Wireless Communications
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Lecture No. # 28
Modulation Techniques for Mobile Communications (Continued)

Welcome to the next lecture on modulation techniques for mobile communications. The outline for today's talk is as follows.

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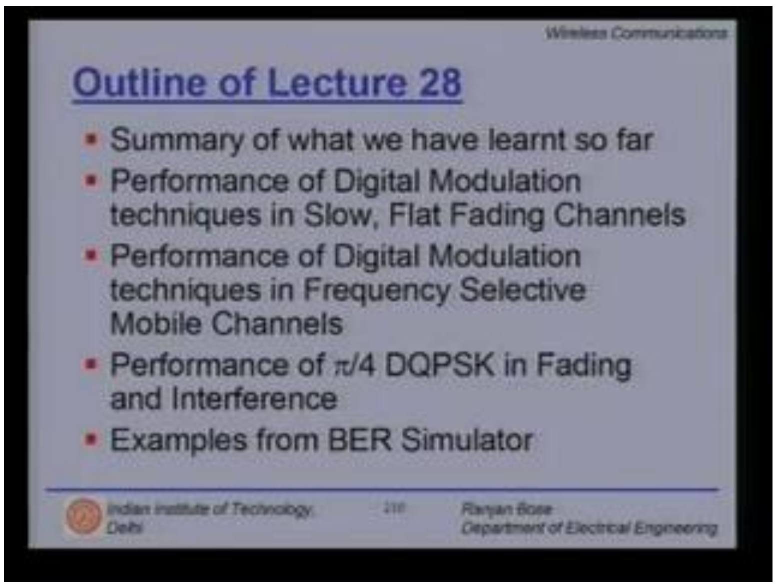

We will start with the brief recap of what we have learnt in the past few lectures, followed by the performance analysis of digital modulation techniques in slow flat fading channels. What we'll learn today is that modulation techniques get adversely effected in fading channels as oppose to additive wide Gaussian noise channel. So we must come up with some fading counter measures so as to ensure that the existing modulation techniques still hold good for fading channels. We will then look at the performance of digital modulation techniques in frequency selective mobile channels. So those which are using large bandwidth; in specific we look at the performance of pi by 4 DQPSK in fading channel with interference. Finally will take up some examples from bit error rate simulator called BER SIM.

First the recap. We have looked at a series of linear digital modulation techniques. The prominent ones were binary phase shift keying or BPSK. Differential phase shift keying or DPSK followed by the quadrature phase shift keying or the popular QPSK. We then looked at the two versions or slight variations of QPSK called the offset QPSK and the pi by 4 QPSK. We then moved over to the domain of constant envelope modulation techniques. We studied the binary frequency shift keying and looked at the minimum shift keying as a special case of FSK followed by the Gaussian minimum shift keying or GMSK which is popularly used in GSM standards.

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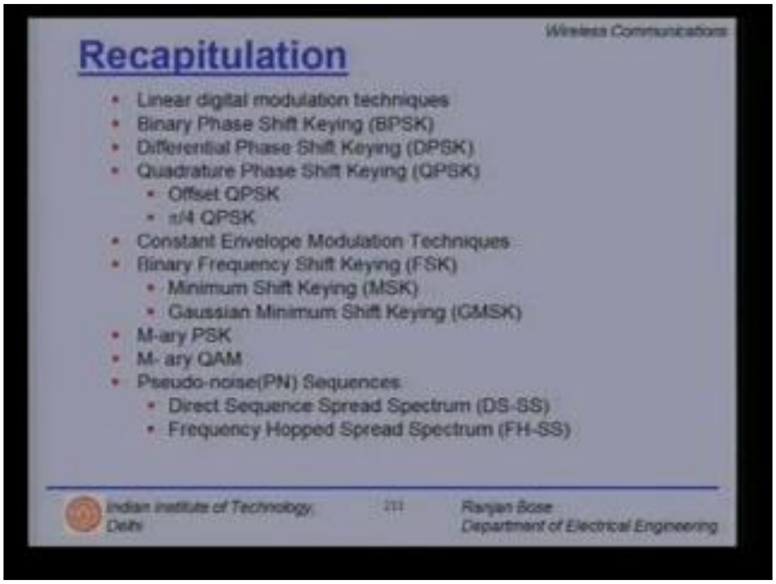

We then talked about M-ary PSK followed M-ary QAM or quadrature amplitude modulation where we play both with the amplitude and phase of the modulator signal. Finally we looked at pseudo noise PN sequences and talked about certain spread spectrum techniques. The first one we studied was direct sequence spread spectrum technique called DS SS followed by the frequency hopped spread spectrum technique or FH SS. Specifically we looked at the performance of each one so this is what we have done so far and today what we want to do is look at the performance of certain known digital modulation techniques in fading environments. What kind of fading environment? Both frequency flat and frequency selective channels. So let’s look at the introductory part.

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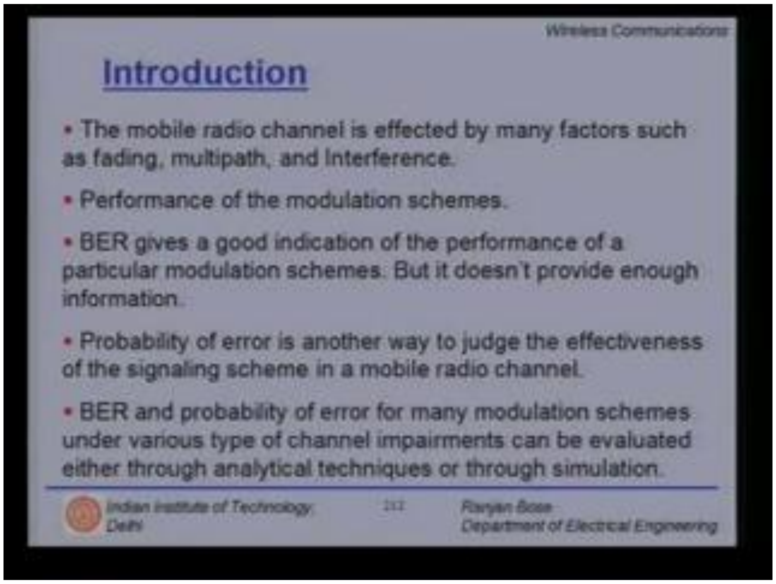

The mobile radio channel is effected by many factors such as fading, multipath and interference. In the earlier part of the course we have seen how fading effects the received signal. Multipath fading is the most common cause of fading and if the frequency bandwidth of the channel is not very large we cannot resolve the various multipath components. On top of that we sometimes work in multi user scenario where we have interference. We then have observed that the performance of different modulation techniques in the additive wide Gaussian noise channels.

Somehow we have found that most BPSK, QPSK, FSK kind of modulation techniques fall off exponentially with increasing signal to noise ratio that is the bit error rate goes down as we increase the SNR. So bit error rate gives a good indication of the performance of a particular modulation scheme but it does not provide enough information. The probability of error is another way of judging the effectiveness of a given signaling scheme in a mobile radio channel. Bit error rate and probability of error for many modulation schemes under various types of channel impediments can be evaluated either through analytical techniques or through simulations. Today we will look at both of them, the analytical method as well as through simulations.

