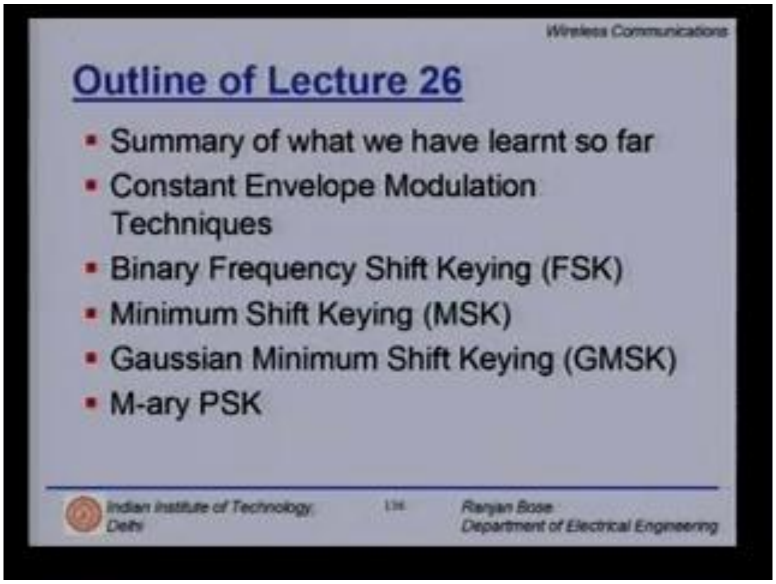
Wireless Communications
Dr. Ranjan Bose
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture No. # 26
Modulation Techniques for Mobile Communications (Contd.)

Welcome to the next lecture on modulation techniques for mobile communications. The brief outline for today's talk is as follows.

(Refer Slide Time: 00:01:26 min)



Wireless Communications

Outline of Lecture 26

- Summary of what we have learnt so far
- Constant Envelope Modulation Techniques
- Binary Frequency Shift Keying (FSK)
- Minimum Shift Keying (MSK)
- Gaussian Minimum Shift Keying (GMSK)
- M-ary PSK

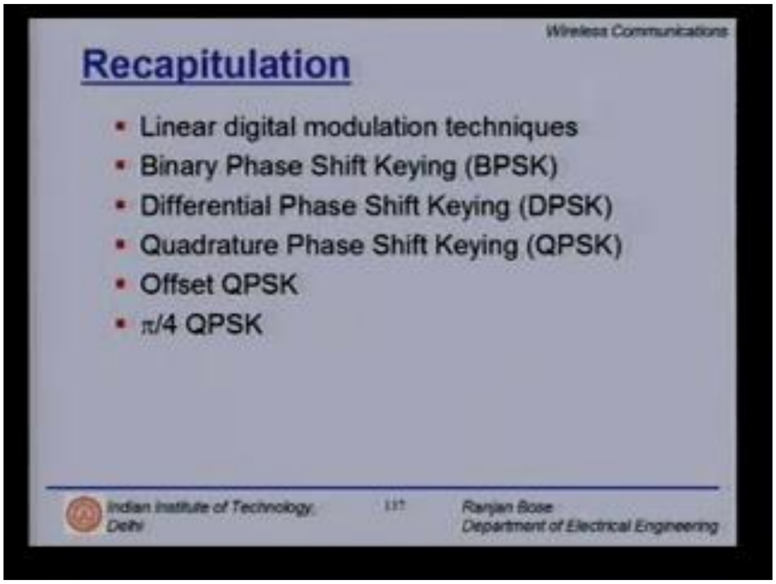
Indian Institute of Technology Delhi

Ranjan Bose
Department of Electrical Engineering

We will first summarize what we have learnt in the previous classes then we look at constant envelop modulation techniques. We will look at binary frequency shift keying or BFSK followed by minimum shift keying and then something which is used in modern GSM systems, Gaussian minimum shift keying or GSMK. Finally we will have a small glimpse at M-ary PSK. So this is the outline for today's talk.

First a brief recap; last time we learnt the linear digital modulation techniques and why they are better than analog modulation techniques. In particular we talked about binary PSK or phase shift keying then we learnt that you can use differential phase shift keying in order to do the same thing without the use of the carrier. You don't have to get the phase information to use DPSK. We then moved on to quadrature PSK or QPSK one of the very popular modulation techniques.

(Refer Slide Time: 00:02:03 min)



Wireless Communications

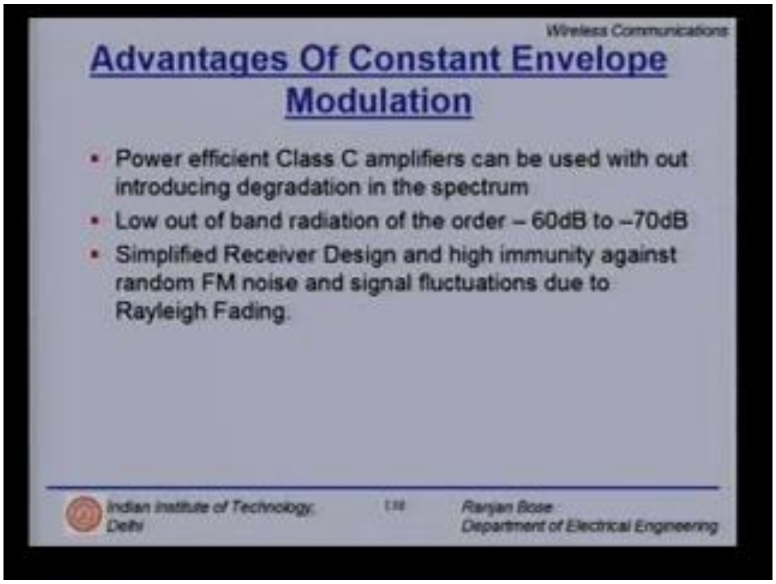
Recapitulation

- Linear digital modulation techniques
- Binary Phase Shift Keying (BPSK)
- Differential Phase Shift Keying (DPSK)
- Quadrature Phase Shift Keying (QPSK)
- Offset QPSK
- $\pi/4$ QPSK

Indian Institute of Technology, Delhi
117
Ranjan Bose
Department of Electrical Engineering

Then there were problems with QPSK because of the switching and there were possible switching's of 180 degrees in the symbols and hence we proposed another technique called offset QPSK which only allows a maximum of 90 degree phase shift between symbols and then a compromise between QPSK and offset QPSK called the pi by 4 QPSK wherein we use not one but two signals constellations of QPSK shifted mutually by pi by 4 and you use alternately each one of the constellation diagrams. Now today we are going to focus on constant envelop modulation techniques. What are the advantages of constant envelop modulation techniques?
First and the foremost is power efficient, class C amplifiers can be used without introducing degradation in the spectrum.

(Refer Slide Time: 00:03:27 min)



Wireless Communications

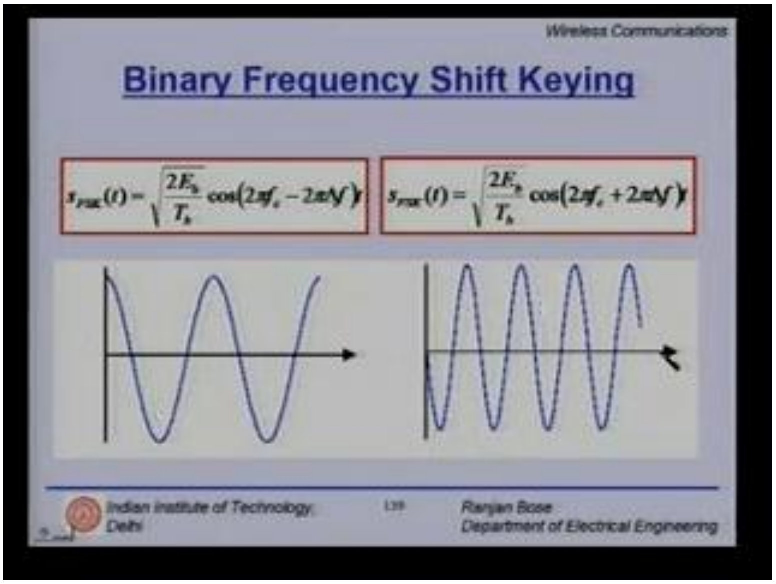
Advantages Of Constant Envelope Modulation

- Power efficient Class C amplifiers can be used with out introducing degradation in the spectrum
- Low out of band radiation of the order – 60dB to –70dB
- Simplified Receiver Design and high immunity against random FM noise and signal fluctuations due to Rayleigh Fading.

Indian Institute of Technology, Delhi
118
Ranjan Bose
Department of Electrical Engineering

Please remember the basic requirements for any modulation technique is a, the spectral efficiency, b the power efficiency and c how easy or difficult it is to design the receiver. Here you can have very cheap power efficient class C amplifiers that can be made use of for constant envelop modulation. The other important feature of constant envelop modulation techniques is low out of band radiation of the order of as low as minus 60 to minus 70 dB. So really low out of band emissions which is very good because my receiver low pass filters and band pass filters do not have to be very strict. Then all this leads to simplified receiver design and high immunity against random FM noise and signal fluctuations due to Rayleigh fading. So in general the constant envelop modulation techniques fair much better in the Rayleigh fading environments and hence a favorite choice of many wireless communication systems.

(Refer Slide Time: 00:04:59 min)



Wireless Communications

Binary Frequency Shift Keying

$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c-2\pi\Delta f)t$

$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c+2\pi\Delta f)t$

Indian Institute of Technology, Delhi

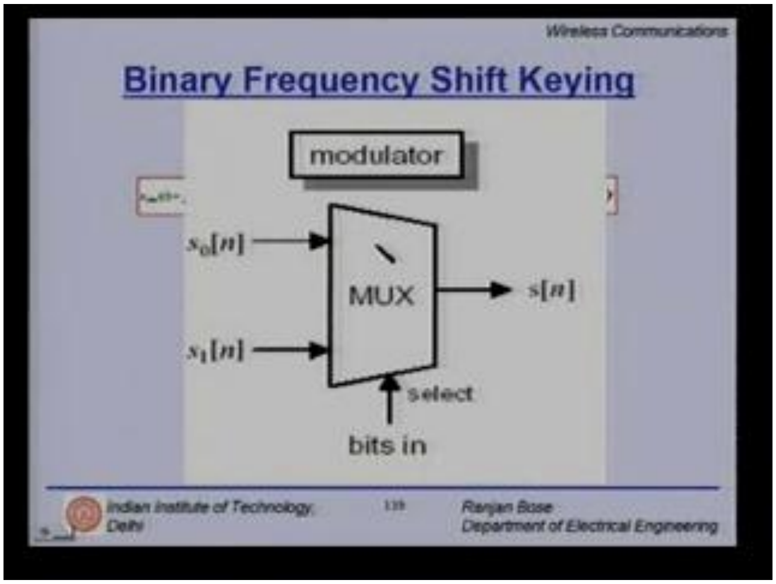
L19

Ranjan Bose
Department of Electrical Engineering

First let us start with binary frequency shift keying or BFSK. clearly we have one bit per symbol and we denote frequency one or FL lower frequency to send symbol one which is representative of say 0 and frequency f_c plus delta f for bit 1. So 0 is denoted by this symbol on the left and 1 is denoted by the symbol on the right. If you plot it a lower frequency represents 0 and a higher frequency represents 1.

