

Wireless Communications

Dr. Ranjan Bose

Department of Electrical Engineering

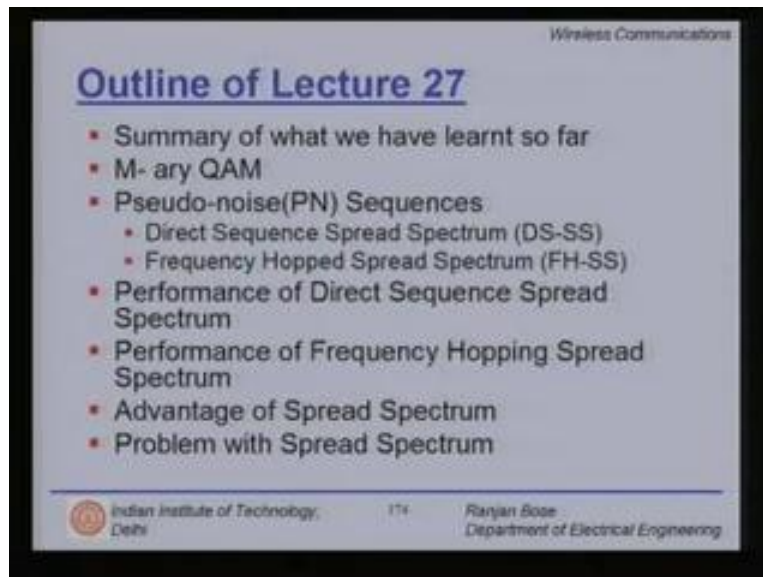
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Lecture No. # 27

Modulation Techniques for Mobile Communications (Continued)

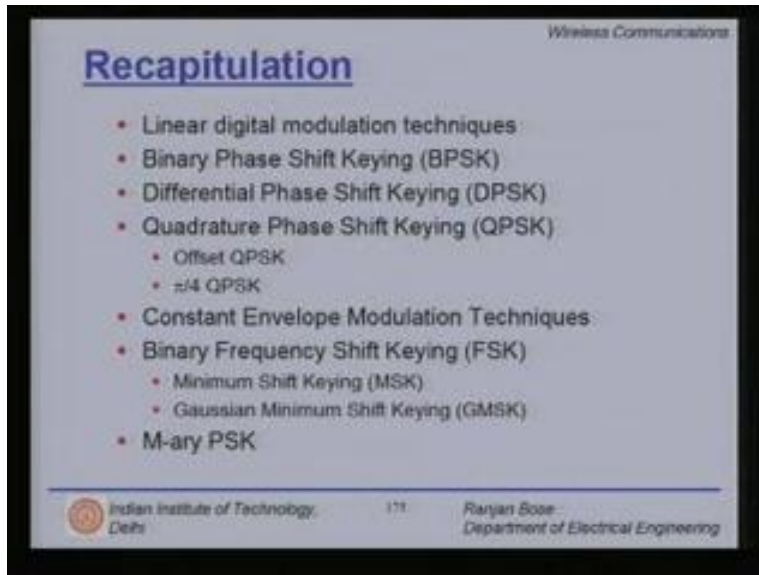
Welcome to the next lecture on modulation techniques for mobile communications. The outline for today's talk is as follows.

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We will start with a brief recap of what we have learnt so far. We will then look at M-ary Quadrature Amplitude Modulation or QAM. We then go into PN sequences and look at some spread spectrum techniques. The first one will be Direct Sequence Spread Spectrum (DS-SS). Then we will look at Frequency Hopped Spread Spectrum (FH-SS). We will then evaluate the performance of direct sequence spread spectrum followed by that of frequency hopped spread spectrum. We will look at the advantages and disadvantages of spread spectrum systems. So this is the outline for today's talk. Let's take a recap. In the past couple of lectures we have looked at linear digital modulation techniques.

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The slide is titled "Recapitulation" and is part of a presentation on "Wireless Communications". It lists several digital modulation techniques:

- Linear digital modulation techniques
 - Binary Phase Shift Keying (BPSK)
 - Differential Phase Shift Keying (DPSK)
 - Quadrature Phase Shift Keying (QPSK)
 - Offset QPSK
 - $\pi/4$ QPSK
- Constant Envelope Modulation Techniques
 - Binary Frequency Shift Keying (FSK)
 - Minimum Shift Keying (MSK)
 - Gaussian Minimum Shift Keying (GMSK)
 - M-ary PSK

The slide footer includes the Indian Institute of Technology Delhi logo, the slide number 171, and the name of the presenter, Ranjan Bose, from the Department of Electrical Engineering.

In specific, we have looked at the binary phase shift keying, the differential phase shift keying, the quadrature phase shift keying or QPSK. Within that we looked at two variations. One was the offset QPSK and then the $\pi/4$ QPSK. These two were intended to ensure that phase changes are limited in the first case of offset QPSK to 90 degrees and in $\pi/4$ QPSK to 135 degrees. We then graduated to constant envelop modulation techniques. Specifically we looked at binary frequency shift keying and two special cases the minimum shift keying and the Gaussian minimum shift keying. We learnt how GMSK is generated and received. It is used for GSM communication systems. Finally we looked at M-ary PSK systems. Now today we will start where we left of last time with the quadrature amplitude modulation signals and then go on to study the spread spectrum techniques.

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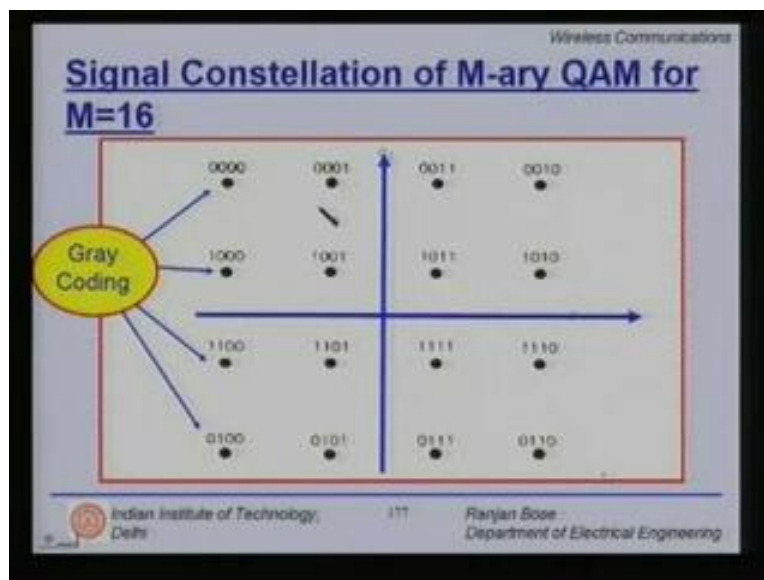
M-ary Quadrature Amplitude Modulation (QAM)

- It's a Hybrid modulation
- Both the amplitude and the phase varies
 - called quadrature amplitude modulation (QAM).
- The constellation diagram of 16-ary QAM consists of a square lattice of signal points.

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What are M-ary quadrature amplitude modulation techniques? First of all it's a hybrid modulation technique. So far we have either dealt with an amplitude or phase variations. Here we have for the first time both amplitude and phase variations in order to encode the data. The constellation diagram of a 16-ary QAM consists of a square lattice of signal points. Let us see how it looks.