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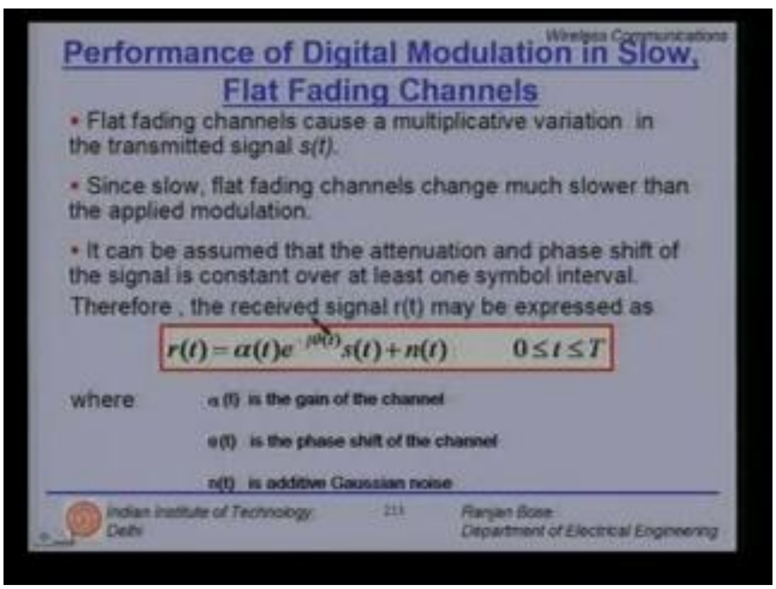

So now let us look at the performance of digital modulation techniques in slow fading channels. Flat fading channels cause a multiplicative variation in the transmitted signal s (t). If you look at the received signal here put in a box r (t) it is a multiplicative factor which has both in amplitude variation and a phase variation times s (t) plus the noise n (t). Since slow flat fading channels change much slower than the applied modulation, it can be assumed that the attenuation and phase shift of the signal is constant over at least one symbol interval.

What is that mean that over one signal interval, we have this alpha (t) is a constant alpha and theta (t) is a constant theta. Therefore the received signal r (t) may be expressed as follows; here we have put alpha as a function of time and theta as a function of time, n (t) is an additive wide Gaussian noise.

So please note the multiplicative factor coming out as a result of fading. In a previous lecture we have seen why it is a multiplicative factor.

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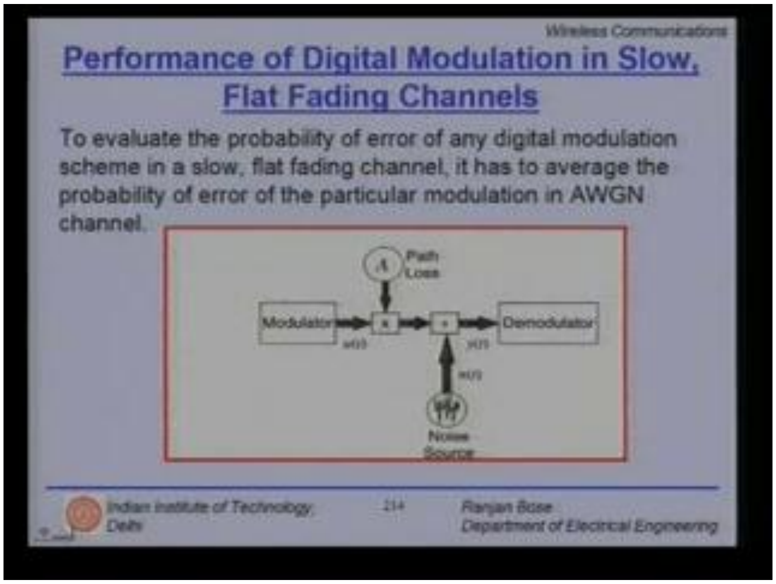

Performance of digital modulation in slow flat fading channels, to evaluate the performance of error of any digital modulation scheme in a slow flat fading channel it has to be averaged over the probability of error of the particular modulation technique in additive wide Gaussian noise channel. What does it mean that we have a modulator which is our particular modulations key and then there is a path loss followed by noise source and you give it to the demodulator. So this is the basic model and the averaging is done over additive wide Gaussian noise channel.

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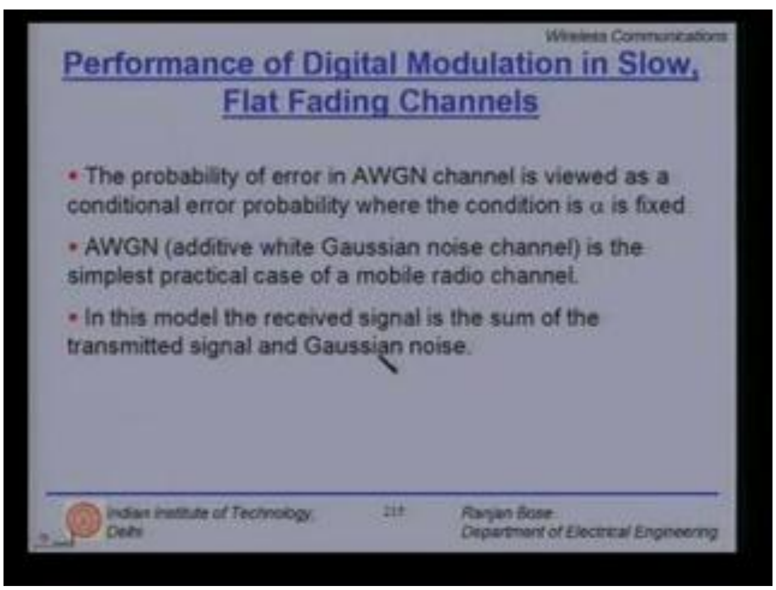

So let us look at the probability of digital modulation in slow flat fading channels. The probability of error in AWGN or additive wide Gaussian noise channel is viewed as a conditional error probability where the condition alpha is fixed. The AWGN channel is a simplest practical case of a mobile radio channel. So please note the two kinds of impediments that are coming into the picture, one is the additive wide Gaussian noise channel and the other is the fading effect. In this model the received signal is a sum of the transmitted signal and the Gaussian noise.

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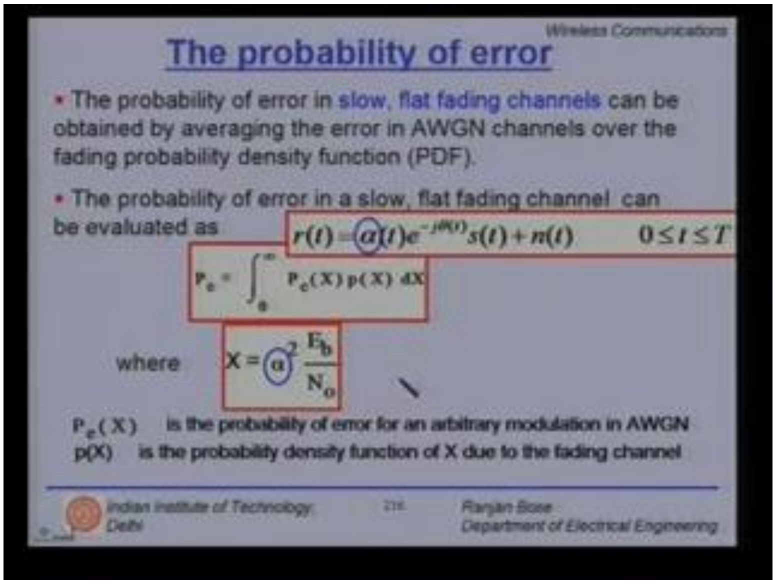

Let us talk about the probability of error in this slide. The probability of error in a slow flat fading channel can be obtained by averaging the error in AWGN channels over the fading probability density which is denoted by the PDF. What is the probability of error in a slow flat fading channel? It can be evaluated simply by this equation P_e equal to integration zero to infinity P_e (x) p (x) dx. So we have to average this P_e (x). What is x? The x is alpha squared E_b over N_0. What is alpha? Alpha is this multiplicative factor which you get when you have the r (t) received signal equal to alpha (t) e raised to power j theta (t) s (t) plus n (t) in the noise. So P subscript ex that is P_e (x) is the probability of error for an arbitrary modulation scheme in additive wide Gaussian noise channel. Whereas P (x) is the probability density function of x due to fading channel. Please note x has already been defined as alpha squared E_b over N_0, E_b denotes the energy per bit and N_0 is a measure of the noise power.