So what is the modulator doing? Modulator is just a multiplexer where you have a select bit in and it chooses between S_0 and S_1, lower frequency and higher frequency depending upon whether the bit is 0 or 1 and clearly there is a problem. The problem is the moment you switch the transitions may be abrupt and anytime I have abrupt transition, abrupt phase changes I will have out of band emissions spurious emissions, this is bad news.

(Refer Slide Time: 00:05:44 min)



Wireless Communications

Binary Frequency Shift Keying

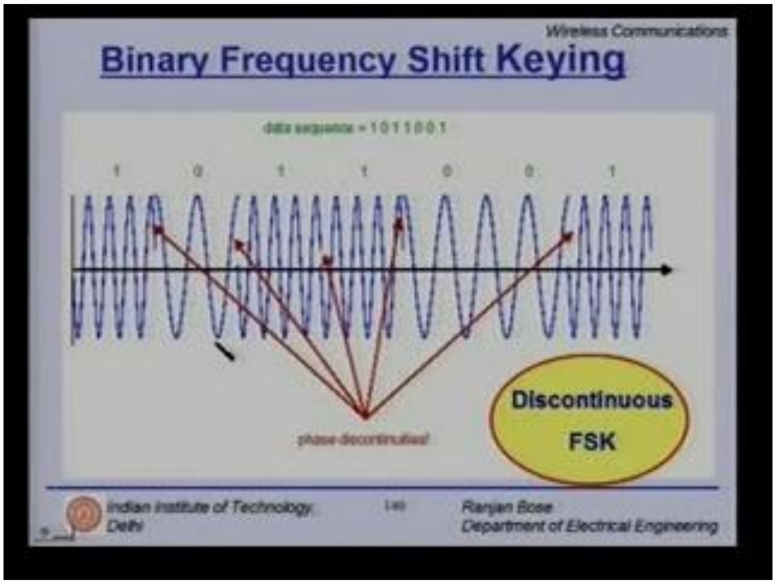
Indian Institute of Technology, Delhi

139

Ranjan Bose
Department of Electrical Engineering

So suppose we take an example for binary FSK and the data sequence that I am trying to send is 1 0 1 1 0 0 1. So one is depicted by a higher frequency, zero is depicted by a lower frequency high frequency, high frequency low, low, high and so and so forth but the moment the symbol interval which is also the bit interval in this case gets over you shift to the next symbol but at the time of transition you have phase discontinuities. Question? Is there any fix ration between higher frequency and lower frequency? Question being asked is can there be any fixed ratio between the higher frequency and lower frequency, the answer is yes actually we need the higher frequency and lower frequency to be orthogonal in some sense.

(Refer Slide Time: 00:06:20 min)



Wireless Communications

Binary Frequency Shift Keying

Discontinuous FSK

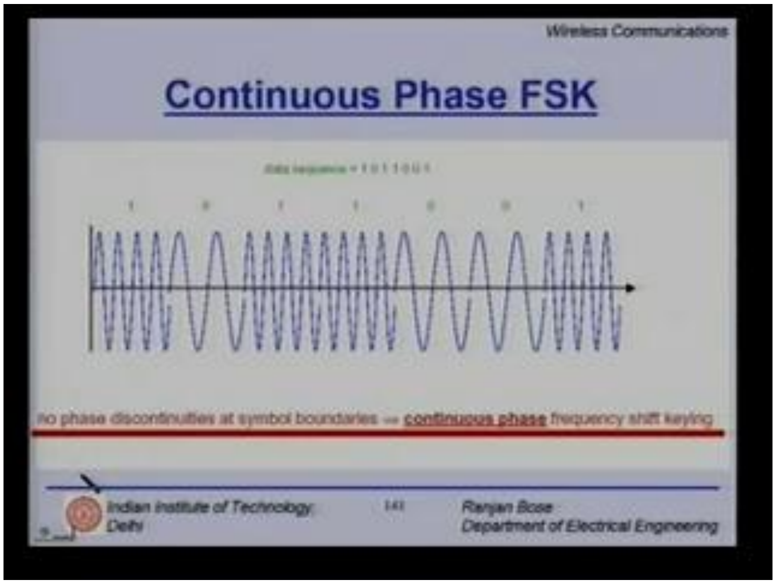
Indian Institute of Technology, Delhi

140

Ranjan Bose
Department of Electrical Engineering

That is integrated over one symbol interval, the integration should yield zero if I integrate v_1 t, v_2 t. So there will be some possible spacing and as you have higher and higher spacing you can reach a number of solutions. The minimum shift possible which will still render it orthogonal will lead us to a minimum shift keying situation but here in a very simplistic scenario we are showing that we have two symbols one denoted by a lower frequency and one denoted by a higher frequency. Also we can put another constraint as to the phase continuity part which will talk about in the subsequent slides today. The point to be noted from this slide is that phased discontinuities will occur if we use a simplistic binary FSK modulator as shown in the previous slide and this is not acceptable, this is called discontinuous FSK. So we have to work around this, it is clearly not an acceptable solution.

(Refer Slide Time: 00:08:34 min)



Wireless Communications

Continuous Phase FSK

data sequence = 1 0 1 1 0 0 1

no phase discontinuities at symbol boundaries → continuous phase frequency shift keying

Indian Institute of Technology Delhi

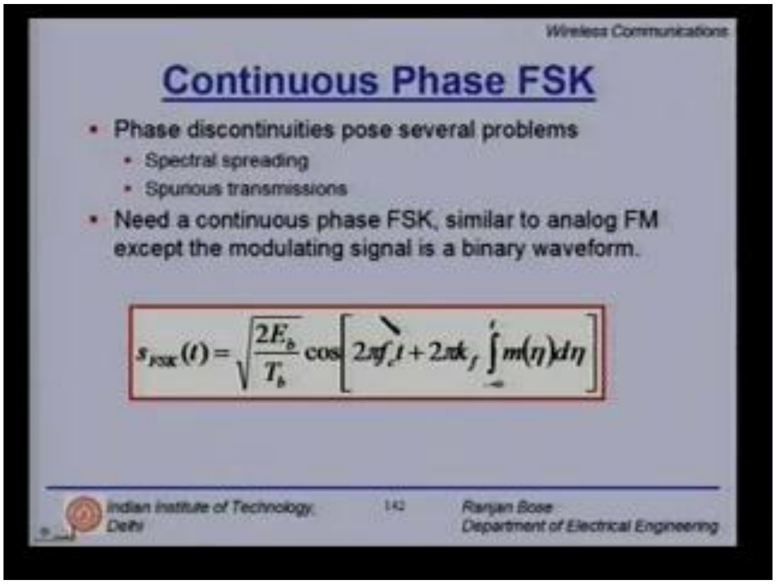
141

Ranjan Bose
Department of Electrical Engineering

The answer comes in the form of continues phase FSK or also called CP FSK, here the solution is simpler. We have a constraint on the number of cycles that we have and the continuity at the transition is maintained. So there must be a relationship between the number of cycles here and a number of cycles here. Clearly if you see the lower frequency has two complete cycles and the higher frequency has 1, 2, 3 and 4 it is logical. So if frequency f_1 is twice of f_0 we need this constraint. Even if we have the frequency f_1 4 times f_0 again the constraint will be met and so and so forth. So you can have as much separation between f_1 and f_0 as possible but you would clearly interested in the minimum possible separation because we do not want to be very expensive in terms of the bandwidth requirement. What we understand from this slide is for the same data stream as discussed previously I can have an FSK which is continuous phase. So no phase discontinuities at symbol boundaries hence continuous phase frequency shift keying.

Now let us see what are the problems associated with phase discontinuities. Phase discontinuities pose several problems like spectral spreading and spurious transmissions both are highly undesirable specially in a regulated environment. So we need a continuous phase FSK something similar to analog FM except the modulating signal is a binary waveform.

(Refer Slide Time: 00:10:05 min)



Wireless Communications

Continuous Phase FSK

- Phase discontinuities pose several problems
  - Spectral spreading
  - Spurious transmissions
- Need a continuous phase FSK, similar to analog FM except the modulating signal is a binary waveform.

$$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos\left[2\pi f_c t+2\pi k_f\int_{-\infty}^{t}m(\eta)d\eta\right]$$

Indian Institute of Technology Delhi

142

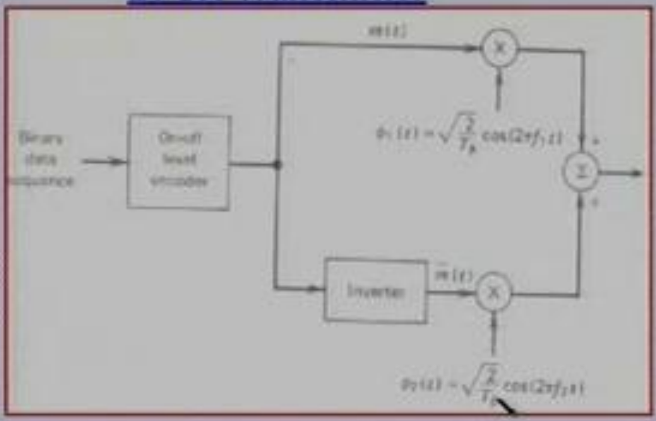
Ranjan Bose
Department of Electrical Engineering

If you remember from one of your previous lectures you have an FM which has the frequency proportional to the input signal. Here instead of having a continuously varying message signal we have a binary waveform consisting of one's and minus one's and we modulate the FM using that, so this will ensure that I do not have phase discontinuities. So my signal S subscript FSK (t) can be represented as under root 2 E_b over T_b, E_b is energy per bit, T_b represents the bit duration, cosine two pi f_ct this is my base frequency plus this is the modulating signal and m (eta) can be binary plus 1 or minus 1. This will yield a continuous phase FSK please note even though m (eta) can be discontinuous, the integration minus infinity to m (eta) d eta multiplied by 2 pi f_k is not discontinuous, this whole thing will be a continuous phase signal.

