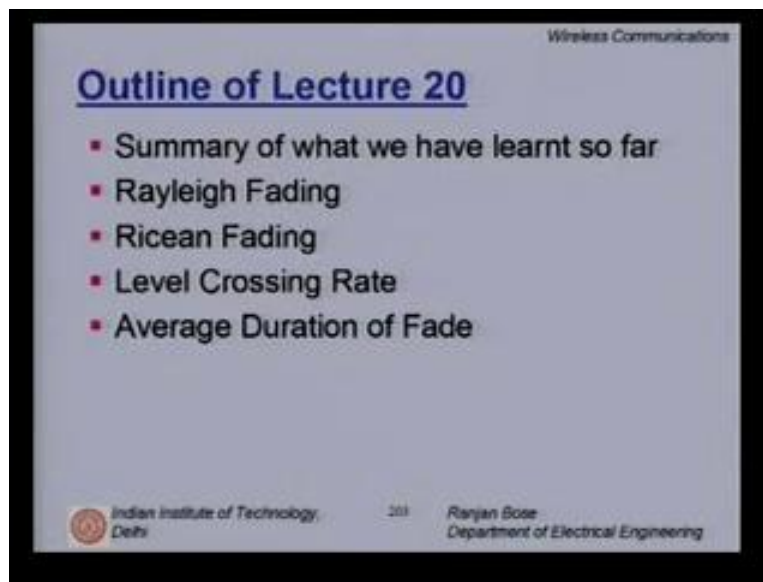


Wireless Communications
Dr. Ranjan Bose
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Lecture No # 20
Mobile Radio Propagation -11- Multipath and Small Scale Fading

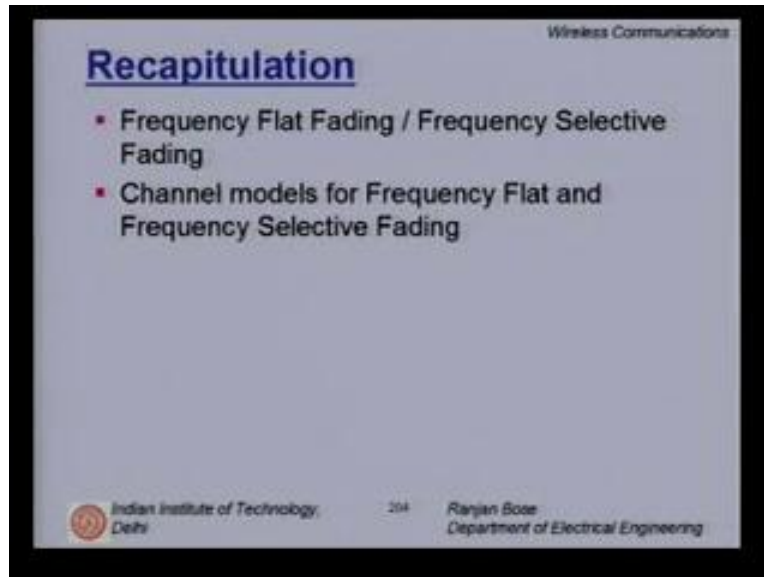
Welcome to the next lecture on mobile radio propagation.

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The brief outline of today's talk is as follows. We will start with the summary of what we have learnt so far followed by a brief description about the Rayleigh fading and the Rayleigh distribution. Then we will look at the Ricean Distribution followed by two real life useful parameters: the level crossing rate and the average duration of fade. Both these things will somehow translate to your quality of service.

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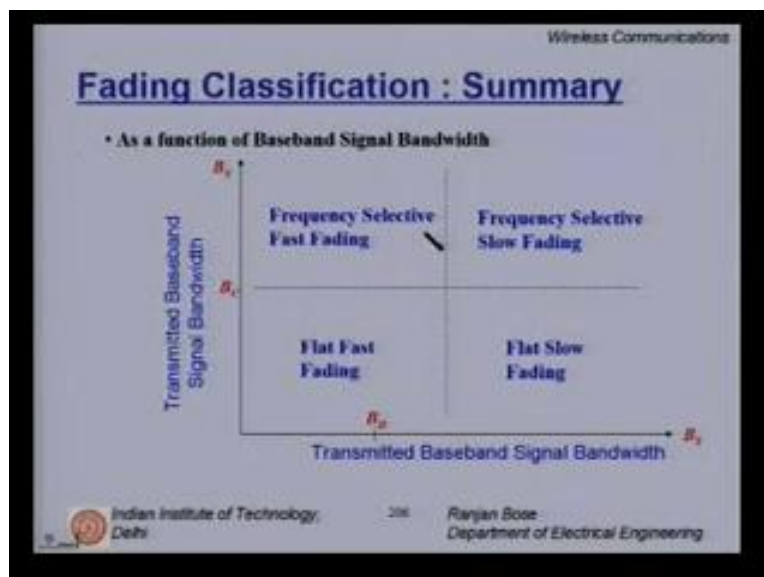
Recapitulation

- Frequency Flat Fading / Frequency Selective Fading
- Channel models for Frequency Flat and Frequency Selective Fading

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So a brief recap. We have learnt so far that there can be two kinds of channels. The frequency flat as well as frequency selective channel. They depend somewhat on the coherence bandwidth. Then we have looked at various channel models for the frequency flat as well as frequency selective fading followed by slow and fast fading which has something to do with coherence time.

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Fading Classification : Summary

• As a function of Baseband Signal Bandwidth

Transmitted Baseband Signal Bandwidth

Transmitted Baseband Signal Bandwidth

Frequency Selective Fast Fading

Frequency Selective Slow Fading

Flat Fast Fading

Flat Slow Fading

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In this slide, we have put on one single sheet the notion of flat slash slow fading, flat fast fading, frequency selective flat fast fading and frequency selective slow fading. On the two axes we have the transmitted symbol period T_S . Here is a vertical line which marks the coherence time T_C . On the y axis again, we have the symbol T_S and this line marks the RMS delay spread. so if your T_S is larger than the root mean square delay spread, you are in the domain of flat fading and again at the same time, if your ' T_S ' the symbol time is greater than the coherence time, then you enter the domain of flat fast fading. Similarly, you have the other options. You can look at the same slide from a different perspective. That is, from the baseband signal bandwidth, on the x axis I have the symbol bandwidth B_S . so also on the y axis, however the vertical line approximately relates to the ' B_D ' - the maximum Doppler shift.

