Wireless Communications

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Lecture No. # 15

Mobile Radio Propagation - II (Continued)

Welcome to the next lecture on mobile radio propagation. The outline of today's talk is as follows.

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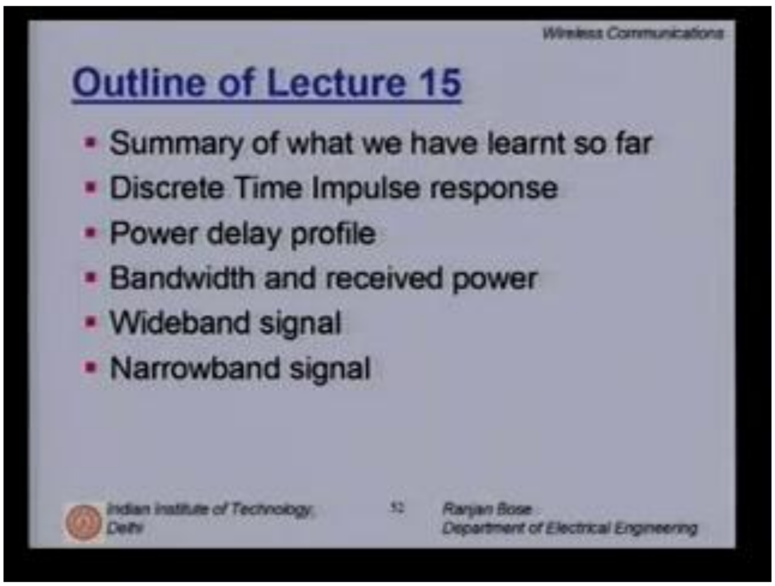

We will first begin by summarizing what we learnt in the previous lecture followed by a brief discussion on discrete time impulse response of the wireless mobile fading channel. We will then talk about power delay profile followed by an analysis of the relationship between bandwidth and the received power. We will look at wideband signals and how they behave in multipath fading environments as well as narrowband signals. What will be interesting to know is that the same multipath channel treats wideband signals and narrowband signals differently. Therefore it is important while designing a wideband versus a narrowband system, what kind of a channel we are using and what kind of treatment the channel gives to the wideband signal versus the narrowband signal.

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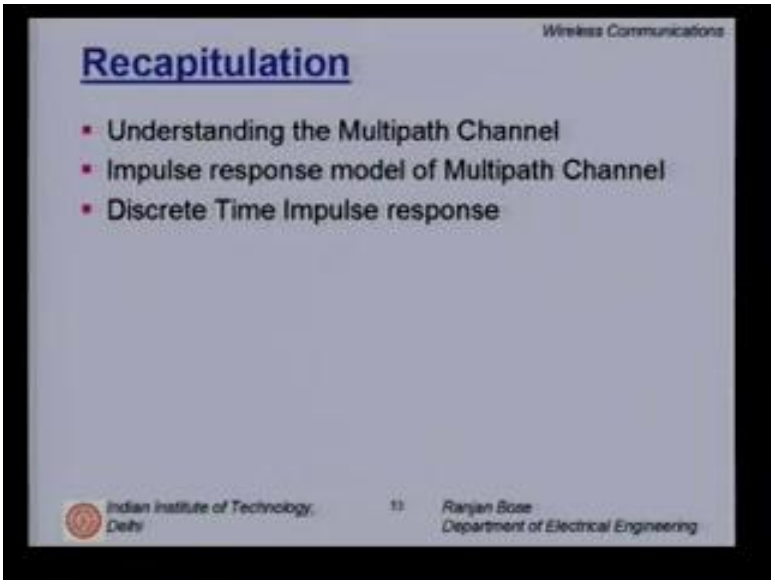

A brief recap of what we learnt. We started with a deeper understanding of the multipath channel. We know that the wireless channel is a multipath channel. It causes delay spread and fading. we then looked at the impulse response model of the multipath channel because if we have a good impulse response model, then we can use it to predict what kind of received signal we will get. We then modified the impulse response to a discrete time impulse response model wherein we divided the received time into bins.

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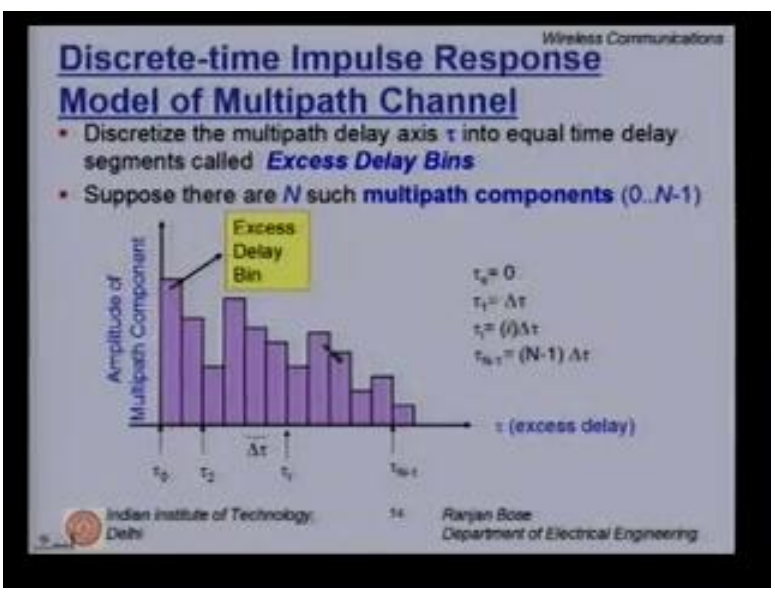

The discrete time impulse response model of a multipath channel works by discretizing the multipath delay axis tau into equal time delay segments called ‘excess delay bins’. Please note our objective here is to understand and then come up with a good model for a multipath channel. So it is important to understand how we fix, how many bins to put and what is the width or resolution of every bin. Suppose there are N such multipath components. So, on the x axis I put the excess delay. Why is it called the excess delay? Because it is with respect to the first incoming ray. On the y axis we put the amplitude received of the multipath component. As we can see that along the time axis which is the excess delay axis, we have various bins. On each of the bins, we get a different amount of energy which is depicted by the amplitude of the multipath component. Some bins have more energy than the others. It is coming from a stronger reflection whereas other bins are weaker.

