Digital Signal Processing

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Lecture – 41

Solving problems on DSP structures

This is the 41st lecture and this is a problem solving session on DSP Structures; refer to Chapter VI of Mitra. The first problem I would solve is 6.29.

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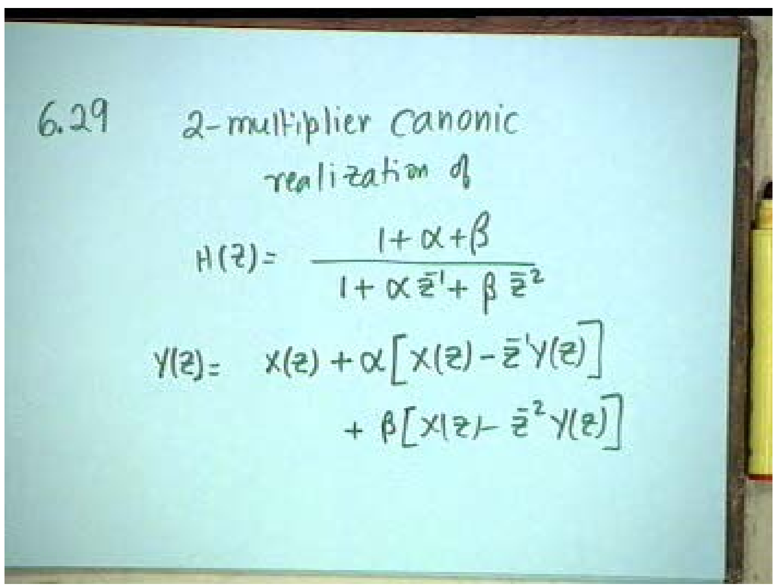

Problem 6.29 is as follows: Develop a 2–multiplier canonic realization of the 2nd order transfer function $H(z) = (1 + \alpha + \beta)/(1 + \alpha z^{-1} + \beta z^{-2})$. Normally it would require 3, α, β and 1 + α + β but since α and β occur in the numerator also, there is a possibility that we may be able to extract these multipliers. You can use only 2 multipliers and of course 2 delays. And now you write Y(z) in terms of X and Y but bringing the multipliers together. The result is $Y(z) = X(z) + \alpha [X(z) - z^{-1}Y(z)] + \beta[X(z) - z^{-2} Y(z)]$.

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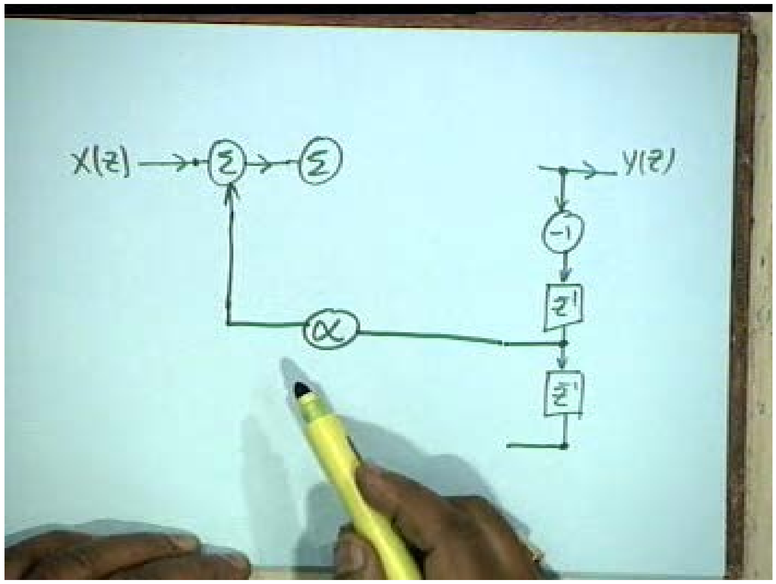

Refer to the diagram. Here is X(z) and here is Y(z). Y(z) is to be multiplied by – 1; then you have a z^{-1} and another z^{-1}. Therefore, you get – z^{-1} Y(z) and – z^{-2} Y(z). X(z) is to be added to – z^{-1} Y(z) and then multiplied by α. So one of the terms is obtained; then we have to obtain the second term and use another summation. Finally, one can redraw the diagram to look like that shown in the next slide.

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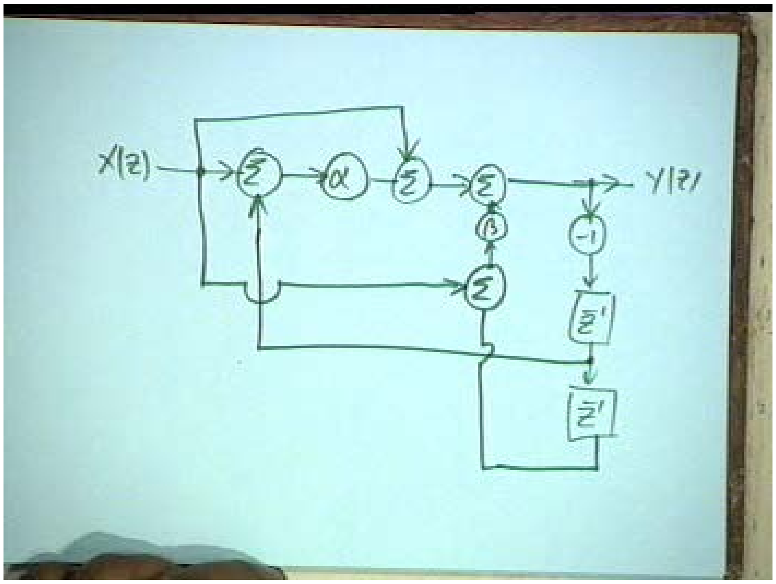