(Refer Slide Time: 00:12:05 min)

Wireless Communications

BFSK Transmitter



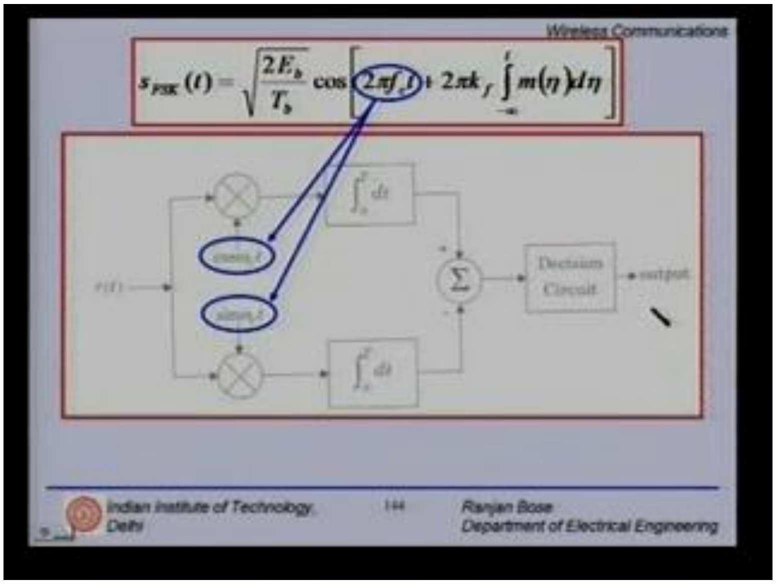
Indian Institute of Technology Delhi

143

Ranjan Bose
Department of Electrical Engineering

Now how do we build a transmitter; a very simplistic transmitter is as follows. We have 2 modulators cos 2 pi f_1 (t) and cos 2pi f_2 (t) pertaining to 0 and 1 and we have a binary data sequence coming in and we have an on off level encoder and then here we pass it through an inverter which will multiply it to cosine with f_2 as a frequency, the other part is f_1 add it up and sent it out. So it's a clear implementation of the formula for binary FSK.

(Refer Slide Time: 00:12:52 min)



Wireless Communications

$$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos\left[2\pi f_c t+2\pi k_f\int_{-\infty}^{t}m(\eta)d\eta\right]$$

Indian Institute of Technology Delhi

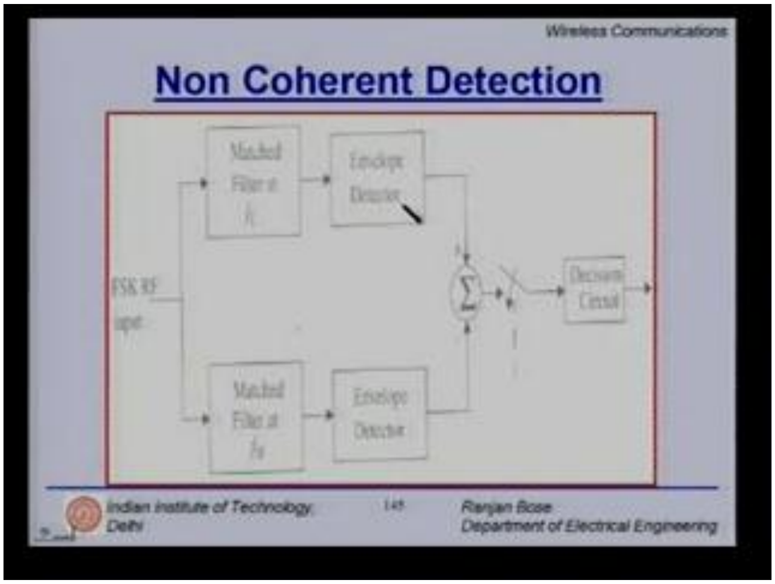
144

Ranjan Bose
Department of Electrical Engineering

What about the detection part? It is both possible to have a coherent detection and non-coherent detection of binary FSK, first let us talk about the coherent detection part. So the coherent detector is just the inverse, logically the received signal is divided into two parts. The first part is multiplied by cosine 2 pi f_ct and second one from sin 2pi f_ct integrate, add pass it through a decision circuit and get an output. Why are we multiplying this? Look at the FSK waveform and we have this 2 pi f_ct being split here, multiplied, integrated and decision circuit is being passed through it.

Let us talk about the non-coherent detection. This is the block diagram for non-coherent detection of binary FSK. Here on the left is the FSK input, again you split it into two parts but this time we have logically a band pass filter at lower frequency and a matched band pass filter at higher frequency, f subscript L stands for the lower frequency part and f subscript H stands for the higher frequency part. Clearly we have two frequencies one of the frequencies representing zero. The higher frequency representing one.

(Refer Slide Time: 14:42)



Wireless Communications

Non Coherent Detection

FSK RF input

Matched Filter at f_L

Envelope Detector

Matched Filter at f_H

Envelope Detector

Σ

Decision Circuit

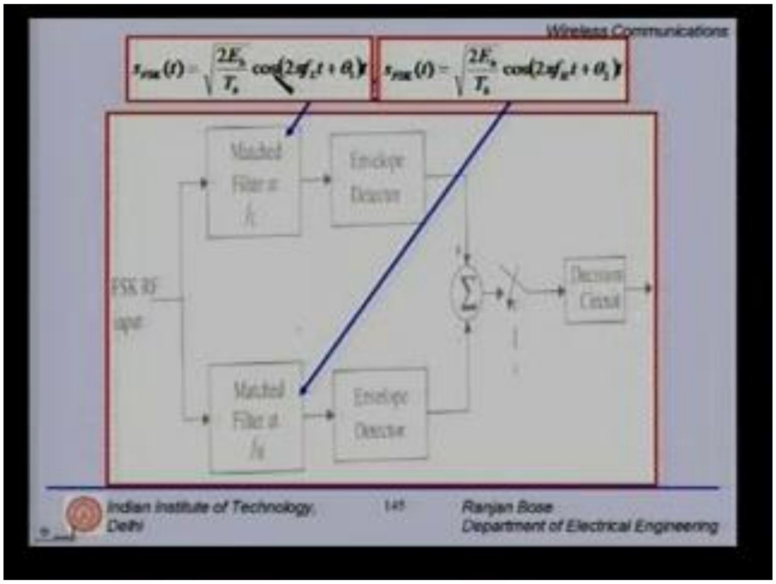
Indian Institute of Technology Delhi

145

Ranjan Bose
Department of Electrical Engineering

Once you pass it through, pass it through an envelope detector. So there is no need to know the carrier, this is non-coherent detection, add it up pass it through a decision circuit. So please note that the two symbols 1 and 0 pertain to the cos 2 pi f_L t and cos 2 pi f_H t pertaining to the lower frequency and the higher frequency terms. Now how does the FSK spectrum look like? Please remember this is also an important part in choosing a modulation technique. How does the spectrum look like? Where are the main lobes ending, how high are the side lobes, what are the locations of the nulls? These are the questions that have to be addressed before we choose any particular modulation technique.

(Refer Slide Time: 00:13:51 min)



Wireless Communications

$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_L t+\theta_1)$

$s_{FSK}(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_H t+\theta_2)$

FSK RF input

Matched Filter at f_L

Envelope Detector

Matched Filter at f_H

Envelope Detector

Σ

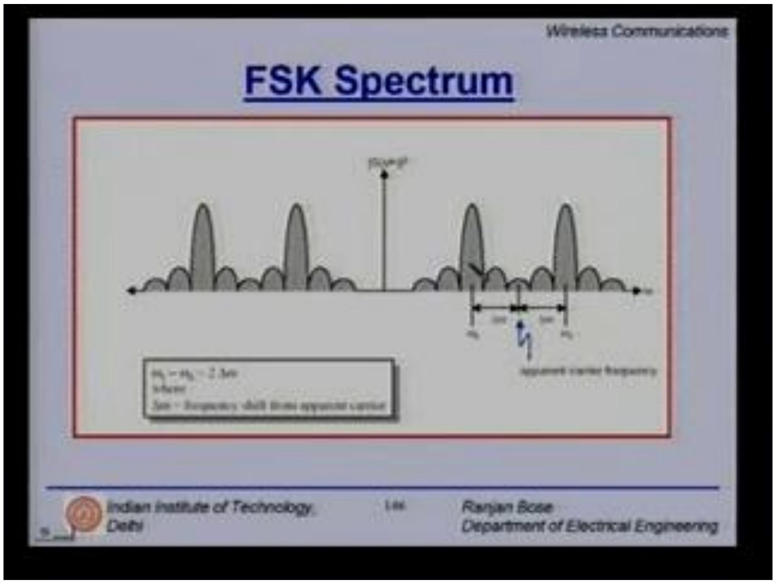
Decision Circuit

Indian Institute of Technology Delhi

145

Ranjan Bose
Department of Electrical Engineering

(Refer Slide Time: 00:15:09 min)



Wireless Communications

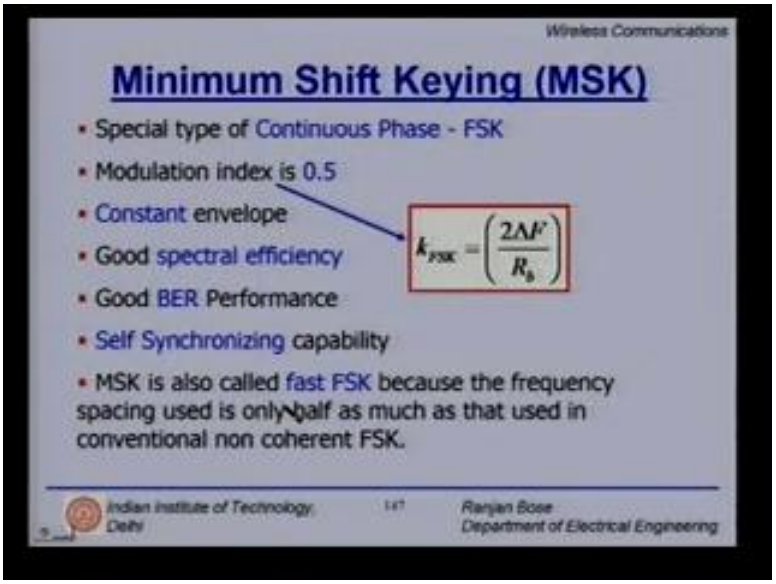
FSK Spectrum

Indian Institute of Technology, Delhi

Ranjan Bose
Department of Electrical Engineering

So for FSK very simplistically, if you are using binary FSK you have one concentration of a main lobe here at omega$_1$ pertaining to frequency one and one at omega zero, the spacing omega minus omega zero is 2 delta omega. It is called the frequency shift, frequency shift from an apparent carrier. So if you pass your FSK spectrum through a non-intelligent detector and decoder and asked to determine the carrier frequency, it might mistakenly give you this center point as the carrier frequency. So this is called the apparent carrier frequency so when you have to specify any FSK modulator or demodulator you specify in terms of the center frequency the phase shift and the frequency bandwidth of f_1 and f_2.