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Here in this slide, we have 16 points in the constellation diagram. let's look at the axes. these are two orthogonal axis. let us label them phi one and phi two. this x axis could be cos omega ct. the y axis could be sin omega ct or any two orthonormal functions. each of the points here in the signal constellation represents one of the possible signal in the signal set. Now, what is interesting is each of the signals carry with it a label. there are 16 possible symbols and hence 4 bits per symbol can be assigned now. there are very many ways to assign 4 bits per symbol. Here we have done one particular method of encoding. this is called gray coding. the important feature of gray coding is that for any adjacent symbols in the signal set, the number of bits that change is only one. that is, for example, if I consider this one where the bits mapped are 1 0 1 1, then to right of it we have a mapping 1 0 0 0. please note that the hamming distance for the number of bits that are different here is only one at the last place. if you consider this symbol upper, there again we have flipt only one of the bits and we come to 0 0 1 1 and so and so forth. on the left we have flipt the third bit only. so the nearest change the closest Euclidean distance wise symbol is also the closest having distance wise separation. the logic is as follows. the probability of error increases as you move to signals closer to each other in the signal space. if the probability of error increases and if error does happen, then the number of bits in error should be minimum. so if you consider two extreme points, for example, this one and this one (Refer Slide Time: 06:47), the probability of these two be an error. that is, you send this one and you interpret it as this one as very low though non-zero. then almost all four bits need to be flipped or at least two bits need to be flipped. so the number of bits changes they having distance is much larger for these two. this method is called gray coding wherein you only change one bit at a time where you move from one signal to the nearest next symbol.

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M-ary QAM

The general form of an M-ary QAM signal can be defined as

$$S_i(t) = \sqrt{\frac{2E_{\min}}{T_s}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_{\min}}{T_s}} b_i \sin(2\pi f_c t)$$

$0 \leq t \leq T \quad i = 1, 2, \dots, M$

where

- E_{\min} is the energy of the signal with the lowest amplitude and
- a_i and b_i are a pair of independent integers chosen according to the location of the particular signal point.

- In M-ary QAM energy per symbol and also distance between possible symbol states is not a constant.

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the general form of an M-ary QAM signal can be defined as follows. here as you can see, I have the two axis cos 2 pi fct and sin 2 pi fct. these are the normalizing factors under root 2 E min over Ts. I have ai's and bi's. here I can vary from one through M and these actually tell you

where exactly on the x and the y axis your signal point is located. E_{\min} is the energy of the signal with the lowest amplitude. Please remember from the constellation diagram different signals are further away from the origin differently. The closest ones pertain to the E_{\min} scenario. a_i 's and b_i 's are a pair of independent integers chosen according to the location of the particular signal point. In M-ary QAM, energy per symbol and also distance between possible symbol states is not a constant.

Conversation between student and professor: the question being asked is: should M, here in M-ary QAM be a power of 4? The answer is, if you're putting it in an xy coordinate, yes, it has to be either 4 or 16. But it is also found that you can have thirty-two QAM wherein it is not a perfect square lattice. So if you go back to the lattice, instead of having 16 points, if I put some off centered points, then also it is my QAM. Traditionally will have 16 QAM & 64 QAM but it is also possible to find 32 QAM. So not necessarily that it has to be a power of 4. But power of 2, yes, because we would like to have them in pairs especially. We are mapping the bits. So if its five bits per symbol, then you'll have 32 symbols. I cannot have 17 QAM. I cannot map my bits.

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M-ary QAM Wireless Communications

- The signal $S_i(t)$ may be expanded in terms of a pair of basis functions defined as

$$\phi_1(t) = \sqrt{\frac{2}{E_{\min}}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_s$$

$$\phi_2(t) = \sqrt{\frac{2}{E_{\min}}} \sin(2\pi f_c t) \quad 0 \leq t \leq T_s$$
- The coordinates of the i th message point are $a_i \sqrt{E_{\min}}$ and $b_i \sqrt{E_{\min}}$ where (a_i, b_i) is an element of the L by L matrix given by

$$(a_i, b_i) = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \dots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \dots & (L-1, L-3) \\ \vdots & \vdots & \ddots & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \dots & (L-1, -L+1) \end{bmatrix}$$
 where $L = \sqrt{M}$.

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The signals $S_i(t)$ can be expanded in terms of a pair of basis functions. These basis functions ϕ_1 and ϕ_2 are given as follows. $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ normalized. The coordinates of the i th message point are $a_i \sqrt{E_{\min}}$ and $b_i \sqrt{E_{\min}}$. Here as you can see I can label a_i 's and b_i 's in the form of a matrix. It is simply the definition of the location in the constellation phase. I have two axes. So I need an x location and a y location.

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M-ary QAM

- For the example M=16-QAM the L by L matrix is

$$\{a_p, b_p\} = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix}$$

- The average probability of error in an AWGN channel is given by

$$P_e \cong 4 \left(1 - \frac{1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{2E_{min}}{N_0}} \right)$$

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For example for M = 16 QAM, the L by L matrix looks simply like this (Refer Slide Time: 10:51). It tells me the leftmost point is located at (-3,3). Then (-1,3) and so and so forth, up to (3,-3). Now let us talk about the average probability of error in an additive wide Gaussian noise channel. Clearly there are some points in the constellation diagram which are closer to each other than other parts. So those are the most likely points to be in error. The average probability of error is approximately $4(1 - 1/\sqrt{M}) Q(\sqrt{2E_{min}/N_0})$. So here, approximately the probability of error is given as a function of M for a M-ary QAM.

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M-ary QAM

- In terms of average signal energy, E_{avg}

$$P_e \cong 4 \left(1 - \frac{1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{3E_{avg}}{(M-1)N_0}} \right)$$

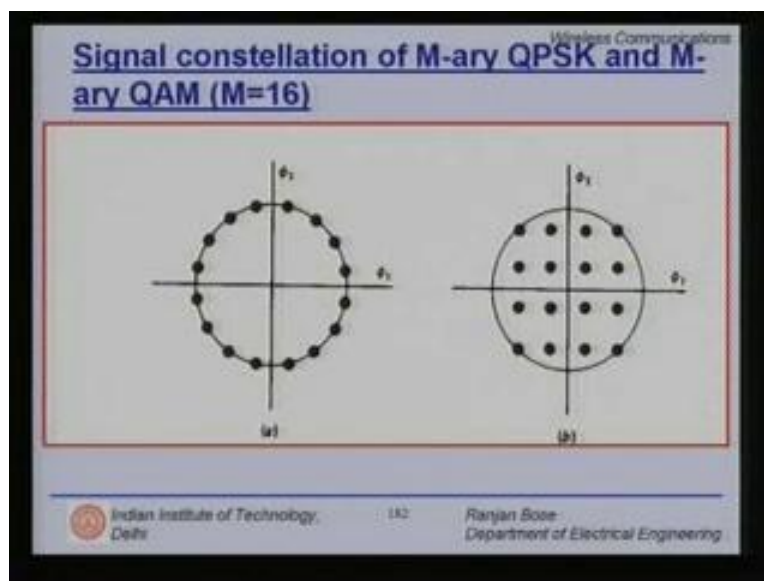
Power Efficiency and Bandwidth :

- Power efficiency of QAM is superior to M-ary PSK.
- Bandwidth efficiency of QAM is identical to M-ary PSK.