On the y axis, we have the ' B_C '- the coherence bandwidth. Now please remember coherence bandwidth was a function only of the multipath components in the channel and had nothing to do with the signal. So I can shift and vary my B_S independent of the B_C . Doppler depends upon how fast the vehicle is moving with respect to the base station or how fast the transmitter and receiver are moving with respect to each other and has nothing to do with B_S . So in that case, again I can move independently B_S - the symbol bandwidth independent of B_D and again I work either in the domain of flat slow fading or frequency selective fast fading or any other combination. Now what we want to do today is slightly different we would like to look at the actual fading profile. look at what kinds of distribution actually comes up when you have a line of sight or you do not have a line of sight and then how does this affect your level crossing rate or average duration of fade because if you are sitting in a fade for too long, you will tend to lose not only the signal but also the synchronization .ultimately it will lead to dropping your call.

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Fading Distributions

- Statistical characterization of the variation of the envelop of the received signal over time.
- Two most common distributions
 - Rayleigh Fading
 - Ricean Fading

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What do we mean by fading distributions? The statistic characterization of the variation of the envelop of the received signal over time is captured by the fading distribution. Please note we are only talking about the envelop. Some notion of the signal power being received here. Of course you have some distributions regarding the phase but usually the phase is uniformly distributed over 0 to 2π .

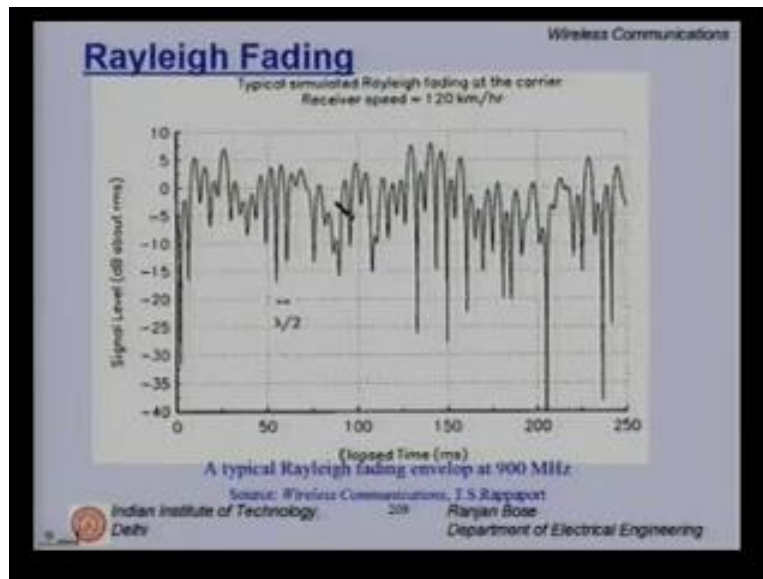
What is interesting is the variation of the envelop of the received signal. We will find that this envelop undergoes deep fades once in a while depending upon what kind of a fading distribution you have. The two most common distributions are the Rayleigh fading and the Ricean fading. You have generalizations of Rayleigh fading in terms of a Nagakami Fading distribution but we will talk about it later. These are by far the most common fading distributions. We will in the subsequent slides today talk about the Rayleigh fading and the Ricean fading distributions.

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When does Rayleigh fading occur? if all the multipath components have approximately the same amplitude that is, when the mobile station is far away from the base station and also doesn't have a clear line of sight, of course that will be definitely the strongest component. So in the absence of a line of sight and when all the multipath components have roughly the same amplitude, then the envelop of the received signal is Rayleigh distributed. Please note we are only commenting on the envelop although what you receive will have an amplitude and phase. Clearly we do not have any dominant signal component such as the line of sight or anyone reflector which is a strong reflector. it is assumed all reflectors are equally strong or equally weak depending upon whether you're looking at it pessimistically or optimistically.

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Let us now look at how Rayleigh fading actually appears when you carry out an experiment. Suppose you have a receiver with an envelope detector and a means to store the data and you drive away from the base station. On the x axis we have plotted the elapsed time in milliseconds for a car which is travelling at about 120 km/hr. so clearly using the fact that the speed of the vehicle is 120 km/hr and this is the elapsed time. You can find out how much distance you have travelled away from the transmitter or the base station. on the y axis is plotted the signal level in dB above the rms. now please note that we start from 10 dB but it goes down way up to -40 dB and in certain cases you have these deep nulls also called the fades. They have called fades because if you listen to a radio, for example and you are sitting in a fade, the sound will diminish to a point that you can hardly hear it. so it's a faded signal that you receive.

Please note that all the edges are pretty much rounded because we are talking in terms of a log plot. It's a dB scale. Otherwise you will get a fairly jagged variation as you move in a car with the receiver at 120 km/hr. now what is of concern are two things. How many times per second? Do we get into the fade? For that we must define what is the acceptable signal strength below which we say that the signal is not usable. So we need a threshold and then we need to come up with a parameter which tells me number of crossings per second. Later today we will try to define a level crossing rate. Now as I mentioned before, the y axis in this graph plots the envelop given. This distribution which is found out to be Rayleigh, if you plot the histogram of the values here, you will come up the distribution which is resembling the Rayleigh distribution. If you have this distribution, then statistically if you draw a horizontal line here and define a threshold, then you can statistically calculate the number of crossings per second. That will be the level crossing rate. The other thing of importance is what the average duration of fade is. For that I need to know again a threshold. so let's draw a line here then. How much time do we stay below that line that will give me the average duration of fade. Clearly if I give you the pdf- probability density function of the Rayleigh distribution, then I can statistically calculate the average duration of fade. We will find that out also. We will also look at some examples.