It’s possible that some of the bins do not have any multipath component. We will learn today that it is possible that two or more reflected components arrive in the same bin. In that case they cannot be resolved and what will be put here is the vector sum of the received signals. This means the model will change if you narrow your bin resolution. That is how broad the bins are will determine what kind of an impulse response model you come up with. There is a relationship between the resolution of the bins and the bandwidth of the signal you can reasonably model it with. We will look at an example in the later part of the class. Now for the sake of completeness, let us label the x axis. So we start with tau 0. Then each one of the bin is delta tau. So you have tau 1, tau 2, so on and so forth till tau $_i$. assuming you have N multipath components. This N can be faded out by taking some measurements. Then we have here tau N - 1 as a starting of the last bin. Here I have put tau 0 is equal to 0. tau 1 is at a distance delta tau. tau $_i$ is at a distance i times delta tau and tau N - 1 is equal to N - 1 times delta tau. These are the excess delay bins.

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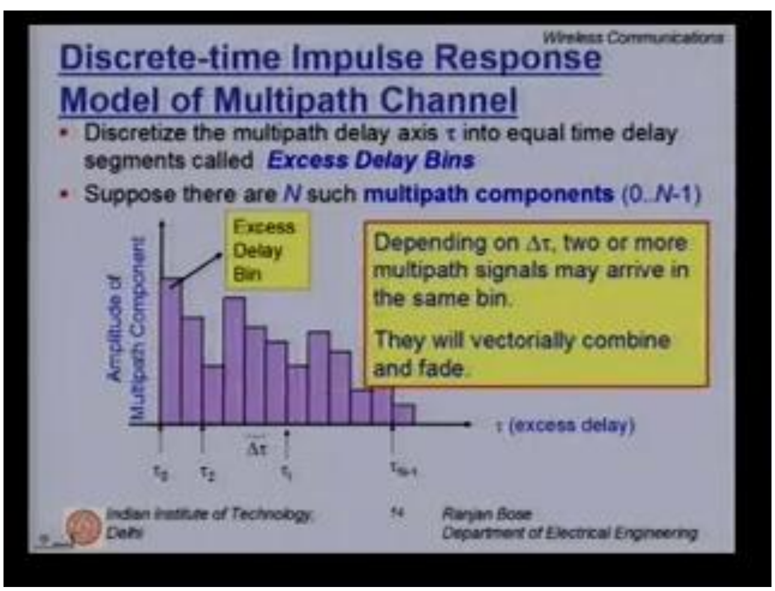

This is important. Depending on tau, 2 or more multipath signals may arrive in the same bin. It will change your model whether you can resolve the two components or not. These will vectorially combine and fade. Not only the amplitude changes but there is a change in phase as well.

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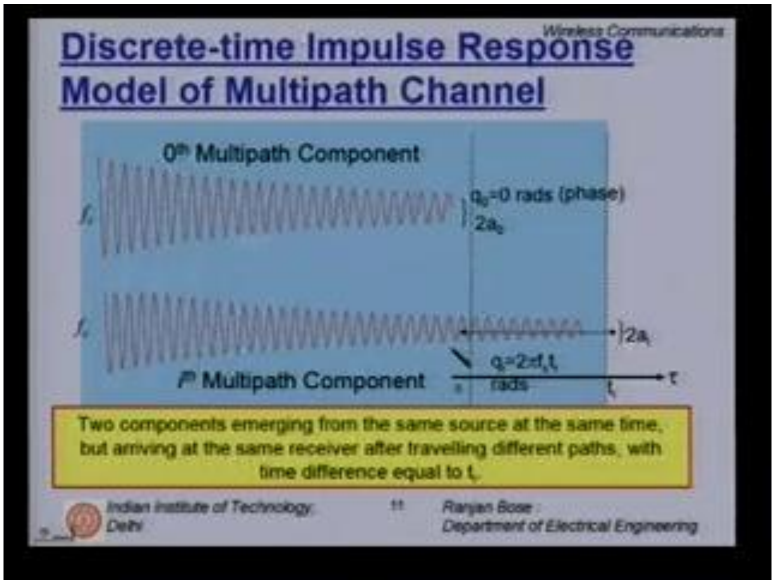

Here is a graphical explanation. Suppose two multipath components are arriving in the same bin, clearly when you receive them at the receiver, the amplitudes will be different and the phases will also be different. They will add up vectorially to give you the resulted value.

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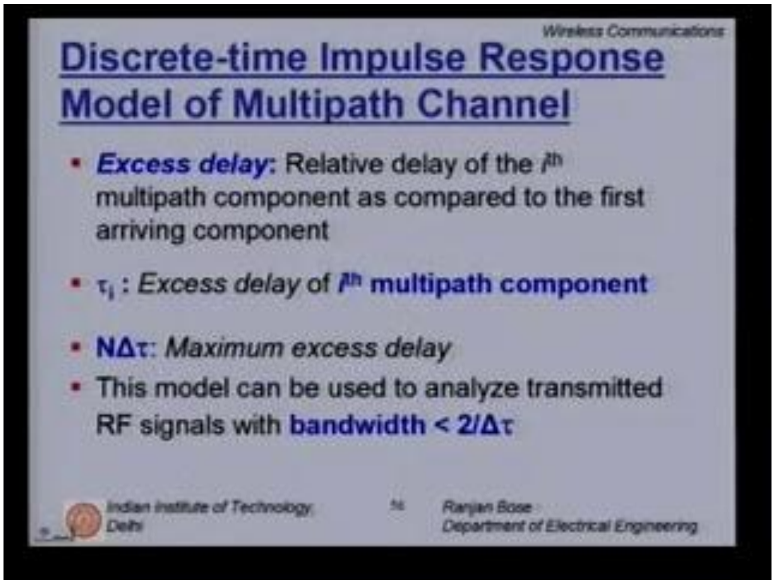

So the excess delay is the relative delay of the ith multipath component as compared to the first arriving component. ti is the excess delay of the ith multipath component. N times delta tau is the maximum excess delay. Delta tau as you know is the resolution of the bin. So if you have a measured excess delay data which gives you the maximum excess delay of say, 100 ms, then you can calculate N and delta tau using it. It should be noted that this model can be used to analyze transmitted RF signals only with a bandwidth up to 2 over delta tau. So if you want to increase your bandwidth of analysis, you have to decrease your delta tau. You must increase the resolution of the bin. You must start resolving more and more multipath components. If you do so, then that impulse response model will be more realistic for your RF bandwidth.