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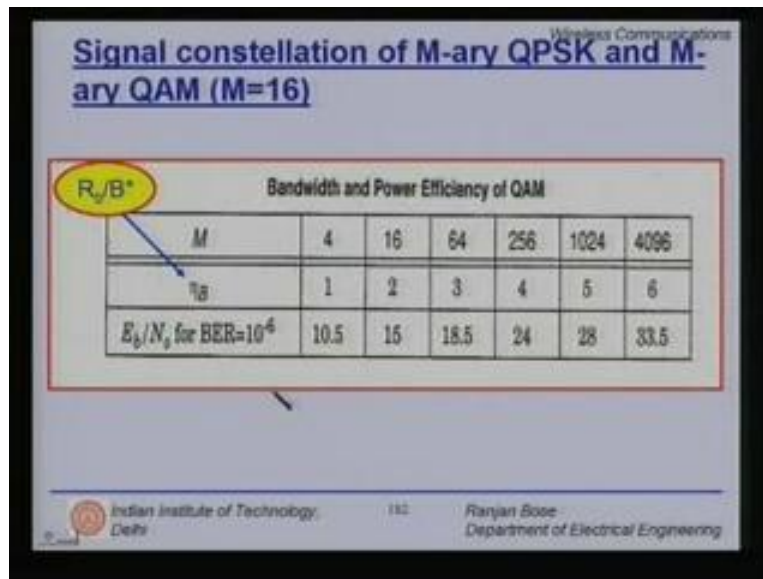
If you want to define it in terms of average signal energy, the probability of error expression comes out as follows. Now let us talk about power efficiency and bandwidth. As always, whenever you have to pick and choose a particular modulation scheme you have to talk about the power efficiency, the bandwidth efficiency and how simple or complex is it to implement it in transmitter and receiver. So these are the three factors which must be considered for any modulation schemes. The power efficiency of a QAM is superior to M-ary PSK. The bandwidth efficiency of QAM is identical to M-ary PSK. In fact, the moment you start going for higher modulation schemes, then you kind of switch to QAM. It is not wise to go beyond 16 PSK and you rarely hear about 32 PSK because you have to be in the circle whereas 32 QAM, 64 QAM, 128 QAMs are well known.

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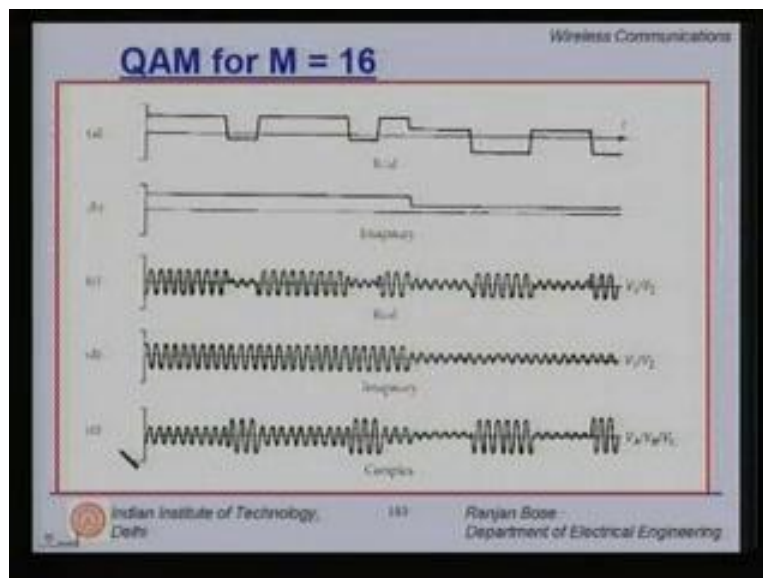
So just a graphical feel for the signal constellations for M-ary QPSK. On the left hand side, it should be M-ary PSK and on the right hand side M-ary QAM. Please note here, the more points are try to fit in, I constrain myself. Here, clearly all of them have equal amplitude. But here, as you can see, the 16 points are located here and they are less densely packed. Hence the probability of error will be better in this case.

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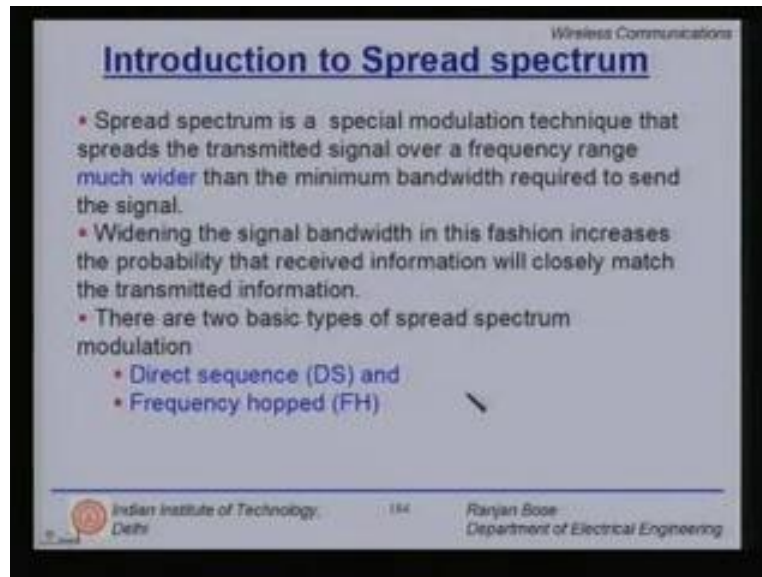
Now let's just talk about the bandwidth and power efficiency for quadrature amplitude modulation. The first row represents M for the M-ary QAM. What is interesting is that you start with 4, 16, 64 and go right up to 4096. Such numbers are unheard of for FSKs and ESKs. Now what is eta B? It is Rb over B star. B star is the first null bandwidth. The Eb over the N0 required for bit error rate= 10⁻⁶ is also listed. So you can compare and find out and predict the bandwidth efficiency for a certain performance.

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It is a simple example which tells us that how the waveform of a QAM for $M = 16$ will look like. so if I have the two axis the x and the y axis and the data is coming as a combination of these two, then you have the real and imaginary) waveforms looking like this (Refer Slide Time: 15:16). When you add it up, the complex waveform looks like this. So the point to be noted is that there are clearly four amplitude levels and the phases can also vary. Hence the name quadrature amplitude modulation.

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Now we move on to another method called the spread spectrum technique. What is spread spectrum? It is a special modulation technique that spreads the transmitted signal over a frequency range much wider than the minimum bandwidth required to send the signal. For the first time we are trying to be greedy in terms of bandwidth. But we know from channels theorem that if you have to invest, it is better to invest in bandwidth than increasing your signal to noise ratio because the capacity of a system grows linearly with bandwidth and logarithmically with SNR. So it intuitively follows from the channels theorem that it is better to have a larger bandwidth and send noise like signals. Here we wish to exploit that fact. The frequency range which will actually be used for transmission will be much more than the signal bandwidth. What else does it do? Widening the signal bandwidth in this fashion increases the probability that the received information will closely match the transmitted information. There are two basic types of spread spectrum modulation techniques. The first one and probably the more popular one is the direct sequence technique. The other one is frequency hopped technique which was fairly popular in the military domain and now has come out in the civilian application as well. But the more common is the direct sequence DSSS.