Conversation between student and professor: the question being asked is: suppose we approximate the cell by a circle with the base station at the center of the circle and the mobile station is moving along the circumference, that is, it is equidistant from the base station, so we are not really translating across. This axis which is moving away from the base station. We are just moving in a circle. Clearly the path loss is not changing because the distance 'D' is not changing. The question being asked is: will we still undergo fading? The answer is: yes. Suppose we are really at the circumference of the base station, the probability of having a line of sight is very low. so you will have components which are coming by reflections and the moment you have multipath and then they add up either destructively or constructively.

We get a fading. So fading will be there and in this case, most likely Rayleigh fading. so clearly if you move around the circle of the base station, you will see similar effects. What this means is that if you draw an average line somewhere, the average signal strength will gradually go down. The curve has not moved very much into 50 ms actually. But if you have not moved at all, that is, you are translating across a circles circumference you have not moved in terms of the distance D but still you will have this kind of a variation. You will get fading. Now let us look at the other realistic scenario that you are indeed closer to the base station or you have a single dominant reflector. in either of the cases not all of the reflected components are equal in strength. So even when they add up, they do not form a Rayleigh distribution but something called as the Ricean distribution.

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Ricean Fading

- When there is a **dominant, stationary (non-fading)** signal component present (such as LOS, which is usually possible when MS and BS are close to each other), the fading envelop is **Ricean**.
- The Ricean distribution **degenerates** to Rayleigh when the dominant component fades away.

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So when there is a single dominant stationary which is non fading signal component present, such as the LOS which is usually possible when the mobile base station are relatively close to each other the fading envelop is not Rayleigh but Ricean. Now the Ricean distribution slowly degenerates to the Rayleigh distribution when the dominant component fades away. Clearly in the Rayleigh fading scenario, if we add one dominant component, we get the Ricean distribution.

In fact if we somehow have a measure to see how much signal strength is in the direct component as opposed to the reflected components from other reflectors, that ratio will tell us how far away is a Ricean distribution from the Rayleigh distribution.

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Rayleigh Distribution (1)

- The Rayleigh pdf is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\left(\frac{r^2}{2\sigma^2}\right)} & (0 \leq r < \infty) \\ 0 & (r < 0) \end{cases}$$

- where σ is the rms value of the received voltage signal before *envelope detection*, and
- σ^2 is the time-average power of the received signal before *envelope detection*.

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Let's go back to the Rayleigh distribution and see what it is mathematically. The pdf or the probability density function of the Rayleigh distribution is given by the following P_r pdf is r over σ squared e raised to power r squared over 2σ squared. This a minus sign here. This is valid for r between 0 and infinity and it is zero for $-ve$ values of the r . please remember this is the amplitude to the envelop. There is no question of this becoming negative. This is obtained by taking the square root of the squares of two version random variables. Now this σ is of importance as in Gaussian distribution, σ is a measure of the spread of the distribution. Here σ is the root mean square value of the received voltage signal before the envelop detection. σ squared clearly is the time average power of the received signal before the envelop detection. The mean the median and other distributions will depend on this σ .

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Rayleigh Distribution (2)

- The probability that the envelope of the received signal does not exceed a specified value of R is given by the CDF:


$$P(R) = P_r(r \leq R) = \int_0^R p(r) dr = 1 - e^{-\frac{r^2}{2\sigma^2}}$$

Also,

$$r_{\text{mean}} = E[r] = \int_0^{\infty} r p(r) dr = \sigma \sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

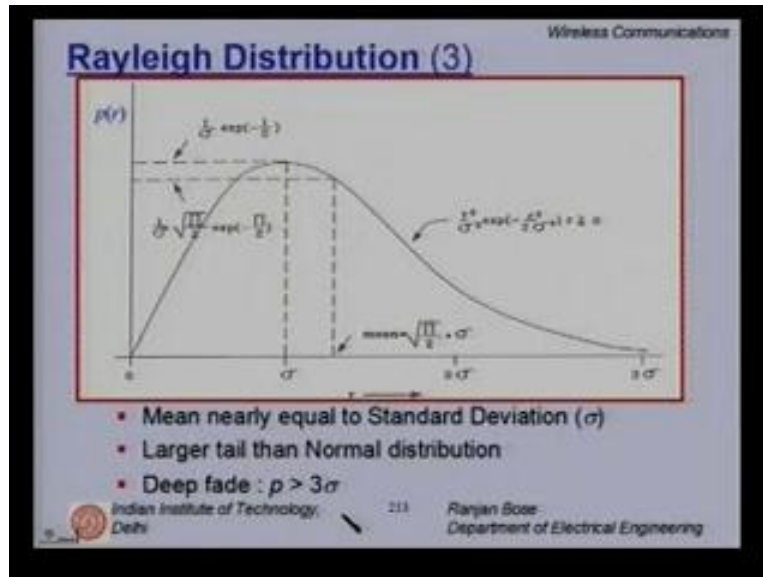
$$r_{\text{median}} = 1.177\sigma \quad \text{found by solving } \frac{1}{2} = \int_0^r p(r) dr$$