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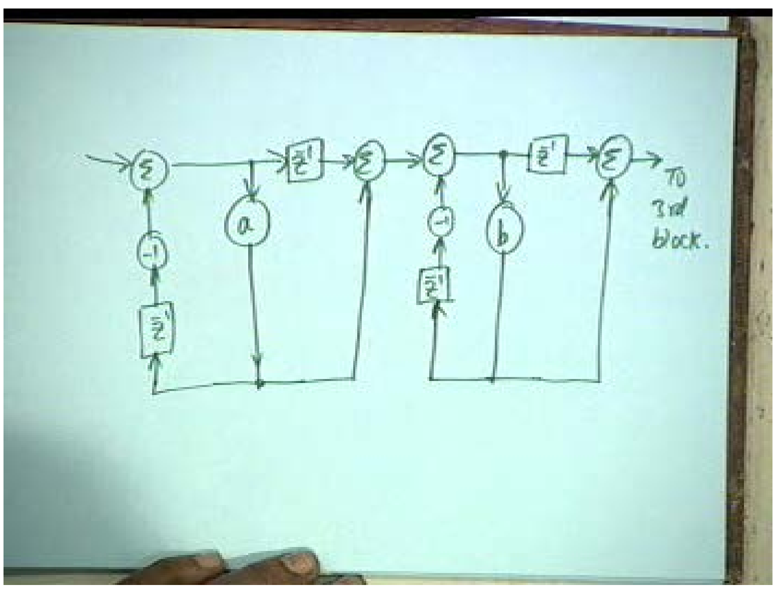

The next problem is 6.36. Given $(a + z^{-1})/(1 + az^{-1})$, it can be realized in the form shown in the first part of the above diagram with a single multiplier but with 2 delays. The problem is to find a cascade realization of the 3rd order transfer function $[(b + z^{-1})/(1 + bz^{-1})]\ [(a + z^{-1})/(1 + az^{-1})]\ [(c$

+ z^{-1})/(1 + cz^{-1})] by sharing delays. Realization is of no problem, you just have to draw three such blocks, as shown above.

The problem is to share delays between adjacent all pass sections and show that the total number of delays can be reduced to 4 instead of 6. In the above diagram, one can interchange the two adjacent summers. If you do that, then the two can be combined into one. Thus two summers are being replaced by one and this can be done in the second block also. Finally, you get only four delays because z^{-1} block cannot be combined in the last section which does not contain two adjacent summers. You have to draw the circuit carefully such that overlaps are minimized. Overlaps are not useful for fabrication.

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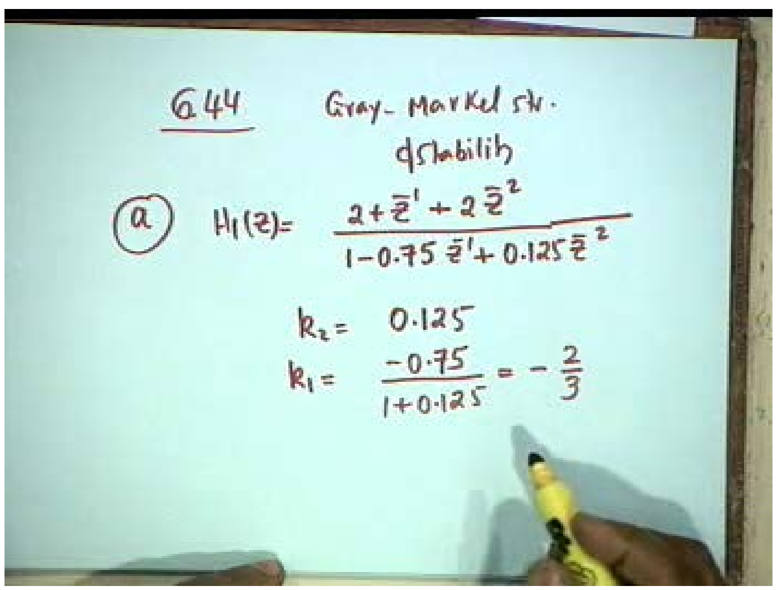

The next problem is 6.44. It has five parts: a, b, c, d and e and the problem is to realize each of the IIR transfer functions in Gray Markel form i.e. with taps from delays, and to check the BIBO stability of each transfer function. In a), $H_1(z) = (2 + z^{-1} + 2z^{-2})/(1 - 0.75z^{-1} + 0.125z^{-2})$. By now you should be quite efficient in obtaining lattice realizations of 2nd order functions in particular. The 2nd order functions require minimal amount of calculations; here, $k_2 = 0.125 = 1/8$ and $k_1 =$

$d_1/(1 + d_2)$ i.e. – 0.75/(1 +0.125) = – 2/3. k_2 and k_1 magnitudes are < 1 and therefore the structure is stable.

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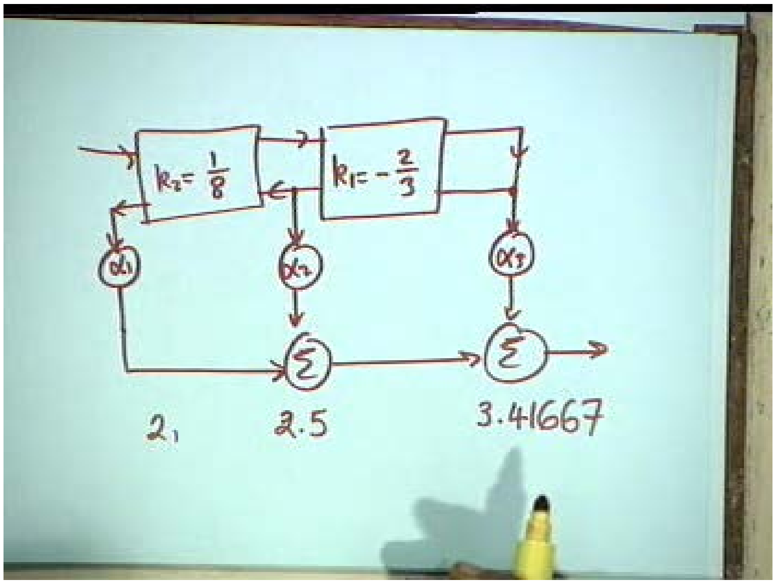

In the above slide, these two lattices, $k_2 = 1/8$ and $k_1 = – 2/3$ have been shown as blocks. The end is connected together and then you have to take taps α_1, α_2 and α_3 as shown. The lattice sections here could be single multiplier or 2 multipliers each. If you complete the example, you get $\alpha_1 = 2$, $\alpha_2 = 2.5$ and $\alpha_3 = 3.41667$.

