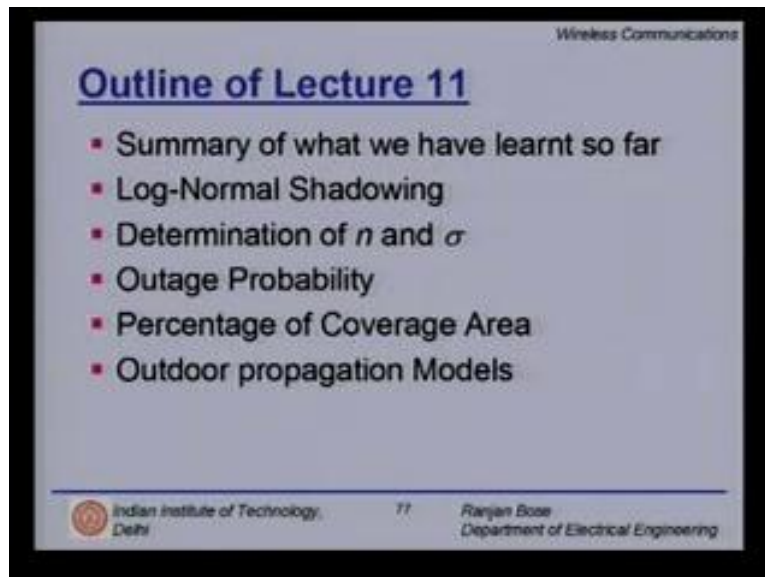


**Wireless Communications**  
**Dr. Ranjan Bose**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**  
**Lecture No. # 11**  
**Mobile Radio Propagation (Continued)**

Welcome to lecture 11 which deals with mobile radio propagation specifically outdoor propagation models. Let us look at the outline of today's lecture.

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We will start by summarizing what we have learnt already in the previous lectures. We will then discuss Log-Normal Shadowing in greater detail. We will look at an example to explain how we calculate the different parameters related to Log-Normal Shadowing. We will also see how to find out the path loss exponent  $n$  and  $\sigma$ . We look at another interesting parameter called the outage probability which is important for cell site planners. The other important thing to learn is what the percentage of coverage area is. That is, how many users can actually be covered with your base station placement. We would see if we can calculate and predict that. Then we will have a brief introduction to outdoor propagation models. So as you can see, today's lecture is geared towards outdoor measurement and estimation of how much area we can cover and what is the probability of outage as well as what are the other possible propagation models. First let us begin with a brief recap.

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The slide is titled "Recapitulation" and is part of a presentation on "Wireless Communications". It lists five propagation models:

- Free Space Propagation Model
- Small Scale Propagation Model
- Large Scale Propagation Model
- Log Distance Path Loss Model
- Log Normal Shadowing

At the bottom of the slide, there is a footer with the Indian Institute of Technology Delhi logo, the number 78, and the name of the speaker, Ranjan Bose, from the Department of Electrical Engineering.

We started off last time by looking at the free space propagation model – “The Friss free space model and we realized that the inverse square law holds, provided you are not taking any ground reflections. Then we looked at small scale propagation model which actually takes into consideration the reflection, diffraction and scattering effects. Then we looked at large scale propagation model which primarily works on the basis of reflection. Only then we came up primarily based on empirical data that log distance path model actually fits well for the path loss model. However the Log Distance Path Loss Model is deterministic. It doesn’t take into consideration the shadowing effects. so what signal you actually receive is a random variable and it depends on where you are actually sitting, whether you are on a shadow of a building or are you being scattered well enough and things like that. Therefore you have to have some kind of a random variable built into your equation so as to model the Log Normal Shadowing better.

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### Log-distance path loss model

- The average large scale path loss for an arbitrary transmitter-receiver separation is expressed as a function of distance by using a path loss exponent  $n$ .
- $n$  characterizes the propagation environment
  - For free space, it is 2
  - When obstructions are present it has a larger value.

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n \quad d \geq d_0$$
$$\overline{PL}(dB) = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$

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So we start with this Log Distance Path Loss Model. We remember that the average large scale path loss for an arbitrary transmitter to receiver separation is expressed as a function of the distance by simply using 'n', the path loss exponent. We learnt last time that this n can go from 2 to even 6 depending upon how dense your environment is. In fact, we look at a special case where n was less than two. The free space path loss exponent in line of site inside room scenarios where the hallways inside buildings can give you a guiding effect. So it's better than two. However 'n' which characterizes the propagation environment usually is two for free space and when obstructions are present that is, no line of site, you have a larger value of 'n'. Today we will look at some examples to find out how to calculate 'n' from measured data. We came up with the following two equations. One was a proportionality that the path loss at a distance 'd' can be expressed in terms of a path loss at a known distance  $d_0$ . Remember this equation holds true only in the far field. Thus value of  $d_0$  is also measured in the far field. 'n' is the path loss exponent. Usually it is comfortable to express the whole thing in dB as explained here.

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## Log-distance path loss model

$\overline{PL}(d)$  denotes the average large - scale path loss at a distance  $d$  (denoted in dB)

$d_0$  is a reference distance

$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$   $d \geq d_0$

is computed assuming free space propagation model between transmitter and  $d_0$  (or by measurement).

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$

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So just to summarize this equation this will form the starting point, PL bar denotes the average. If you remember the actual measured data fluctuates about a mean. The fluctuations are important but what really characterizes the path loss is the average value. Hence this PL bar denotes the average large scale path loss at a distance 'd'. You have this  $d_0$  as your reference distance. When we do an example we will make sure how to use this 'd' and of course  $\overline{PL}(d_0)$  is computed using free space propagation model. That's different so this equation is mix and match where  $\overline{PL}(d_0)$  has been computed using a free space model where 'n' is actually equal to two. It can be measured or calculated. It's usually calculated and the rest of it assumes value of 'n' which may be larger than two.

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### Log-normal Shadowing (1)

In practice  $n$  and  $\sigma$  are calculated from measured data.

$$PL(d) [dB] = \overline{PL}(d) + X_{\sigma}$$
$$PL(d) [dB] = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right) + X_{\sigma}$$

- $X_{\sigma}$  is a zero mean Gaussian (normal) distributed random variable (in dB) with standard deviation  $\sigma$  (also in dB).