Let us look at the following slide where we talk about the various probability of errors for the different modulation schemes in additive white Gaussian noise channel. So let us now decouple the two effects, so let us list here the probability of error for some modulation schemes only in additive white Gaussian noise channel. So all this we have seen before and having put here for the sake of comparison.

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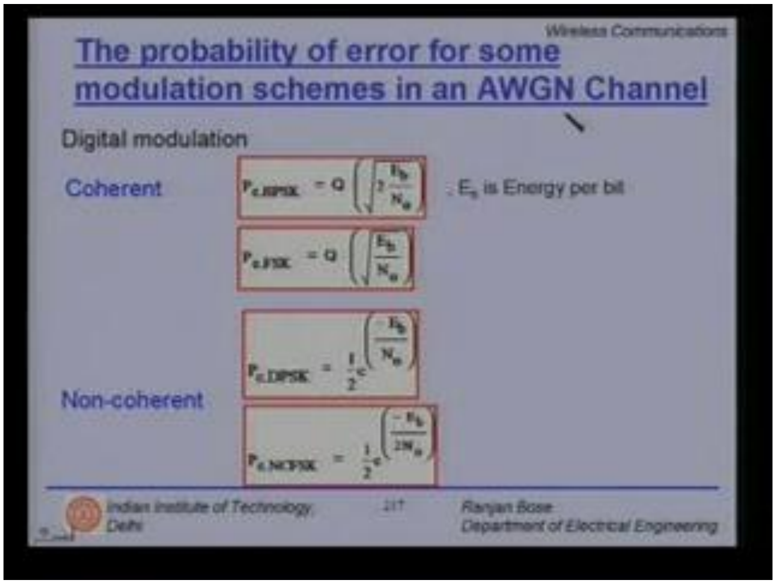

So either you talk about coherent or you talk about non coherent modulation techniques, in all these expressions we have a common platform for comparison which is E_b energy per bit. Now please note that we have for BPSK equal to Q function under root two E_b over N_0 whereas for FSK is a 3 dB difference Q under root 2 E_b under root E_b over N_0 instead of 2 E_b over N_0 these are the coherent techniques. Similarly for DPSK and NCFSK non coherent FSK it is given by these expressions. What is important here is to note that with respect to E_b over N_0 which is a measure of SNR or signal to noise ratio, the fall of probability of error is exponential, it is E raise to power minus SNR.

So it really falls down very fast so it makes sense that if I am working only in additive white Gaussian noise channels I rather invest some resources in increasing the signal to noise ratio and I get orders of magnitude improvement in my probability of bit error rate. So it makes sense here. What we are trying to show here is that it makes sense only in additive white Gaussian noise channels but this logic may not hold for fading channels which is typical of mobile communications. So we will refer back to this slide in terms of the exponential fall off of bit error rates of common modulation techniques in pure additive white Gaussian noise channels.

Let us now talk about the probability density function (PDF) of X in a Rayleigh fading channel. Now just to recap the Rayleigh probability density function is given by P (r) equal to r over sigma squared e raise to power minus r squared over 2 sigma squared, please note r is greater than or equal to zero.

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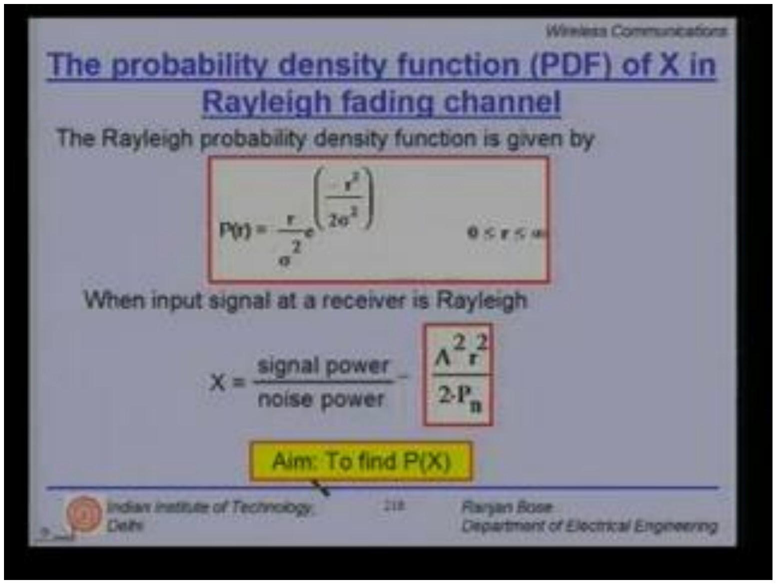

When the input signal at a receiver is Rayleigh then if you define X is equal to signal power over a noise power and what is that mean; A squared r squared over 2 P_n, P_n is the noise power the aim is to find P (X). Please remember what we are trying to do, we have Rayleigh fading environment and we have defined X is equal to signal power over noise power as follows in a Rayleigh fading environment and we have to find P (X). Once we have that we can go over and derive the probability of error expression.

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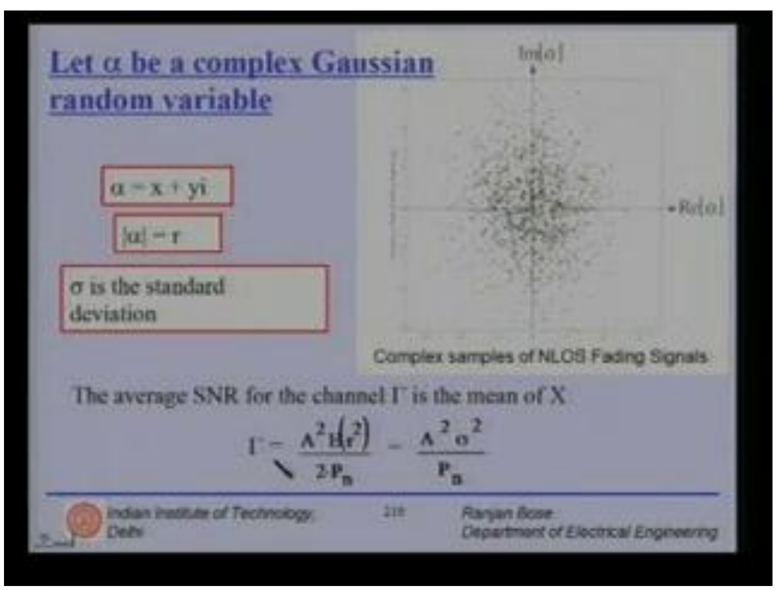

Let us see. Let alpha be a complex Gaussian random variable. So let’s denote alpha is equal to x plus yi and let’s say the absolute value of alpha is equal to r.

Let's say sigma is a standard deviation because we are talking about Gaussian random variables. So if you want to plot on this axis, the imaginary part of alpha. The complex samples of a non-line of sight fading signal does look like something like this. The average signal to noise ratio for the channel gamma so let's denote the average SNR by this capital gamma is the mean of X let's say gamma is equal to A squared expected value of r squared over 2 P_n noise power which is nothing but A squared sigma squared over P_n this is coming from the Rayleigh distribution. So this gamma is nothing but the average SNR for the channel. So our expressions the bit error rate expressions must be functions of this gamma in a Rayleigh fading environment.

