Wireless Communications
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Lecture No. # 29
Equalization and Diversity Techniques for Wireless Communications

Welcome to the next lecture on wireless communications. Today we will look at equalization and diversity techniques. First the brief out line for today's talk.

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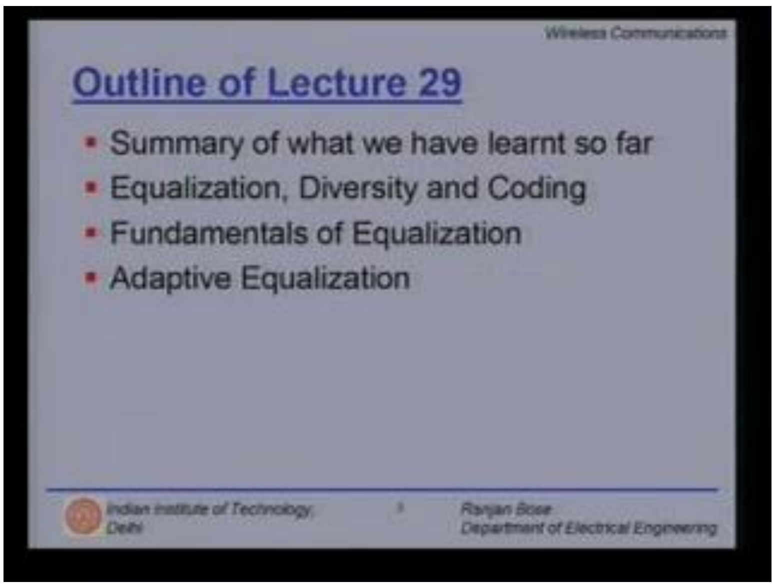

We will briefly summarize what we have learnt in the previous lectures followed by a very brief introduction to equalization, diversity and coding. The three techniques used to overcome the detrimental effects of fading channels. Specifically we will look at the fundamentals of equalization followed by adaptive equalization so this is the agenda for today's talk.

First a brief recap. We started off with modern wireless communication systems providing us the motivation to study the various techniques for wireless communications. We looked at 2 G systems, 2.5 G and 3 G networks. Then we moved over to the cellular concept and the concepts of frequency reuse as well as co-channel interference that comes with it. We then looked at cell coverage and cell capacity. Then we studied the mobile radio propagation where we looked at the large scale fading which involved outdoor and indoor propagation models as well as small scale fading wherein we looked at Rayleigh fading and Ricean fading.

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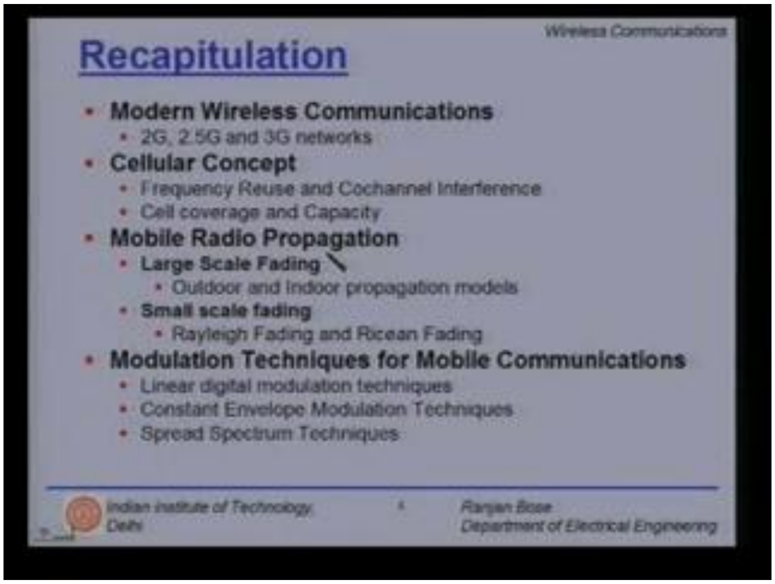

We will see that fading causes a lot of degradation in the performance of any modulation technique in wireless communication and therefore we have to device ways and means to overcome the effects of fading. Lastly we looked at modulation techniques for mobile communications wherein we studied linear digital modulation techniques followed by constant envelop modulation technique and then spread spectrum techniques. Today we will focus at how to overcome the effects of fading in some way or the other.