(Refer Slide Time: 00:16:41 min)



Wireless Communications

Minimum Shift Keying (MSK)

- Special type of Continuous Phase - FSK
- Modulation index is 0.5
- Constant envelope
- Good spectral efficiency
- Good BER Performance
- Self Synchronizing capability
- MSK is also called fast FSK because the frequency spacing used is only half as much as that used in conventional non coherent FSK.

$$k_{FSK} = \left(\frac{2\Delta F}{R_b}\right)$$

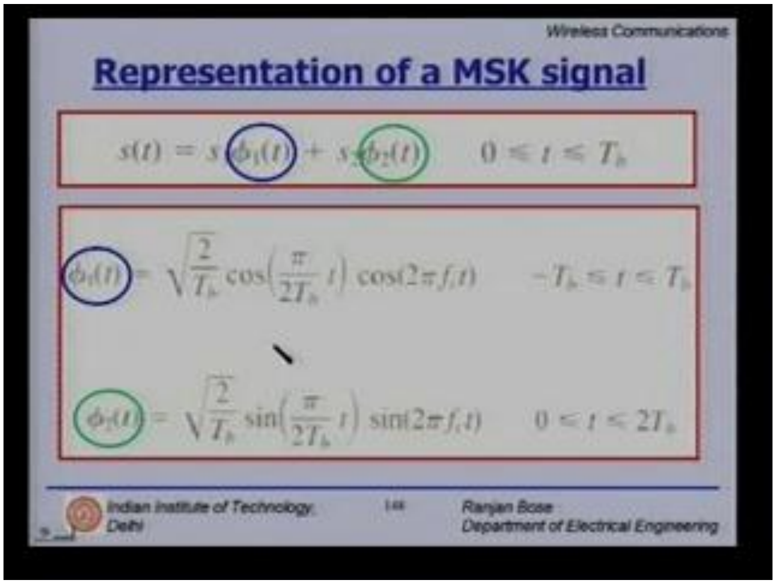
Indian Institute of Technology, Delhi

Ranjan Bose
Department of Electrical Engineering

Now let us shift gears and move into something called as minimum shift keying. What is minimum shift keying? It’s a special type of continuous phase FSK so we have entered the domain of continuous phase modulation techniques and we will stay there. The modulation index for minimum shift keying is 0.5. How do you define a modulation index for the case of FSK? It is given by k subscript FSK equal to 2 delta F over R_b, delta F is a frequency separation, R_b is the bit rate. So minimum shift keying has a property that the modulation index is 0.5. The other important characteristics of minimum shift keying are constant envelop, highly desirable works well for Rayleigh fading environments, good spectral efficiency we will find out how good is it in the subsequent slides, good bit error rate performance it is scoring high on all of these things, spectral efficiency, bit error rate performance, continuous phase. No doubt MSK is one of the popular modulation techniques.

It also has a self-synchronizing capability. So let us now go deeper into minimum shift keying. Just one more point, minimum shift keying is also called fast FSK. Why, because the frequency spacing used is only half as much as that used in a convention non-coherent FSK. We are talking about the frequency shift between f_1 and f_2, the two frequencies used to denote 1 and 0. Let us see how we can represent an MSK signal. Let us say S (t) can be represented as S_1 phi_1 (t) plus S_2 phi_2 (t) where t lies between zero and the bit interval T_b.

(Refer Slide Time: 00:18:50 min)



Wireless Communications

Representation of a MSK signal

$$s(t) = s_1\phi_1(t) + s_2\phi_2(t) \qquad 0 \le t \le T_b$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos\left(\frac{\pi}{2T_b}t\right)\cos(2\pi f_c t) \qquad -T_b \le t \le T_b$$

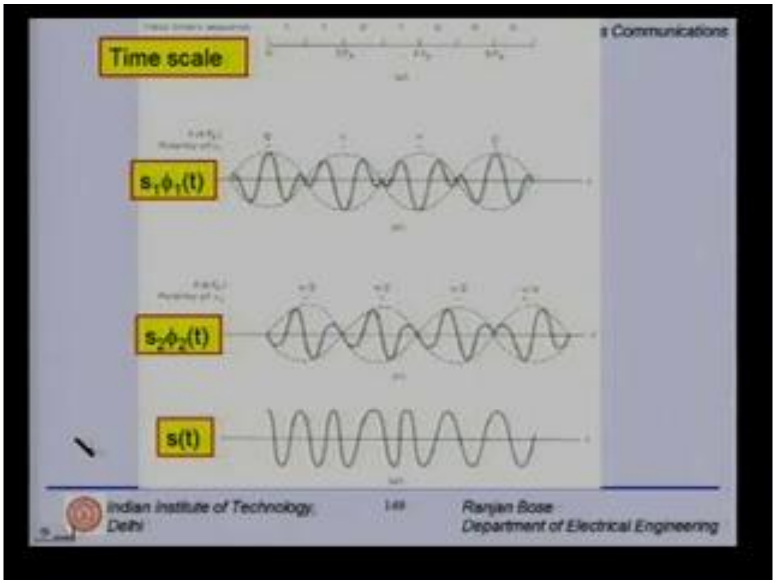
$$\phi_2(t) = \sqrt{\frac{2}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right)\sin(2\pi f_c t) \qquad 0 \le t \le 2T_b$$

Indian Institute of Technology, Delhi 148 Ranjan Bose Department of Electrical Engineering

Now what are phi one’s and phi two’s? Phi_1 for MSK’s given by under root 2 over T_b cosine pi over 2 T_b t times cosine 2 pi f_c t and it lasts from minus T_b to T_b but please note phi_2 is shifted in time. It is under root 2 over T_b sin of pi over 2 T_b t sin 2 pi f_c t but please note this time is from 0 to 2 T_b, please note there is a time shift between phi_1 and phi_2. In some sense we can equate it to offset QPSK. In fact in offset QPSK we use rectangular pulses shifted by T_b by 2, here instead of rectangular pulses we are using the half cycle of cosine; other than that it is very similar to your offset QPSK.

Here is phi_1 and here is the phi_2 that will plug in and multiply to get your S (t). Note this method of representation of an MSK signal will lead to how you build the modulator and then the demodulator. We will have a simple multiplication of cos 2 pi f_c t with cos pi over T_b (t) to begin with, to create the basis functions. Then you create this S of t multiplied with message bits and send it up.

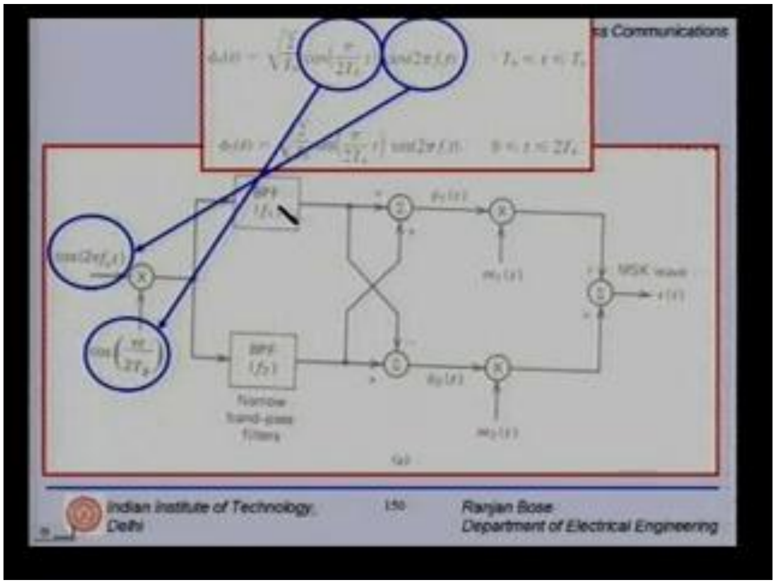
(Refer Slide Time: 00:21:02 min)



Here is a simple representation in the time domain. The first figure shows the times domain input binary sequence. So 1 1 0 1 0 0 0 suppose this is an example we are taking, when you use the previous slide to find out phi_1 and phi_2 and multiply it, you get this kind of a waveform for S_1 phi_1(t) and clearly a time shifted S_2 phi_2 (t). When you add it up magically you reach a continuous wave FSK. Please note that the places where there is a one there is a higher frequency, the moment you enter the domain of zero correspondingly there is a lower frequency and then again higher frequency and then lower frequencies. So this is how MSK works, it is spectrally one of the most efficient continuous phase modulation techniques.

Now let us use our understanding of the MSK signals and built a transmitter. We first start with cos 2 pi f_c t and multiply with what, cos pi t over T_b these are the two things we multiplied with. If you remember phi_1 (t) is cos (pi over T_b t) cos 2 pi f_c t and we multiply it. Here is what you get and so this is how we are trying to build the phi_1. Pass it through the band pass filter centered it around f_1, pass this one through the filter centered around f_2. So these are narrow band pass filters, add it up and you get your phi_1 and phi_2. Now all you have to do is multiply by m_1 and m_2 (t) and send out the MSK waveform. This is a simple transmitter for MFSK.

(Refer Slide Time: 00:22:16 min)



Communications

$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos\left(\frac{\pi}{2T_b}t\right) \cos(2\pi f_c t)$ $\quad -T_b \le t \le T_b$

$\phi_2(t) = \sqrt{\frac{2}{T_b}} \sin\left(\frac{\pi}{2T_b}t\right) \sin(2\pi f_c t)$ $\quad 0 \le t \le 2T_b$

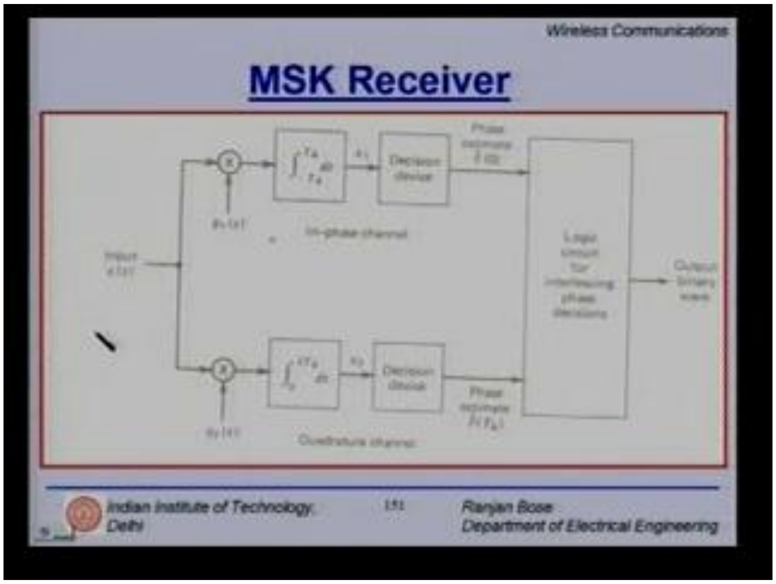
Indian Institute of Technology, Delhi

150

Ranjan Bose
Department of Electrical Engineering

Now how does the MSK receiver look like? It is just the reverse; you again start with your phi$_1$ and phi$_2$ (t) which you can generate locally. Take the inputs split it up, multiply it integrate it pass it through the decision devise, estimate the phase logic circuit and find out the binary output waveform. So this is the MSK receiver it’s fairly easy to construct.

