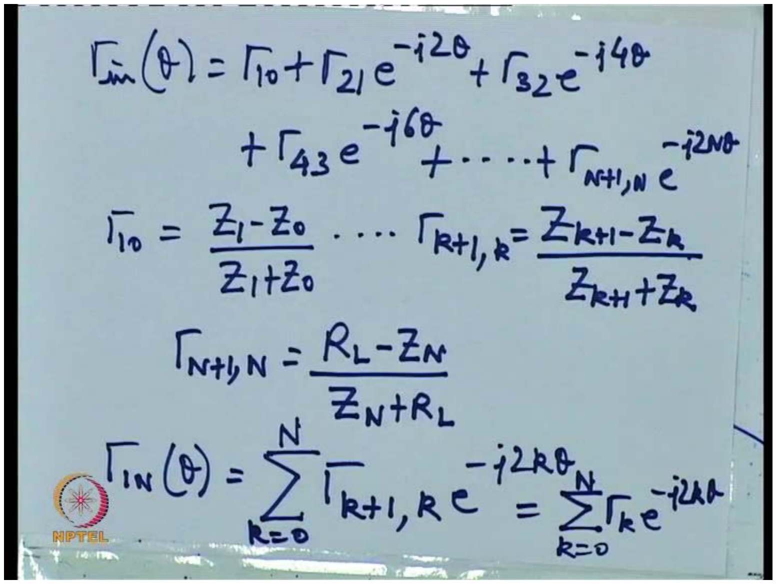

The only difference will be that this power, this exponent of this e will keep on increasing. So then for n sections I can write this general formula, where these gamma10 terms are, these gamma n terms are given like this. Or simply I can write it in a summation format. So this is the formula, finally we’ve obtained. Here we have simply, instead of writing this gammak+1,k I've replaced this with k, that is the mismatch expression is replaced by this, just for convenience, we shall see this later. So this is the general formula for the input reflection coefficient when we have n sections connected in cascade.

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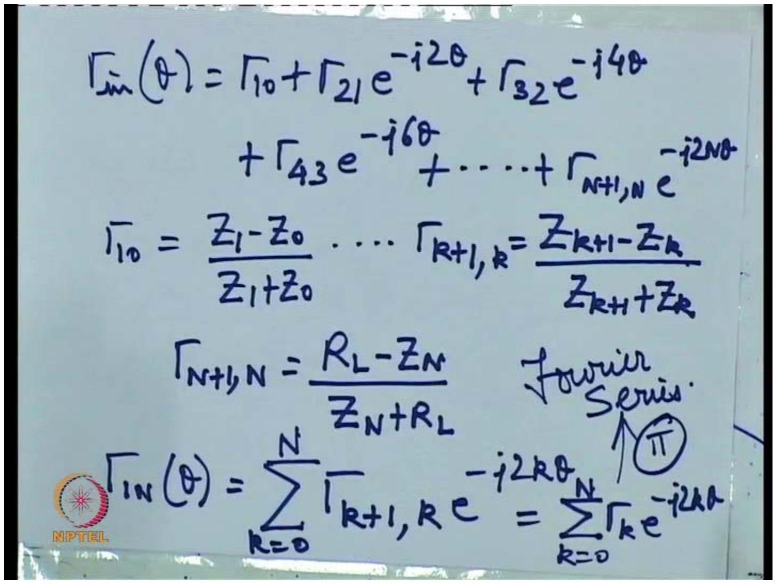

Let us see what are the implications of these expressions. So if we can go back to our slide, first thing that we see is that, this is the expression of a Fourier series, this has the same expression as the Fourier series. So this is a Fourier series. And that's true because as you can see this gamma k which is a function of theta, itself is in periodic with gamma k replacing itself after every increment of theta by pi.

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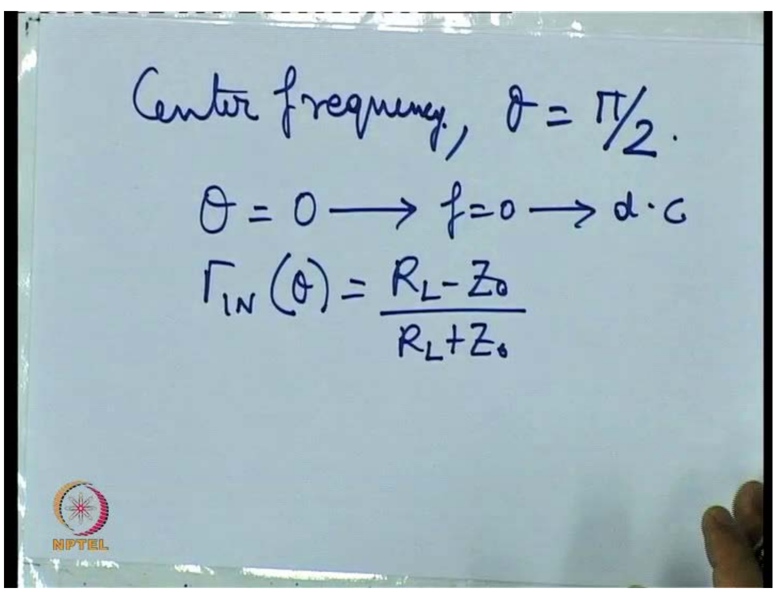

Now the 2nd conclusion we can draw from this conclusion is the centre frequency will be at theta equal 5 upon 2 & at theta equal 0 which corresponds to f equal 0 i.e. bc, at bc we should have gamma in theta equal to rl-z0 upon rl+z0.