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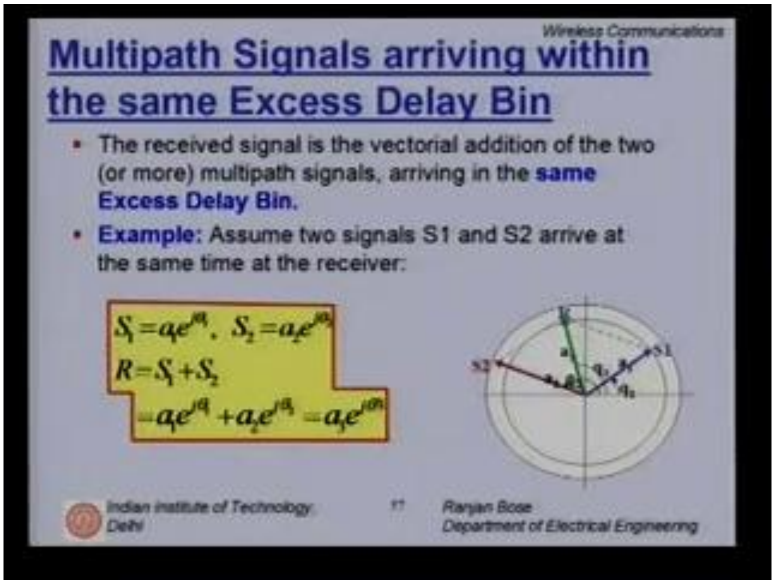

Now here, in this slide let us graphically look at the multipath signals arriving within the same excess delay bin. Because no matter what you do there will be some bins where two or more multipath components arrive. The received signal is vectorially added in the same bin and the arriving signals in the excess delay bin combined vectorially either to change their amplitude or phase. Suppose I have two signals S 1 and S 2 and then they arrive at the same time or within delta tau. That is important. They are in the same bin. So if I represent my S 1 as a e raised to the power j theta 1 a 1, clearly is the received amplitude and theta 1is the angle which depicts the phase e raise power by j theta 1. Similarly S 2 is a 2 e raise to the power j theta 2. Clearly the received signal in that bin is S 1 + S 2. vectorially if you add them up, you get another a 3 which has a different amplitude and a different phase. Here I show that you have S 1 and S 2 vectorially adding up to give a resultant R.

Conversation between a professor and student:

The question being asked is: is it possible that the vector sum leads to a received amplitude much larger than what either S 1 or S 2 is? The answer is yes. Clearly if you are fortunate enough to have them arriving in phase or very close in phase, then the vector sum will give them a much larger resultant vector. On the other hand it is also possible that they destructively combine and then what you receive is much smaller. Here what I have shown is roughly S 1 S 2 and R are of equal amplitudes. So you get both larger or smaller as we will discuss in the next slide.

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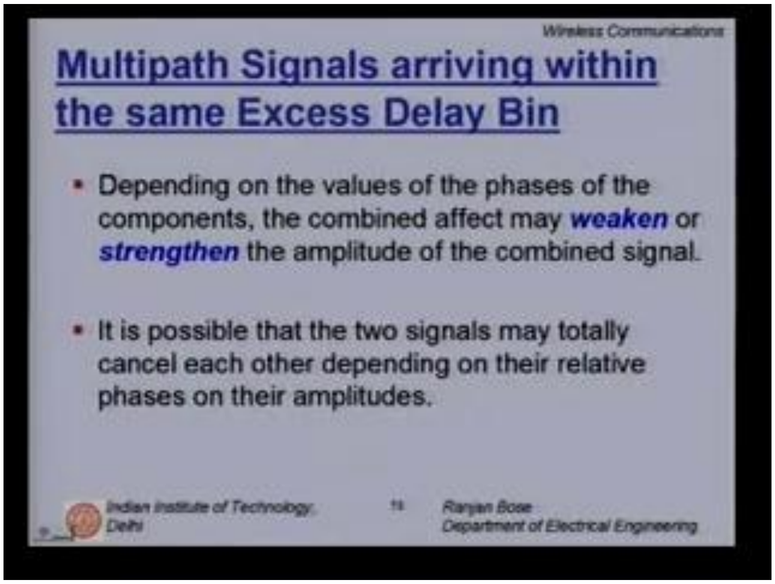

So depending on the values of the phases of the components, the combined effect may weaken or strengthen the amplitude of the combined signal. In fact, if you assume that the arriving phases are uniformly distributed between zero and 2 pi for all the arriving multipath components, then you can find out the probability of the resultant signal being stronger or weaker than either S 1 or S 2. Now the condition becomes more complicated if we have more than two arriving multipath components in the same bin. It depends how many you get depending upon how broad is your bin. It is also possible that the two signals may totally cancel each other depending on the relative phases and their amplitude. So, many times you receive very weak or no signal in certain bins. It’s possible that you get nothing because of there are no reflections in that time duration or two signals have unfortunately cancelled each other out. In that case also, I will not get anything in my bin. however if I increase the resolution that is, have a narrower bin it’s possible that in that new model, I will not have a perfect cancellation but two of the signals will appear in their own bins.

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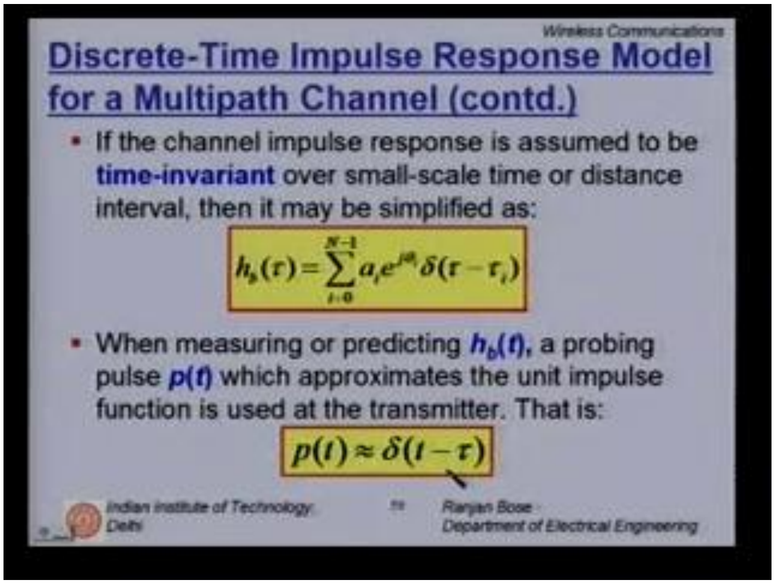

So let us now talk about the discrete time impulse response model for a multipath channel. If the channel impulse response is assumed to be time invariant, we learnt last time that there are two time axes and the channel impulse response is characterized by tau and t. t is responsible for the motion of the mobile station whereas tau relates to the variations in the channel. Suppose we assume that a channel impulse response is time invariant over small scale time or the distance interval. So the mobile has not travelled much than h_b (tau) is given by summation i is equal to zero through N - 1 ai e raised to the power j theta i times delta t - tau_i. It's a much more simplistic model. When measuring or predicting h_b (t), a probing pulse pt which approximates the unit impulse function is used at the transmitter. So it is basically a very narrow pulse. That is p(t) approximately delta (t – tau).