$$r_{\text{rms}} = \sqrt{2}\sigma$$

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 212
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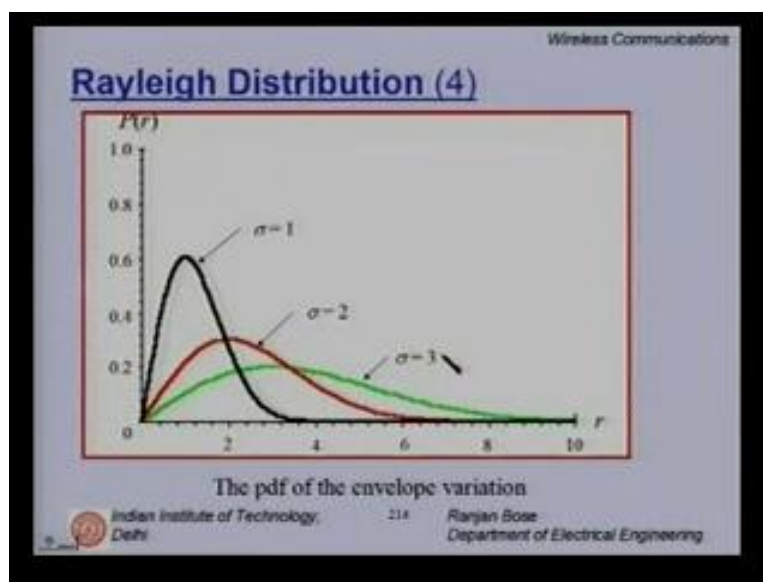
The probability that the envelop of the received signal does not exceed a specified value of R is given by the cumulative distribution function – ‘CDF’. now this is important because you can have a pdf but at the end of the day, you are interested in finding out how often does it fade, what is the probability that the envelop received at the receiver is above a certain threshold level, how often is it above the receiver sensitivity, etc. so it is important to find out the probability that r is less than R and then to find out the other case, we will find out $1 - P_r$. this is nothing but from 0 to R . it’s a threshold level $P_r dr$. if we integrate it, you get this expression $1 - e$ raised to power $- R$ squared over 2 sigma squared. Also you can calculate the r_{mean} , the r_{median} and the r_{rms} -root mean square. There are certain properties of the Rayleigh pdf. Let us quickly look at it. The mean value is given by sigma under root pi over 2.it’s just 1.25 times sigma. It’s fairly close to this sigma value. The median on the other hand is close to 1.177 sigma and the root mean square rms value is given by root 2 sigma. Please note that the mean and the rms value are fairly close to each other. They are 1.25 sigma and 1.414. Some properties of the Rayleigh distribution it’s by far one of the most common distribution use for mobile radio propagation.

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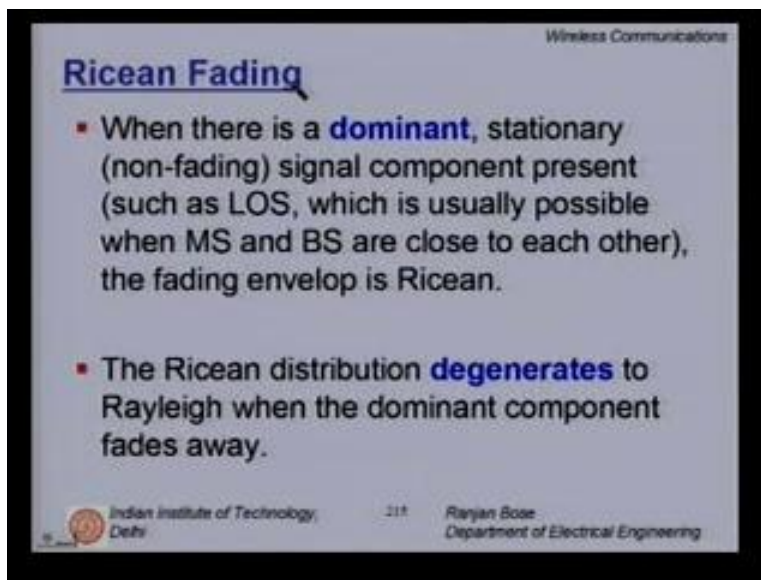
How does it look graphically? On the y axis, we have plotted the pdf and this is the Rayleigh distribution. The mean is nearly equal to the standard deviation sigma. The tail is much larger than the normal distribution than the Gaussian distribution. The spread is much larger and we can say that you are in a deep fade when p is greater than 3 sigma. So there is a mean value here given by under root pi by 2 sigma and this is the standard deviation sigma fairly close. It's a very slowly falling tail. Clearly if you change the sigma, then the pdf will become more constrained to the initial values, so it will become the spread will reduce.

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Let's look at three typical values. Sigma is equal to 1, 2 and 3 and you can see that the spread increases as you increase the sigma. It's a typical Rayleigh pdf. What you have to see is when you take into consideration the Rayleigh pdf. You have to find out what is the sigma for your case and then based on that sigma, you have to take your calculations and later on the average durations of fade and the level crossing rates. Both will depend upon the actual value of sigma. So if the number of scatterers increase, then your sigma will tend to increase. Also there will be an effect on how fast you are moving. You have to do the measurement. Plot a histogram, determine your sigma and do the further calculations. Now let us spend some time talking about the Ricean fading.

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Ricean Fading

- When there is a **dominant**, stationary (non-fading) signal component present (such as LOS, which is usually possible when MS and BS are close to each other), the fading envelop is Ricean.
- The Ricean distribution **degenerates** to Rayleigh when the dominant component fades away.

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211
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Clearly we need to have one dominant line of sight. When there is a dominant stationary non fading signal component then the fading envelop is Ricean. The Ricean distribution as seen before degenerates to the Rayleigh when the dominant component fades away. What will happen when there is not one but two dominant components or just three dominant components from different directions, it is not Ricean anymore. It's not Rayleigh anymore. You will have to look at certain other kinds of distributions like the Nakagami fading distribution to capture these effects but right now, let us limit ourselves to a single dominant signal component.

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Ricean Distribution (1)

- The Ricean distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\left(\frac{r^2 + A^2}{2\sigma^2}\right)} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0, & \text{for } (r < 0) \end{cases}$$

- Where A denotes the **peak amplitude** of the dominant signal, and
- $I_0(\cdot)$ denotes the zeroth order **Bessel function** of the first kind.