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Let's go back to Log Normal Shadowing. Why do we need log normal shadowing? In order to take into account the shadowing effects due to cluttering on the propagation path, a factor has to be added. How is it added? Well, we have a same PL path loss average at a distance 'd' plus this factor  $X_{\sigma}$  in the terms of your PL  $d_0$  with respect to reference distance. PL (d) at a distance d is nothing but the path loss measured at  $d_0$  plus  $10 n \log d$  over  $d_0$  plus  $X_{\sigma}$ .  $X_{\sigma}$  is interesting because it is a zero mean Gaussian random variable in dB with a standard deviation  $\sigma$  which is also in dB. So please remember that this n and  $\sigma$  are usually calculated using measured data.

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## Log-normal Shadowing (2)

- Determination of  $n$  and  $\sigma$ 
  - In practice the values of  $n$  and  $\sigma$  are computed from measured data using *linear regression*
  - The difference between the measured data and estimated path losses are minimized in a *mean square error* sense.

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So how do we determine  $n$  and  $\sigma$ ? In practice, the values of  $n$  and  $\sigma$  are computed from measured data using linear regression. So that is, we take some measured data but somehow we have to do some kind of a curve fitting. Now there many ways to fit a curve. What is done is, we use something called as “a minimizing mean square error”. So the difference between the measured data and the estimated path losses are minimized in a mean square error sense – ‘mmsk’.

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## Example: Determination of $n, \sigma$ (1)

- Let  $P_r(d_0) = 0$  dBm at  $d_0 = 100$ m.
- The following table gives the measured values of received power,  $P_r$  at various distances.

Distance from Transmitter	Received Power
100m	0 dBm
500m	-15 dBm
1000m	-21 dBm
3000m	-36 dBm

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Let us look at an example. So the scenario is as follows. I am interested in modeling a sub-urban environment where I have transmitted and I have a receiver with a car and a power meter. So what I do is I measure the values of the received power as I move along and I don't take just one measurement. In fact, I take tens of measurements, average them and find out the average received power. The first step must be to find out the reference power at a certain distance. Let the distance  $d_0$  be 100 meters. So I move away from the base station at about 100 meters. Remember these 100 meters is from antenna to antenna not from the base of the base station to the receiver. Suppose I get 1 mW of received power which is nothing but 0 dBm. If you remember, dBm is nothing but the relative power with respect to 1 mW. Now as we move along radially away from the base stations, I keep on taking measurements and I tabulate it. Just for the sake of illustration, I take four measurements. One at the reference distance, one at 500 meters, 1000 meters and about 3 kilometers. At each of these points using a power meter, I have obtained the following readings. In reality, if I have to really measure distances up to 3 kilometers, I would take more than 20 measurements here.

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### Example: Determination of $n, \sigma$ (2)

- Compute the estimates for received power at different distances using *long-distance path loss model*.
 
$$\overline{PL}(dB) = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$
- Since  $P_r(d_0) = 0$  dBm, the equation reduces to
 
$$PL(dB) = 0 + 10n \log\left(\frac{d}{d_0}\right)$$
- Use this equation to compute estimates of power levels at  $d = 500m, 1000m, \text{ and } 3000m$ .

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So what do we have to do now? We have to somehow fit our measured data in the best possible manner in the mean square error sense into this equation. Since your  $P_r d_0$  'the reference distance' is 0 dBm, my equation  $PL(dB) = PL d_0 + 10 n \log (d / d_0)$ . This simply reduces to  $PL(dB) = 10 n \log (d / d_0)$ . Now this equation may be used to compute the estimated power levels for a certain value of 'n'. I know the value of  $d_0$ . I know the value of d because I have 3 more entries in my table. I can have a sequence of numbers.

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### Example: Determination of $n, \sigma$ (3)

Distance	Measured Value of $P_r$ (dBm)	Estimated Value of $P_r$ (dBm)
100m	0	0
500m	-15	-6.99 $n$
1000m	-21	-10 $n$
3000m	-36	-14.77 $n$

- Now calculate the Mean Square Error (MSE) between the estimated and the measured values.
- Then choose a value for  $n$  such that the MSE is minimized.

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So let's do that. I have another column in my table now where for every distance corresponding to the every measured power, I have an estimated value. Now the first row is zero of course because it is based on the  $d_0$  itself. But rest of the three entries has to be a function of  $n$ . the question before us is: what value of  $n$  do we choose so that these three points best fit this curve? So the next step is to calculate the mean square error between the estimated which is this column and the measured which is this column (Refer Slide Time: 12:48). The objective is to choose a value of  $n$  such that the mean square error is minimized.

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### Example: Determination of $n, \sigma$ (4)

- Mean Square Error:  
$$MSE = \sqrt{\sum_{i=1}^k (p_i - \hat{p}_i)^2}$$

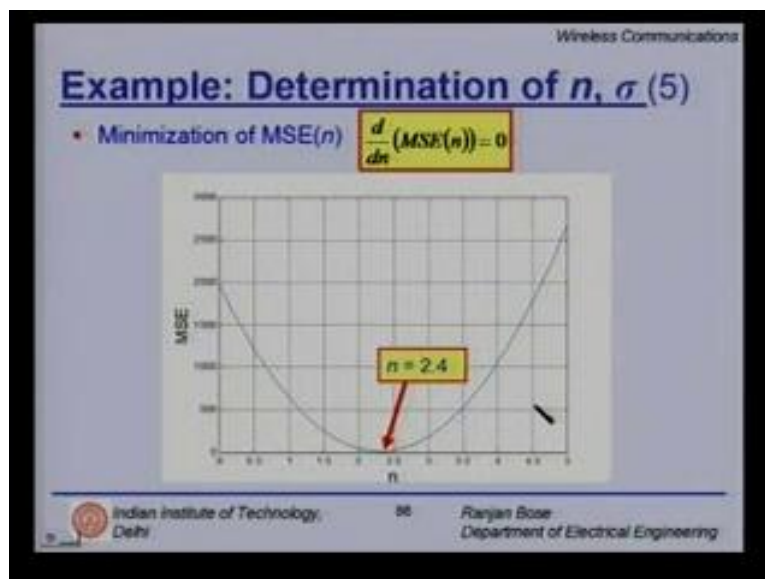
$p_i$  is the actual measured value of power at some distance  
 $\hat{p}_i$  is the estimate of power at that distance  
 $k$  is the number of measurement samples
- $(MSE)^2 = 0 + (6.99n - 5)^2 + (10n - 11)^2 + (14.77n - 16)^2$
- Note that MSE is a function of  $n$  :  $MSE(n)$
- Minimization of  $MSE(n)$   $\frac{d}{dn} (MSE(n)) = 0$