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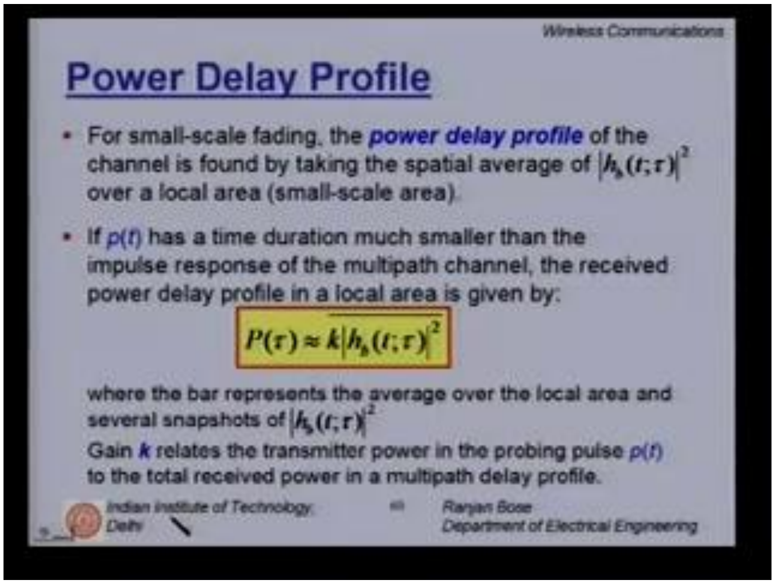

Let us now talk about the power delay profile. When we carry out channel measurements in order to determine a good channel model, we usually sound the channel. That is, we send either a narrow pulse or a continuous wave and do a lot of measurements at different places. What we measure is usually the power of the received signal. So it makes sense to start talking about something called as a 'power delay profile'. As the name suggests, we will have certain delays because of the multipath nature of the channel. So what is power delay profile? For small scale fading, the power delay profile of the channel is found by taking the spatial average of h_b (t, tau) over a local area say, a small scale area.

Please remember these lectures pertain to small scale fading. We are only talking about a very local area here. Also note the words spatial average. So the power delay profile is related to absolute value h_b (t; tau)^2. That's what we measured. if pt, the pulse that we are using to sound the channel has a time duration much smaller than the impulse response of the multipath channel, the received power delay profile in a local area is given by p tau approximately k (average h_b (t; tau)^2. so the over bar represents a spatial average.

So we take a lot of measurement at close by grid points. So we divide the local area into small grid points. Then we carry out measurements at all of these grid points and take a spatial average. Later on today, we will realize that these measured values are interestingly different for wideband signals and narrowband signals. Here in the equation, the bar represents the average over the local area and several snapshots of hb (t ; tau)^2. What we measure has to be the power. The k gain relates the transmitter power in the probing pulse pt to the total received power in a multipath delay profile.

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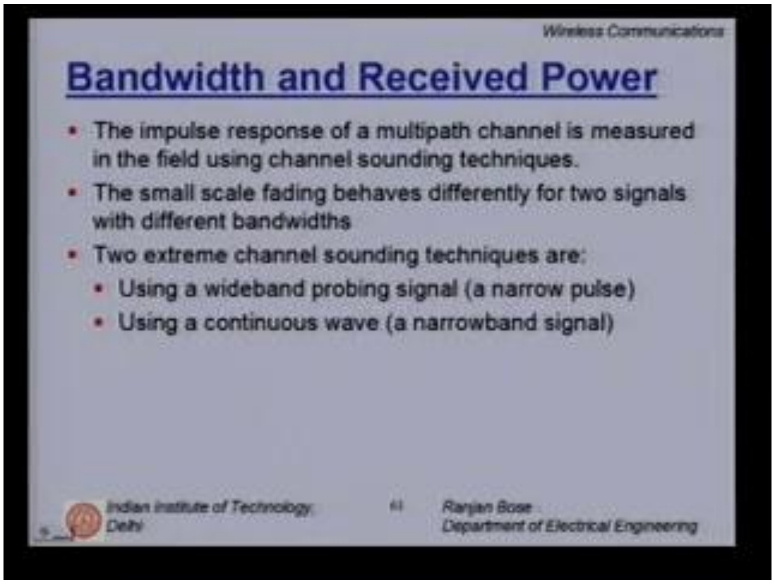

Now let us talk about the relationship between bandwidth and the received power. The impulse response of a multipath channel is measured in field using channel sounding techniques. so we probe the channel either with a narrow pulse or with a narrowband signal which is a continuous wave kind of a signal and then we take measurements over spatial area. We then try to figure out what is the impulse response. The small scale fading behaves very differently for two signals with different bandwidths. This is important. Now two extreme channel sounding techniques that can be used are a wideband probing signal that is, a narrow pulse or a continuous wave. For various applications, we can either use the wideband probing signal for example, in ultra-wideband application UWB, I would rather use a wideband probing signal to find out the impulse response of the multipath channel. Whereas, for GSM operations, many times I would like to use a continuous wave to probe the channel. A lot of channel measurements have already been done using a continuous wave.

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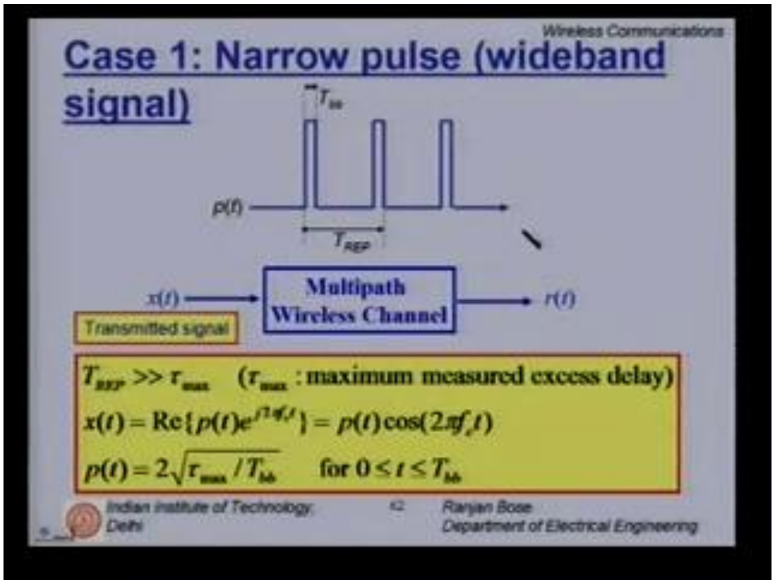