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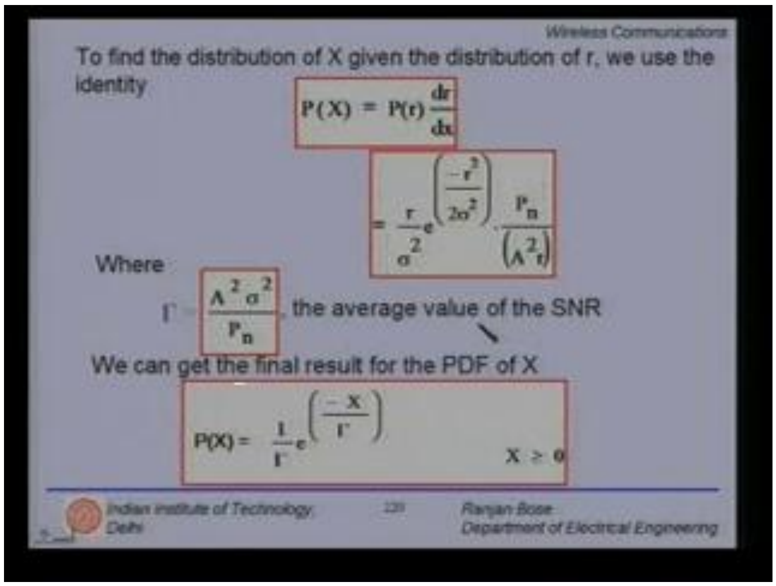

Now to find the distribution of X, given the distribution of r we use the following identity P (X) is equal to P (r) dr over dx this is well known. Here this is nothing but r over sigma squared e raise to power minus r over 2 sigma squared, for the Rayleigh distribution P_n over A squared r. This is dr over dx coming from the previous expression. Now gamma is nothing but A squared sigma squared over P_n is the average value of the SNR. So we get the final result for the probability density function of X as P (X) equal to one over gamma e raise to power minus X over gamma where X is greater than or equal to zero. So we have been able to obtain the PDF of X as a function of gamma which is nothing but the average value of the SNR. We can use this to calculate the probability of error for different kinds of modulation techniques.

So performance of digital modulation in slow flat fading channels, let us talk about coherent BPSK. The probability of error for coherent BPSK in additive white Gaussian noise channel is given by using this expression and probability of error for PSK is Q under root 2 E_b over N_0 we have seen is given by this expression. Substituting the two equations above in the following equation, what is the following equation? Probability of error X over the times P (x) dx zero to infinity will give you the probability of error.

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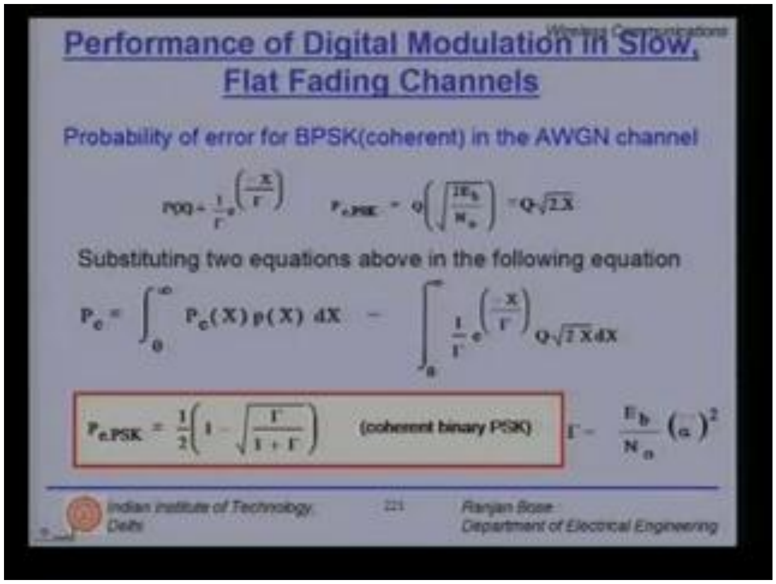

So averaged the probability of error expression over rho AWGN channel and if we do that P_e (X) can be put as one over gamma e raise to power minus X over gamma coming exactly from here and this P (X) which pertains to the coherent BPSK comes from Q under root 2 E_b over N_0 is Q under root 2 X comes out here dx. In fact this gives you a standard method to evaluate the probability of error bit error rate for all the different kinds of modulation schemes; you just have to substitute P (X) properly. So here we have done it for the BPSK. If you evaluate this expression you get P_e for BPSK as half one minus under root gamma over one plus gamma. This is the solution for coherent binary PSK, please note what is gamma; gamma is E_b over N_0 alpha bar squared.

So this is for half fading channel, the performance of coherent BPSK. This is different; this is very different from this Q function under root 2 E_b over N_0 kind of thing. This is an exponential fall off; this is nothing even close to exponential follow off. In fact it is an inverse relation only. This is a disturbing fact because it tells us that the probability of error in fading channels really does not fall off exponentially with increasing SNR here average SNR that means we have to work much harder, we have to increase the SNR much higher to obtain the same level of drop in bit error rates.

Let us look at the probability of error for other modulation techniques in fading channels. Rayleigh fading is what we have assumed and additive white Gaussian noise is present. Now already for coherent BPSK we have obtained this expression half one minus under root gamma over one plus gamma. Let us see how it compares with coherent binary FSK where we have probability of error for FSK is half one minus under root gamma over two plus gamma. Slightly different is a two in the denominator here, for DPSK it is very clear it is 1 over 2 into 1 plus gamma so it is clearly an inverse relationship with respect to gamma; for non-coherent FSK it compared to be 1 over 2 plus gamma.

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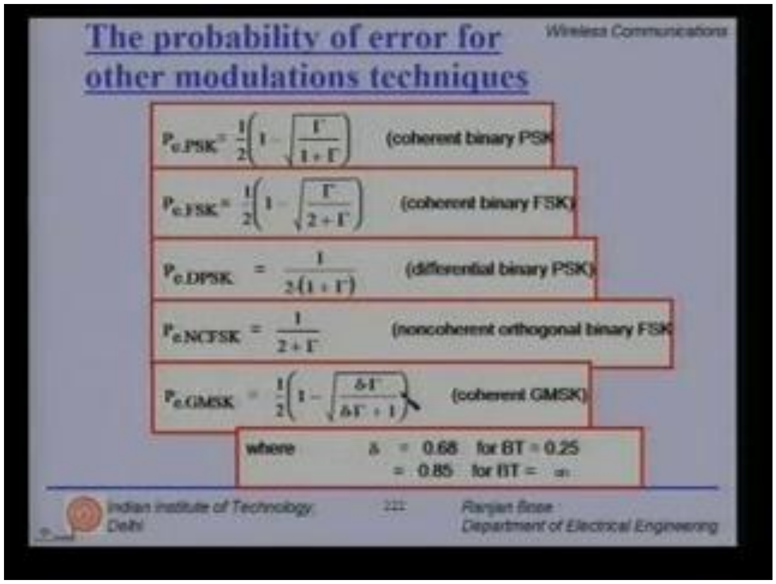

For GMSK the modulation technique that we use for mobile communication GSM standard it is half one minus under root delta gamma over delta gamma plus one where delta has these two values for different bandwidth time products. We have talked about this BT product for GMSK earlier. It tells you how sharp or how narrow is the Gaussian pulse used to pulse shape the MSK signal. What is important to note is that in all of these cases? The probability of error falls off inversely with respect to the signal to noise ratio this is the story for all of them. So performance of digital modulation in slow flat fading channels, for large values of signal to noise ratio the error probability of equations may be simplified say in most of the denominators we saw earlier that is one plus gamma or two plus gamma kind of expressions.