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Pseudo-noise(PN) Sequences

- A binary sequence with an autocorrelation that resembles, over a Period, the autocorrelation of a random binary sequence.
- Usually generated using sequential logic circuit.

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now to do spread spectrum techniques, you have to have something called as pseudo noise sequences or PN sequences. a binary sequence with an autocorrelation that resembles over a period the autocorrelation of a random binary sequence is called the pseudo noise sequence. it is usually generated using a sequential logic circuit. one example could be as follows. so we have various stages. this could be flip flops. then there is a feedback logic simple EX-ORs and then you output a PN sequence how you tap this points depends on what kind of sequence you get.

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Direct Sequence Spread Spectrum (DS-SS)

The received spread spectrum signal for a single user can be represent as

$$S_{SS}(t) = \sqrt{\frac{2E_s}{T_s}} m(t) p(t) \cdot \cos(2\pi f_c \cdot t + \theta)$$

$m(t)$ is the data sequence
 $p(t)$ is the PN spreading sequence
 f_c is the carrier frequency
 θ is the carrier phase angle at $t = 0$

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Now let us talk about direct sequence spread spectrum also known as DS-SS. the received spread spectrum signal for a single user can be represented as follows. $S_{ss}(t)$ - ss stands from direct sequence spread spectrum under root 2 E_s over T_s . please note a product $m(t) p(t) \cos 2\pi f_c t + \theta$. In the subsequent slides, we will talk about what do $m(t)$ and $p(t)$ represent. This is of course a normalizing factor. $m(t)$ here is the data sequence. $p(t)$ is the PN pseudo noise spreading sequence. f_c here is the carrier frequency and θ is a carrier phase angle at time $t=0$. This is a general depiction of the received spread spectrum signal for a single user. We will soon see that the spread spectrum technique is very popular for multi user scenario where different users will have different PN sequences as the spreading codes.

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Direct Sequence Spread Spectrum

$m(t)$

- The data waveform is a time sequence of non-overlapping rectangular pulses
- An amplitude equal to +1 or -1.
- Time duration is T_s

$$S_{ss}(t) = \frac{\sqrt{2E_s}}{T_s} m(t) p(t) \cos(2\pi f_c t + \theta)$$

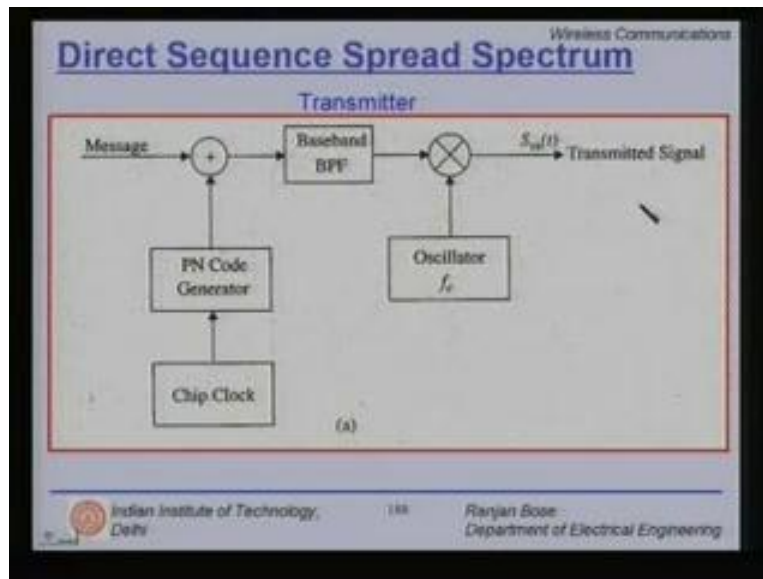
$p(t)$

- Represents a chip
- The data waveform is a time sequence of nonoverlapping rectangular pulses
- An amplitude equal to +1 or -1.
- Time duration is T_c

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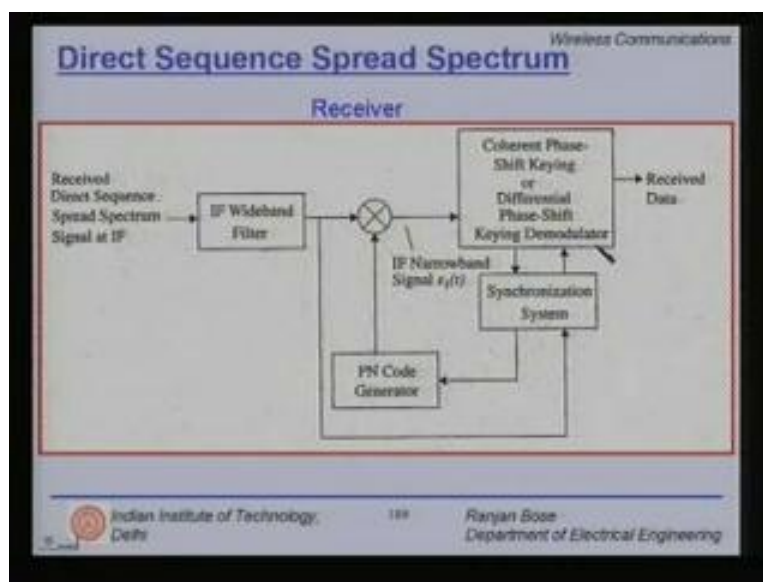
So let's start with the received waveform $S(t)$ as $m(t)$ times $p(t)$ times cosine $2\pi f_c t + \theta$ normalized. Let's talk about $m(t)$ in a little bit more details $m(t)$ is a data waveform is actually a time sequence of non-overlapping rectangular pulses. Now please note that the amplitude is +1 when I want to transmit bit 1 and -1 when I want to transmit bit zero. so it's a bipolar depiction. The time duration is T_s . Let's now talk about this $p(t)$. $p(t)$ is interesting. it is called a chip. chip is much narrower in time than the bit. this chip is actually causing the bandwidth to extend. The data waveform is a time sequence of non-overlapping rectangular pulses and amplitude of +1 and -1 is used for $p(t)$ as well. But the time duration T_c for $p(t)$ is much smaller than T_s for $m(t)$. So you will have several chips in one bit interval.

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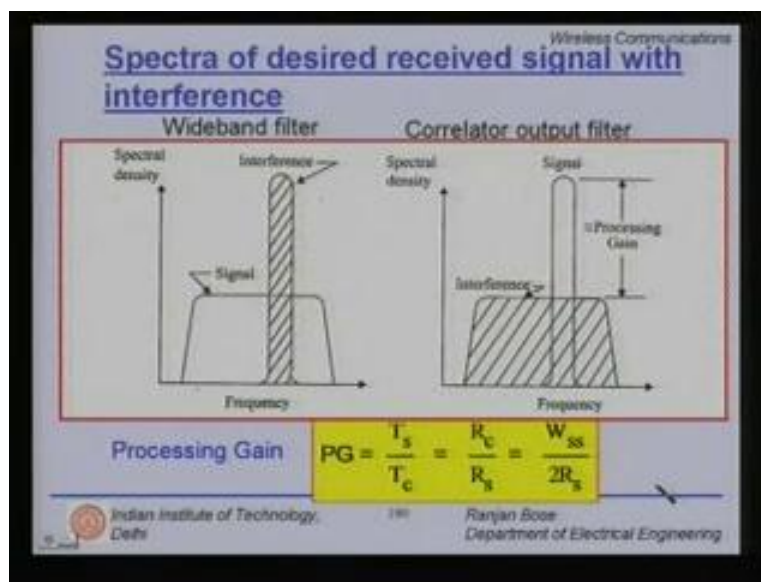
Now let us look at the block diagram of the transmitter for the case of direct sequence spread spectrum. Suppose I get a message $m(t)$. Clearly it is bipolar switching between + 1 and - 1 and then you have a pseudo random noise code generator (PN code generator) and a chip clock which makes it run. You combine them together; pass it through a baseband oscillator. This should be a product. So messages multiplied with a PN code generator. Pass it through a band pass filter, put it through an oscillator and send out your transmitted signal. This is the block diagram.