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Mathematically, the Ricean distribution is given by the following. P_r is again r greater than or equal to 0. r over sigma squared e raised to power minus r squared plus A squared divided by two sigma squared I_0 Ar over sigma squared and 0 for the negative values of r . clearly it is the envelop of the signal. I shouldn't have any distribution for r less than zero. Here in this equation A denotes the peak amplitude of the dominant signal. it had to come somewhere. If I put A is equal to zero I should go back to my Rayleigh distribution. I_0 denotes the 0th order Bessel function of the first kind. $I_0(0)$ is 1. if you put $A = 0$ here, then you get r over sigma squared e raised to power - r squared over 2 sigma squared is exactly the Rayleigh distribution. so clearly you have filtered in the effect of the dominant signal into the probability density function.

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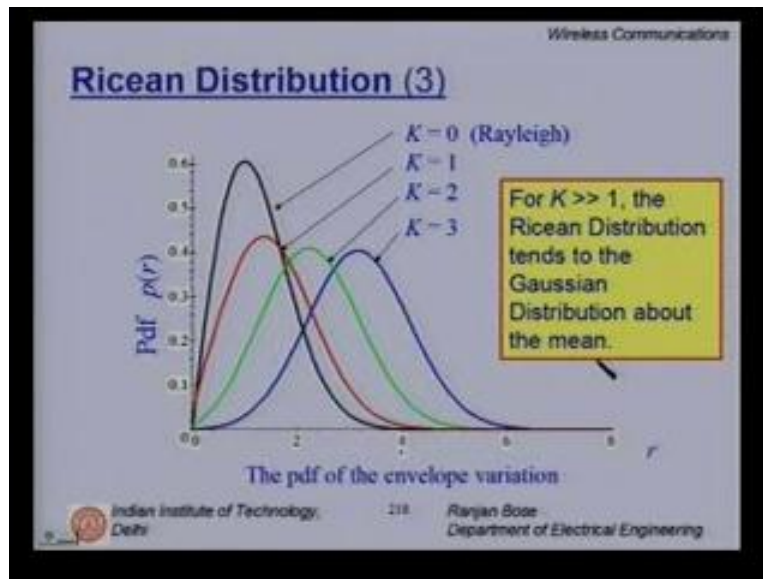
Ricean Distribution (2)

- The Ricean distribution is often described in terms of a parameter K
- $K = A^2/(2\sigma^2)$
- In terms of dB, $K(\text{dB}) = 10 \log \left(\frac{A^2}{2\sigma^2} \right) \text{ dB}$
- For $K \gg 1$, the Ricean Distribution tends to the Gaussian Distribution about the mean.

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The Ricean distribution is often described in terms of a parameter 'K' where K is A squared divided by 2 sigma squared. Please remember A represents the most dominant component and sigma is the strength of all the other components. So it tells us the energy or the ratio of the energy in the dominant component with respect to all other components if we have A tending to zero, we must go back to the Rayleigh distribution. on the other hand, if A is much larger than sigma squared, that is, all the other reflections are kind of negligible with respect to the signal which is in the dominant component, then you should somehow tend to the Gaussian distribution. so there are two extremes of the Ricean distribution. Sometimes K is represented in terms of dB. So K in dB is nothing but 10 log A squared over 2 sigma squared dB. Please note we are talking about the power here A squared. A is the signal strength. A squared represents the power components. Therefore we are using 10 and not 20 log here. Now for K much greater than one, when the signal in the dominant component is much stronger than the other components, then the Ricean distribution tends to the Gaussian distribution about the mean. Then your analysis of the fading channel will just become the analysis of the additive white Gaussian noise channel. You have no fading. You cannot have fading if you have just a dominant line of sight. If there is only one component which is strong and all the multipath components are negligible, there is no fading. Fading comes from multipath components. On the other hand if K tends to zero which means there is no line of sight, what dominates is the multipath components and then you must have fading.

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How does it look? Well here we have plotted different Rayleigh pdfs for various values of K . please note the sigma is one for all of them. What we have done is started with no clear line of sight component to a weak dominant signal to a stronger dominant signal and so on and so forth. So as we move along this axis, I am getting a stronger and stronger and stronger dominant line of sight component.

Clearly, when K is equal to zero, we have the Rayleigh distribution and as you move away you get away from the Rayleigh and quickly move to a distribution which somehow starts looking like a Gaussian distribution. However this distribution is centered around the mean. So what is this? This is the pdf of the envelop of the received signal. Clearly if you have to do a calculation for the level crossing rate or the average duration of fade for a Ricean distribution, you have to now use the pdf for the Ricean distribution as opposed to the Rayleigh distribution. Please observe that for K much greater than one, the Ricean distribution is really tending towards the Gaussian distribution about the mean. This is the second most popular distribution that we use when we have the line of sight.

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Level Crossing and Fading Statistics

- **Level Crossing Rate (LCR)**
 - The expected rate at which the Rayleigh fading envelop, normalized to the local rms signal level, crosses a specified threshold level in the positive going direction.
 - The number of level crossings per second is given by:

$$N_x = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

where:

$$\rho = R / r_{rms} \text{ (specified envelope value normalized to rms)}$$

N_x : crossings per second

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Now let us get into some of the practical aspects. What do we do with this distribution? What kind of information do we gather or infer from this distribution? First of all if I have to comment something about how many times do I dip below a certain threshold and what is the average duration, I stay below a threshold. I must come up with either the level crossing rate or the average duration of fade scenarios. How do I carry out this experiment? First I will take a lot of measurements. I will get into a moving vehicle with a receiver and an envelope detector and I'll find out how my signal strength is varying. I'll take a lot of measurements, I'll plot the histogram of the envelop. If I am in a Rayleigh fading channel, I will get something which resembles the Rayleigh distribution. Based on that I will find out the sigma. Then I have the complete specification of my pdf. Once I have that then I will find out how many times I go below a threshold or how many times I stay below the threshold. Let us look at it mathematically. the level crossing rate or LCR is defined as the expected rate at which the Rayleigh fading envelop normalized to the local root mean square signal level crosses a specified threshold level in the positive going direction. So it's a very clear cut definition. Please note a couple of things. Here we are talking about the Rayleigh fading envelop. However you can have LCR for other distributions as well.