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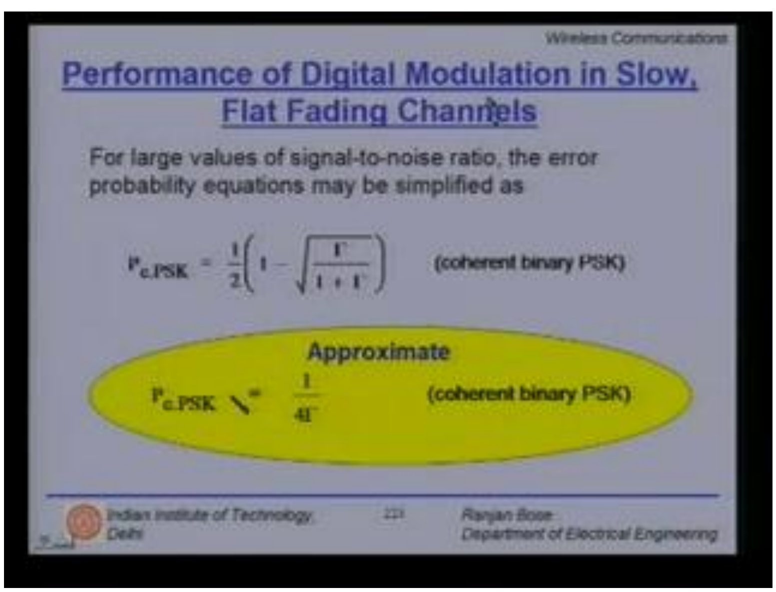

So consider a case of binary PSK coherent which was half one minus under root gamma over one plus gamma for large values of gamma is a signal to noise ratio. You can approximate it to obtain one over 4 gamma for coherent PSK which clearly tells you that the probability of error drops off inversely. How good or bad is this assumption?
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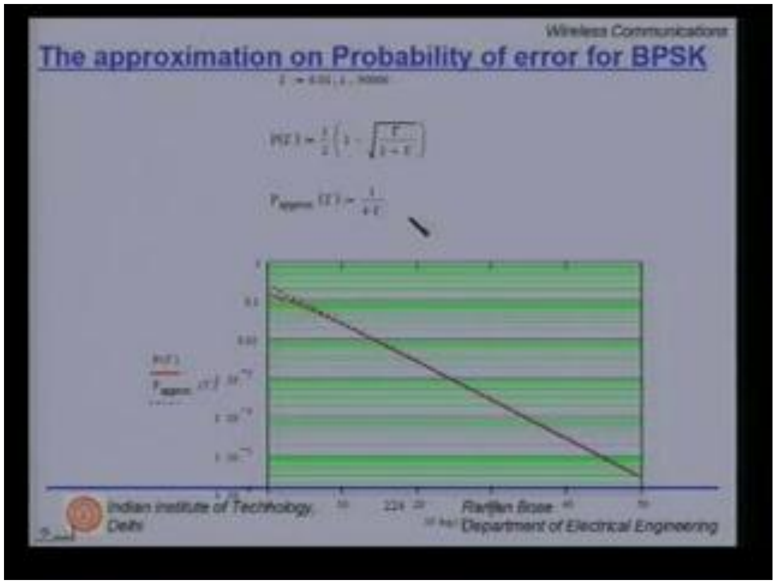

So for again coherent BPSK where the exact expression is half one minus under root gamma over one plus gamma and the approximate expression is one over 4 gamma. We have plotted in red the actual value, the exact expression and dotted blue is the approximate expression and there is a clear match for high SNR's and its start deviating at low SNR's but fairly a good match so for all practical purposes we can use the practical approximate value. This is the result for BPSK, it doesn't stop us from approximating all other digital modulation techniques that we have seen so far.
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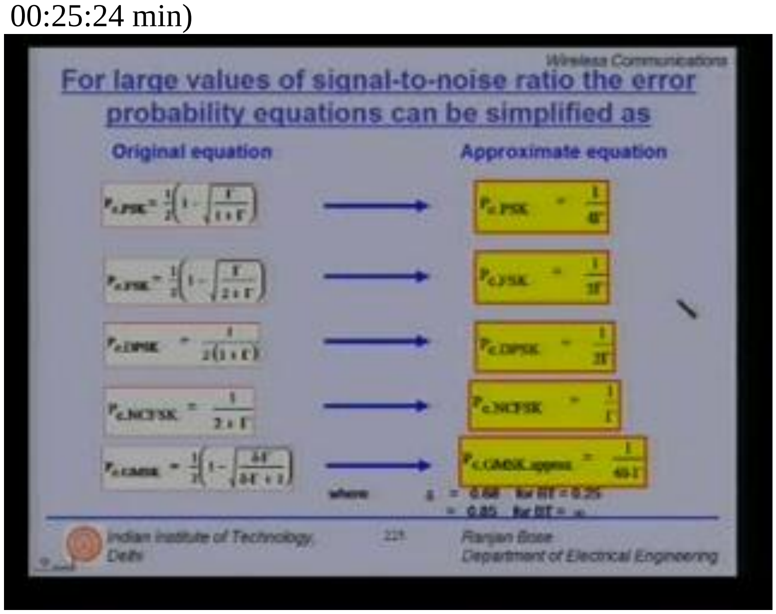

So let us say for large values of signal to noise ratio, the error probability equations can be simplified as follows. For BPSK we have carefully seen it comes out to be one over 4 gamma; for FSK it is one over 2 gamma. So there is a 3 dB kind of effect here then DPSK again one over 2 gamma, so again you can see that in fading channels the binary FSK and DPSK perform almost equally well in high SNR scenarios. Then non-coherent FSK falls up as one over gamma so it verses by a factor of 2 and then for GMSK it is one over 4 delta gamma where delta we have earlier seen it is 0.68 for bandwidth time product of 0.25 and 0.85 for the BT product very large. So what we learnt from this slide is that we have parallel expressions, approximate equations for the better probability for the different common modulation techniques while we perform in fading channels.

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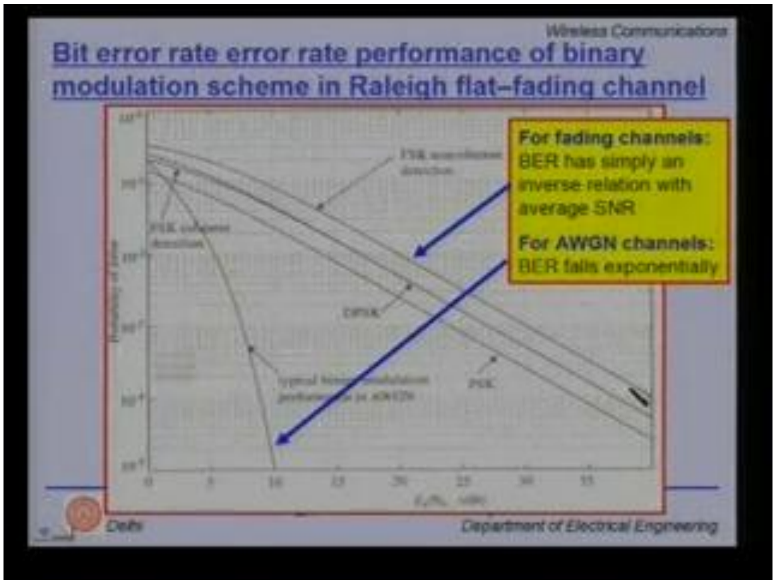

So let us look at it graphically in this slide we want to know the bit error rate performance of binary modulation schemes in Rayleigh flat fading channels. As you can see that two categories of curves here, one the gradually following curves, on the x axis we have plotted the E_b over N_0. The energy per bit over noise power which is an indication of the signal to noise ratio so x axis marks the signal to noise ratio in dB, the y marks the probability of error. Please note here it's a log's scale also. So x axis also logarithmic dB, y axis is also logarithmic. Here it is almost inverse linear fall of the first curve is FSK non coherent.