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Now let's move over to the receiver end. The received direct sequence spread spectrum signal at IF is first faded into the IF wideband filter. Please note that the signal received is a wideband signal even at the IF. Here again, we have a synchronization system which gives feedback to the PN code generator. now you should have the same code that was used to spread the message signal to despread it. if you use the same code, you despread your signal. if you use a wrong code, then you spread it further. you multiply it the moment you despread it, the bandwidth reduces and now it is an IF narrow band signal. now you can do detection and obtain the received data. here we can have either the coherent phase shift keying or differential phase shift keying demodulator.

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now let us see how does a spectrum of the desired received signal look like with respect to the interference. suppose I have this (Refer Slide Time: 23:46) as my signal, x axis denotes the frequency and the y axis denotes the spectral density. suppose I have an interference, clearly I am depicting a narrow band interference. it could be a jammer. now at the receiver when you have the correlator, when you despread the signal, the signal despreads, its bandwidth reduces but at the same time, the interference gets spread. since the power is constant it is at a much lower level at this. so this shows that a strong interference to weak signal once despread gives you a strong signal and a weak resultant interference. in fact this is the processing gain. It's very simple technique but very effective. what it also tells me is that I can increase my processing gain more and more and thereby we will be able to handle stronger and stronger interferers or jammers. so inherently this technique is secured against jamming to some extent. what is processing gain? processing gain is defined by T_s over T_c . remember T_s is kind of the bit duration. T_c is the chip duration which is much smaller than the bit duration and it is written as R_c over R_s . It is approximately equal to W bandwidth R_{ss} over $2 R_s$.

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Frequency Hopped Spread Spectrum (FH-SS)

Definitions

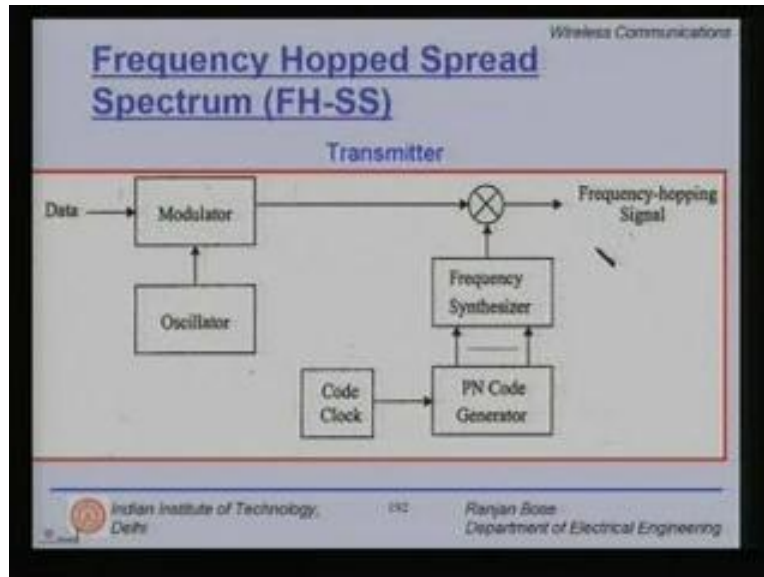
- The set of possible carrier frequencies is called the *hopset*
- Hopping occurs over a frequency band that includes a number of channels.
- The bandwidth of a channel used in the *hopset* is called the *instantaneous bandwidth*, W_{ss}
- The bandwidth of the spectrum over which the hopping occurring is called the *total hopping bandwidth*, B
- If only a single carrier frequency (single channel) is used on each hop, digital data modulation is called *single channel modulation*.
- Time duration between hop is called the hop duration or hopping period, T_h
- Processing Gain = W_{ss}/B

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Ranjan Bose
Department of Electrical Engineering

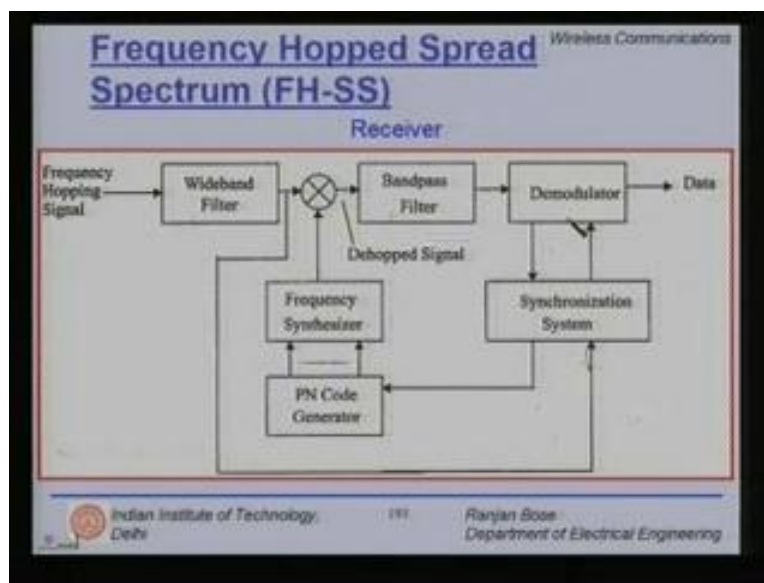
now let us look at the other kind of spread spectrum technique called frequency hopped spread spectrum technique. the set of possible carrier frequencies is called a hopset. so the whole philosophy here is as follows. you have a large spectrum. you subdivide it into sub bands and your hop on these sub bands that is you picked one of the sub bands randomly or as per a pseudo noise random number generator and transmit it and then pick another subband and then transmit it on that subband and so and so forth. so basically you will jump or hop along the frequency spectrum across the different bands. the hopping occurs over a frequency band that includes a number of channels. what are these channels? these are frequency subband channels. the bandwidth of a channel used in the hopset is called the instantaneous bandwidth ' W_{ss} '. the bandwidth of the spectrum over which the hopping occurs is called the total hopping bandwidth ' B '. so clearly there is a big large chunk of bandwidth and then there are sub bands which are narrower and we are jumping and picking and choosing the different sub bands. if only a single carrier frequency that is, signal channel is used on each hop, the digital data modulation is called a single channel modulation. the time duration between hop is called the hop duration or hopping period ' T_h '. so these are some of the definitions related to FH-SS. the processing gain for FH-SS is W_{ss} over B .