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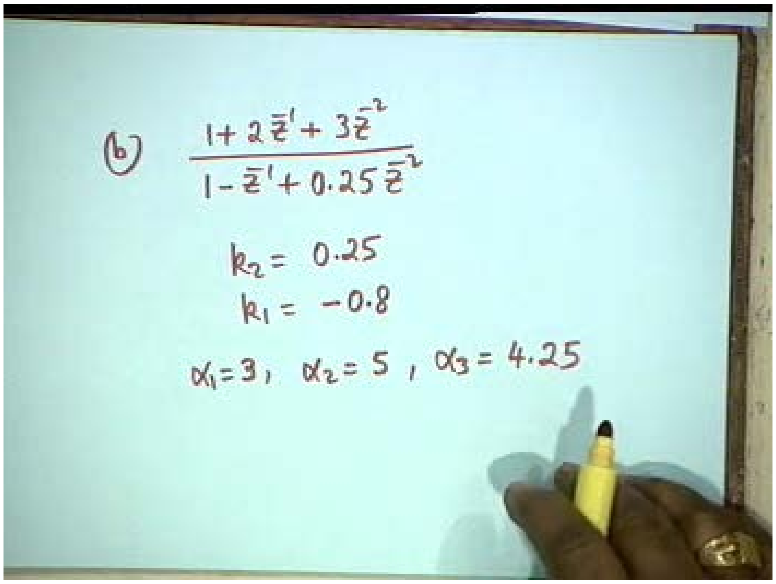

Since second order is very simple I will give you the results, only for part b. Here $H_2(z) = (1 + 2z^{-1} + 3z^{-2})/(1-z^{-1} + 0.25z^{-2})$ and the results are $k_2 = 0.25$, $k_1 = -0.8$, $\alpha_1 = 3$, $\alpha_2 = 5$, $\alpha_3 = 4.25$.

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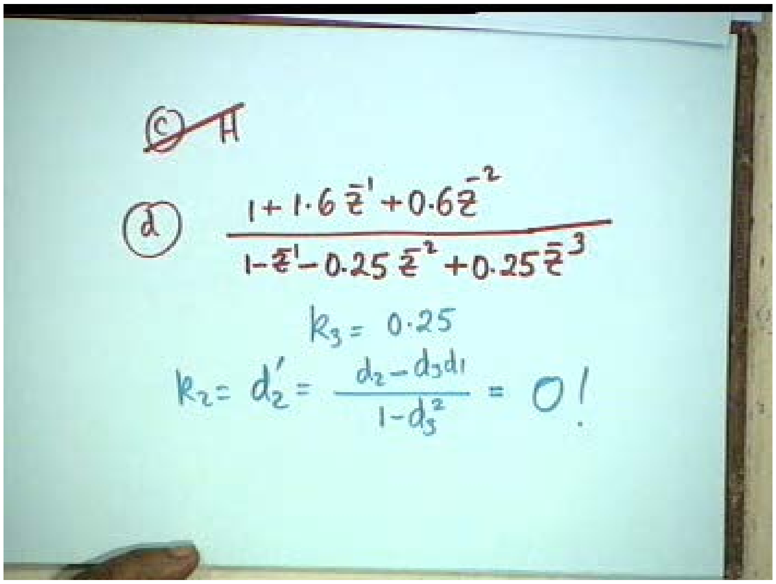

There is no twist in this problem; it is a straightforward one. In part d, $H_4(z) = (1 + 1.6z^{-1} + 0.6z^{-2})/(1 - z^{-1} - 0.25z^{-2} + 0.25z^{-3})$. Obviously $k_3 = 0.25$. And $k_2 = d_2' = (d_2 - d_3d_1)/(1 - (d_3)^2) = 0$.

Therefore $k_2 = 0$ and k_1 comes out as -1. Note that $k_2 = 0$ simplifies the calculation. Obviously, the system is BIBO unstable, and has no realization.

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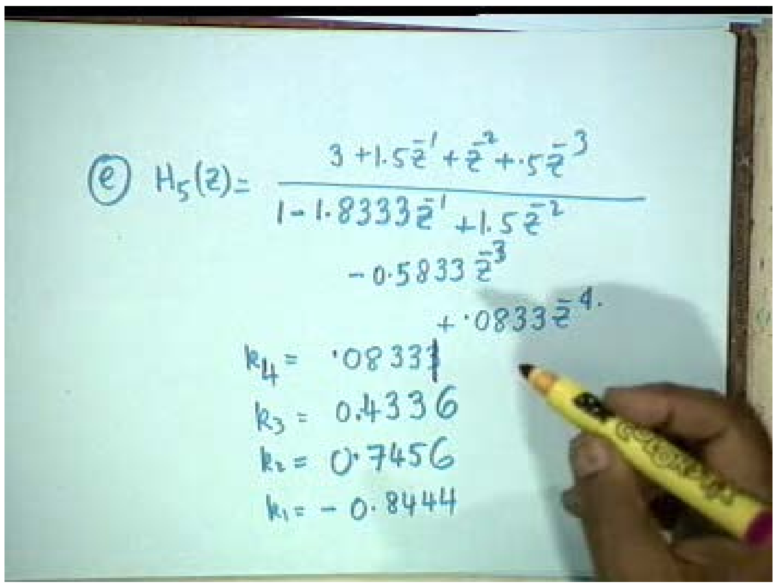

The next one, part e has $H_5(z) = (3 + 1.5z^{-1} + z^{-2} + 0.5z^{-3})/(1 - 1.8333z^{-1} + 1.5z^{-2} - 0.5833z^{-3} + 0.833z^{-4})$. This occurrence of 33 in the 3 coefficients of the denominator indicates that it is a practical situation where the numbers had to be truncated. In other words, the designer has not taken care to keep them as fractions. Nevertheless, the results here are $k_4 = 0.08331$, $k_3 = 0.4336$, $k_2 = 0.7456$, $k_1 = -0.8444$. In addition $\alpha_1 = 0$, $\alpha_2 = 0.5$, $\alpha_3 = 1.8986$, $\alpha_4 = 3.6060$, and $\alpha_5 = 4.8460$.