Then is the DPSK and then is the PSK. Here is a typical binary modulation performance in a pure additive white Gaussian noise channel in the absence of fading environment. The simple additive white Gaussian noise channel, it really falls off exponential, there is a lot of gap and this gap increases at high SNR's. So as expected for fading channels the bit error rate has simply an inverse relation with the average SNR gamma for additive white Gaussian noise channel, the BER falls exponentially.

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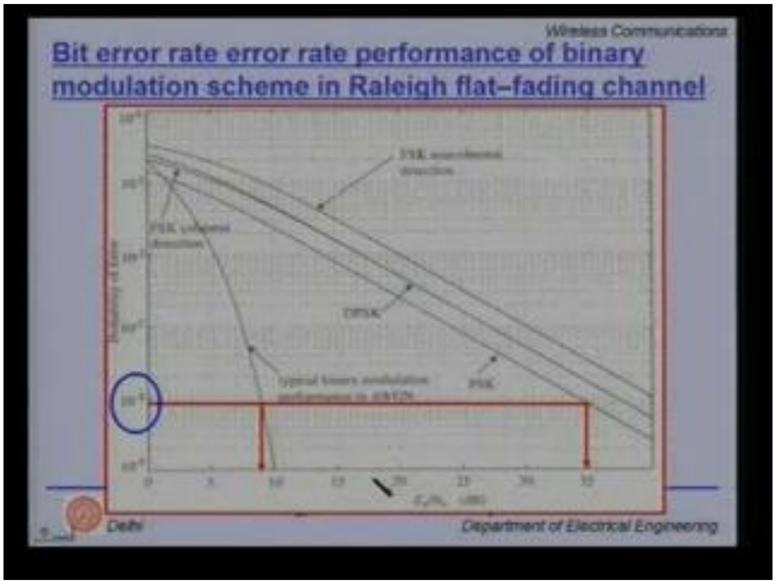

Suppose we want to compare let us say for a particular application the desired bit error rate is 10 raise to power minus 4. It is not a very demanding environment I am expecting a bit error rate of 10 raise to power minus 4. If I am working only in a pure additive wide Gaussian noise channel without any fading then I require a mere 8 dB E_b over N_0 to achieve this effect. We are talking about any kind of a binary modulation technique in additive wide Gaussian channel but look at what happens if you move over to the fading environment. Here even for the best case you need humongous 35 dB E_b over N_0. That's a lot for a simple requirement of 10 raise to power minus 4, clearly my battery will run out in no time to achieve the same probability of error but please note this is the average probability of error we are talking about. We will soon see that the reason why the performance degrades to this extent is because of the presence deep fades and when you are doing your average it shakes the balance.

In fact if you can overcome the effects of this deep fades then you are back into business. That is you don't have to work on increasing the signal to noise ratio, you can work around by using some kind of a fading counter measure. It could be frequency diversity or antenna diversity or polarization diversity or angle diversity or time diversity or code diversity any kind of diversity technique which helps you to overcome the effects of fading, deep fades will work very well and will help us get back into shape. Therefore diversity techniques are very important for design of any communication system which is going to work in mobile scenarios. It also tells us that by increasing the signal to noise ratio thereby trying to bring down your bit error rate performance is not the best way to move about in fading environments.

In some cases where you do not want to put in fading counter measures or do not have the luxury to do so but you have the luxury to increase your signal to noise ratio, normally you put a fade margin while designing your communication systems. So if you think about what is the maximum fade that will happen if it is on an average 10 dB or 15 dB you just add your working value by increase it by 10 to 15 dB that is called adding a fade margin.

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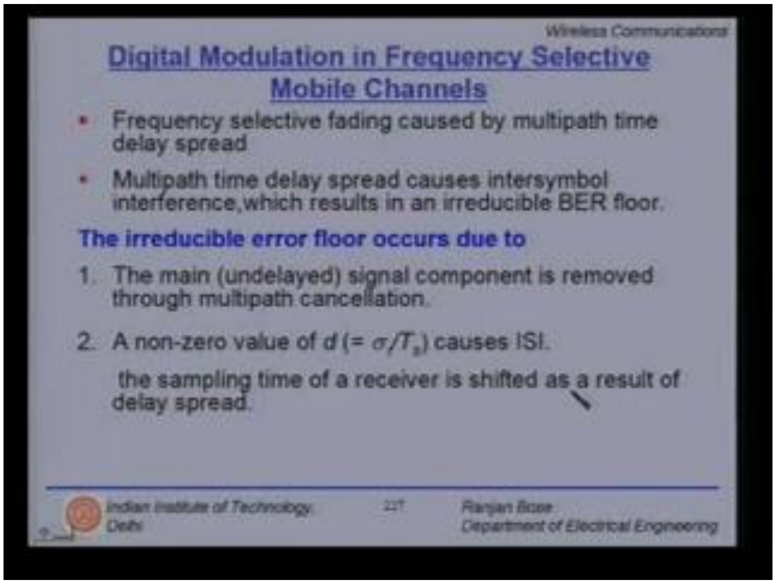

Now let us move over from frequency flat to frequency selective mobile radio channels. Let us look at digital modulation in frequency selective mobile channels. The frequency selective fading caused by multipath time delay spread is what we have learnt so far. The multipath time delay spread causes inter symbol interference which results in an irreducible bit error rate floor. The irreducible error flow occurs because of these two reasons. The main or the un delayed signal component is removed through multipath cancellation and a non-zero value of d which we defined as sigma$_t$ over T_s. This is the RMS delay spread that we have learnt in earlier lectures and T_s is the symbol duration, this d causes ISI. The third point is the sampling time of a receiver is shifted as a result of delay spread.

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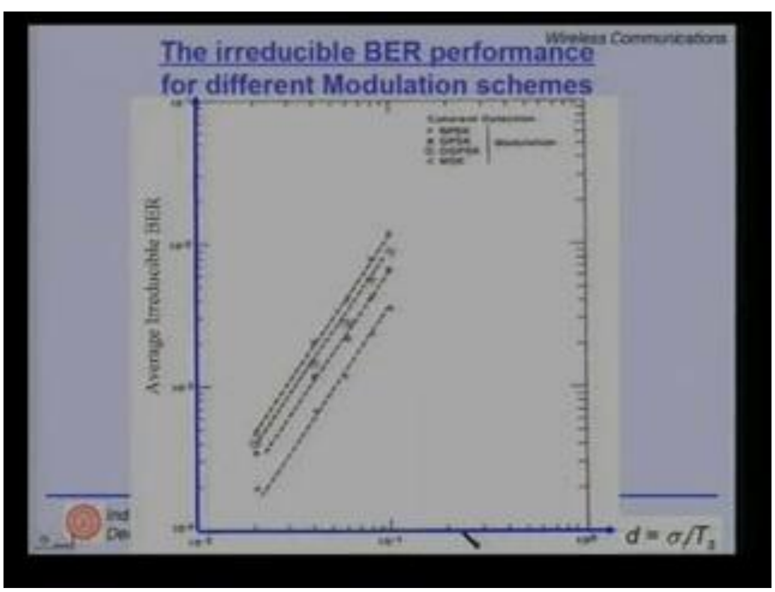

In this slide let us look at the irreducible bit error rate performance for different modulation schemes in frequency selective fading channel. On the x axis we have this parameter d equal to sigma$_t$ over T_s this is the RMS delay spread sigma$_t$, T_s is the symbol interval. On the y axis we have the average irreducible bit error rate. Here we have the different kinds of modulation techniques DPSK, QPSK, OQPSK and MSK and you note that as we increase the d the irreducible bit error rate floor increases.