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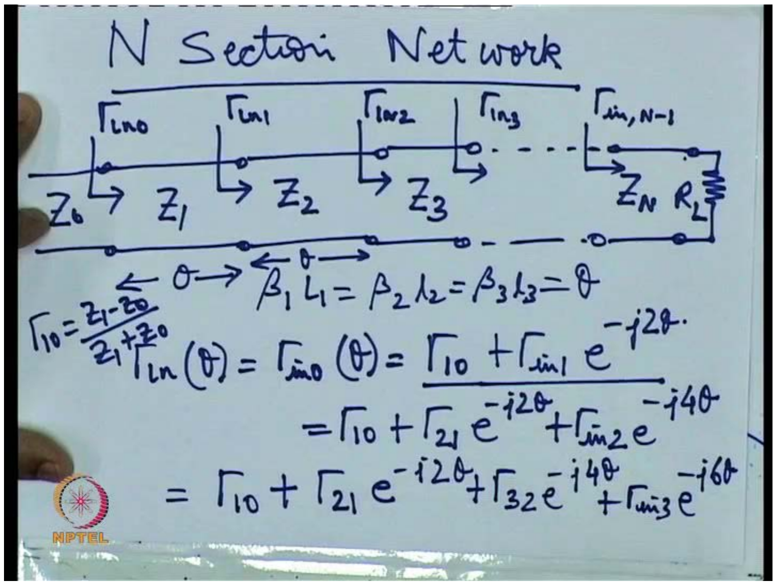

This is because, as we know, even if we have n sections if we go back to the previous slide. If at bc from the input to the output it will be one shorted line, so the input reflection coefficient at bc will simply be rl-z0 upon rl+z0.

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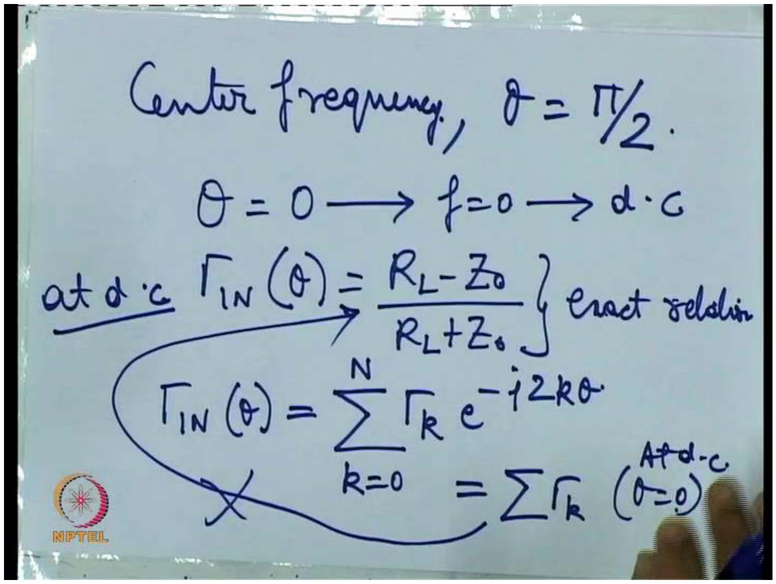

That’s why I've written here at bc gamma in theta should be this simple expression. This must be satisfied, if this is not satisfied then there must be something wrong with our theory & we shall use it as a boundary condition, this is an exact relationship which must be satisfied.

So let’s see whether our expression is satisfying this or not. As we can see from the previous line, at bc if we go with the expression that we have just derived, gamma in theta equal sigma k 02n, gamma k e raised to –j2k theta, then at bc this is simply equal to gamma k. and if we do this summation we will find that this is not equal to this expression. Something is wrong, is something wrong with our theory.

The reason this is happening is because this small single reflection approximation that we are have done & because of that if we had the complete closed form expression then this anomaly would not have happened, but this is happening because of this single reflection expression that we have considered, so we will not change our concept of single reflection theory, but we will modify the expression for this gamma k.