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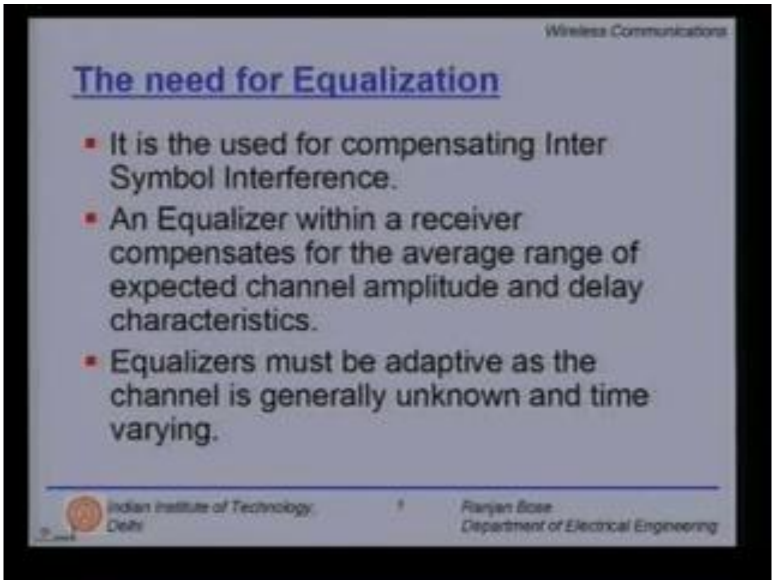

Let us look for the motivation first? What is the need for equalization? Why do you need to equalize at all? Equalization is used for compensating inter symbol interference. We work in a multipath environment wherein most of the time the received signal comes from several multiple paths thereby causing inter symbol inference. We have studied this before, the need for equalization comes in so as to overcome ISI. Earlier we had looked at certain pulse shaping techniques which can also overcome the effects of ISI but it also comes with an extra burden. Here equalization can help overcome the effects of inter symbol interference that is the primary use of equalization and equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics.

In some way undoing the effects of the channel, it is equalizing the distortions put in by the channel. So in some case you can think of it as an inverse filter. Of course the inverse filter may have certain problems we will also look at those problems. The most important point for mobile communications is that equalizers must be adaptive. Why? The channel is generally unknown and even if it is measured it will change with time. So we must have a method to have adaptive equalization.

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Now another popular technique other than equalization to overcome the effects of multipath channel is diversity. Diversity is primarily used to counter act fading effects and we have seen fading causes a big degradation in the performance of any modulation techniques. What is diversity? It is a technique used to compensate for the fading channel impairments. The different kinds of diversity techniques. What are these? The popular ones are antenna diversity, frequency diversity, time, polarization, angle and code diversity. Of this antenna diversity is one of the more popular ones where in we use multiple antennas. Now this multiple antennas can be either at the transmitter or at the receiver or both. Frequency diversity requires you to use frequency signals which are separated by the coherence bandwidth of the channel, we have looked at frequency diversity in our previous lectures.

Time diversity we must separate the transmission more than the coherence time of the channel. We will revisit the coherence bandwidth and the coherence time briefly to refresh our memories. Polarization diversity, the horizontal and the vertical polarize signals fade differently and independently, angle diversity and used different codes so we can have code diversity.

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So the popular antenna diversity is implemented by using two or more receiving antennas and as I mentioned before you can also have them at the transmitter. Now while equalization is used to counter the effects of ISI or inter symbol interference, diversity is usually employed to reduce the depth and duration of it's that is the fundamental difference. So the depth of fade and the duration of fades experienced by a receiver in a flat fading scenario can be overcome by using diversity. These techniques can be employed both at base station and mobile receivers, any of the diversity techniques can be deployed either at the base station or at the mobile receivers.

Spatial diversity is the most widely used diversity technique; you can have multiple antennas at the base station because having multiple antennas in your handset is a little inconvenient or you can even have antennas on different base stations. So you really get a good spatial diversity. The key point in spatial diversity is the separation of the antenna elements depending upon how clattered is the multipath environment, your antenna separations may have to be increased or decreased. Let us talk about spatial diversity.

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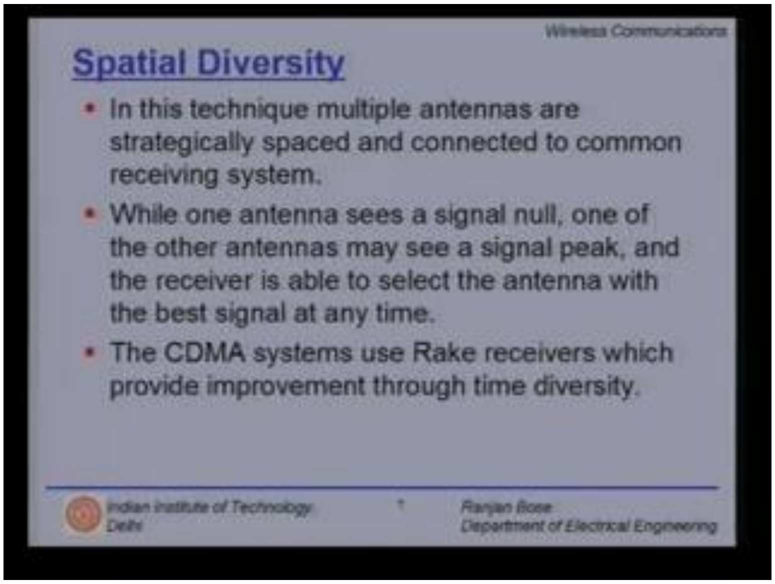