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Let us look at the mean square error first. mean square error is defined as under root, summation from  $i = 1$  through  $k$  where  $k$  is the total number measured data points, ' $P_i$ ' the measured data, ' $\hat{P}_i$ ' squared which is the estimated data. Inside the parenthesis is the error part squared and this is the averaging (Refer Slide Time: 13:35) part and this under root is taken. In our case, we go by the measured value and the estimated value squared. In the first case it is zero. If you go back to your first equation here there is no error between the measured value and the estimated value. However, next time this is the measured value of -5 dBm and this is the estimated value. When we square it, we have mean square error. Please note as expected the MSE is a function of  $n$ . now what do we have to do? We have to minimize it. So the best way to minimize is of course to take a simple derivative. If we have the luxury to plot the data, you can plot the data with respect to different values of ' $n$ ' and you know ' $n$ ' can vary from 2 to 6 and find out the value of  $n$  for which MSE is minimum. So we have got two ways to do it.

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You either take a derivative or you do a plot. So if we plot the mean square error with respect to  $n$ , the minimum point is reached close to 2.4. For this specific set of data points, the path loss exponent is 2.4 which means it is a kind of an urban scenario but not densely populated urban area. We do not have very many concrete buildings here. Otherwise  $n$  would have been 3 or larger. Clearly it is not free space. There are a few obstructions like foliage, a little bit of scattering and things like that. But it is a good estimate because your mean square error is fairly close to zero. Now what do we do with this? We have got a handle on your  $n$ , 'the path loss exponent'. If I had taken a larger set of measurements, then my error would have been larger. The minimum error would have also been larger. It all depends upon the total number of data points you had taken how easier or difficult it is to fit the curve.

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### Example: Determination of $n, \sigma$ (6)

- Sample variance calculation ( $\sigma^2$ )  
$$\sigma^2 = \frac{\sum_{i=1}^k (p_i - \hat{p}_i)^2}{k}$$

$p_i$  is the actual measured value of power at some distance  $d$   
 $\hat{p}_i$  is the estimate of power at that distance  $d$   
 $k$  is the number of measurement samples
- Clearly,  
$$\sigma^2 = \text{MSE}(N)/k$$
$$\sigma^2 = \text{MMSE}/k$$
$$\sigma = \sqrt{\text{MMSE}/k}$$

where  $N$  is the value that minimizes  $\text{MSE}(n)$   
MMSE is Minimum Mean Square Error.

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Now let us focus our attention on the sigma part. What is the sigma part? Sigma is this standard deviation. Now the sample variance sigma squared is defined as in the numerator summation I is equal to one through k where k is the total number of measured data points. for our case it was  $(4P_i - \hat{P}_i)$  where  $P_i$  is the actual measured power,  $\hat{P}_i$  is the estimate of the power at that distance 'd' connected from our equation squared whole divided by k where k is the number of measured samples. so it's a measure of your sample variance from this and if you remember your mean square error equation in the last slide, your sample variance sigma squared is equal to mean square error for that value 'n' which minimizes the MSE.

So if N represents that value of path loss exponent that minimizes the mean square error that minimum value of that mean square error divided by k is sigma squared. This is obtained simply by combining this equation and the equation for mean square error. It can also be represented as that minimum error divided by k. the sigma which is your standard deviation is under root MMSE divided by k. here, if the value of k is larger than from the weak law of large numbers, the estimate of the value of sigma squared and hence sigma will be better. In practice, k is not 4. You can take 20 to 100 measurements and remember each of these measured points are averaged themselves. So just to give you a feel, we are doing some in-house data measurements for ultra-wideband frequencies from 3.1 to 10.6 GHz.

We do the measurements in frequency domain, find out the  $h$  of the transfer function and then take the inverse Fourier transform to get the impulse response of the channel. So this measurement is not to measure the sigma squared but I am just talking from the perspective of channel measurements. We take about 1600 measurements for every point and then average it. The channel is fast changing. It is a time variant channel that we are talking about. So at every location, we are taking close to 1600 measurements and then averaging it. Here we can average just 20 measurements or so at every point and so you can get  $P_i$  by averaging say 20 measurements at a certain point. Averaging is very important because we have seen that there is a lot of fluctuations that take place.

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### Example: Determination of $n, \sigma$ (7)

- Sample variance calculation ( $\sigma^2$ )

$$\sigma^2 = \frac{\sum_{i=1}^k (p_i - \hat{p}_i)^2}{k}$$

$p_i$  is the actual measured value of power at some distance  $d$   
 $\hat{p}_i$  is the estimate of power at that distance  $d$   
 $k$  is the number of measurement samples

- For  $n = 2.4$  we have  $\sigma^2 = 3.11$  ( $\sigma = 1.76$ )
- A Gaussian random variable having zero mean and variance  $\sigma^2$  can be added to the Log-distance Path Loss model to simulate the Shadowing Effects.

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So let's continue with our example. We have this nice relationship between sigma squared, the measured value, the estimated value and the number of measurements 'k' for our case when 'n' is 2.4, we substitute this value. So our M is 2.4 which minimizes the mean square error. We get sigma squared as 3.11 which is very low. This is nothing but sigma equal to 1.76. If you remember, the equation sigma is also expressed in dB. So you can take 10 log sigma squared which will give you the expression in dB. Now what do we do? We enhance our log normal path loss model. What we do is, a Gaussian random variable having a zero mean and variance sigma squared can be added to the log distance path loss model to simulate the shadowing effects. Please remember shadowing is very important even if you are sitting in a cell fairly close to the base station. However because of the shadowing effect, you may not get enough power whereas a person sitting farther away from you may enjoy power more than the required threshold. That's because of the shadowing effect. It is a random variable. That will help us determine the coverage. Not all points in the cell are covered because of the shadowing effect. You may be closer to the base station and still enjoy less power as opposed to somebody whose farther away from the base station. Let us now look at something called "outage probability".