At the end of the day, it only deals with how many times do the signal level crosses a specified threshold in a positive going direction. So you got into a fade and then you are going up after sometime. You got into the fade and you are going up. Each time you cross that threshold you count one and at the end of a certain duration and time you find out how many times you did that cross that threshold line divided by the total number of amount of time. You took the measurement for and you get the level crossing rate in terms of numbers per second. The number of level crossings per second is given by this formula which is easily obtained from the Rayleigh fading distribution. It's NR given by under root 2 pi fm rho e raised to power - rho squared. Now please note the two interesting parameters here. One is the rho, the other one is fm or the maximum Doppler spread. What is rho? Rho is what we define in terms of a ratio.

R is that special threshold value and r_{rms} is a characteristic of the Rayleigh distribution. If you go back and check, r_{rms} depends on the sigma. So it will be root 2 times the sigma. So your rho is simply given by the threshold. You defined divided by the r_{rms} . f_m on the other hand deals with how fast you are travelling and at what angle does the ray form the base station coming towards you. So f_m can be found out by that. Once you have these two numbers, then you have the Rayleigh for the Rayleigh the level crossing rate in terms of crossings per second. How strong is the effect of rho? How sensitive is the number of crossings per second to your threshold? These are the things we will look at today with some examples. Clearly if you fix up a high R , then the rho will increase but it is e raised to power - rho squared. So number of crossings will go down. It's intuitive. In a distribution if you increase the level of your threshold, then the number of times you cross it per second will decrease. On the other hand if you have a very low threshold, you will tend to cross it many number of times per second but it is e raised to power - rho squared.

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Level Crossing Rate (Example)

- **Scenario**
 - Rayleigh Fading environment
 - Carrier Frequency = 900 MHz
 - Maximum vehicular speed = 50 km/hr
 - $\rho = 1$ (i.e., Threshold = RMS level)
 - Find the level crossing rate.
- $\lambda = 3 \times 10^8 / (900 \times 10^6) = 0.33 \text{ m}$
- $f_m = v/\lambda = (50000/3600)/0.33 \approx 41.67 \text{ Hz}$

$$N_x = \sqrt{2\pi} f_m \rho e^{-\rho^2} = 38.42 \text{ crossings/scc}$$

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Let us look at an example. Let us describe a scenario. Maybe a typical GSM scenario. Let us say we have a really Rayleigh fading environment. So we are in an urban environment, fairly far away from the base station. Clearly no line of sight. The carrier frequency let us say is 900 MHz. it is important to observe that the carrier frequency will play a role in your level crossing rate. How sensitive it is to carrier frequency, we will look at some other examples also. but for this example, let us say it is 900 MHz because this will translate to your wavelength which will translate to your f_m , the Doppler shift which will translate to your level crossing rate, let us say that the maximum vehicular speed is 50 km/hr. clearly it's a congested urban scenario. The rho is one. Who has specified rho? We ourselves as design engineers can specify a rho. It will depend really on the quality of service you wish to provide and of course your receiver sensitivity. But for the sake of illustration only, let us say rho is equal to one. That is, your threshold r is equal to the RMS and r over r_{rms} is nothing but one. What do we have to find? We have to find out the number of crossings per second going in the positive direction.

So first we have to calculate the wave length at which we are operating. So it is the speed of electromagnetic waves divided by the frequency. That is equal to about 33 cm. the maximum Doppler shift is $V \cos \theta$ divided by λ . But let's assume that θ is equal to zero. So it's V over λ 50000 m/s divided by 3600 sec/hr. so V over λ gives me 41.67 Hz. this is the maximum Doppler shift. Now at this stage, we can quickly put these values in this equation. Under root $2 \pi f_m \rho e$ raised to power - ρ squared and we get 38.42 crossings per second. Is this good or bad? What do we do with this value? It tells me that for this particular situation and here please note that the σ has not come into the picture because I have specified ρ is equal to one. Otherwise your answer should and will depend on the σ . But assuming ρ is equal to one. Then you are close to 40 crossings per second. 40 times you are dipping below the fade and going up again most likely. You have to do something about it. Not a very healthy scenario. But then you are calculating it for 50 km/hr.

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Level Crossing and Fading Statistics

- **Average Fade Duration**
 - The average period of time for which the received signal is below a specified level R .
 - For Rayleigh fading signal, this is given by:

$$\bar{\tau} = \frac{1}{N_x} \Pr[r \leq R] = \frac{1}{N_x} (1 - e^{-\rho^2})$$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}, \quad \rho = \frac{R}{r_{\text{rms}}}$$

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Now let us look at the next important parameter which is the average fade duration sometimes also referred to as the average duration of fade in the literature. This is the average period of time for which the received signal is below a specified level R . this R can be specified by us and has to be a function of the receiver sensitivity or the minimum quality of service. We wish to give for Rayleigh fading distribution. first of all when you calculate it is τ average is one over NR probability that r is less than a specified threshold R and if you carry out the simple calculations using the Rayleigh distribution you get τ bar is equal to $e^{\rho^2} - 1$ in the denominator ρf_m . f_m being the maximum Doppler spread under root 2π . ρ is defined as before $R -$ the threshold fixed bias divided by the r_{rms} comes from the σ of the pdf of the Rayleigh distribution. So let us look at it again. Clearly it's a very strong function of ρe raised to power ρ squared. If ρ is greater than one, this one is usually neglected. In the denominator in the numerator, you have e raise to power ρ squared. The denominator has also a ρ and f_m . see if you travel much faster, then your f_m should increase. The value of the denominator increases. Your average duration of fade will go down you will get out of the fade very fast.