In this technique multiple antennas are strategically spaced and connected to common receiving system; while one antenna sees a signal null, one of the other antennas may see a signal peak provided the signals are not correlated. In this case the receiver is able to select the antenna with the best signal at any time or it can do some kind of an intelligent combining. This CDMA or code division multiple access systems use rake receivers which provide improvement through time diversity.

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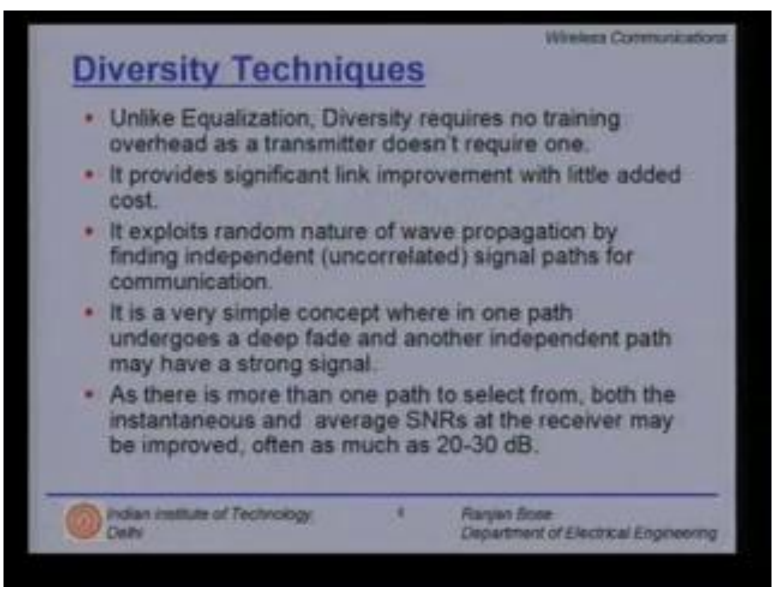

Now let's look at the diversity techniques unlike equalization diversity requires no training overhead as a transmitter doesn't require one. Today we'll see that equalization has two modes, the training mode and the tracking mode. The training mode will require sending a known signal and it is used to set the weights of the equalizer whereas diversity doesn't have that issue. What else these diversity techniques do? It provides significant link improvement with little added cost, it also exploits random nature of wave propagation by finding independent and hence uncorrelated signal paths for communication that's how diversity works. It is a very simple concept wherein one path undergoes a deep fade and another independent path may have a strong signal component that's it.

If you cost several nets you're bound to catch fish in one of them that is the basic philosophy. As there is more than one path to select from both the instantaneous and average SNR's at the receiver may be improved often as much as 20 to 30 dB. Please recall from one of your earlier lectures that when you have to have an improvement in bit error rate as compared to additive white Gaussian noise channel fading channels required 20 to 30 db additional SNR for the same performance. Here in diversity techniques you overcome that.

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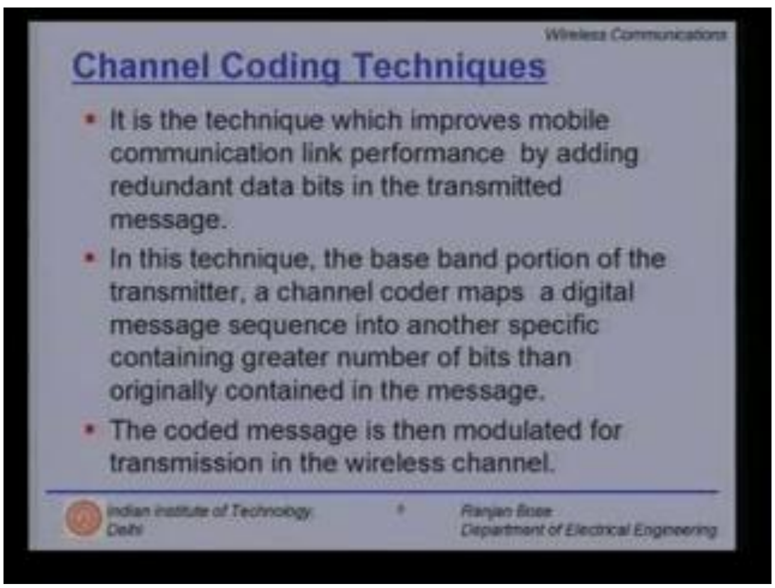

Now the third and also a very popular method to overcome the impairments put in by the channel is called the channel coding. It sometimes also refers to as error control coding because the objective is to detect and recover from errors introduced by noise and fading. So what is channel coding? It is a technique which improves mobile communication link performance by adding redundant bits in the transmitted message in a known manner that is key. We add redundancy in a known manner so that there is some kind of a structure built in to the message, at the receiver because of the noise or fading the structure gets disturbed but we can kind of guess and hence recover from errors based on the algebraic structure of the code. In this technique the base band portion of the transmitter, a channel coder maps a digital message sequence into another specific containing greater number of bits than originally contained in the message.