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### Outage Probability

- What is the probability that the receiver power  $P_r(d)$  (in dB) at distance  $d$  is **above** (or **below**) some fixed value  $\gamma$ ?
- That is,  
**Prob** ( $P_r(d) \geq \gamma$ ) or **Prob** ( $P_r(d) \leq \gamma$ )?
- For a normally distributed random variable,  $X$

$$\Pr(X > x_0) = \int_{x_0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

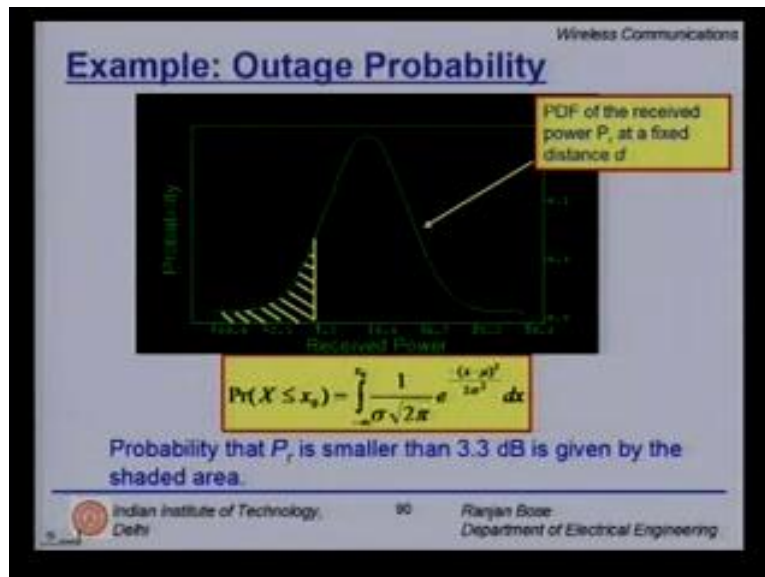
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What we would like to know is: what is the probability that the receiver power which is denoted by  $P_r(d)$  at a distance 'd' of course measured in dB at a distance  $d$  is above or conversely below some fixed value  $\gamma$ ? Now let us understand the objective behind the statement. We have fixed at threshold value  $\gamma$  now. The threshold value can be fixed because of several reasons either my receiver sensitivity decreases the  $\gamma$  or in that particular application, my modulation scheme requires certain signal strength. That is, if my level is below  $\gamma$ , I will have to resort to a lower modulation scheme which will lead to a lower system capacity in the overall cells. Or I might be prone to interference in the system and I need a certain signal strength level so that I need to combat the effects of interference and interference limited scenario.

Again my received strength should be greater than  $\gamma$ . Otherwise my system will fail because I am troubled down by interference. All these reasons require me to find out somewhere how I can decide the probability that my received strength should be greater than this threshold  $\gamma$ . Of course if you are below the threshold, then you are out. We are trying to calculate the outage probability. So we would like to know the probability that the received power at the distance 'd' is greater than or equal to  $\gamma$  conversely less than equal to  $\gamma$ . Remember these have to do something with your shadowing effect. Otherwise based on a clearly deterministic situation, we cannot come up with these values. Now for a normally distributed random variable  $X$ , we have probability that  $X$  is greater than a value  $X_0 = \int_{X_0}^{\infty}$  to infinity. This is your famous Gaussian random variable pdf.  $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$   $d(x)$ .  $\mu$  is the mean  $\sigma$  in the standard deviation  $\sigma^2$  is a variance. If you know  $\mu$  and  $\sigma$  for a Gaussian random variable you know everything about it. That's the beauty of the Gaussian pdf. This is one of the most favorite of all the pdf's in communication engineering. So for normally distributed random variable, the probability  $X > X_0$  is given by this expression.

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Let us now look at an example of outage probability here. We have plotted the Gaussian random variables pdf. If you look on the X axis, I have plotted the received power. Now clearly there is a mean. So if you look at the mean, the average power is centered around here but because of the shadowing effect, you may get some power below it or above it. So on the y axis is the probability and this bell shaped inverted bell is the pdf of the received power 'Pr' at a fixed distance 'd'. Now how broad is this and what is the sigma for this one will be determined by the measured data if you remember. This equation that  $X < X_0$ . So I have reversed the situation here. I am trying to find out the probability for outage. Outage will occur when my received power is below a threshold so if my threshold is  $x_0$  the probability that the value of the received power  $x$  is less than  $x_0$  is equal to - infinity to  $x_0$  area under the curve. So let's take a value for the sake of illustration. Suppose I would like to have my  $P_r > 3.3$  dB for it to function value.

So what I do is, I draw a line here by now. You have already learnt that all the received power is always being expressed in dB's. So I have drawn a line here and the area under the curve to the left of it represents the probability that the received power is less than 3.3 dB. So whatever be the area under the curve, the value is the probability that you will have the outage. So outage power probability is given by the area shown by the shaded figure. So you can easily find out if, instead of having 3.3 dB, my line was centered around the mean then the outage probability is 0.5. Clearly, if I improve my receiver sensitivity that is, my receiver is more and more sensitive to the received signal. That is, I can work it lower received power. I move the yellow line to the left and my outage probability goes down it is not linear. It changes exponentially.