(Refer Slide Time: 00:23:32 min)



Wireless Communications

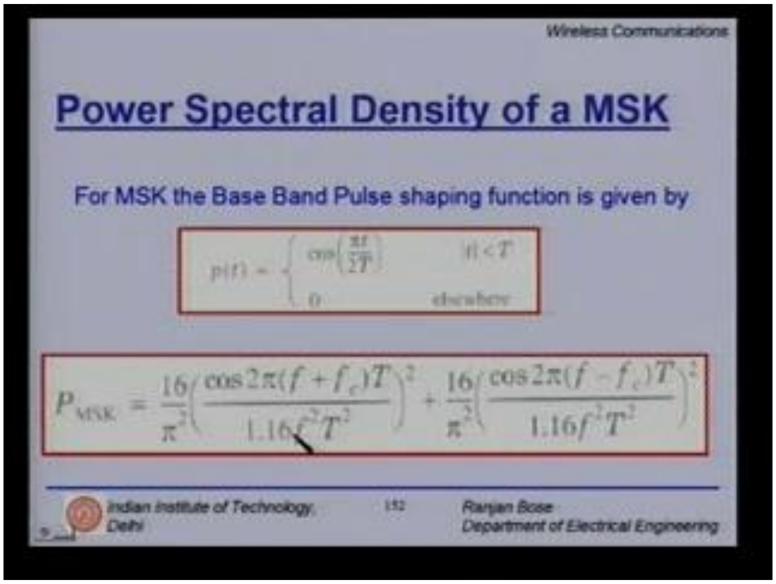
MSK Receiver

Indian Institute of Technology, Delhi

151

Ranjan Bose
Department of Electrical Engineering

(Refer Slide Time: 00:24:05 min)



Wireless Communications

Power Spectral Density of a MSK

For MSK the Base Band Pulse shaping function is given by

$$p(t) = \begin{cases} \cos\left(\frac{\pi t}{2T}\right) & |t| < T \\ 0 & \text{elsewhere} \end{cases}$$

$$P_{\text{MSK}} = \frac{16}{\pi^2}\left(\frac{\cos 2\pi (f+f_c)T}{1.16 f^2 T^2}\right)^2 + \frac{16}{\pi^2}\left(\frac{\cos 2\pi (f-f_c)T}{1.16 f^2 T^2}\right)^2$$

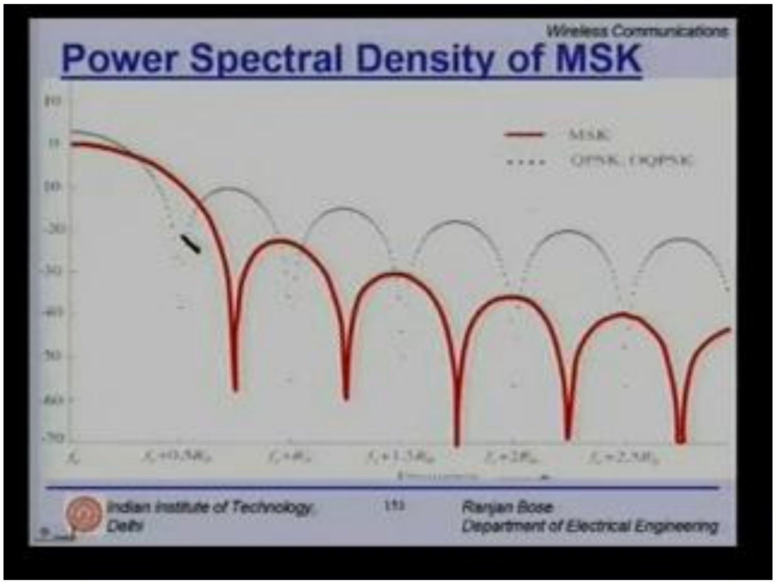
Indian Institute of Technology, Delhi

152

Ranjan Bose
Department of Electrical Engineering

Now the other aspect is what is the power spectral density of an MSK? For MSK the baseband pulse shaping function is given by p (t) which is nothing but cos pi t over 2 capital T and it is zero elsewhere. In this case if we use this pulse shaping function then you have the power spectral density of MSK given by the following equation. Please note it is here f plus f_c and here f minus f_c and the denominators are same, so there is a symmetry around f_c.

(Refer Slide Time: 00:24:56 min)



Wireless Communications

Power Spectral Density of MSK

Indian Institute of Technology, Delhi

153

Ranjan Bose
Department of Electrical Engineering

How does it look like? Only on the positive side here is your MSK and to compare it with we have the power spectral density in the background for the QPSK are also the offset QPSK which has the similar power spectral density.

So couple of things that can be noted, please note this is the x axis representing frequency and y axis is in dB. The first thing that we observe is that the main lobe of your MSK is broader than QPSK or offset QPSK which means it requires a little bit more bandwidth but this comes at an advantage. The advantage is that the side lobes are greatly reduced, when I say greatly please note that the y axis is in dB. So here there is almost a 20 dB reduction with respect to the first side lobe of QPSK which means most of the energy is present in the main lobe and usually the bandwidth is measured with respect to the main lobe only, so it's a good property. So for your MSK the power spectral density is fairly conducive.

(Refer Slide Time: 00:26:36 min)

Wireless Communications

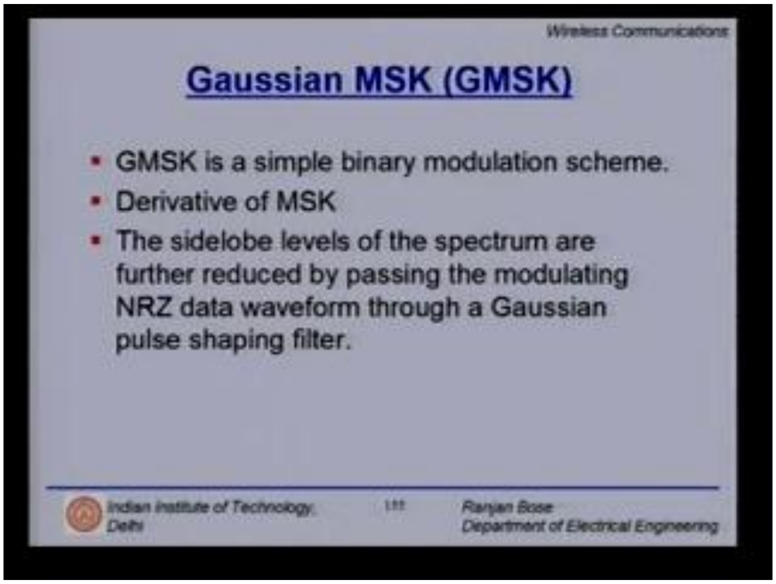
Advantages of MSK over BFSK

- MSK and BFSK produce constant envelope carrier signals with no amplitude variations.
- This is a desirable characteristic for improving the power efficiency of transmitters
- Amplitude variations can exercise non-linearities in an amplifier's amplitude-transfer function, generating spectral re-growth, a component of adjacent channel power
- Therefore, more efficient amplifiers (which tend to be less linear) can be used with constant-envelope signals, reducing power consumption.

Indian Institute of Technology, Delhi 154 Ranjan Bose Department of Electrical Engineering

Now what are the advantages of MSK over binary FSK? See both MSK and binary FSK produce constant envelop carrier signals with absolutely no amplitude variations. This is a desirable characteristic for improving the power efficiency of the transmitters can use the class C amplifiers. The amplitude variations can exercise non-linearity's in an amplifier's amplitude transfer function. What does it do? It generates spectral re-growth, a component of the adjacent channel power. Therefore more efficient amplifiers which tend to be less linear can be used with constant envelop signals reducing power consumption.

(Refer Slide Time: 00:27:32 min)



Wireless Communications

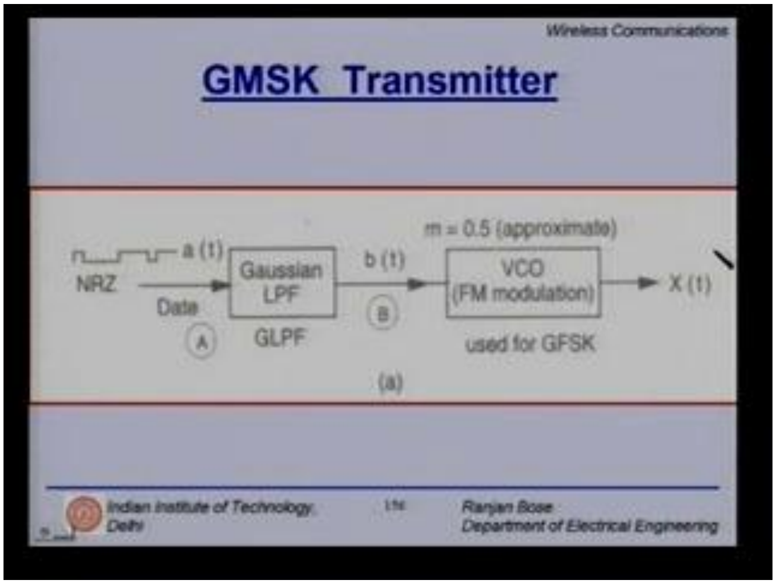
Gaussian MSK (GMSK)

- GMSK is a simple binary modulation scheme.
- Derivative of MSK
- The sidelobe levels of the spectrum are further reduced by passing the modulating NRZ data waveform through a Gaussian pulse shaping filter.

Indian Institute of Technology, Delhi 111 Ranjan Bose Department of Electrical Engineering

Now let us look at one variant of MSK called the Gaussian MSK. This is popular because it has been adopted by the GSM standard also. So how does it differ from MSK? GMSK is a simple binary modulation scheme. The beauty simplicity, it is a derivative of MSK we will soon see and how is it derived from MSK? The side lobe levels of the spectrum are further reduced. Which spectrum? The spectrum of the MSK are further reduced by passing the modulating non-return to zero data waveform through a Gaussian pulse shaping filter. So you first pass the data waveform through a Gaussian pulse shaping filter and then use the MSK. It will clearly give you better spectral properties because you are doing pulse shaping but we will see that the price is not too much. Gaussian baseband pulse shaping smooth's the phase trajectory of the MSK signal.