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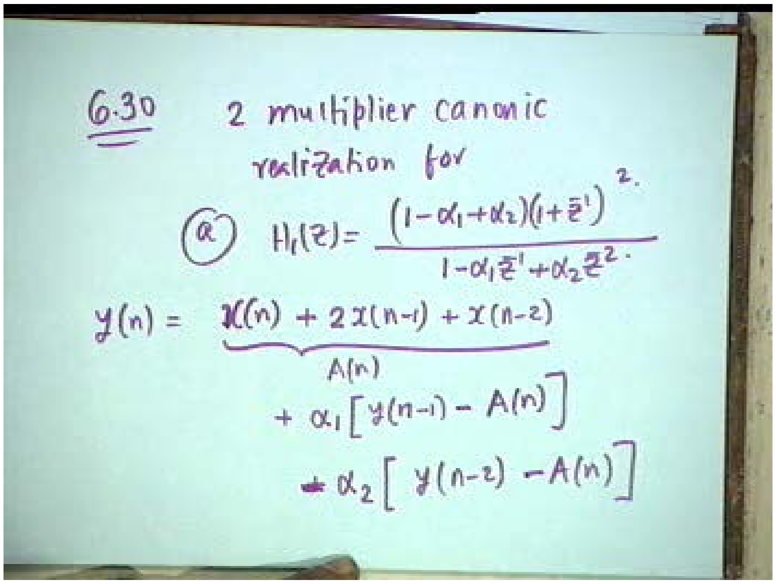

The next problem is 6.30: It says: develop a 2 multiplier canonic realization for two transfer functions: The first one is: $H_1(z) = (1 - \alpha_1 + \alpha_2)(1 + z^{-1})^2/(1 - \alpha_1 z^{-1} + \alpha_2 z^{-2})$. If I write the difference equation, I get y(n) = x(n) + 2x(n – 1) + x(n – 2) (As this will be occurring every time, I call this some function A(n)) + α_1[y(n – 1) – A(n)] – α_2[y(n – 2) – A(n)] All that now you have to do is to construct is A(n) = x(n) + 2x(n – 1) + x(n – 2) and obtain y(n) delayed by 1 sample and 2 samples, and then make appropriate additions and multiplications. But first, you must reduce the number of delays in the overall structure to 2. This can be done by physically lifting the A(n) realization and putting it on the right side.

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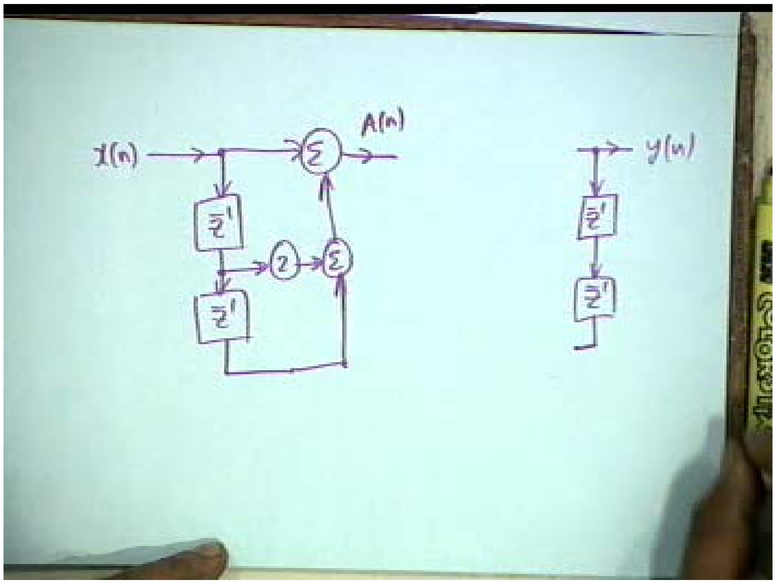

Then these two delays can be shared and therefore you get a canonic realization. Try to draw the diagram yourself.

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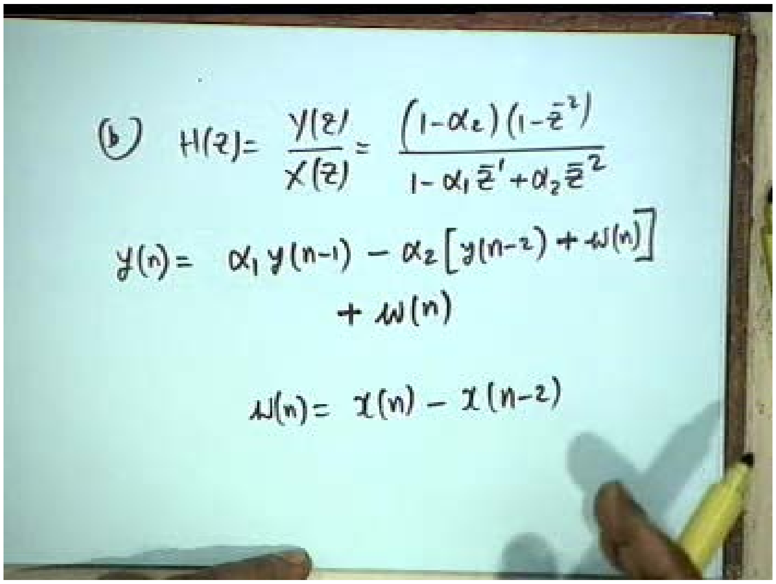

The next transfer function is $H(z) = Y(z)/X(z) = (1 - \alpha_2)(1 - z^{-2})/(1 - \alpha_1 z^{-1} + \alpha_2 z^{-2})$. Here, the difference equation is $y(n) = \alpha_1 y(n-1) - \alpha_2[y(n-2) + w(n)] + w(n)$. Here $w(n) = x(n) - x(n-2)$. First I construct $x(n) - x(n-2)$; I require two delays. Then I make appropriate combinations. From $y(n)$, we require 2 delays in order to find $y(n-1)$ and $y(n-2)$. We apply the same trick as in the previous example. Again, you can draw the diagram yourself.