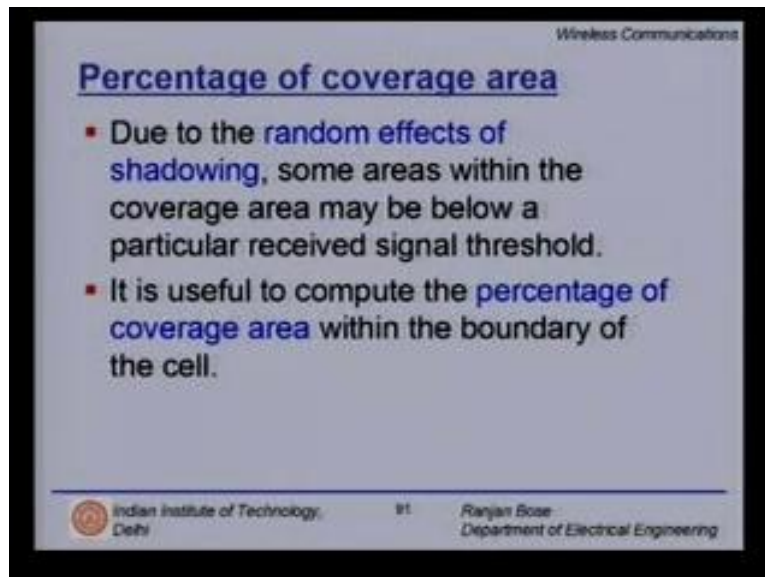
Conversation between Student and Professor: the question being asked is as follows. What is the relationship between the threshold value and the hand off threshold? The answer is: yes. We would like to use this to design our system in such a way that if it drops below a certain threshold, you like to hand off but the hand off should happen when truly you are in a shadowed region and not because of the random fluctuations. So far the whole theory is based on average power bands. If it falls below this threshold, I would definitely like to hand off.

So it's possible. It is a probabilistic quantity which you are not moving. You are sitting at one location in the cell but because of the random nature of the shadowing, some scatterers have moved. Some cars are moved around as the environment has changed in such a manner that you are getting lesser power than your threshold. You would like to hand off another.

Conversation between student and professor: the question being asked is: what relationship does this threshold have with the design of the amplifier? So the way people calculate it is as follows. First of all here, outage probability is a selling feature. Your customer who is going to subscribe wants to know what the coverage area is. What is the probability of outage? They are not known in this term but they are some buzz words which can be told to you. You can claim that it is 99% coverage which is also taken practice into account the outage probability. The first thing that the service provider must understand is what the outage probability the person has to guarantee is. Once the outage probability is decided, your average received power threshold is decided. Of course, it also depends on the cell sizes, the interference requirements and other effects. You fix what kind of an amplifier you want to choose. However amplifier strength for standard handsets also comes kind of fixed. May be a better receiver, a better handset. A more expensive handset might have a better amplification facility. It is possible. But in general, all you can play with as a service provider is to manage this size of the cell. Answer is we can change the amplifier.

Maybe we can go into a more expensive model and it will lead to a better outage probability. But that has nothing to do with the service provider. The service provider is not selling with the handsets. But he is designing the cell for standard handsets available off the shelf. So in that sense, you will have nothing to do with the amplifier. If we improve your amplification, your outage probability will go down. A service provider should use this curve to actually plan the cell sizes and will shortly look at an example to relate somehow these equations to the coverage area. What is the percentage of area within the cell that I can provide service to? That's another important feature. So this outage probability can give you a sense as to the percentage of time. That's the fact you can come up. Or the percentage of people who will fall into the outage category. But a more realistic prospective is to find out the percentage area within the cell which is covered or which is not covered at a certain time. So let us now focus our attention on the percentage of coverage area.

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### Percentage of coverage area

- Due to the **random effects of shadowing**, some areas within the coverage area may be below a particular received signal threshold.
- It is useful to compute the **percentage of coverage area** within the boundary of the cell.

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Due to the random effects of shadowing, some areas within the coverage may be below a particular received signal threshold remembered. My intension was good when I designed my cell. I ensured base to the average power measurement that everybody within the cell has enough power. Unfortunately because of the random scattering effects shadowing, some people, despite my calculation will be at a lower level. Is that point is clear? So even if I designed myself for average values, I still have people within the cell who do not receive enough power. It is useful to compute the percentage of coverage area. Please remember there is a Gaussian random variable that has been placed at the average received power level and the Gaussian random variable extends to minus infinity on one side. So no matter how much signal strength you pump into the cell, they will be an extremely low but non-zero percentage of area which is not being covered unless your sigma is zero. For a non-zero sigma you will always have some fraction of the cell which is not being covered. None the less it could be very small.

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### Percentage of coverage area

- Consider a circular coverage area with radius  $R$  from a base station and a minimum threshold power level,  $\gamma$ .
- Need to determine the percentage of area where the received signal level greater than or equal to  $\gamma$ .
- The percentage area is given by:

$$PA(\gamma) = \frac{1}{\pi R^2} \int P[P_r(r) > \gamma] dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R P[P_r(r) > \gamma] r dr d\theta \quad (1)$$

- where  $P[P_r(r) > \gamma]$  is the probability that the received power at a distance  $d = r$  is greater than  $\gamma$ , and is constant within the incrementally small area  $dA$



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So consider a circular coverage area for the sake of simplicity with a cell radius  $R$  from a base station and a minimum threshold power level  $\gamma$ . Let's draw it. I have a cell. I have put a base station here and the radius of the cell is  $R$ . what we do need to find out is to determine the percentage of area. Remember the percentage of area where received signal level is greater than or equal to  $\gamma$ . So we are trying to find out the 1- outage probability. I am looking at the optimistic perspective. What percentage of the cell is covered? The percentage area is given by  $PA(\gamma)$ . Clearly it will be a function of  $\gamma$ . If I reduce my  $\gamma$ , more and more area will be covered and that is nothing but the probability that the received power is greater than  $\gamma$ , the threshold  $dA$  integrated over the whole cell area normalized with respect to the entire cell area. So the area which is covered divided by the total cell area is easier to express in polar coordinates. I have this double integral 0 to  $2\pi$  all around the cell, 0 to  $R$ . so moving radially away from a certain cell section  $r dr d\theta$  is a small area where the probability of received power is greater than  $\gamma$ . That is not formulation. So if you notice, this analysis is simplistic. In the sense that, I am only walking for circular cell. I can extend that for a hexagonal cell or a square cell by doing numerical integration. Let us label this as equation 1. We will use this equation to come up to the final expression.