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let us now look at the transmitter for a frequency hopped spread spectrum system. so you have a data coming in, it goes to a modulator and the oscillator input gives you this data. now the question is on which of the sub band? which channel must it be sent? so you have a code clock which generates a PN code sequence. then according to the PN code sequence, you generate the frequency, up convert and send it. as I jump along my PN code, I jump along my frequency band. the PN code tells me which sub band to choose for onward transmission.

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let us look at the receiver for the frequency hopped spread spectrum technique. you have the frequency hopped signal coming. I have to pass it through a wideband filter. so I need something which is broad wideband at the RF. Now, again I must have a synchronization system and I must use the PN code in order to synthesize the same frequency. I multiply, pass it through a band pass filter and demodulate it. if I use a wrong frequency I will get junk data. so it is important that the hopping sequence be the same. hence we are getting a processing gain there by.

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Frequency Hopped Spread Spectrum (FH-SS)

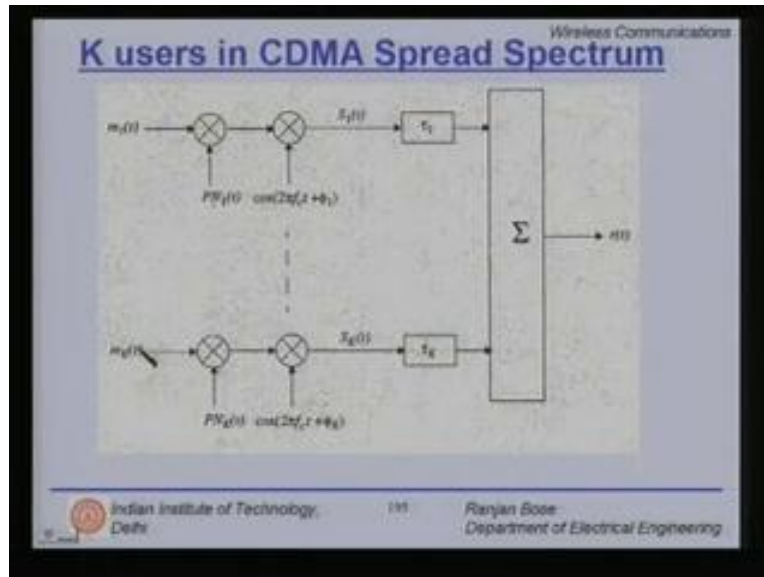
Two types of Frequency hopping:

1. **Fast frequency hopping** occurs if there is more than one frequency hop during each transmitted symbol.
 - Thus frequency hopping implies that the hopping rate equals or exceeds the information symbol rate.
2. **Slow frequency hopping** occurs if one or more symbols are transmitted in the time interval between frequency hops.

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Ranjay Bose
Department of Electrical Engineering

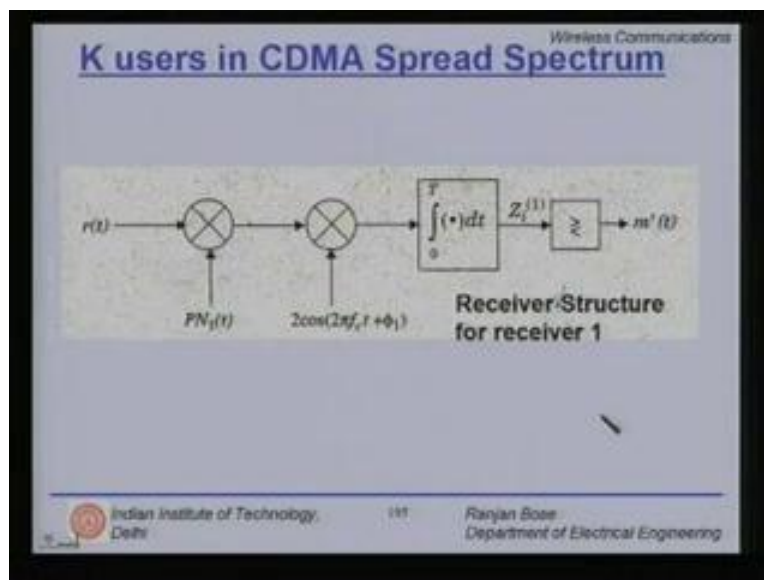
now what are the different types of frequency hopped spread spectrum systems? they are basically two classes. one is the first frequency hopping technique and the other one is the slow hopping technique. fast frequency hopping occurs if there is more than one frequency hop during each transmitted symbol. that is, while I am transmitting my symbol, within the symbol duration I have already hopped several times. it's called fast hopping sequence. That is, the frequency hopping implies that the hopping rate equals or exceeds the information symbol rate. on the other hand, in the case of slow frequency hopping, I send several symbols over the same frequency and then hop at a later time. clearly synchronization is an issue in the fast hopping case.

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Now, spread spectrum techniques have become very popular for the code division multiple access technique where multiple users can be supported by giving them different orthogonal or pseudo orthogonal codes. let us consider the case when we have K users in the system and how they can use the CDMA. so here, I have message 1 emanating from user 1. user 1 has his or her own PN code multiplied with the cosine $2\pi f_c t + \phi_1$. you get your signal S 1. similarly for m 2 and so and so forth, till the K^{th} user m_k . take the signal. these are the bits coming in multiplied with the PN sequence multiplied with the $\cos 2\pi f_c t + \phi_K$. you have the $S_k t$. these are the time delays summation and this is the received signal. so at the receiver, I have a summation of all these signals coming in. this is a multiuser scenario. what you want to do is extract say, user one's data and reject m 2, m 3, up to $m_k t$. so we have to treat the other ones as interference.

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how does the receiver look like? so let us talk about just receiver 1 which is a summation of all the various kinds of messages being spread differently by different PN codes added together with a delay that is coming in. you take that and you multiply with user 1's PN sequence. what will happen is only user 1's sequence will despread. the rest of the other ones will spread even further. so you go on. this is the despread signal, multiply, pass it through the band pass filter or integrate and dump and take a decision to get the received signal. so it's a fairly simple receiver structure is a selling feature of CDMA systems.

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Performance of Direct Sequence Spread Spectrum

Assume: K multiple users
 Each user has a PN sequence with N chips per message symbol period T such that $NT_c = T$
 Thus transmitted signal of the k^{th} user can be expressed as

$$S_k(t) = \sqrt{\frac{2E_s}{T_s}} m_k(t) p_k(t) \cos(2\pi f_c t + \phi_k)$$

where $p_k(t)$ is the PN code sequence of the k^{th} users
 $m_k(t)$ is the data sequence of the k^{th} user

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let us now spend a few slides on the performance of direct sequence spread spectrum technique. what is our assumption? K multiple users. so each user has a PN sequence with N chips per message symbol period T such that NT_c is T. thus the transmitted signal of the kth user can easily be expressed as follows. we have seen this expression before. it is the kth message and the kth sequence P_{kt} and other things remain the same. They are $\cos 2\pi f_c t + \phi_k$.