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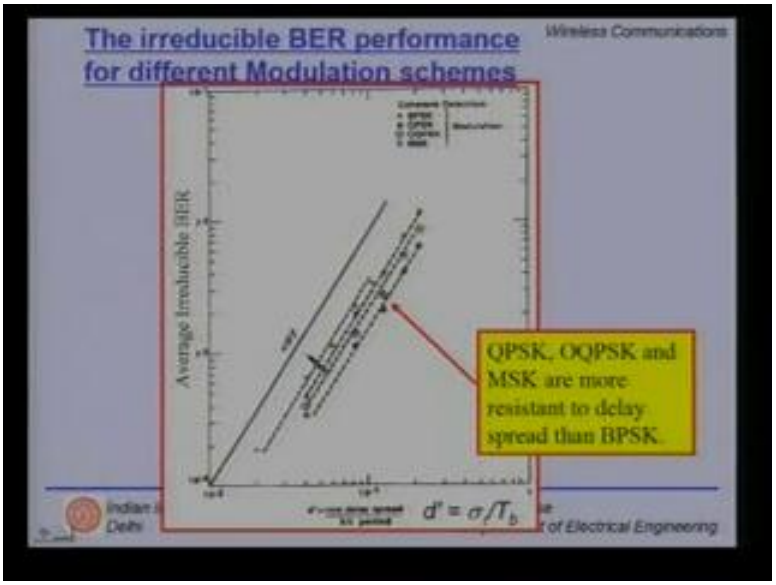

Now let us look at another parameter d prime which is being defined as sigma$_t$ RMS delay spread over T_b and the bit duration. On the y axis again we have average irreducible bit error rate, I mean no matter what you do, you cannot go below this error floor it's an irreducible bit error rate. Again we have plotted and then fairly close to each other BPSK, QPSK, OQPSK and MSK. So typically what we can learn from the slide is if a value of d prime which is equal to sigma$_t$ over T_b is 0.1 then we are roughly in the range of 10 raise to power minus 2 between 10 raise to power minus 2 and 10 raise to power minus 3 is your irreducible bit error rate.

You cannot do any better but when you plot these curves the lower two curves belong to the QPSK and OQPSK. So we can conclude from this at QPSK, offset QPSK and minimum shift keying are more resistant to delay spread than BPSK which is at the top of the line. So some schemes perform better inherently than other schemes. Now let us talk about pi by 4 DQPSK in fading channels in the presence of interference. We can study the bit error rate performance of pi by 4 DQPSK by using computer simulations. So in the next couple of slides we can use simulations to study the effects of modulation techniques in fading channels.

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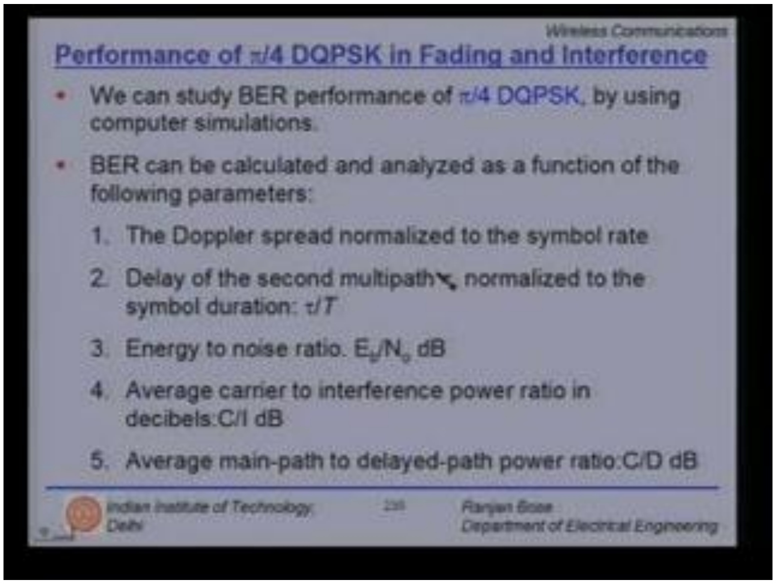

The bit error rate can be calculated and analyzed as a function of the following parameters. What are these? The Doppler's spread normalized to the symbol rate, the delay of the second multipath tau normalized to the symbol duration. The energy to noise ratio, the average carrier to interference power ratio in decibel C to I and average main path to delayed path power ratio C over D. So this is the main path and this is the delayed path. So if you have these parameters then you can write or make a computer program or buy a computer program which will be able to predict the bit error rate performance in fading channels. So one of the programs is the bit error rate simulator of version, here you have a data source and it could be a real data from a TV camera optical or other computer terminal passes through a modem.

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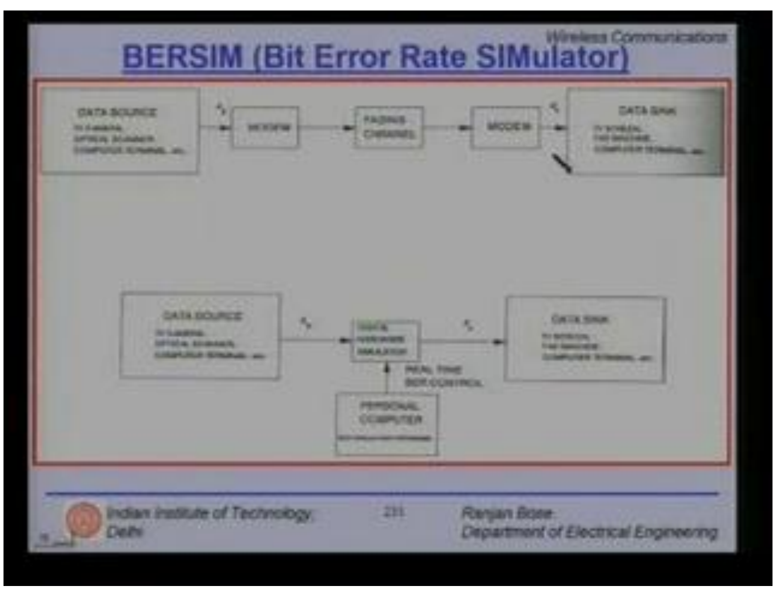

Then we have the fading channel, you can have different models for the fading channel then at the receiver there is a modem and there is a data sink. Even have an alternate way data source, digital hardware simulator which you can have a PC to do real time bit error rate control and then a data sink. This is a general model for the bit error rate simulator, you can change this type of data, the data rate, the signal to noise ratio, the fading effects whether it is a frequency or flat frequency selective fading, change the signal to noise ratio and get the different kinds of performance.

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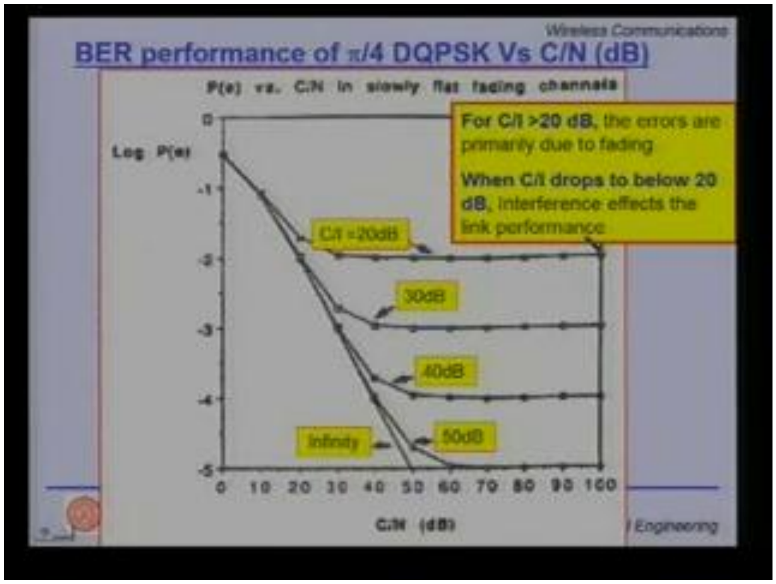

In this slide we have the bit error rate performance of pi by 4 DQPSK versus C to N. On the x axis we have the C to N ratio carrier to noise. So here we have the various curves for different carrier to interference, here C to I is 20 dB, 30 dB 40, 50 and infinity and here you have the bit error rate. It is just telling you, the power of 10 so 10 raise to power minus 1, 10 raise to power minus 2 and so and so forth there is 10 raise to power minus 5. Clearly when you have higher C to I ratio as you move down, your error rate goes down. For C to I greater than 20 dB the errors are primarily due to fading but when C to I drops below 20 dB the interference effects the link performance.