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### Percentage of Coverage Area (2)

- Note that
$$P(P_r(d) > \gamma) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{\gamma - P_r(d)}{\sigma\sqrt{2}}\right) \quad (II)$$

where

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$$

- Now, the path loss at distance  $d$  ( $PL(d)$ ) can be broken down as :
$$PL(0 \text{ to } d) = PL(0 \text{ to } d_0) + PL(d_0 \text{ to } R) - PL(d \text{ to } R)$$
where  $d_0$  is a free space reference distance ( $r > d_0$ )

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n \log(R/d_0) + 10n \log(d/R) \quad (III)$$

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So we note that the probability that the received power at a distance 'd' greater than gamma is nothing but  $1/2 - (1/2)$  half error function  $\gamma - P_r(d)$  average over  $\sigma$  under root 2. This comes from the Gaussian random variable. This is a standard result. It is the area under the curve in the Gaussian tale. Error function is defined as  $1/\sqrt{\pi} \int_0^z e^{-x^2} dx$ . So these expressions are coming because we have a normally distributed received power. Now the path loss at a distance d which is given by  $PL(d)$  can be broken down into three parts. So path loss from 0 to d is path loss from 0 to  $d_0$ . What is  $d_0$ ? This is the reference distance. So time and again, my reference distance is popping up and hence the important of a measured value plus the path loss  $d_0$  to R - path loss d to R. so if you combine them together,  $PL$  average d is nothing but  $PL$  average ( $d_0$ ) +  $10 n \log (R/d_0) + 10 n \log (d/R)$ .


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### Percentage of Coverage Area (3)

- Substituting (II) in (III), we get
 
$$P(P_r(r) > \gamma) = \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left( \frac{\gamma - [P_t - (PL(d_0) + 10n \log(r/d_0) + 10n \log(r/R))]}{\sigma \sqrt{2}} \right)$$
- Substitution in (I) gives
 
$$PA(\gamma) = \frac{1}{2} - \frac{1}{R^2} \int_0^R \operatorname{erf} \left( a + b \ln \frac{r}{R} \right) r dr$$
 where
 
$$a = (\gamma - P_t + PL(d_0) + 10n \log(R/d_0)) / \sigma \sqrt{2}$$

$$b = (10n \log e) / \sigma \sqrt{2}$$
- Simplifying, we get
 
$$PA(\gamma) = \frac{1}{2} - \left( 1 - \operatorname{erf} \left( a \right) + e^{\left( \frac{1-2ab}{b^2} \right)} \left[ 1 - \operatorname{erf} \left( \frac{1-ab}{b} \right) \right] \right)$$

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If you combine equation 2 and 3 and do some basic mathematics, the probability Pr greater than gamma or threshold can be expressed like this. Again we do a little bit more mathematics. If you go back to your original equation which finds out the area where your received power is greater than gamma, you get this expression where a and b are expressed as follows. b only depends on n and sigma. By now we know how to calculate ‘n’ and sigma by measured data which is not difficult to calculate. ‘a’ here depends on your threshold gamma. it has to depend on transmit power and your n and sigma k. now once you simplify everything and when the dust settles, you get the final expression of the percentage of area which will get the received power higher than the threshold gamma as this following expression. There are two ways to use the expression. Either you calculate ‘n’ sigma. Figure out what is the transmit power PT and then substitute into this equation or you can plot a family of curves and quickly look at the parameters and find out what is the percentage of area. What are your parameters? Your parameters are that the probability that your received power is greater than some gamma. That is one probability. The other probability is the coverage area and then there are two parameters, sigma and n. you can use something called as a normalized sigma over n variables. So there are three parameters with which we can have a family of curves which will capture the essence of this equation. To do this equation, let us plot the family of curves.

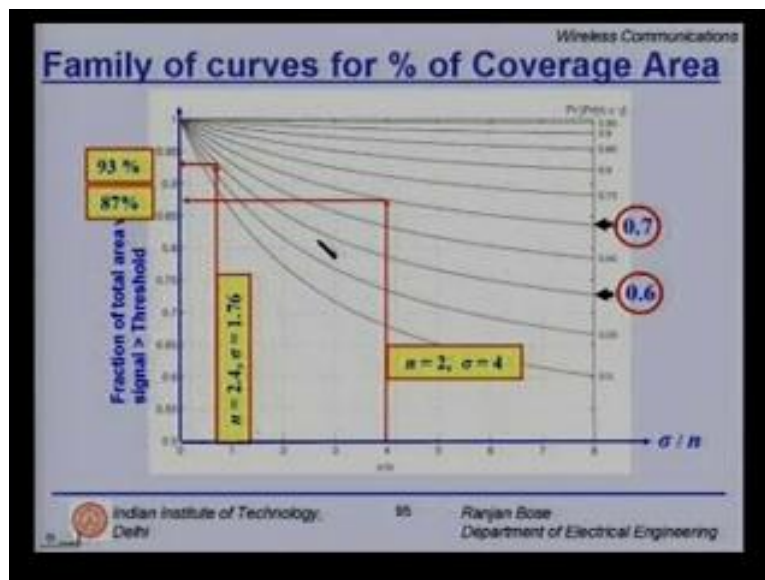
Conversation between Student and Professor:

Student: what is “erf”?

Professor: erf is “error function”. Error function has been defined previously as error function of z which is nothing but  $2/\sqrt{\pi} \int_0^z e^{-x^2} dx$ . This is a handy way to find out the area under the curve of a Gaussian tale. A q function is sometimes used. In some text book, you will find error function. In some text books, you will find the q function and in others,

you will find the error function compliment. They are all related to the area under the curve for the Gaussian random variable.