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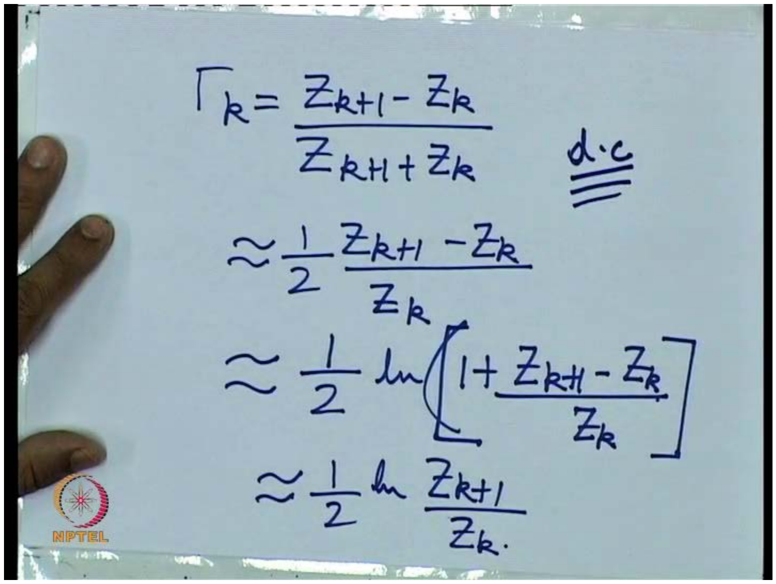

What I mean is the gamma k that we have so far used, the expression for gamma k is like this, & this we can approximate as like this, again if the mismatch between every consecutive stays i.e. zk 7 zk+1 is not too much then we can do this approximation & this can further be simplified like this. We use the first term of the logarithmic series. Ln here represents the logarithmic series & so if I just add this up it comes out to ln zk plus 1 upon zk. So it’s a mathematical manipulation, but since we see from experiments that if there is a small mismatch, our gamma k indeed can be approximated by this relationship. We shall take it in order to get a self consistent solution that is the solution should be valid at bc.

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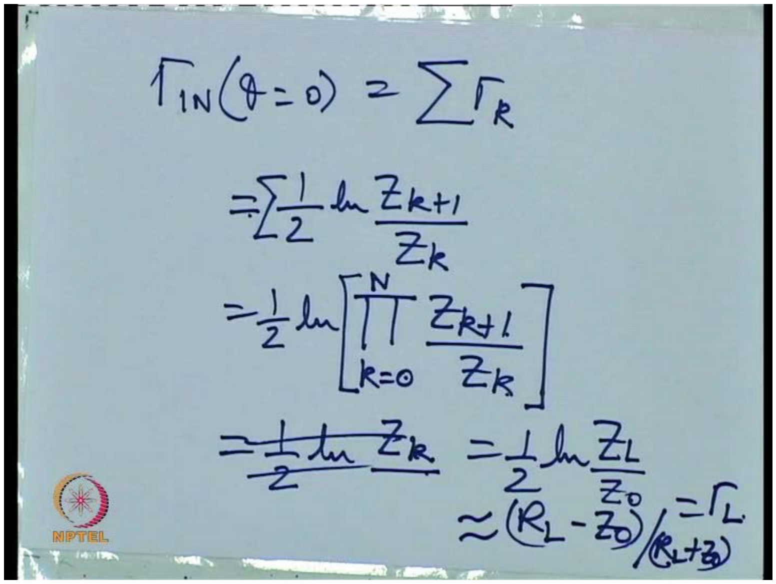

Now if we use this expression for gamma Sk & then if we go back to our expression for gamma in, if we can go back to the slides, so this becomes the product. The summation for the logarithmic series will convert to a product. And this is equal to half; the terms will cancel each other if we take this product. And we will be left simply with lnzl upon z0 which is gain approximately equal to this expression, which is equal to gamma l. So now we have obtained self consistent solution.

In summary we see 2 things from this module. First is the concept of single reflection at various junctions i.e. only the incident wave get reflected, not the reflected wave will not again get reflected. And the second based on this single reflection theory we have obtained a very simple expression for the input reflection coefficient of the cascade of n transmission line stages. Then while doing the solution, while finding out the expression for gamma in theta we had to do a small mathematical manipulation on the value of gamma k.

Instead of taking just the by linear transform of z we had to do a logarithmic approximation of the gamma k & the reason we had to do this is we had to obtain a self consistent solution for gamma k. And then with this we now obtain a complete expression for gamma in theta. But our design process is not yet done, we have just found out an expression for gamma in theta when we have n sections with characteristic impedance z1, z2, z3 & zx, but our main problem is the reverse that is given a certain reflection coefficient, how can we find out z1, z2, z3 & zx.

So that is the basic problem of a design & the way to do it, what we usually follow is to have some prototype functions. And based on these mathematical prototype functions we will be comparing those functions with our expressions that we have just derived & equating the 2 so that we can satisfy the values of the zk from the prototype function, so that is something we will cover in the next module.

Thank you.