So let us consider these two cases separately. Let us first talk about a narrow pulse. Please remember we are actually trying to find out the impulse response of a channel by exciting a very narrow pulse which almost mimics an impulse. It is a wideband channel measurements and can be used for a variety of frequency band. So a wideband channel characterization which is nothing but the impulse response of the channel can be used for any desired frequency band of choice. So what are we using here? Suppose we have a train of pulses. Now pulses are interesting in the sense that, there is a pulse width defined by t_{bb} and then there is a pulse repetition frequency prf which is given by T_{REP}.

Since I am sounding the channel, I am trying to send a train of pulses and each one of the pulses will generate several received multipath components. What I must be careful about is that the pulse repetition frequency should be such that the delay spread does not over run into the duration of the next coming pulse. So this pulse repetition frequency will be different for various kinds of environments. If I have an outdoor environment then my pulse repetition frequency should be lower. So consider a multipath wireless channel. Again this can be used both indoors and outdoors. Let x(t) be your transmitted signal and what you receive is r (t). Now T_{REP} the pulse reputation time should be greater than the tau $_{max}$ which is the maximum measured excess delay. So you first carry out a rough measurement figure out then fix your pulse repetition frequency. x(t) is nothing but real part of the pulse p(t), the pulse that is being used to probe the channel, e raise to power j 2 pi f_ct. So it is a modulated pulse as shown here p(t) times cosine 2 pi f_ct.

The narrowness of the pulse, i.e. the pulse width will force you to choose the certain frequency f_c. You cannot have too low a frequency. You have to capture a couple of cycles at least into your p (t). Now we also put another constraint. The constraint is p(t) is 2 times root tau $_{max}$ divided by T T $_{bb}$. This constraint is being put later on to normalize the energy of the pulse as we will do a derivation later. This when integrated over a pulse duration would even out and give unity. So this is only for the sake of simplicity that we are putting this constraint. We will use it in one of our subsequent slides. This is true only for the pulse duration. So this is defining the height of the pulse. So now we are going to sound the channel with this pulse train.

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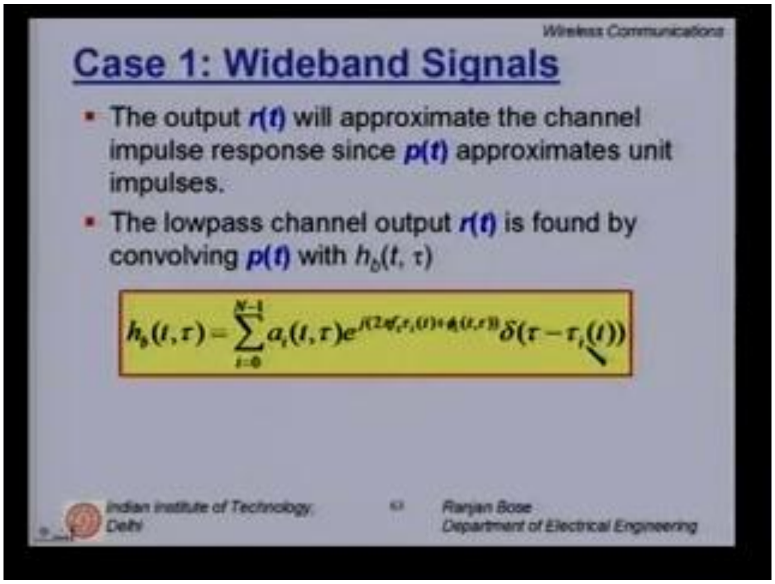

The output r(t) which is the received signal will approximate the channel impulse response. This is because p(t) approximate unit impulses. So we are directly trying to measure the impulse response of the channel. The lowpass channel output r(t) is found by convolving p(t) with h_b(t , tau). Now just to remember what was our h_b(t , tau). we had put h_b(t , tau) is equal to summation i is equal to zero through N – 1, N being the number of excess delay bins, a_i (t , tau). So the amplitude of each of the multipath component is both a function of t and tau as expected. Then there is a phase. So we can roughly put whole of this as e raise to power j phi i or even j theta i and then impulses delta t - tau i. again tau i must be a function of t as we saw in the 2 D mesh diagram last time.

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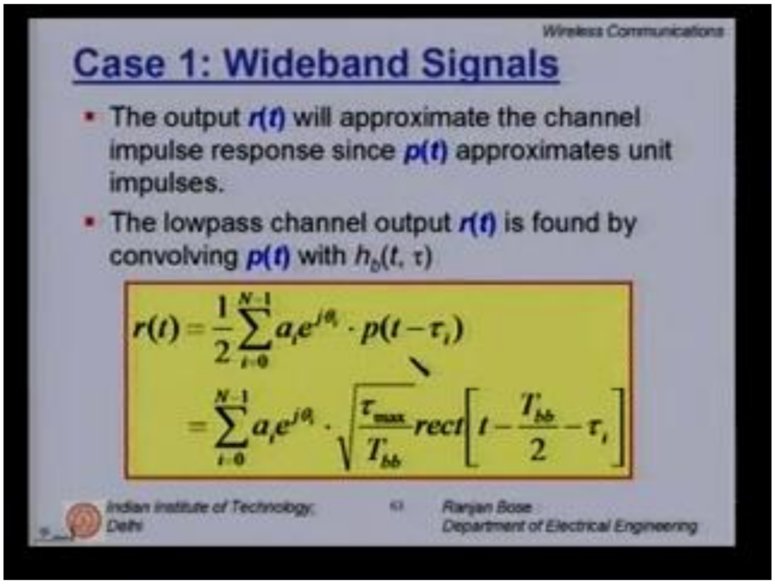

So having remembered that, now the lowpass channel output r(t) is simply found by convolving the p(t), our favorite pulse with h_b (t , tau). We just saw that now. If you carry out that convolution, you get r(t) is equal to ½ i = zero to N – 1, a_i e raised to the power j theta i. remember we plugged all of that phase terms as j theta i times p(t - tau i). Because in the convolution process, the p(t) would be placed in the locations of the deltas. But we have put a constraint on p(t). It is a train of pulses and each pulse is a kind of a stretched up rectangle. So if you carry out further, you can write a_i e raise to power j theta i multiplied by the pulse train. So this is what you expect to receive. The condition is that the p(t) is fairly narrow and impulse like.