(Refer Slide Time: 00:29:00 min)



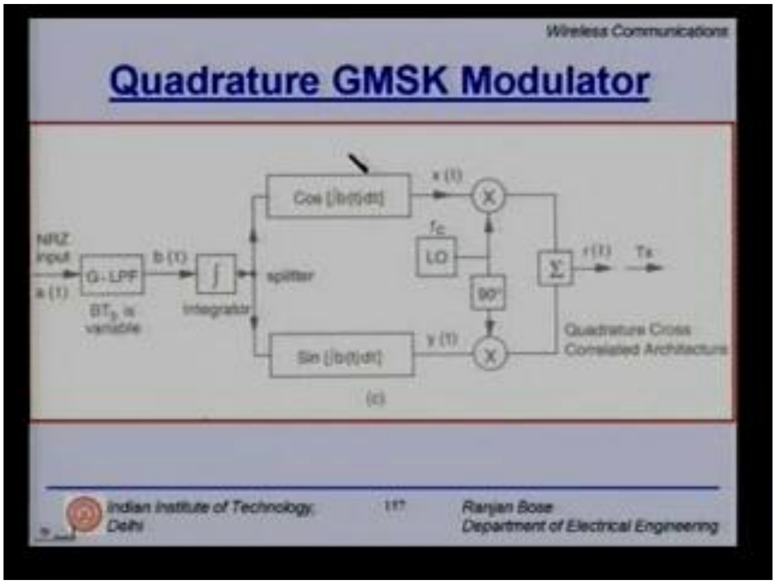
Wireless Communications

GMSK Transmitter

Indian Institute of Technology, Delhi 116 Ranjan Bose Department of Electrical Engineering

Let us see how a GMSK transmitter looks like. So the implementation is fairly simple, you have a non-return to zero data 1 0 0 0 1 1 0 1 1 which is coming in. It is first passed through a Gaussian low pass filter which actually smooth's the edges and what you obtain is b of t. Now in this case your m is equal to 0.5 so you use a FM modulation it's a fairly technology and then what you send out is Gaussian FSK. Please note the implementation is very simple and hence very inexpensive.

(Refer Slide Time: 00:29:59 min)



Wireless Communications

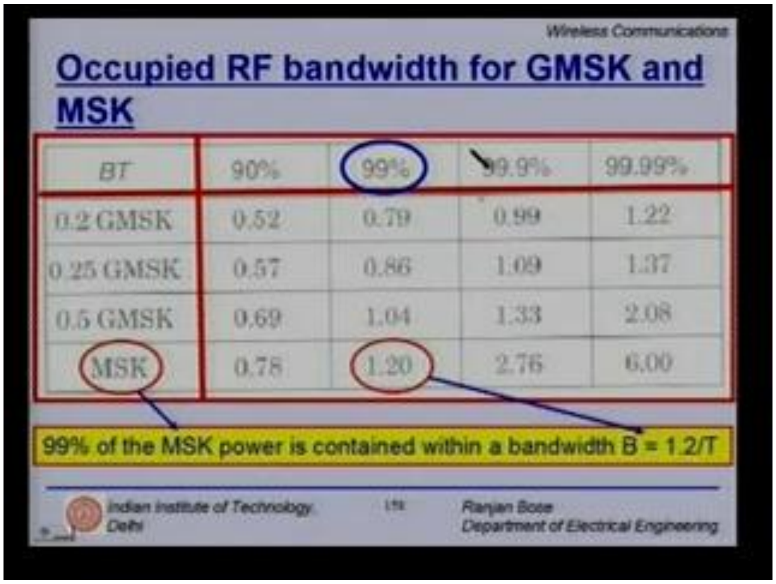
Quadrature GMSK Modulator

Indian Institute of Technology, Delhi
117
Ranjan Bose
Department of Electrical Engineering

You can also go to the MSK root where the GMSK modulator uses the quadrature components to generate the quadrature GMSK. Again you have the non-return to zero input f (t), first you pass it through the Gaussian low pass filter. Now the Gaussian low pass filter will also be characterized by a parameter called alpha which decides how broad or narrow is a low pass filter. Basically the role of factor, that is a designed parameter. Once you pass it you will get b (t) you integrate it, split it pass it through cos and sin filters get x (t)'s and y (t)'s multiplied by the local oscillator sum and send it. So this is the simple implementation of quadrature Gaussian minimum shift keying.

Now let's compare how does GMSK stands with respect to MSK. Now here on the left most column is the product of time and bandwidth. The time bandwidth product given by BT, here it denotes the different kinds of GMSK possible for the different role of factors alpha. This comes from the Gaussian filter. So it's 0.2, 0.25, 0.5 and when it becomes very large say infinity it becomes MSK. So here let's look at an example, how much power is contained within a certain bandwidth? So if I want to take this example of MSK, what it means is that 99% of the power is contained within a bandwidth B equal to 1.2 divided by T or BT is equal to 1.2.

(Refer Slide Time: 00:31:06 min)



Wireless Communications

Occupied RF bandwidth for GMSK and MSK

BT	90%	99%	99.9%	99.99%
0.2 GMSK	0.52	0.79	0.99	1.22
0.25 GMSK	0.57	0.86	1.09	1.37
0.5 GMSK	0.69	1.04	1.33	2.08
MSK	0.78	1.20	2.76	6.00

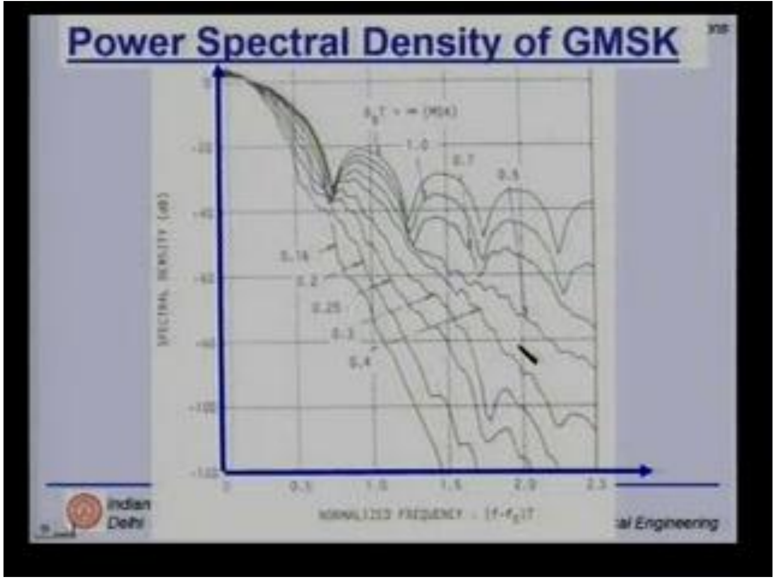
99% of the MSK power is contained within a bandwidth B = 1.2/T

Indian Institute of Technology Delhi

Ranjan Bose
Department of Electrical Engineering

It means for MSK 90% of the power is contained in a bandwidth equal to 0.78 divided by T and so and so, you can become as stringent as possible. At the same time please compare it with some of the GMSK schemes. Consider for example 0.5 GMSK. If you use the same parameter how much power of 0.5 GMSK is contained within 1.04 by T of the bandwidth. The answer is 99%. Clearly a much smaller band contains the same amount of power. If you choose to move further up and you have a more sharper role of factor. So if you look at 0.2 GMSK then hardly 0.8 over T is the bandwidth requirement for 0.2 GMSK to contain 99% of the power. So it's a very spectrally efficient schemes. This table gives you how much strict you want to be on the x axis and how lenient you want to be in terms of choosing your GMSK. So in this slide you can also see that MSK is a special case of GMSK.

(Refer Slide Time: 00:34:07 min)

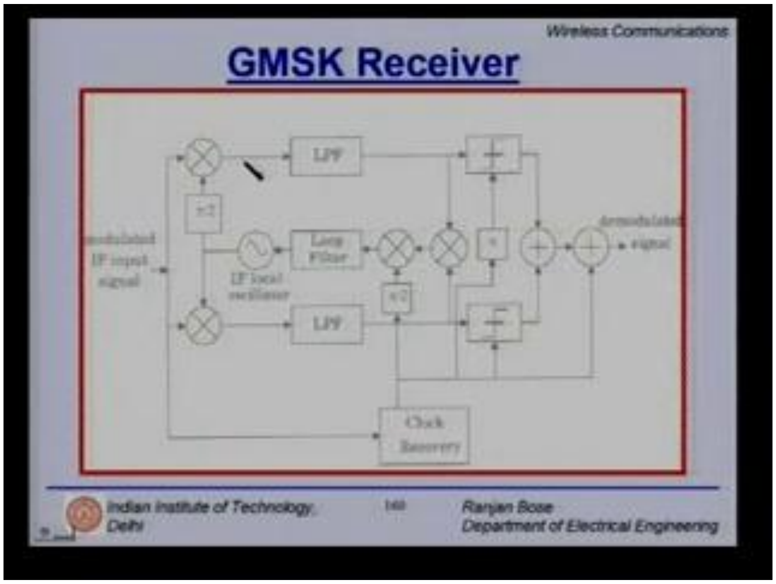


Power Spectral Density of GMSK

Now lets us get a better feel of power spectral density of GMSK. Please note that other than having analytical expression, a lot of stimulations are carried out to determine the power spectral density of GMSK because actually it depends upon what kind of data that you get. It is data dependent so you average out over a lot of data sequences and then come up with some kind of a power spectral density depiction for GMSK. So the curves are not that smooth please note that the top most curve corresponds to MSK, MSK where BT is equal to infinity and then you come down as you increase, as the product decreases from infinity to if you have 1, 0.7, 0.5 and so and so forth up to 0.16 is depicted. This is the role of factor which we are talking about.

So as you decrease your alpha, you start packing in more power within the main lobe and you decrease the value of the side lobes. Please note that the y axis is in dB and x axis is in normalized frequency given by f minus f_c t. So this diagram gives us some feel for the power spectral density, this also shows that your MSK is a special case of GMSK.

(Refer Slide Time: 00:36:06 min)



Wireless Communications

GMSK Receiver

Indian Institute of Technology, Delhi

160

Ranjan Bose
Department of Electrical Engineering

Let us now consider how does a Gaussian minimum shift keying receiver look like. Let us say you have a modulated IF input signal coming in, you pass it through a splitter then you multiply it with a sin and a cosine. So pi by 2 shifted here, sin and cosine this is a local oscillator please remember that the input signal has itself been taken through a clock recovery circuit and then generated the lobe filtering and which will tell you where the face of the local oscillator should be. Now once you multiply, you pass it through low pass filters. Do some kind of a thresholding, add and get the demodulated signal. So exactly how you construct it you take it apart.

(Refer Slide Time: 00:37:05 min)

Wireless Communications

Advantages of GMSK over MSK

- GMSK is a derivative of MSK where the bandwidth required is further reduced by passing the modulating waveform through a Gaussian filter
- The Gaussian filter minimizes the instantaneous frequency variations over time
- GMSK is a spectrally efficient modulation scheme and is particularly useful in mobile radio systems
- It has a constant envelope, spectral efficiency, good BER performance and is self-synchronizing.