The next problem is 6.48 which says: Develop a realization of the given 1st order complex coefficient transfer function with real multipliers only. One might ask: why real multipliers? Why can we not do with complex ones? You can do it with complex multipliers, but the problem is that there is no hardware for realizing square root of –1.

So, you find the real part and the imaginary part separately, and then combine them. The signal will also be a complex signal, this happens in FFT for example. The real part is stored in one storage and imaginary part in another storage. Wherever required you will either require the magnitude or the phase or both so you shall have a software to calculate real part square plus imaginary part square and square root of that, and the other is tangent inverse of imaginary part divided by real part and then utilize the result.

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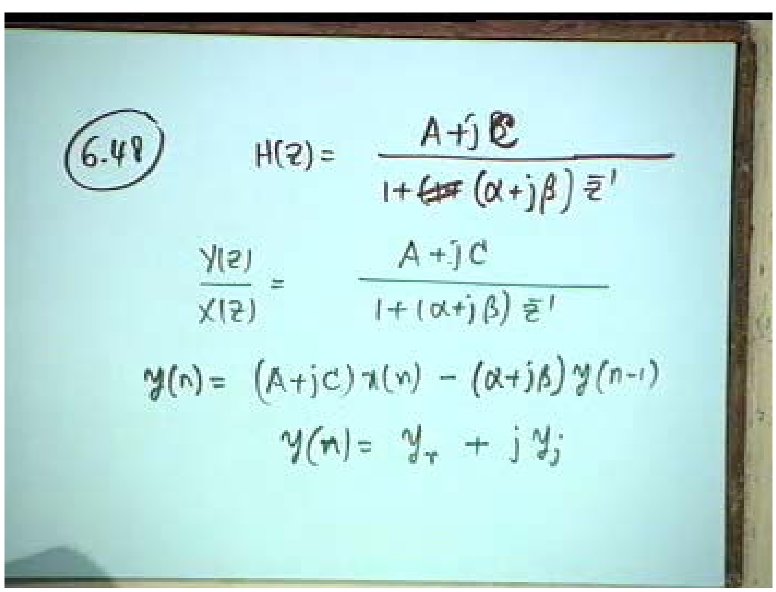

The problem transfer function is $H(z) = (A + j C)/[1 + (\alpha + j\beta)z^{-1}]$. So we write this as $Y(z)/X(z) = (A + jC)/[1 + (\alpha + j\beta)z^{-1}]$. Then $y(n) = (A + jC) x(n) - (\alpha + j\beta) y(n - 1)$. We assume that x(n) is real, y(n) shall consist of a real part and an imaginary part. So write this equation in terms of y_r and y_j.

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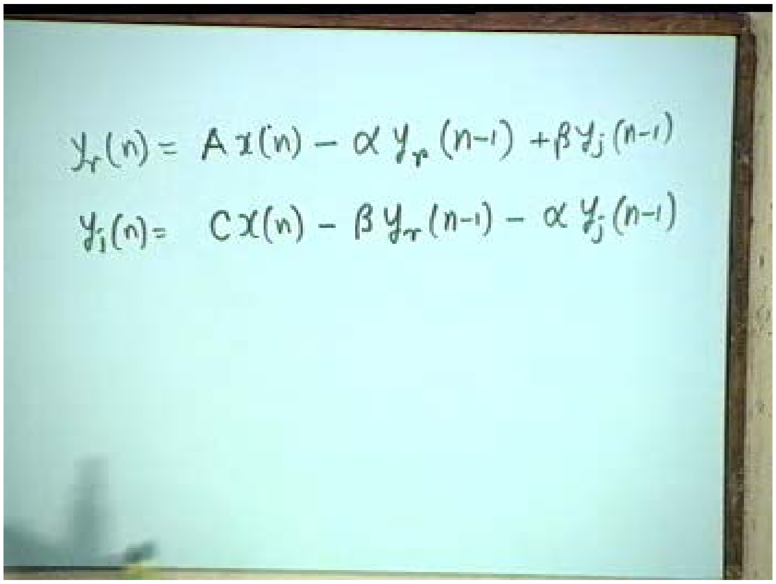

Then you get two equations; obviously $y_r(n) = Ax(n) - \alpha\, y_r(n - 1) + \beta y_j(n - 1)$. And the second one is $y_j(n) = Cx(n) - \beta y_r(n - 1) - \alpha y_j(n - 1)$. These are the two equations that you have to realize. And the diagram is to be drawn carefully.

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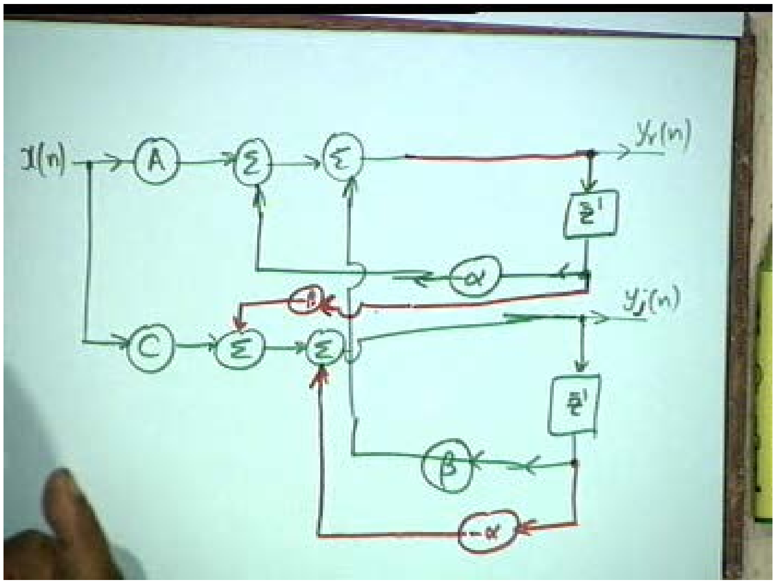