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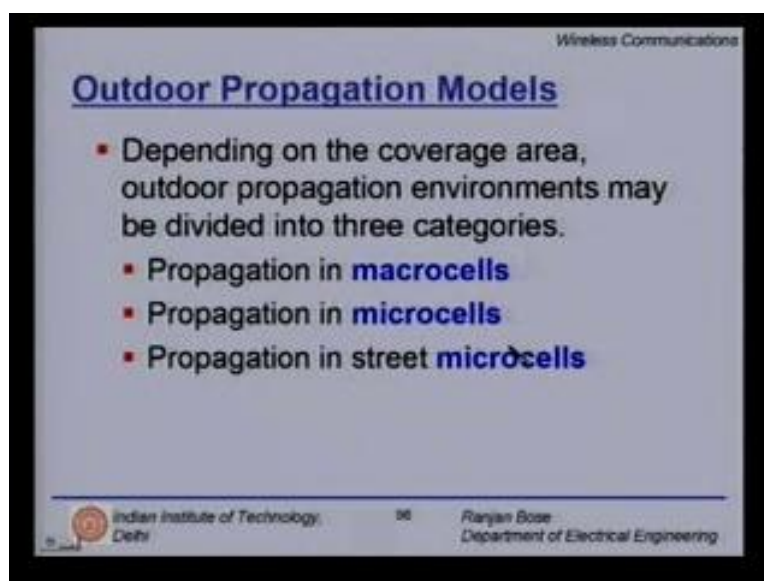
So going back to the family of curves which will help us relate the coverage area sigma and n. here I have drawn some curves. Let us look at the axes first. On the x axis, I have tried to put a normalized value of sigma, sigma over n. I have not separated out sigma nm. This is also done in literature. Please remember if you have a small value of sigma, then you will have a larger and larger coverage area because the randomness goes. What you designed for is what you get. On the y axis ordinate I put the fraction of the total cell area where the signal is actually is greater than threshold. So it's my useful area. This is my selling feature as a service provider. The fraction of cell area where the signal strength is better than the required threshold. So one is the best possible scenario. Zero is bad. As you know, intuitively if you move towards the left on the x axis, that is, we go for a lower value of sigma for the same n, you move higher and higher up in terms of the fraction of area that you cover.

This is intuitive. I have less randomness. On the other hand if you fix the sigma and you increase n, this is hypothetical because sigma and n themselves are related as you saw in the previous calculation. For example, if you fixed sigma and increased n, then you again move towards the left. However that's not a fair comparison because sigma itself will be a function of n. if they are more scatterers, n will go up and this sigma will also go up. So we cannot fix one with respect to the other. On this side, I have put probability that your received power is greater than gamma. Now you would like to have the probability that you received power is always greater than threshold. You want to be absolutely sure in your cell design that you are exactly at the top level. That much can be done about your strict requirement. On the other hand, I want 90 % of the time that my received power is greater than my threshold. So 10 % of the time you may drop your call. Then I am on a curve which is represented by 0.9. Move along this curve. If on the other

hand, I am a service provider who is cutting corners and I said, “No look! 75% is good enough. Let 25% of the calls get dropped once in a while because the received power is exceeding the threshold value”. That probability is only 0.75. Then I move along this curve. Now let’s consider an example or continue with our previous example where we calculated  $n = 2.4$  and  $\sigma = 1.76$ . In this case, if you take a ratio, then this is close to 0.76. See if we divide  $\sigma$  1.76 by 2.4, you are somewhere here. Suppose I say, “Look. I would like to work at a level where my probability required that my received power exceeds the required threshold 3.6. This is a poor design but let’s see where we are. Under this constraint I would like to know what fraction of the cell area will be covered.

Please note I have not talked about how big my cell is. Those things are not coming into the picture. Then if I follow this curve, I see that 93% of the people in the cell will have the received power greater than the threshold with probability 0.6. It’s a way to design your cellular system. Let’s look at a slightly different scenario and let’s look at another set of values,  $n = 2$  and  $\sigma = 4$ . So I have moved more into the rural area instead of having 2.4. I am in more free space environment and  $\sigma$  is four. So under the situation, this curve is for four. It should be possibly for  $\sigma$  is equal to a value of eight where I have  $\sigma/n = 4$ . So let’s go by this example. But for this set of parameters, I should have moved somewhere along this line of two. So when the ratio of  $\sigma$  to  $n$  is 4, if you modify this example of  $\sigma = 8$  and  $n = 2$ , then we say, “Look! Let us look at 0.7 as a probability that your received strength is greater than the threshold”. So it made my things stricter. In that sense, I will have this time the percentage coverage as only 87%. Typically, I would like to work over 95 % coverage if I am going to make an economically viable system with this family of curves. It gives you a very handy way to relate your  $n$ , your  $\sigma$ , the probability that you want to give your signal receive signal greater than the threshold and the coverage area. This is true for the circular derivation. I can have another family of curves for hexagonal where I will calculate this values based on numerical integration.

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Wireless Communications

### Outdoor Propagation Models

- Depending on the coverage area, outdoor propagation environments may be divided into three categories.
  - Propagation in **macrocells**
  - Propagation in **microcells**
  - Propagation in **street microcells**

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At this time, let us now look at certain outdoor propagation models. I would like to introduce the concepts today and in subsequent lectures look at the exact models. Depending on the coverage area, outdoor propagation environments may be divided into three broad categories. Category 1 is propagation in macro cells. We'll describe what a macro cell is. The next is propagation in microcells and propagation in a specific subset of micro cells called street microcells. Please note we are talking about outdoor propagation models. After this as we move inside indoors, we have home cells and we also have picocells within the homes and also body cells where we have a body area network. But all those are much smaller. So as you can guess, these are in the decreasing sizes: large cells & smaller cells.

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## Macrocells

- Base stations located at high points
- Coverage of several kilometers
- The average path loss in dB has **normal distribution**
  - Average path loss is a result of many forward scatterings over a large number of obstacles
    - Each contributing a random multiplicative factor
    - Converted to dB, this gives a sum of random variable
  - Sum is normally distributed because of **Central Limit Theorem**

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What are macro cells? Here, the base stations are located at high points on top of a tower, on top of a building or on top of a tall building itself. The coverage could be of several kilometers. So I'm talking about a standard cellular mobile network. The average path loss in dB has a normal distribution. This we have measured, seen and done the examples with. So these are all characteristics of a macro cell. The average path loss is a result of many forward scatterings over a large number of obstacles, each contributing a random multiplicative factor. Hence the notion of fading converted to dB gives us a sum of random variables. Thus if you use the central limit theorem, the sum is normally distributed. Hence you have the average path loss in dB being distributed as normal. Those are characteristics of macro shells.