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Performance of Direct Sequence Spread Spectrum

The decision variable for the i^{th} transmitted bit of user 1 is

$$Z_i^{(1)} = \int_{(i-1)T + \tau_1}^{iT + \tau_1} r(t) p_1(t - \tau_1) \cos(2\pi f_c(t - \tau_1) + \phi_1) dt$$

Can be rewritten $Z_i^{(1)} = I_1 + \sum_{k=2}^K I_k + \xi$

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Now, the decision variable for the i^{th} transmitted bit of user 1 is given as follows. this is the summation of the different users. but this received signal is being multiplied by the PN sequence of user one with the proper time delay. this is obtained by proper synchronization. if you don't synchronize you cannot recover your data. so it is very sensitive to timing synchronization. \cos two pi $f_c t$ again synchronized ϕ_1 one dt integrate. so this is the decision variable for the i^{th} user. this can be written as $Z_i^{(1)}$ for the first bit user $1 = I_1 + (\text{summation } k = 2 \text{ to } K) I_k + \text{eta}$. what are these terms? in the next slide let us understand what are these terms. clearly and intuitively, I_1 has something to do with the users own despread signal. here from two to k represents the interference due to the other users and this is the only present additive wide Gaussian Noise.

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Performance of Direct Sequence Spread Spectrum

$$Z_i^{(1)} = I_1 + \sum_{k=2}^K I_k + \xi$$

$$I_1 = \int_0^T S_1(t) p_1(t) \cos(2\pi f_c t) dt = \sqrt{\frac{E_b T}{2}}$$

is the response of the receiver to the desired signal from user 1

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so we start with Z_i equal to $I_1 + (\text{summation } k = 2 \text{ to } K) i_k + \eta$. I_1 is integral 0 to T, $S_1(t) P_1(t) \cos(2\pi f_c t)$. η is its own sequence $\cos 2\pi f_c t$. this is the response of the receiver to the desired signal from user 1.

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Performance of Direct Sequence Spread Spectrum

$$Z_i^{(1)} = I_1 + \sum_{k=2}^K I_k + \xi$$

$$I_1 = \int_0^T S_1(t) p_1(t) \cos(2\pi f_c t) dt = \sqrt{\frac{K_1 T}{2}}$$

$$I_k = \int_0^T S_k(t - \tau_k) p_1(t) \cos(2\pi f_c t) dt$$

represent the multiple access interference from user k.

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what is the other term? here I_k represents the multiple axis interference from user K.

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Performance of Direct Sequence Spread Spectrum

$$Z_i^{(1)} = I_1 + \sum_{k=2}^K I_k + \xi$$

$$I_1 = \int_0^T S_1(t) p_1(t) \cos(2\pi f_c t) dt = \sqrt{\frac{K_1 T}{2}}$$

$$I_k = \int_0^T S_k(t - \tau_k) p_1(t) \cos(2\pi f_c t) dt$$

$$\xi = \int_0^T n(t) p_1(t) \cos(2\pi f_c t) dt$$

is a Gaussian random variable representing noise with mean zero and variance $E[\xi^2] = \frac{N_0 T}{4}$

$n(t)$ is additive Gaussian noise

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this term, μ_p is the Gaussian random variable representing noise with mean 0 and variance given by $N_0 T$ over 4. here $n(t)$ is the additive Gaussian noise.

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Performance of Direct Sequence Spread Spectrum

The average probability of bit error given by

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-1}{3N} + \frac{N_0}{2E_b}}}\right)$$

For:

- A single user, $K = 1$, and
- Interference limited case where thermal noise is not a factor, E_b/N_0 tends to infinity,
- The BER expression has a value equal to

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

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so if you calculate the average probability of bit error, it is given by P_e equal to Q function 1 over (root $K - 1$ over $3N + N_0$ over $2E_b$). so please note that the K comes in the denominator with an under root sign. for the case when there is only one user, it's not a multi user scenario. suppose you just want to communicate from point to point in the presence of a jammer, for a single user K is equal to one and in the interference limited case when thermal noise is not a factor E_b over N_0 tends to infinity. under these constraints the bit error rate expression has the value P is equal to the Q function under root $3N$ over $K - 1$. this is coming from this term.

Conversation between Student and Professor: the question being asked is: does it depend on the zeta factor? the answer is: yes. it will depend on the zeta factor here. the zeta depends on the additive wide Gaussian noise and N_0 is coming into the picture here. so this is N_0 over $2E_b$. it is one over SNR. so this signal to Noise ratio is coming into the picture here (Refer Slide Time: 38:15). So please note that for large E_b to N_0 , the N_0 over E_b tends to 0 and you can directly get probability of error as Q under root $3N$ over $K - 1$. Please also note that if you put K is equal to one here, the first term vanishes and you will have P_e is equal to Q under root two E_b over N_0 . the standard modulation technique binary FSK.

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Performance of Frequency Hopping Spread Spectrum

If two users are **not** simultaneously utilizing the same frequency band, the probability of error for BFSK can be given by

$$P_e = \frac{1}{2} e^{\left(\frac{-E_b}{2N_0}\right)}$$

when "hit" occurs. The overall probability of bit error can be

$$P_e = \frac{1}{2} e^{\left(\frac{-E_b}{2N_0}\right)} (1 - P_h) + \frac{1}{2} P_h$$

P_h is probability of the hit, which must be determined.

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if two users are not simultaneously utilizing the same frequency band, the probability of error for binary FSK can be given by $P_e = \frac{1}{2} e^{-\frac{E_b}{2N_0}}$. when the hit occurs, the overall probability of bit error can be given as $P_e = \text{summation of 2 terms multiplied by the probability. one is } \frac{1}{2} P_h$ and here $1 - P_h$ is the probability that the hit doesn't occur. what is P_h ? it is the probability of a hit. this has to be determined. it depends upon how many users are there in the system and various other factors.

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Performance of Frequency Hopping Spread Spectrum

- If there are M possible hopping channels (called slots), there is $1/M$ probability that a given interferer will be present in the desired user's slot.
- If there are $K-1$ interfering users, the probability that at least one is present in the desired frequency slot is

$$P_h = 1 - \left(1 - \frac{1}{M}\right)^{K-1}$$

reduced to $\frac{K-1}{M}$ when M is large

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now let us move to the performance evaluation of frequency hopping spread spectrum technique. if there are M possible hopping channels called slots, there is 1/M probability that a given interferer will be present in the desired users slot. if there are K - 1 interfering users, the probability that at least one is present in the desired frequency slot is given by P_h is equal to 1 - (1 - 1/M)^{K-1}. for large values of M, we can do a simple expansion. you get 1 - K - 1 and neglect the larger terms and you get it as K - 1 over M. this is P_h.

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Performance of Frequency Hopping Spread Spectrum

Substitute p_h into

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0}\right)} (1 - p_h) + \frac{1}{2} p_h$$

gives

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0}\right)} \left(1 - \frac{K-1}{M}\right) + \frac{1}{2} \left(\frac{K-1}{M}\right)$$

If $K = 1$, the probability of error reduces to

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0}\right)}$$

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For the performance of frequency hopping spread spectrum, if you substitute P_h into this equation, it gives probability of error as the following expression. some basic mathematics if K is equal to one, that is, we have only one user, the probability of error reduces simply to the BFSK.

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Performance of Frequency Hopping Spread Spectrum

If E_b/N_0 approaches infinity,

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} P_e = \frac{1}{2} \left(\frac{K-1}{M} \right)$$

- The previous analysis assumes that all users hop their carrier frequencies synchronously.
- This is called slotted frequency hopping.

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if on the other hand E_b over N_0 approaches infinity, the probability of error is given by $1/2 K - 1$ over M . the previous analysis assumes that all users hop their carrier frequencies synchronously. this is called slotted frequency hopping all of them are changing between one sub band to the other sub band at the same time. during the course of changing, they may land up using the same slot. that's when they interfere. that is the probability of a hit.