Start with x(n) and the two outputs $y_r(n)$ and $y_j(n)$. You require $y_r(n-1)$ and $y_j(n-1)$. Obtain them by two delays and then it is a matter of combination. So multiply x(n) by A and add it to $-\alpha Y_r(n-1)$, with another summer, add the result to $+\beta Y_j(n-1)$. The result is $y_r(n)$. Similarly you can construct $y_j(n)$ as C multiplied by x(n) and use two summers as shown in the figure to combine it with $-\beta y_r(n-1)$ and $-\alpha y_j(n-1)$. Now you notice that $-\alpha$ occurs twice, and there are multiplications by β and $-\beta$ ($-\beta$ is nothing but sign changed β). It should be possible to combine each pair into one multiplier. Try it yourself.

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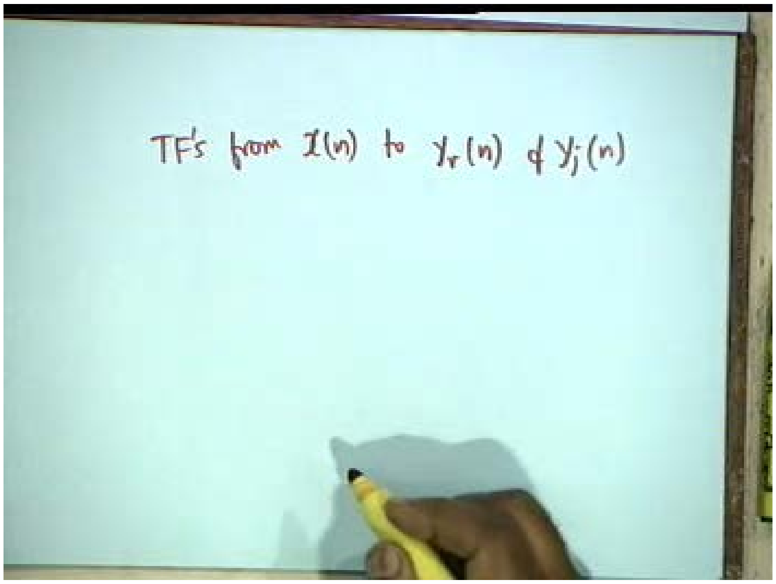

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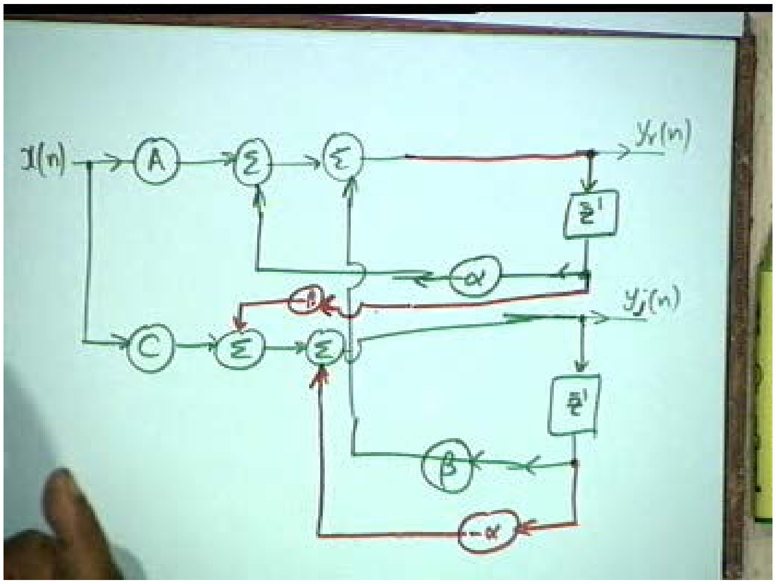

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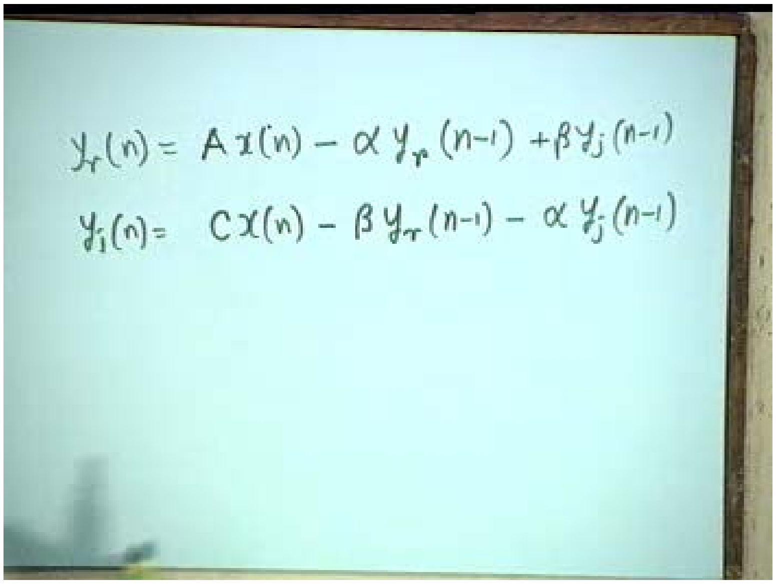

This problem also has another part which asks you to determine the transfer functions from x(n) to $y_r(n)$ and $y_j(n)$. What you do is take the difference equation, find their Z–transforms and evaluate $Y_r(z)$ and $Y_j(z)$.