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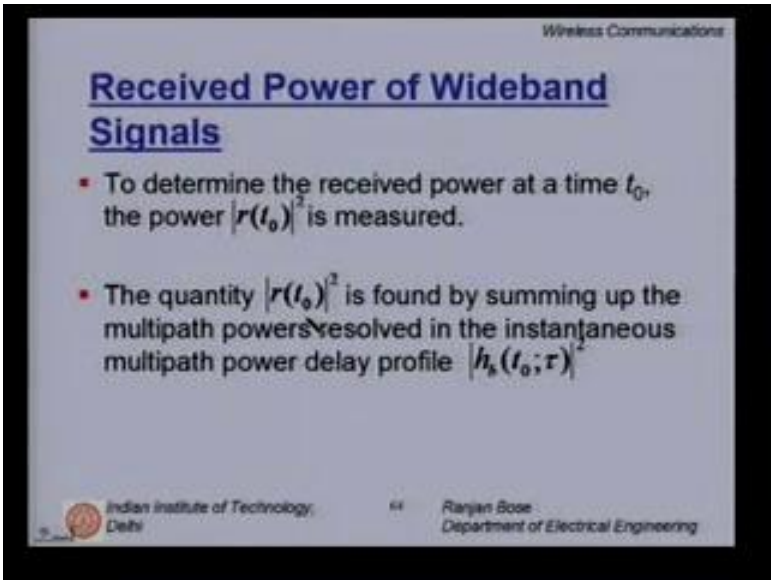

Now let us talk about the received power of the wideband signals. So to determine the received power at time t_0. So I pick a certain time t_0. The power r (t_0) absolute value squared. That's exactly the instantaneous power. This value must be measured. So what you measure is in fact r(t_0) absolute value squared at any time t_0 and this will change from time to time. the quantity r(t_0) absolute value squared is found by summing up the multipath powers resolved in the instantaneous multipath power delay profile given by h_b(t_0 ; tau) absolute value squared. This is fairly simple. So we are getting energy in the various multipath bins. You sum them up.

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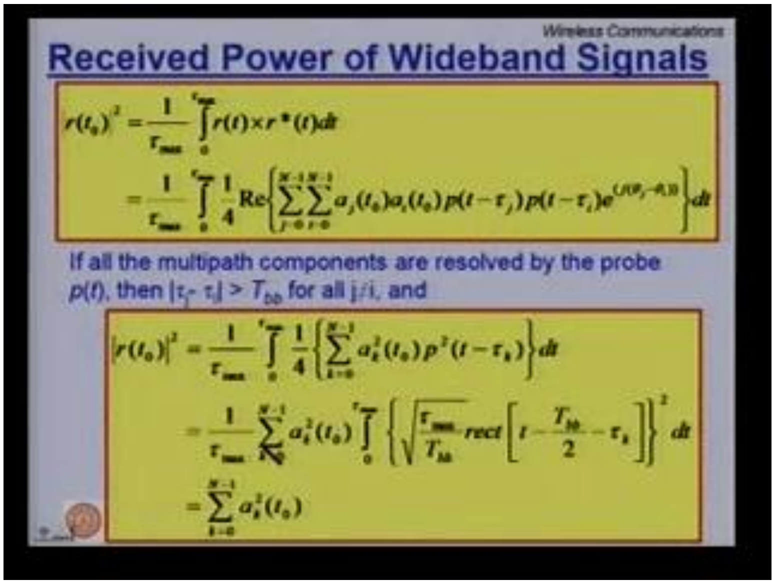

So let's look at it mathematically. What is the received power of a wideband signal through a multipath channel. The received power by a power mind you, is nothing but $r(t_0)$ absolute value squared is equal to 1 over tau $_{max}$ normalizing factor 0 to tau $_{max}$ because that's the maximum excess delay spread. Absolute value of $r(t_0)$ is nothing but rt times r star t complex conjugate dt by definition. but let us plug in the value of r(t) in the previous slide which is nothing but $a_j(t_0)$ pt - tau e raise to power j theta j. if you do this multiplication you get this expression. Now let us see if all the multipath components are indeed resolved by the probe pt. That means we are putting some constraints on the width of pt as well as the resolution of the bins. if this is true, if all the multipath components are indeed resolved, then absolute value tau 1 - tau j is greater than T $_{bb}$ for all j not equal to i.

Under this assumption, we have the same received power of the wideband signal $r(t_0)$ absolute value squared is equal to, coming from this above equation, we get 1 over four summation k is equal to zero to N – 1 a_k squared at time $_0$ p squared t - tau k integrated dt. So it follows from non-overlapping multipath components. If you carry forward by substituting the value of p(t) which is nothing but a train of pulses represented by this rect function and carry out the mathematics and remember the condition on the height of the rect pulse, this value integrates to unit power and we are left with summation a_k squared t $_0$. If we had not taking that into consideration about the pulse height, then there will be a gain factor or a multiplicative factor outside. What is important to note is this a $_k$ squared is coming from the gain of every multipath component. That is where your information lies. A different multipath components coming into the different bins as you have made an assumption about resolving each of the multipath components. The total received power is simply the summation of the powers in those respective bins. It was intuitive but it also comes true mathematically.

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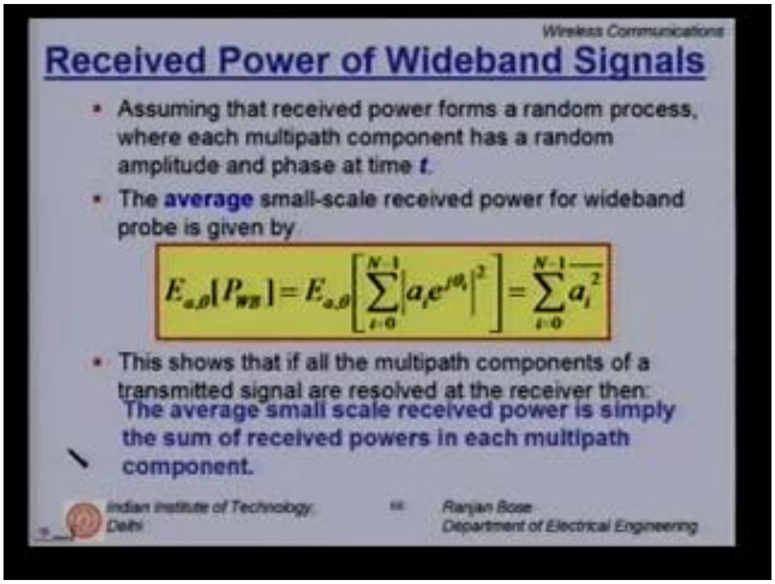