But this is nothing to do with how quickly you get back into the fade. So there are two things if you move fast. This f_m goes up. But you must also remember that. Just staying below a threshold depends on how fast you are travelling but very soon you might get back into the fade that must come from this NR. That is the significance of NR. You couldn't have calculated the average duration of fade without the notion of the level crossing rate. They are interrelated.

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Average Fade Duration (Example)

- **Scenario**
 - Rayleigh Fading environment
 - Carrier Frequency = 900 MHz
 - Maximum vehicular speed = 50 km/hr
 - $\rho = 1$ (i.e., Threshold = RMS level)
 - Find the average duration of fade.
- $\lambda = 3 \times 10^8 / (900 \times 10^6) = 0.33 \text{ m}$
- $f_m = v/\lambda = (50000/3600)/0.33 = 41.67 \text{ Hz}$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} = 16.5 \text{ ms}$$

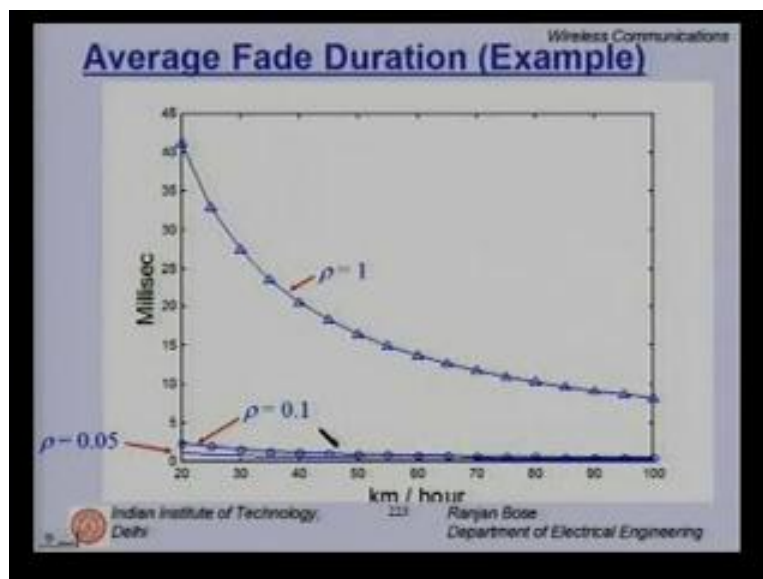
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Let us look at the average fade duration with an example. Let us describe a scenario. Let it be similar to what we did in the previous example a GSM scenario. It is a fading channel the carrier frequency again is 900 MHz. it'll help us calculate the lambda. We stick to our maximum vehicular speed of 55 km/hr. again fixed rho is equal to the root mean square value the task. This time is to find out the average duration of fade.

The lambda or the wavelength is calculated to be 0.33 m or 33 cm. the maximum Doppler shift again comes out to be 41.67 Hz because I have fixed my speed at which I am travelling at 50 km/hr and the lambda is also fixed. If you now plug into the equation for the average duration of fade, you get a value of 16.5 ms. again the question before us is: is it good or bad? What do we do with this number? So one thing is clear if we are designing an inter leaver to overcome the fade duration, then this value will tell us how deep my inter leaver should be. If you are trying to do block coding of some sort for a channel coding to overcome the effects of errors, then we must make the errors look like they are random errors. However the moment you have fades, you wipe out series of bits. So what happens is you have burst error fading channels. Normally it will lead to burst errors. Now if you have a data rate, so if you are sending 1000 bps, then multiply with 16.5 ms and you know how many bits have been erased. Clearly, you work at much high data rates. So you're talking about hundreds of thousands of bits that can easily get erased for this fading situation. It's a very unhappy situation to be in, unless you do something about it.

The easiest way is to put in an inter leaver with an appropriate depth so that these burst errors appear as random errors and hence can be overcome by block codes. So this is the significance. Clearly this single number owes its genesis to your sigma which has been found out by actual measurements or a modeling of the channel. so at the end of the day, the channel models channel measurements will give you a Rayleigh distribution with a certain sigma which will give you the average duration of fade, help you design an inter leaver, help you give you or guarantee a certain bit of a rate. If you do not want to do all of that, you have to change your 'R' the threshold because if you change your R, the sigma changes. If you change your R you can either pump in more power. So you can give a fade margin pump in more power, change your R and consequently change this value. It can be shown that increasing just the signal strength is not always the best way to overcome the effects of fading. It's a wasteful method though a simple one.

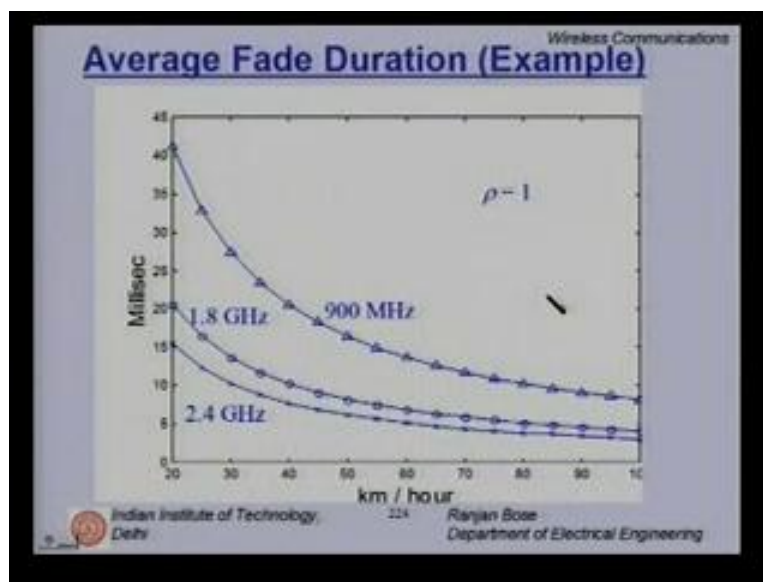
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Let us look at another example. Here on the x axis, I have plotted the vehicular speed in km/ hr. on the y axis, I have plotted the average duration of fade in milliseconds. But I have taken three scenarios because I would like to see the effect of the threshold. So I have rho =1 which tells me that my threshold value R is equal to the root mean square rms value. Here it is 0.1. R is much less than the rms value. In fact it is a 10th and here it is a 20.05. Now on the y axis is the duration of fade in milliseconds as you can see. Let's follow the first curve. As you increase the vehicular speed, the average duration of fade decreases. It's an interesting phenomenon even though as you increase the vehicular speed, the level crossing rate increases. On an average the duration of fade decreases. So it's good news. A vehicle travelling at a faster rate will have less burst errors. It's somewhat intuitive. Whereas if I sit on a red light for a long time or if I go very slowly, I may stay in a fade for a long time. So the average fade duration increases. As I go towards the left side of this axis. Say at 20 km/hr, my average fade duration could be 40 ms. now see what