Now again we are talking about bit error rate performance of the pi by 4 DQPSK versus signal to noise ratio. On the x axis we have E_b over N_0 SNR, on the y axis we have bit error rate going from 10 raise to power minus 1 down up to 10 raise to power minus 6 but the different curves of a different vehicular speeds starting from 10 kilometers per hour up to 150 kilometer per hour where after all working in a mobile situation. So let us see how it affects our performance. So as we go up we are increasing the vehicle speed.

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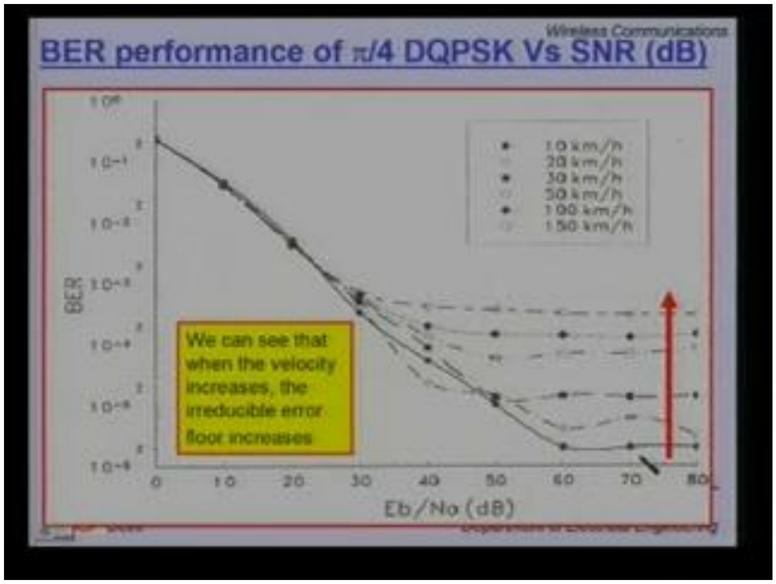

We can see that then the velocity increases, the irreducible bit error rate floor also increases; cannot do better than that. So inherently just by ring mobile and just by moving fast, you have an irreducible bit error rate. These problems do not exist in a simple additive white Gaussian noise channels. So fading environment is very different than what we have learnt so far and we have to come up with ways and means to overcome this effect. Please note if I am on the top most curve at 150 kilometers per hour, for a wide range of E_b over N_0 I have bit error rate minus 10 raise to power minus 3. No matter how much I increase the signal to noise ratio, see the x axis SNR I saturate. This is an irreducible bit error rate. The layer occurs little later point as we slow down.

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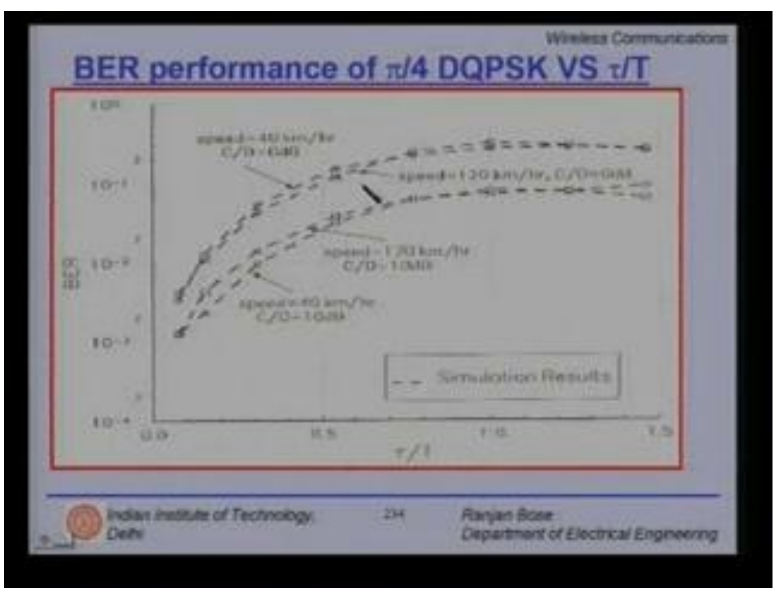

In this slide we have the bit error rate performance of pi by 4 DQPSK versus normalized tau over t. On the x axis we have tau over t, y axis is again the bit error rate. Here we have different speeds this is 120 kilometer per hour for different kind of C to D ratio which is zero dB. We improve the C to D ratio, C is the direct path, D is the delayed path. If you improve that your bit error rate goes down, speed is the same and for the same C to D ratio if you lower your speed your bit error rate goes down also. All these are simulation results, so we are trying to say that if you have a good simulator you can carry out a lot of analysis simply by simulations. So in this diagram we have two parameters the C to D ratio and the speed of the vehicle. The delay in amplitude of the second ray have strong impact on the average. This is what we take home from this slide, the delay an amplitude of the secondary have a stronger impact on the average.

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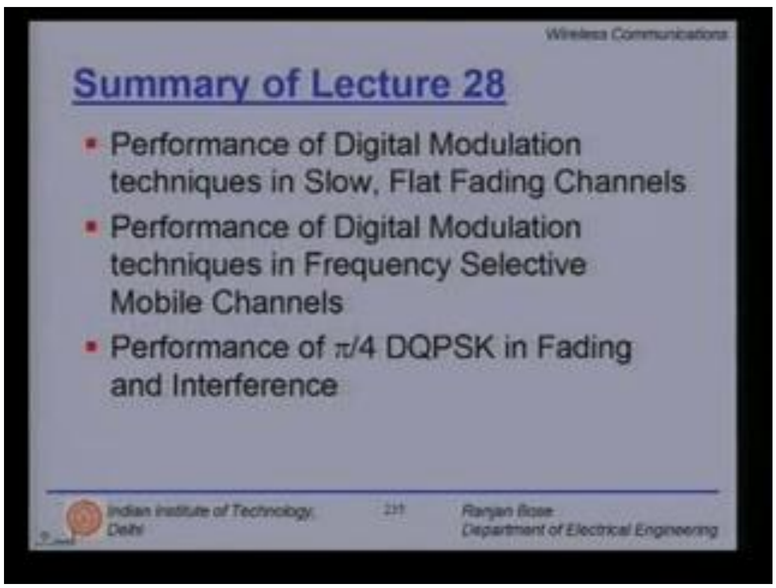

Let us now summarize what we have learnt in today's lecture, we have focused our attention on the performance of digital modulation techniques in slow flat fading channels. We then looked at the performance of digital modulation techniques in frequency selective mobile channels. So both flat fading and frequency selective fading channels were analyzed today. We focus our attention on pi by 4 DQPSK in fading channels in the presence of interference and we used BER's SIM to find out simulation results and the performance of pi by 4 DQPSK. Finally we had couple of graphs showing us examples from the bit error rate simulator. We will conclude our lecture here today.

Thank you