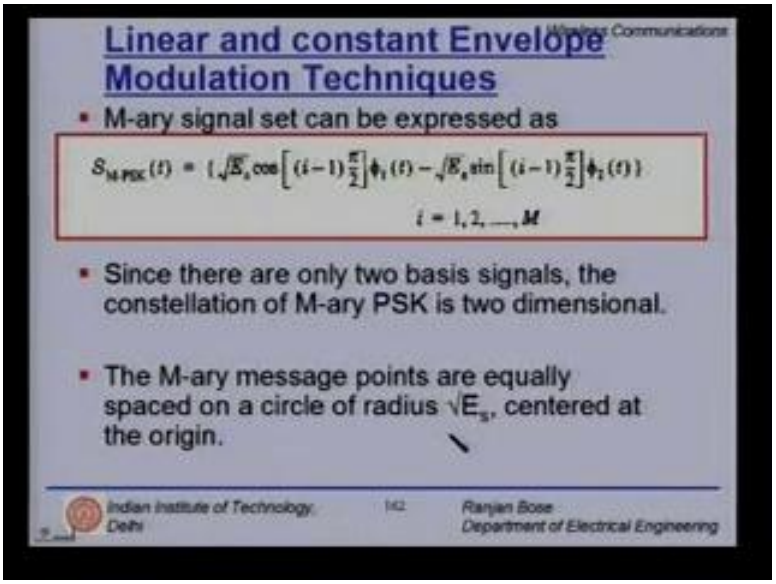
Indian Institute of Technology, Delhi

141

Ranjan Bose
Department of Electrical Engineering

What are the advantages of GMSK over MSK? Clearly spectral efficiency is an advantage we have already seen, what are the other advantages. GMSK is a derivative of MSK where the bandwidth required is further reduced. How? By passing the modulating waveform through a Gaussian filter. The Gaussian filter minimizes the instantaneous frequency variations over time. GMSK is a spectrally efficient modulation scheme and it's particularly useful in mobile radio systems. It is a standard in GSM, it has a constant envelop spectrally efficient, good BER performance and self-synchronizing so a lot of advantages of GMSK.

(Refer Slide Time: 00:38:01 min)



Wireless Communications

Linear and constant Envelope Modulation Techniques

- M-ary signal set can be expressed as

$$S_{M\text{-}PSK}(t) = \{\sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t) - \sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t)\}$$

$$i = 1, 2, \ldots, M$$

- Since there are only two basis signals, the constellation of M-ary PSK is two dimensional.
- The M-ary message points are equally spaced on a circle of radius $\sqrt{E_s}$, centered at the origin.

Indian Institute of Technology, Delhi

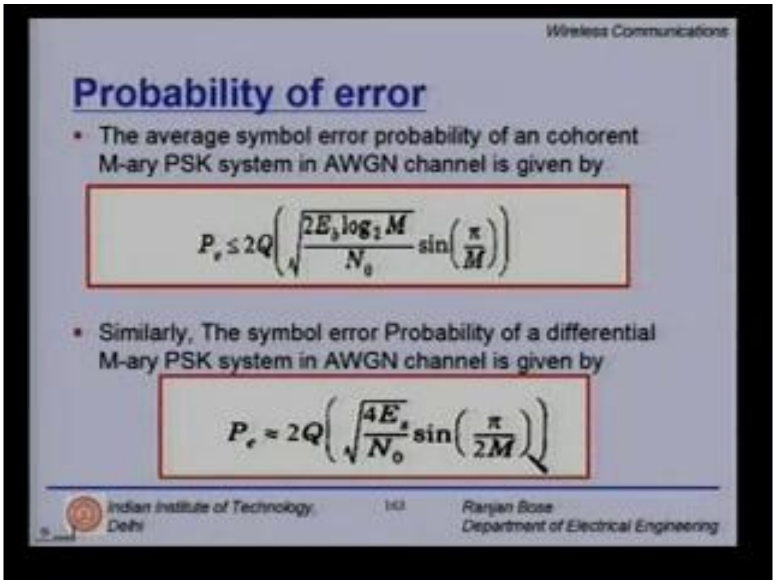
142

Ranjan Bose
Department of Electrical Engineering

Now we shift gears and look at a combination of linear and constant envelop modulation techniques. Basically a sneak preview on MPSK and then in the subsequent lectures we will try to see how we deal with quadrature amplitude modulations, etc and finally the spread spectrum techniques. So let's say the M-ary signal set can be expressed as $S_{M\text{-}PSK}$ in time domain under root E_s, this denotes the energy per symbol cosine i-1 pi by 2 phi_1 (t) plus E_s sin i-1 pi by 2 phi_2 where i goes from 1, 2 up to M because it is an M-ary signal set. So I have 2 phi one's and phi two's and then we have this sinusoids here. Since there are only two basis signals, the constellation of M-ary PSK is two dimensional.

The M-ary message points are equally spaced on a circle of radius under root E_s centered at the origin. So if you increase the radius, you are actually pumping in more energy per symbol and as we increase the radius, the separation between the points in the constellation diagram increases giving you better signal to noise ratio.

(Refer Slide Time: 00:39:40 min)



Wireless Communications

Probability of error

- The average symbol error probability of an cohorent M-ary PSK system in AWGN channel is given by

$$P_e \leq 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$

- Similarly, The symbol error Probability of a differential M-ary PSK system in AWGN channel is given by

$$P_e \approx 2Q\left(\sqrt{\frac{4E_s}{N_0}} \sin\left(\frac{\pi}{2M}\right)\right)$$

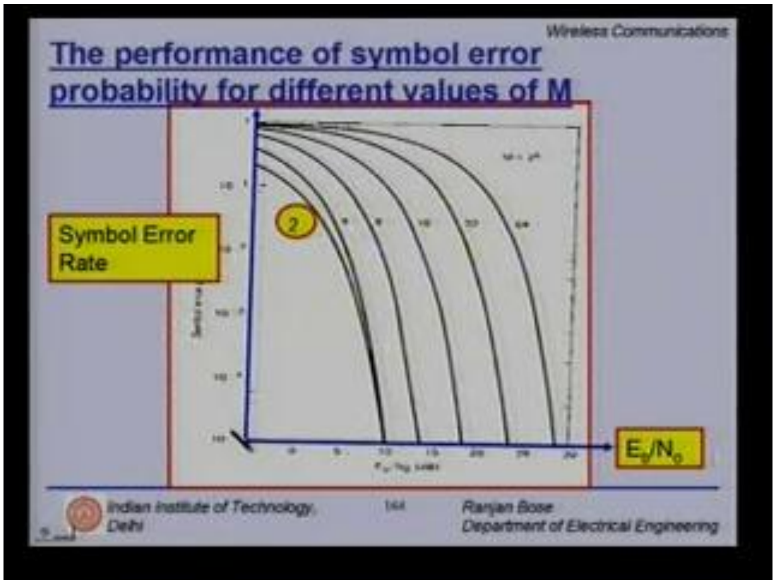
Indian Institute of Technology, Delhi

143

Ranjan Bose
Department of Electrical Engineering

The probability of error, the average symbol error probability of coherent M-ary PSK system in additive white Gaussian noise or AWGN channel is given by the following equation. It is upper bounded by 2 Q function under root 2 E_b log to the base 2 M over N_0 sin pi over M. So you can derive this equation, it is available in standard text books. We are not going into the derivation but it gives you a feel as how as you increase the M, your probability of error increases. The symbol error probability of a differential M-ary PSK system in additive white Gaussian noise channel is given by 2 Q under root 4 E_s over N_0 sin pi over 2M.

(Refer Slide Time: 00:40:46 min)



Wireless Communications

The performance of symbol error probability for different values of M

Indian Institute of Technology Delhi

144

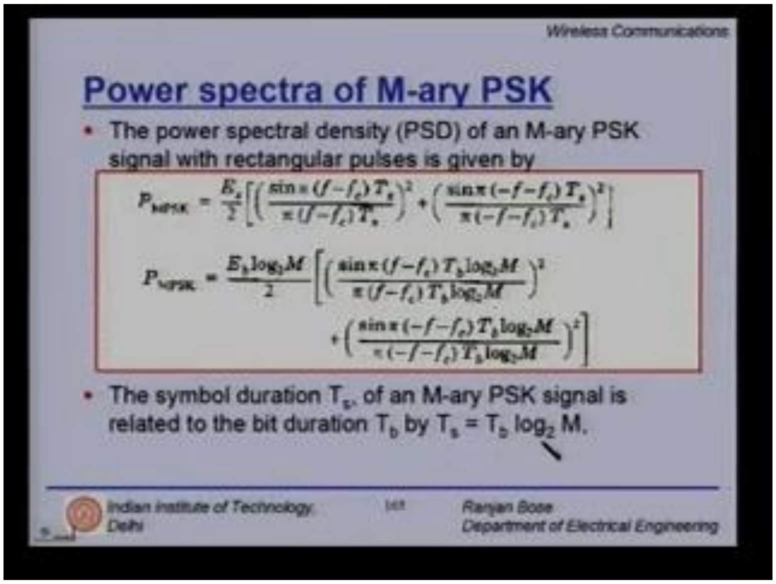
Ranjan Bose
Department of Electrical Engineering

What is the performance of symbol error probability for different values of M. Here let us see on the x axis we have plotted E_b over N_0, on the y axis is the symbol error rate and then lot of curves. These are called the water fall curves because of the way they look and let us look at case 1, M =2. So this is actually M =2BPSK, as you can see up to 10 dB E_b over N_0 it goes to fairly low values, 10 raise to power minus 5 which is the an acceptable performance region. The moment you go from 2 to 4 or 16, you have to have a higher E_b over N_0 to reach the same performance.

So now you are close to about 18 dB E_b over N_0 to reach the same performance level of 10 raise to power minus 5 but clearly since you have M =16, you are sending 4 bits per symbol. If you are more greedy and go in to 64, again for 64 you have to have an E_b over N_0 close to about 27 dB, so as to get the same performance of 10 raise to power minus 5. So there is a tradeoff between the bit rate and the probability of error.

Now let us spend some time looking at the power spectra of M-ary PSK. The power spectral density PSD of an M-ary PSK signal with rectangular pulses is given by this following equation. So you have this sin squared terms here centered around f_c and if you write E_s in terms of E_b log to the base 2 M then you get the following expression. The symbol duration T_s of an M-ary PSK signal is related to the bit duration T_b by T_s is equal to T_b log to the base 2 M.