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Performance of Frequency Hopping Spread Spectrum

The probability of a hit in the asynchronous case is

$$P_h = 1 - \left[1 - \frac{1}{M} \left(1 + \frac{1}{N_b} \right) \right]^{K-1}$$

when N_b is the number of bits per hop

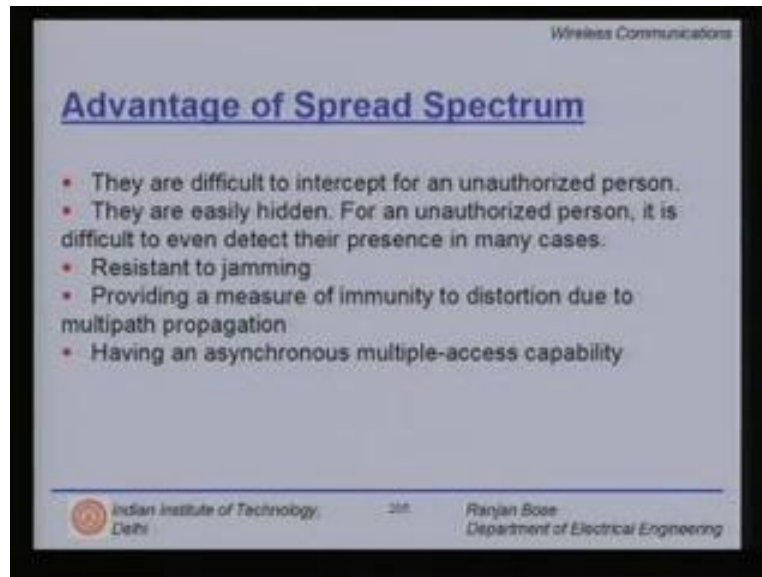
The probability of error for the asynchronous FH-SS case is

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0} \right)} \left[1 - \left[1 - \frac{1}{M} \left(1 + \frac{1}{N_b} \right) \right]^{K-1} \right] + \frac{1}{2} \left[1 - \left[1 - \frac{1}{M} \left(1 + \frac{1}{N_b} \right) \right]^{K-1} \right]$$

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However, for the asynchronous case, the probability of a hit can be shown to be this quantity. here N_b is a number of bits per hop. the probability of the error for the asynchronous frequency hopping spread spectrum case is given by the following expression. so they are have pretty logical to follow from one step to the other. we have an expression for the synchronous case and here is an expression for the asynchronous case. in this case, two users can hop independently also in between the transmission.

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let us now talk very briefly about the advantages and disadvantages of spread spectrum techniques. Let's look at the advantages first. First of all, they are inherently very difficult to intercept for unauthorized persons. we use a large spectral bandwidth but the power emitted is very low. in fact in CDMA systems, we only transmit as much power as required and no more. very good power control is inherent and is compulsory for successful operation of CDMA systems. if we don't, then there is a fear that you'll drown out other users. you will cause so much interference that other users cannot be heard. but in general, they are very difficult to intersect. the other advantage is that they are easily hidden for an unauthorized person, it is difficult even to detect .the presence in many cases. they almost operate at the noise floor. therefore that's the advantage with military systems. It's resistant to jamming. It despreads its own signal but spreads further the jamming signal. in frequency hop case, if you are jamming in a certain band you hop to the next band and you don't have any more problem with jamming. it provides a measure of immunity to distortion due to multipath propagation. so it is inherently wideband. It's usually more than the coherence bandwidth of the channel. so by definition it is resistance to multipath fading to some extent. it also has an asynchronous multiple access capability. It's the multiple access property of the spread system makes it so popular today.

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Problems with Direct Sequence Spread Spectrum

Near-Far problem
Without careful power control of each mobile user, one close-in user may dominate the received signal energy at a base station, making the Gaussian assumption inaccurate.

Stronger signals may bleed into the weaker signals due to imperfect filtering.

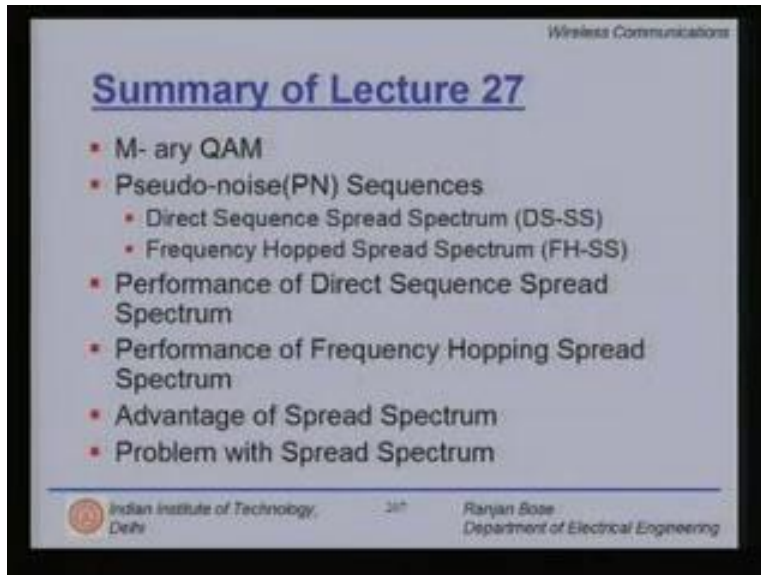
Must use strong Reed-Solomon codes to overcome this problem.

Indian Institute of Technology, Delhi

Ranyan Bose
Department of Electrical Engineering

of course, no free lunch. there are some problems that come with direct sequences spread spectrum. the most important problem is the near-far problem. the problem is that all of the users are separated using their own codes. So, consider for example, the CDMA scenario where user 1 is closer to the base station and user 2 is farther away from the base station. in that case, the signal coming from the user 1 might drown the signal coming from the farther located user 2 unless user 1 closer to the base station lowers its radiated power. this is called the near far problem. in case of the mobile scenario without careful power control, one user which is close in may dominate the received signal energy at a base station making the Gaussian assumption inaccurate. stronger signals may bleed into the weaker signals due to imperfect filtering. if that happens, we are forced to use strong error correcting codes like Reed Solomon to overcome this problem. so this is one of the biggest impediments.

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Summary of Lecture 27

- M-ary QAM
- Pseudo-noise(PN) Sequences
 - Direct Sequence Spread Spectrum (DS-SS)
 - Frequency Hopped Spread Spectrum (FH-SS)
- Performance of Direct Sequence Spread Spectrum
- Performance of Frequency Hopping Spread Spectrum
- Advantage of Spread Spectrum
- Problem with Spread Spectrum

Indian Institute of Technology, Delhi 207 Ranyan Bose
Department of Electrical Engineering

so let us now summarize today's lecture. we started off with M-ary QAM. then we graduated to the PN sequences and the direct sequence spread spectrum technique as well as the frequency hopped spread spectrum technique. we looked at both these techniques in greater detail. we then analyzed the performance of direct sequence spread spectrum followed by the performance of frequency hopping spread spectrum. in the last couple of slides, we looked at the advantages and the disadvantages of spread spectrum techniques. we will conclude the lecture here today. Thank you.