happens when you change rho. Now the moment I would lower my threshold, there is a drastic fall because it is e raised to power rho squared. It's a very strong impact. So even at 20 km/hr, your average duration of fade has gone down to close to about 2.5 ms and it is still further for rho is equal to 0.05. So yes, there is a way to overcome the fading by providing for fade margins and in some communication systems, you do provide for fade margins. You pump in a little bit more power. This will translate to playing with your threshold level. Also beyond a certain speed the decrease in the average fade duration is not significant. If you note, for low values of rho, the function of the vehicular speed doesn't show up prominently. There is not a significant impact on the speed of the vehicle on that average duration of fade for low values of rho. It is more pronounced when rho is one or greater. So these will translate to your design issues. Let us look at another example.

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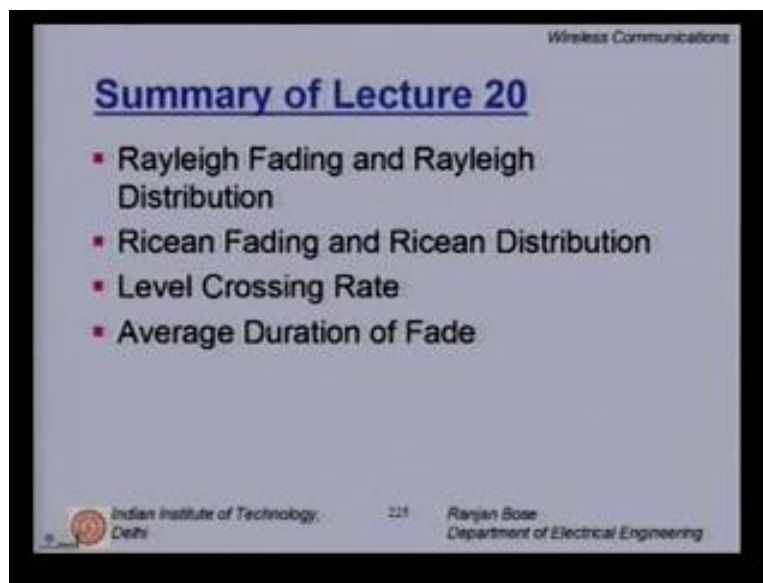


This is relevant for various frequencies of operation. Clearly average duration of fade is important. It'll kill my situation. It'll kill my synchronization and it'll cause burst errors.

It'll cause very poor bit error rates if I do not think of overcoming average fading durations and how to overcome that will first ask me to find out what is the average duration of fade. But the average duration of fade is also a function of the frequency of operation. Just look at the two bands: the 900 MHz band and the 1800 MHz band. Here let us consider the rho =1 scenario. So the threshold R is equal to the RMS value. This one we have seen before 900 MHz for rho =1. so at 20 km/hr, you get average durations of fade up to 40 ms and if you go up to 100 km/hr. you are down to only 10 ms. if you have a dual band and if you just switch from 900 MHz to 1800 MHz. you have drastically lowered your average duration of fade. This is not very intuitive. In fact, what it will show up in your actual measurements is that your level crossing rate increases drastically. But level crossing rate is in the denominator for tau bar. So it has caused a decrease in your average duration of fade just by translating to a higher frequency of operation or a smaller wavelength. You improved your situation. Your burst lengths have decreased.

Of course everything is not so. The moment you go to a higher frequency band, you hope to pump in more number of bps. So even if your direction of fade has gone down the bit rate has gone up. So may be the number of burst number of bits in burst error probably stays the same although. It doesn't decrease linearly. Then when you go from 1.8 GHz to the more familiar ISM band of 2.4 GHz and then clearly, 2.4 was never designed to be for very high vehicular speeds. But it can allow you low portability. Here in this region, the effect is prominent. again the general trend remains, the same the average duration of fade has gone down and for lower speeds, the effect is pronounced whereas if you tend to move to slightly higher speeds. You tend to flatten out. This is an interesting observation for $\rho = 1$. If you now make this $\rho = 0.1$, you go much below. Please note 2.4 GHz scenario is not so as it appears because here you have to go towards the left of this axis. 20 km/hr is much higher than what is expected in the 2.4 GHz range for which we normally design our systems and so the moment you go towards the left, you tend to go up. So it will still be a problem. You have to understand the average fade duration and design your communication systems accordingly so as to overcome the effects of fading.

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Let us now summarize what we have learnt today. We started off with Rayleigh fading and Ricean fading. We looked at not only the fading but the distribution. We looked at the pdf's. Then we looked at one derivative of the Rayleigh fading distribution which is the Rayleigh level crossing rate. How many times per second do you cross a certain threshold in the positive direction. Finally we looked at another important parameter which is the average duration of fade. We also looked at some examples to see how the threshold value which is the function of the ρ makes a difference in terms of the average duration of fade and even how the wavelength of operation or the frequency of operation changes the average duration of fade. We will conclude our lecture here and will continue further in the subsequent lectures. Thank you!