So assuming that the received power forms a random process where each multipath component has a random amplitude and a phase at any time instant t, the average small scale received power for wideband probe is given by averaging over Ea and theta. What are a and theta? This is the gain of each multipath component and the phase factor. So if you are assuming that the received power is indeed a random process, then the average value for wideband power is taking the expectation with respect to a and theta. If you do that and assume theta to be distributed uniformly, then you get equal to summation i is equal to zero through N - 1 a_i squared average. the whole bar represents the average. This is an interesting result. This shows that if all the multipath components of a transmitted signal are resolved, then the average small scale received power is simply the sum of the received powers in each multipath component. This was intuitive what it has been shown.

However if you do not resolve each of the multipath components and two or three of them arrive in the same bin and actually cancel each other out, then you are in trouble. This model will not hold. The question being asked is: is it totally independent from the phase? The answer is yes because we have removed both the phase part and the dependence on phase by averaging it over. Yes. It is independent of the phase. It is only dependent on the gain of each multipath component. So we stress here again that the average small scale received power is simply the sum of the received power in each multipath component. Why is this being stressed because for the narrowband case, we will find something different and that makes the analysis interesting.

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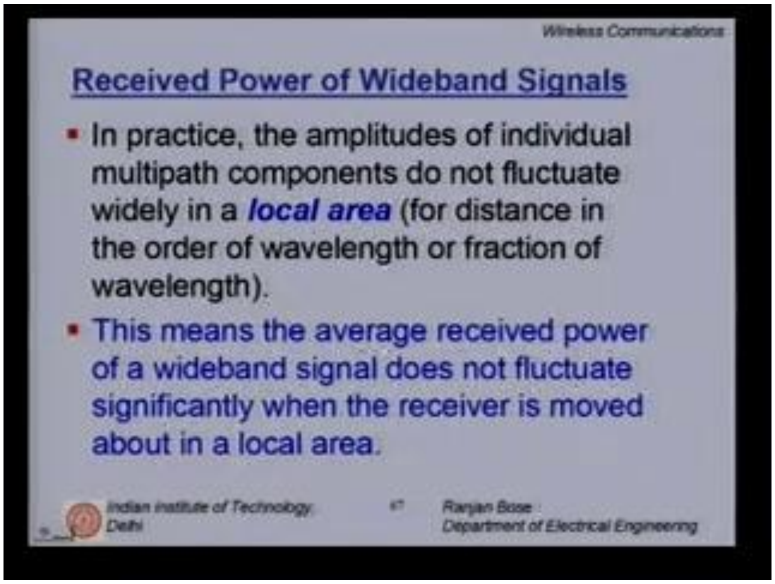

So, continuing with the received power of wideband signals, in practice the amplitudes of individual multipath components do not fluctuate widely in a local area. It cannot. because if the all the multipath components have been resolved, regardless of which value has what, it is the summation of the a_k i squared. After all you are adding up all the received power. So, over a local area, whatever you do if you are probing the channel with a wideband signal, that is, a narrow pulse, you will be amazed to find that the signal power profile doesn't change much this. This will be different in the narrowband case. Now when we say local area, how big or small is that area? We are talking about distance of the order of the wavelength or fraction of wavelengths. This means, the average received power of a wideband signal does not fluctuate significantly when the receiver is moved about in the local area. They are hardly any fluctuations.

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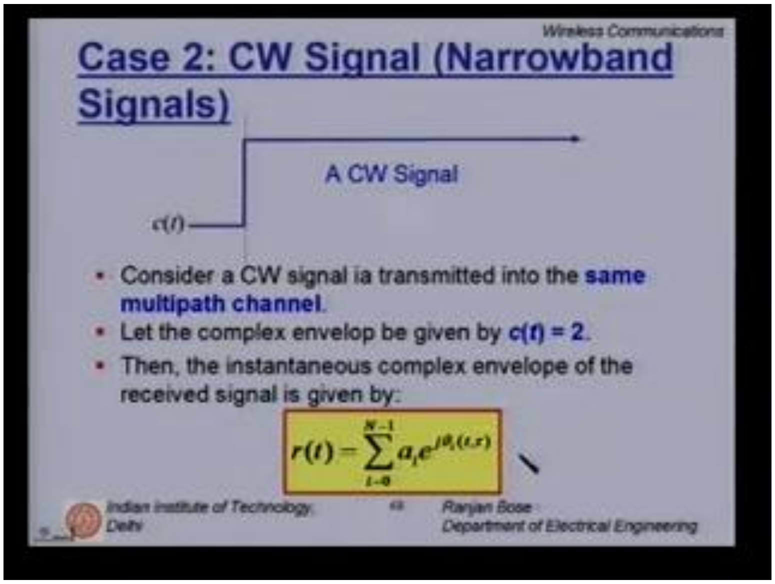

Now let us look at the other popular part which is continuous wave signal. That is, narrowband signals. How does the multipath channel treat the narrowband signals? Here let us probe the channel using a continuous wave, CW signal. That is, at time t_0, we start a CW signal. We are probing the same channel. Let the complex envelop be given by ct =2. We talked about the complex envelop in the previous class. then the instantaneous complex envelop of the received signal is given by r(t) is equal to summation i is equal to zero through N - 1 a_i, the gain of ith multipath component e raised power j theta I, which is both a function of t and tau. This is the instantaneous complex envelop of the received signal.