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The slide is titled "Microcells" and is part of a "Wireless Communications" presentation. It contains a bulleted list of characteristics and uses of microcells. The footer includes the logo of the Indian Institute of Technology, Delhi, and the name of the speaker, Ranjan Bose, from the Department of Electrical Engineering.

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## Microcells

- Propagation differs significantly
  - Milder propagation characteristics
  - Small multipath **delay spread** and shallow fading imply the feasibility of higher data rate transmission
  - Mostly used in **crowded urban areas**
  - If transmitter antenna is lower than the surrounding building than the signals propagate along the streets : **Street Microcells**

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As opposed to microcell, the propagation differs significantly because you have a milder propagation characteristic, the small multipath delay spread and shallow fading implying that the feasibility of higher data rate transmission. We have also seen before that as you grow into smaller and smaller cells, your data sending capacity increases. It is mostly used in crowded urban environments. If the transmitter antenna is lower than the surrounding buildings, an interesting thing happens. The streets start forming guides. The waves tend to become guided along the streets. So the signal actually propagates along the streets and it forms the street microcells. Now many times, you don't want your base stations to be very tall. Clearly in macro cells, base stations must be as high as possible. "As high as possible" means more probability of line of sight to the receiver, less blockage if you increase the base station height. However, the interesting scenario is when we do not want the base station height to be much higher. In fact almost at the level of the receiver. This is because if you can communicate happily with the receiver in your cell or in your microcell without the maximum height, then why increase the height. It will only cause interference to other base stations and other cells where the frequency is being reused.

So your base station height should be only as high as required but not any higher because you have a chance of sending out more interference signals. By design, we may want to put the base stations not at very high locations. It depends on what is a system. So now let's look at this scenario of street microcells. Most of the signal power propagates along the streets. An analogy is inside your building if you do measurements along long corridors and hallway, you have the similar effect.

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### Street Microcells (1)

- Most of the signal power propagates along the street.
- The signals may reach with LOS paths if the receiver is along the same street with the transmitter
- The signals may reach via indirect propagation mechanisms if the receiver turns to another street.

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In one of your earlier slides, you have seen that path loss exponent 'n' can be less than 2 inside the building where you have a line of sight and corridors. The signal may reach with line of sight path if the receiver is along the same street or it may reach via an indirect propagation mechanism if the receiver turns to another street. But the indirect mechanism could be scattering or reflection or even diffraction but once the reflection or diffraction occurs, again you have the guiding effect. Let us look at this graphically.

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### Street Microcells (2)

Building Blocks

A B C D

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So here is a bird's eye view of a densely populated urban area with lot of buildings. We have drawn them uniformly but it could be of irregular shapes and heights. Let A be a location of a transmitter and let C be the location of another transmitter denoted by red circles. Let B be the location of one of the receiver. The mobile station located at B easily gets access to either A or C because of the great guiding effect of the streets. Note A and C are lower than the building height. However if B moves to D, then the signal must propagate along this line. It should diffract or scatter or reflect then again go through the street guiding effect and reach D. so it is a very funny kind of propagation. Where will it be useful? When you plan your base station locations for a dense urban environment, then you have to take into consideration these street effects and also the height of the base station. Your measurement data will also be different in the scenario.

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### Macrocells versus Microcells

	Macrocell	Microcell
Cell Radius	1 to 20 km	0.1 to 1 km
Tx Power	1 to 10 W	0.1 to 1 W
Fading	Rayleigh	Nakagami-Rice
RMS Delay Spread	0.1 to 10 $\mu$ s	10 to 100ns
Max. Bit Rate	0.3 Mbps	1 Mbps

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Now, a slide on macrocells versus microcells. The cell radius, as the name suggests, macro cells are larger as opposed to the microcells. Macro cells can go from 1 km to 20 km. Of course, these days you never have cell sizes of 20 km. Much before that, you either get interference limited or capacity limited but we will still call them macro cells. If my cell is a footprint of a satellite, remember we are looking at wireless communication in general. Let's not think about only mobile systems as the only means of wireless communication. A footprint of a cell is formed because of satellite communication might have a radius of 20 km. Microcells on the other hand are much smaller. 0.1 km to 1 km in 802.16, the IEEE standard 802.16 wireless WAN standards. The cell sizes are by definition in the microcell region. Maximum you have 2 km cells. So it's in between. Clearly if your cell sizes are smaller, you require less transmit power. You should use only as much power as required and no more because your power is interference for somebody else. Transmit power ranges from 1 to 10 W whereas sub-watt 0.1 to 1 W for microcells. Fading characteristics will also be different. Here it is Rayleigh. Clearly you don't have a line of sight. On the other hand, in microcell, the probability of line of sight is much larger because it is closer to the base station and you can have a Rician distribution or a Nakagami-Rice distribution. The



delay spread depends on the multipath environment. If you have the reflectors far apart, you will have a larger delay spread which is true for macro cells. But the delay spread could be 0.1 to 10 ms as opposed to very low 10 to 100 ns for microcells. If you are doing voice transmission, this delay spread might cause a problem. The delay spread will be looked at in greater detail later on when we look at counter measures for fading. Max bit rate also depends on the size of the cell. Here we cannot go very much. However for microcells you can really go up to Mbps or even higher. Actually this will be a function of bandwidth allocation as well. These are token numbers fine.

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## Summary of Lecture 11

- Log-Normal Shadowing
- Determination of  $n$  and  $\sigma$
- Outage Probability
- Percentage of Coverage Area
- Outdoor propagation Models

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So at this point, let us summarize what we have learnt today. We started off with Log Normal Shadowing and then we looked at an example to find out how to determine 'n' and sigma based on measured value. We used the method of minimum mean squared error and we found out value of n and sigma. Then we looked at the definition of outage probability and went onto cover the percentage of coverage area which is an important design parameter for base station allocation and cell sight planning. Again, we realize that the percentage of coverage area depends on n, sigma and a threshold value. We also had an introduction to some outdoor propagation models. We will look at specific propagation models in subsequent lectures. Thank you for your attention.