(Refer Slide Time: 00:42:34 min)



Wireless Communications

Power spectra of M-ary PSK

- The power spectral density (PSD) of an M-ary PSK signal with rectangular pulses is given by

$$P_{MPSK} = \frac{E_s}{2}\left[\left(\frac{\sin \pi (f-f_c)T_s}{\pi (f-f_c)T_s}\right)^2 + \left(\frac{\sin \pi (-f-f_c)T_s}{\pi (-f-f_c)T_s}\right)^2\right]$$

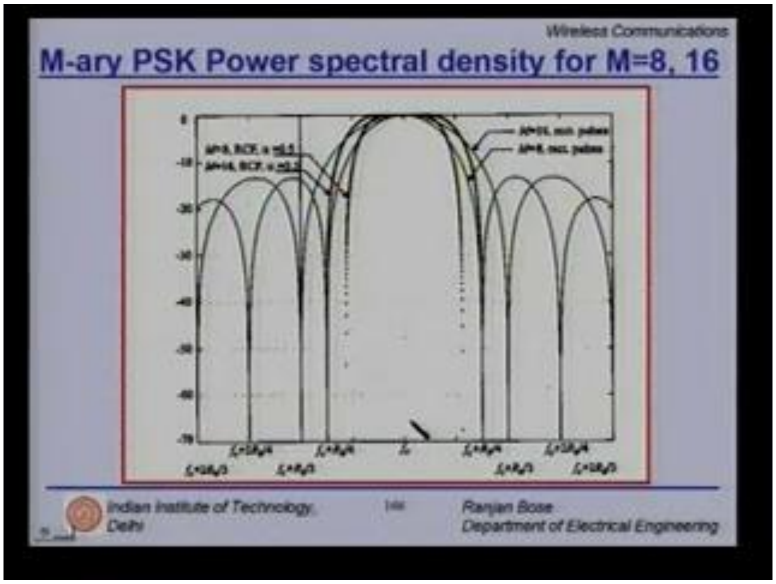
$$P_{MPSK} = \frac{E_b \log_2 M}{2}\left[\left(\frac{\sin \pi (f-f_c)T_b \log_2 M}{\pi (f-f_c)T_b \log_2 M}\right)^2 + \left(\frac{\sin \pi (-f-f_c)T_b \log_2 M}{\pi (-f-f_c)T_b \log_2 M}\right)^2\right]$$

- The symbol duration T_s, of an M-ary PSK signal is related to the bit duration T_b by $T_s = T_b \log_2 M$.

Indian Institute of Technology Delhi | 143 | Ranjan Bose
Department of Electrical Engineering

In this slide we have plotted the power spectral density for two cases M =8 and M =16. Here again with M =8 and raise cosine filter alpha is equal to 0.5 brings it down lower and again M=16 raise cosine filter alpha is equal to 0.5 it brings it down lower. So what can be learnt from this slide is that the power spectral density looks as follows, this is in dB scale therefore the sink functions look much more rounded and the M =16 case is spread out, M is equal to eight’s case is a little narrower in this spectral content, please note they are both centered around f_c.

(Refer Slide Time: 00:43:24 min)

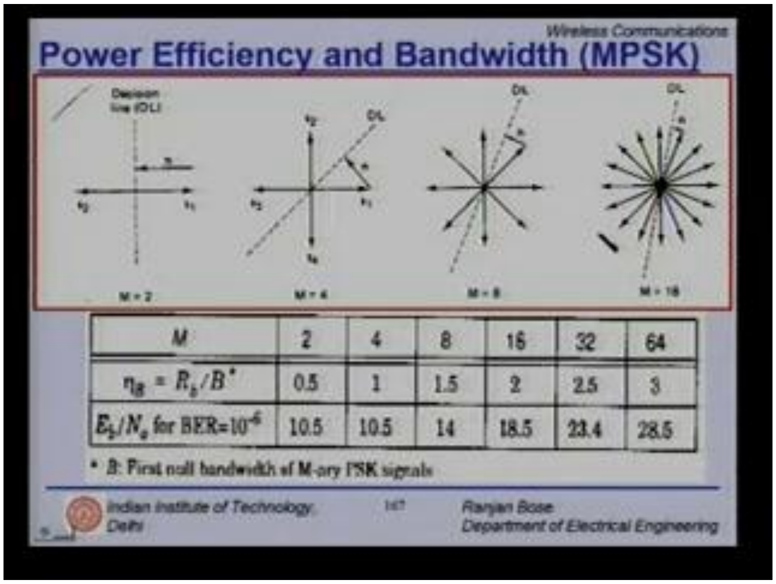


Wireless Communications

M-ary PSK Power spectral density for M=8, 16

Indian Institute of Technology Delhi | 144 | Ranjan Bose
Department of Electrical Engineering

(Refer Slide Time: 00:44:25 min)



Wireless Communications

Power Efficiency and Bandwidth (MPSK)

M	2	4	8	16	32	64
$\eta_B = R_b/B^*$	0.5	1	1.5	2	2.5	3
E_b/N_o for BER=10^{-6}	10.5	10.5	14	18.5	23.4	28.5

* B: First null bandwidth of M-ary PSK signals

Indian Institute of Technology Delhi

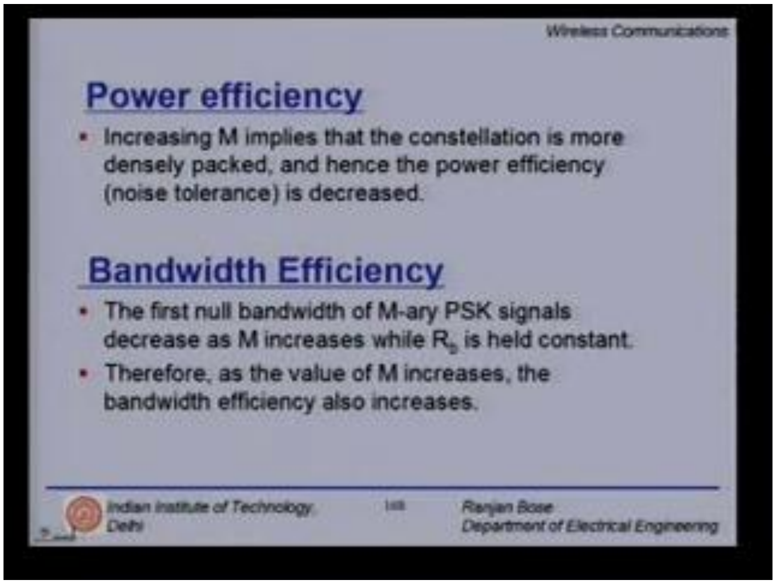
147

Ranjan Bose
Department of Electrical Engineering

Power efficiency and bandwidth efficiency of an MSK signal so let's look at the constellation diagram where you have M =2 scenario, M =4 QPSK, 8 PSK and 16 PSK. If you have not increase the radius of the circle which is under root E_f's then as you are packing in more and more number of symbols on the circumference of the circle they tend to get closer and the probability of error will go up but in this slide we are talking about the bandwidth efficiency. Let us look at this table, here on the first row we have put down the numbers M =2, 4, 8, etc for the MPSK. Second row denotes the eta B which is the efficiency bandwidth given by R_b over B^*. What is B^*? B^* is the bandwidth measured between the first null bandwidth of M-ary PSK signals

So clearly as you increase from M =2 to 4 and then to 8 and so forth gradually your R_b over B^* increases. So bandwidth requirement increases but R_b increases much faster, so therefore you can see general increase. In the previous slide if you see the first nulls are different. So the first null bandwidth is different from for M =8 and M =16 however the R_b also increases leading it to a larger value of eta B. The third row plots for a particular bit error rate, here we are taking 10 raise to power minus 6 the E_b over N_0 requirement. As we saw in the water fall slides earlier for the same bit error rate, we need to increase the E_b over N_0 as we increase your M. So here is the plot of how your requirement for E_b over N_0 increases so clearly there is a tradeoff. If you are working in a low SNR scenario then you have to resort to a smaller value of M but the price you pay is poorer efficiency. If you are lucky enough to have very good SNR's between 25 to 30 dB you can easily choose somewhere between M = 32 and 64.

(Refer Slide Time: 00:47:28 min)



Wireless Communications

Power efficiency

- Increasing M implies that the constellation is more densely packed, and hence the power efficiency (noise tolerance) is decreased.

Bandwidth Efficiency

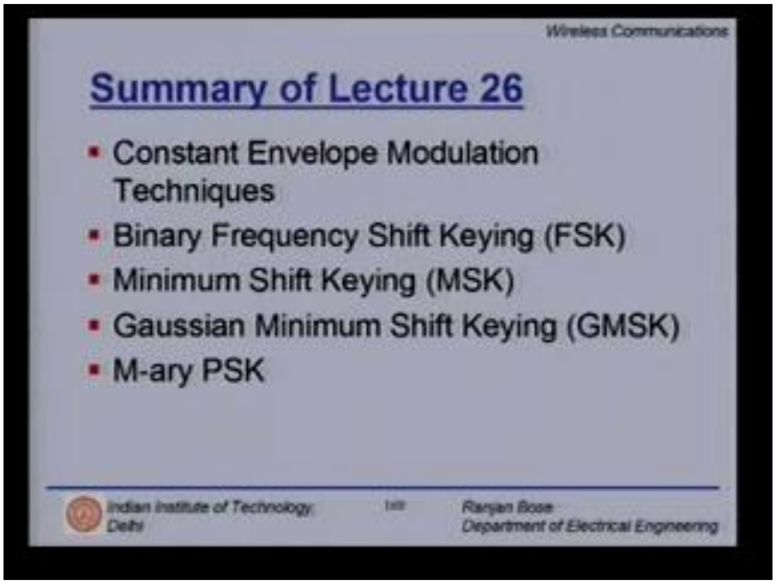
- The first null bandwidth of M-ary PSK signals decrease as M increases while R_b is held constant.
- Therefore, as the value of M increases, the bandwidth efficiency also increases.

Indian Institute of Technology, Delhi

Ranjan Bose
Department of Electrical Engineering

So couple of comments on power efficiency and bandwidth efficiency, increasing M implies that the constellation is more densely packed as seen before and hence the power efficiency or noise tolerance is decreased this is intuitive but the bandwidth efficiency perspective is different. The first null bandwidth of M-ary PSK signals decrease as M increases while R_b is held constant. Therefore as the value of M increases the bandwidth efficiency also increases.

(Refer Slide Time: 00:48:04 min)



Wireless Communications

Summary of Lecture 26

- Constant Envelope Modulation Techniques
- Binary Frequency Shift Keying (FSK)
- Minimum Shift Keying (MSK)
- Gaussian Minimum Shift Keying (GMSK)
- M-ary PSK

Indian Institute of Technology, Delhi

Ranjan Bose
Department of Electrical Engineering

Let us now summarize today’s lecture, we took a look at constant envelop modulation techniques. Specifically we talked about binary FSK or binary frequency shift keying. We then graduated to another technique called minimum shift keying which is spectrally more efficient. A derivative of minimum shift keying called the Gaussian minimum shift keying was studied next. Finally we took a look at M-ary PSK and its power spectrum. We will conclude our lecture here and in the subsequent lectures we will talk about quadrature amplitude modulation and also spread spectrum techniques.

Thank you.