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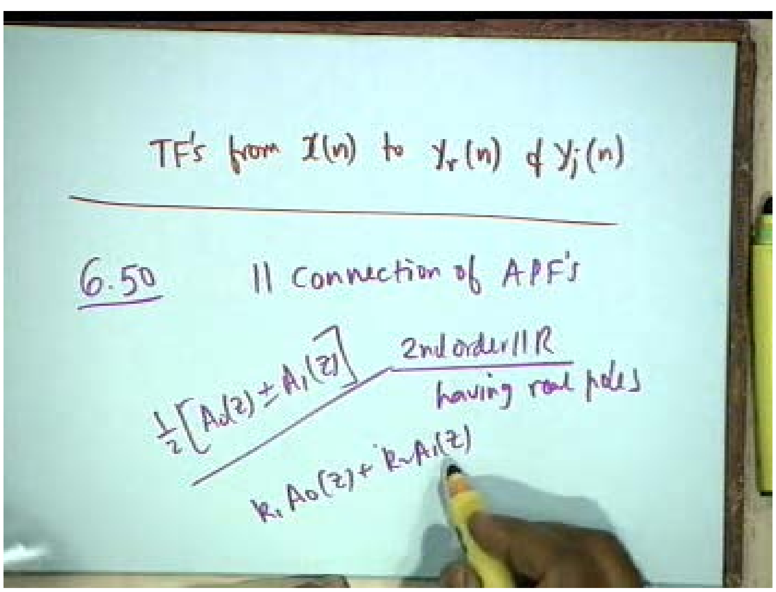

The last problem for today is a problem concerned with an all pass filter. You are to realize the following first and second order transfer functions in the form of parallel connection of all pass filters. The poles will determine what kind of decomposition is possible. If poles are complex, you could decompose into a constant + a 2^{nd} order all-pass. If the poles are real, even then you could do that. If the poles are real, then the simplest thing to do would be decompose them into 1^{st} order all-pass transfer functions.

Suppose you are given a 2^{nd} order IIR with real poles, then both decompositions are possible. You should prefer two first orders because the processing is faster; only one delay is required in both the parallel sections. Given real poles, you may require two constants in general. That is, instead of ½ $[A_0(z) \pm A_1(z)]$, you try $k_1A_0(z) \pm k_2A_1(z)$. Both A_0 and A_1 you can construct very simply by looking at the denominator; the coefficients get interchanged.

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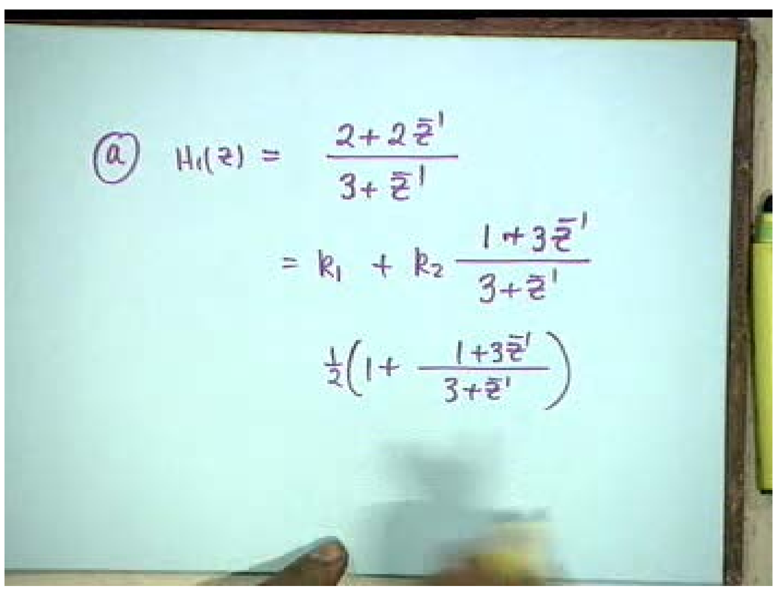

Here, $H_1(z) = (2 + 2z^{-1})/(3 + z^{-1})$. You decompose this as $k_1 + k_2 (3 + z^{-1})^{-1} (1 + 3z^{-1})$. It should always be possible because you have two unknowns and two equations. The result here happens to be ½ $[1 +(1 + 3z^{-1})/(3 + z^{-1})]$, but this is not guaranteed to occur in general.

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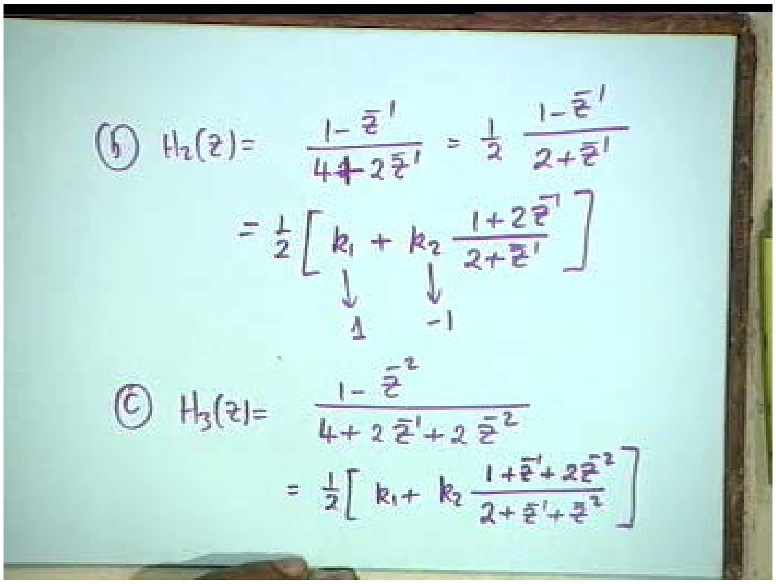

In part b), we have $H_2(z) = (1 - z^{-1})/(4 + 2z^{-1})$. You can write this is as ½ $(1 - z^{-1})/(2 + z^{-1})$ = ½ $[k_1 + k_2 (1 + 2z^{-1})/(2 + z^{-1})]$ and find out $k_1$and k_2. Here k_1 comes out as 1and k_2 comes out as – 1. Part c) has $H_3(z) = (1 - z^{-2})/(4 + 2z^{-1} + 2z^{-2})$. If you find the poles, you see that they are complex and therefore it has to be ½ $[k_1 + k_2 (1 + z^{-1} + 2z^{-2})/(2 + z^{-1} + z^{-2})]$ and find k_1 and k_2 as 1 and – 1 respectively.