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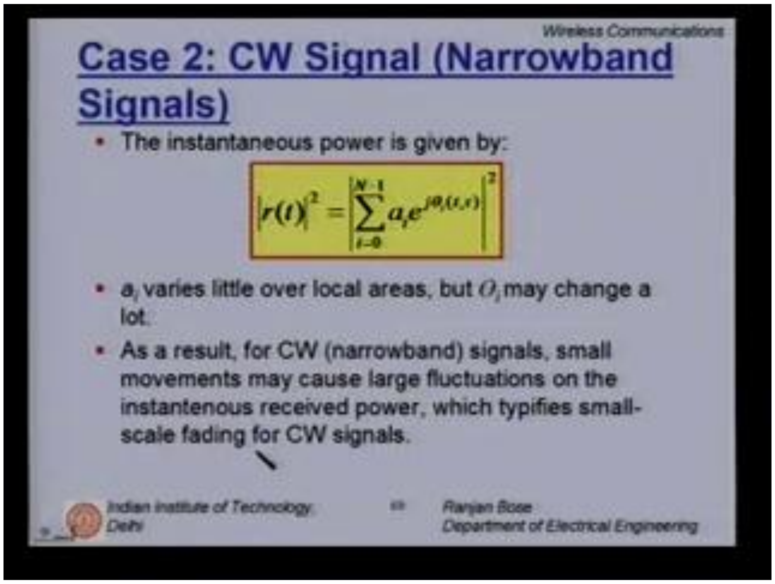

Now what we measure is the instantaneous power. the instantaneous power is given by r(t) absolute value squared is equal to i is equal to zero through N - 1 a_i e raised to power j theta i again function of t and tau absolute value squared. No further simplification. Note that a_i varies little over local areas but theta i may change a lot. In fact, the fluctuations will actually depend on the frequency of operation. Still it's a narrowband case. If I am working in the millimeter wave frequency range, then my theta i's will fluctuate over a few millimeters drastically. Whereas a_i is not that fluctuating. As a result, for continuous wave signals, small movements may cause large fluctuations on the instantaneous received power which is typical of small scale fading for CW signals.

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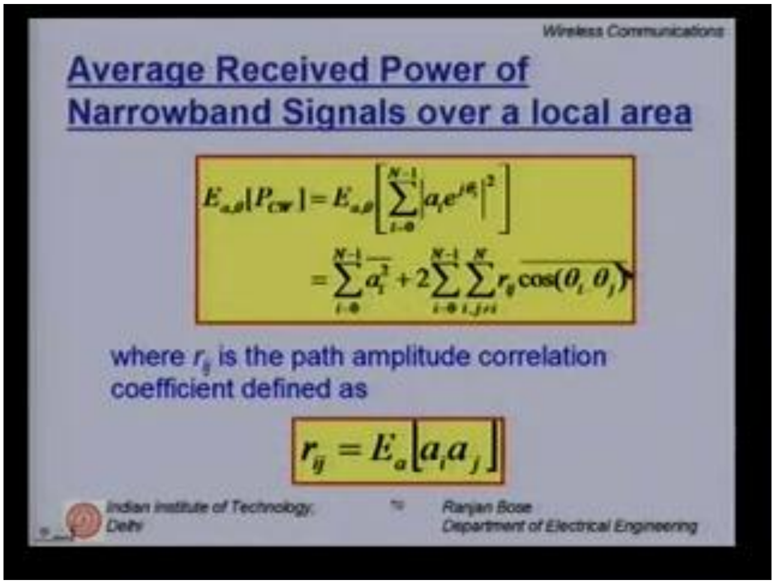

So let us do a local area average with respect to a and theta for the power received for CW signals averaging over a and theta absolute value squared of a_i e raise power j theta i summation i is equal to zero through N – 1. If you carry this out, you are left with two terms. i is equal to zero to N - 1 a_i squared average value. whatever be the assumption on the distribution of ai, based on that this is the average value taken + 2 times summation i is equal to zero through N - 1 and summation j is not equal to i through N, r_{ij} average value of cosine theta i - theta j where rij is the path amplitude correlation coefficient. This is defined as r_{ij} is equal to expected value $E_a[a_i a_j]$.

This is talking about the correlation between the gain of ith path and the jth path. So let us look at it a little bit more carefully. What are we talking about? We are talking about the average received power of a continuous wave signal. On the right hand side, we have two terms. The first term looks very similar to what we got for the wideband signal. But there is another term which is cropped up now. This term depends on two things. One is the correlation coefficient between the two path gains of path i and path j, r_{ij} and interestingly enough, the difference of the phases theta i and theta j for path i and j. both these two terms are extra.

Now if we want to compare and say, in what conditions would a narrowband signal behave similarly to the wideband signal. We have to put this second term to zero. That can be done in two ways either I put r_{ij} zero or I put the average value of cosine theta i - theta j zero. If any of these two conditions are satisfied, again we will have your average received power the same as that of the wideband signal which has a characteristic that it doesn't fluctuate much. Why shouldn't it fluctuate much? Because this is in fact the total power of the summation in the power received in the different bins. That is almost the same as you are not losing any part. However, if we cannot put r_{ij} zero or cos theta i - theta j average value to be zero, then it will have a lot of fluctuations which is normally the case.

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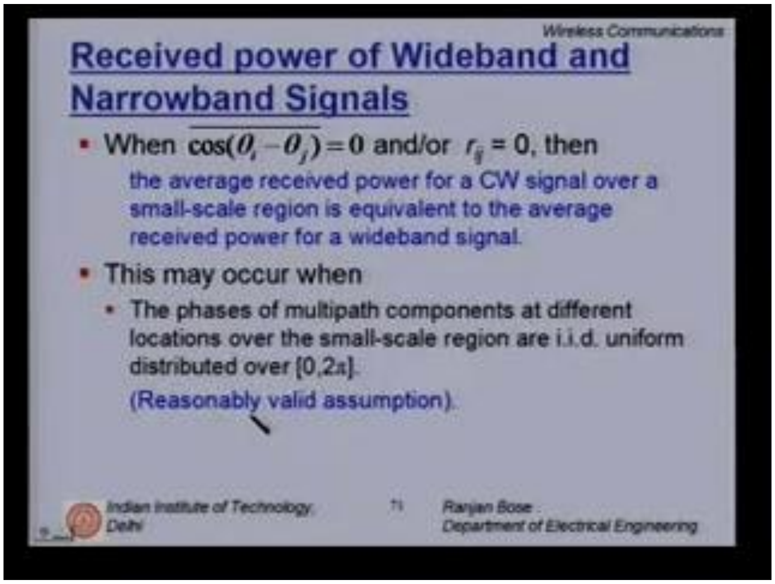

So let us look at these two things. when the average value of cos theta i - theta j is zero or rij is zero, in that case the average received power for a CW signal over a small scale region is equivalent to the average received power for a wideband signal. They behave similarly. We do not see too much of fluctuation by just simply moving your antenna around. Now when can this occur is it possible at all? Are there certain cases when the behavior of the narrowband signal and wideband signal are similar in terms of the average received power? Well, these two conditions may occur when the phases of the multipath components at different locations over the small scale region are i.i.d. - independent identically distributed uniformly over zero to 2 pi. Now this is not a very difficult assumption to make. It's a reasonably valid assumption. So if this assumption holds (incomplete statement)