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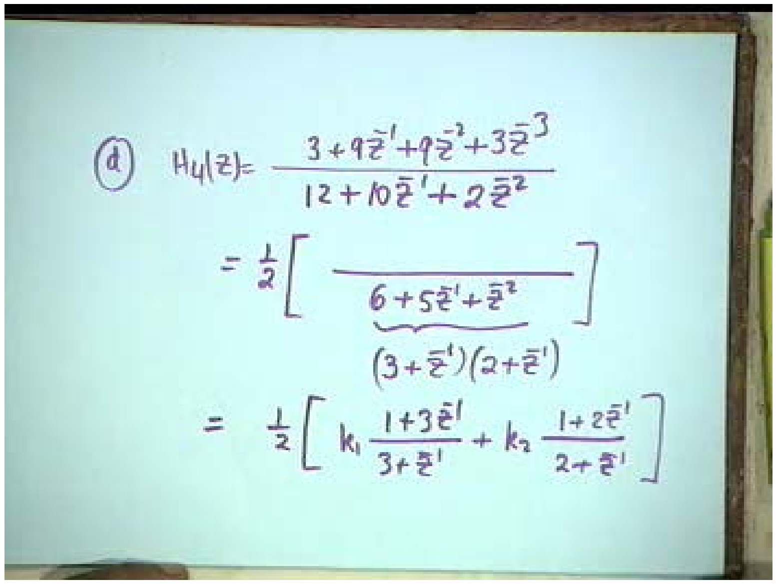

The next problem is d): $H_4(z) = (3 + 9z^{-1} + 9z^{-2} + 3z^{-3})/(12 + 10z^{-1} + 2z^{-2})$. So once again you take 1/2 out and write the denominator as $6 + 5z^{-1} + z^{-2}$ which also makes it obvious that this can be replaced with $(3 + z^{-1})\,(2 + z^{-1})$. Write $H_4(z) = (1/2)\,[k_1\,((1 + 3z^{-1})/(3 + z^{-1})) + k_2((1 + 2z^{-1})/(2 + z^{-1}))] + k_0z^{-1}$. The k_0z^{-1} is needed because the numerator degree is 3, and not 2.

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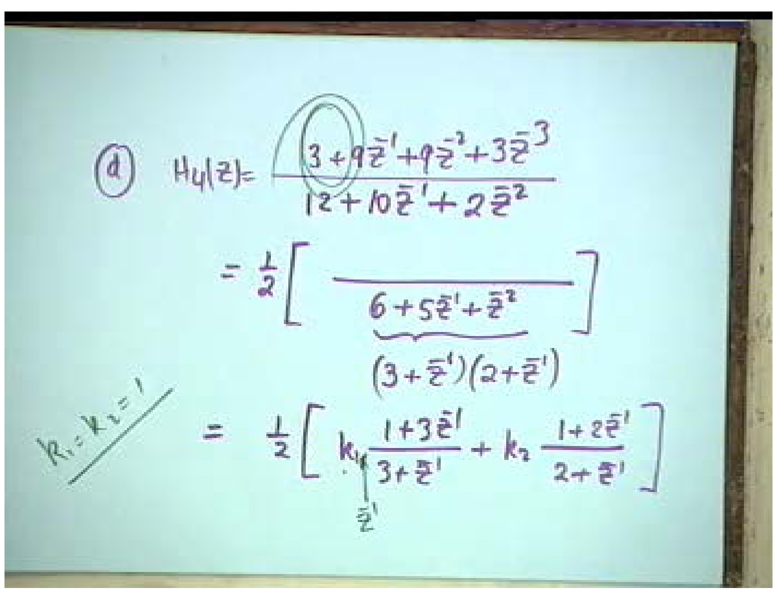

Here there will be three all passes.

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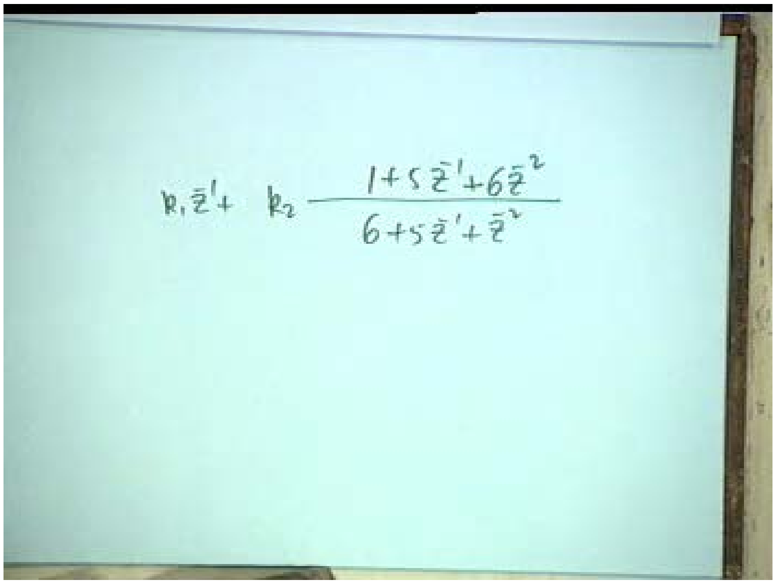

I can also write $H_4(z)$ as $k_1 z^{-1} + k_2(1 + 5z^{-1} + 6z^{-2})/(6 + 5z^{-1} + z^{-2})$.