All it means is that have a mapping from k bits to n bits where n is larger than k there is I add n-k redundant bits but in a known manner. The coded message is then modulated for transmission over the wireless channel.

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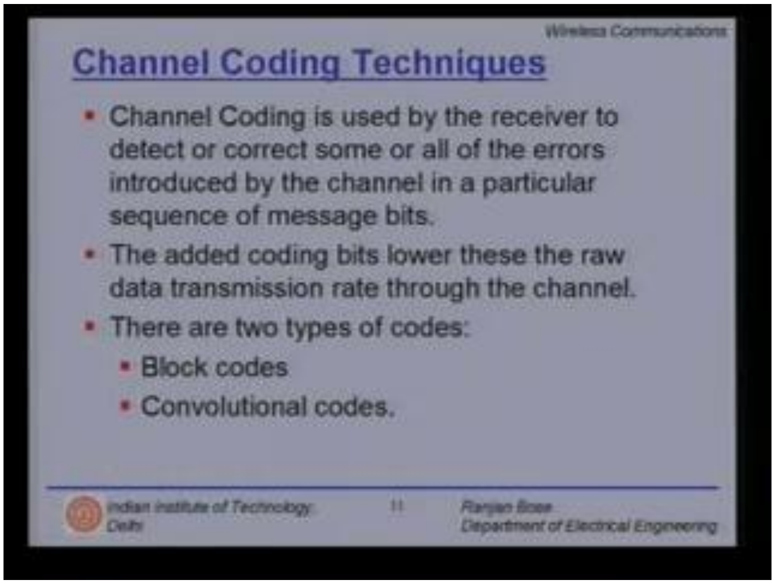

Channel coding is used by the receiver to detect or correct some of or all of the errors introduced by the channel. It depends upon how strong is an error correcting code; in fact it is a design problem. If you have a channel which is a bad case scenario then you must use a stronger code wherein you have to use more number of redundant bits. The added coding bits lower this raw data transmission rate through the channel. In fact if you are mapping k bits to n bits then k over n is given as the code rate of the code. The two types of codes broadly speaking one is block codes and then there is convolutional codes, convolutional codes are codes with memory.

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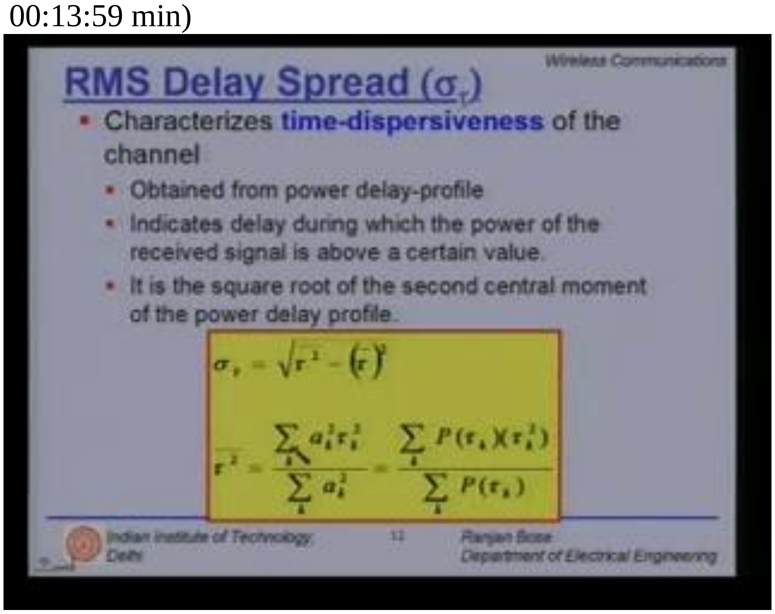

Now let us revisit very briefly the concepts of coherence bandwidth, coherence time and delay spread because we will realize that diversity techniques, equalization techniques and coding techniques individually or in combination must be used in different channel conditions whether it is frequency flat fading, frequency selective fading, fast fading or slow fading. We must come up with a matrix which can help us put things in perspective which technique to use in which fading environment. So let us revisit RMS or root mean square delay spread. What was that? It characterizes the time dispersiveness of the channel obtained from the power delay profile of the channel, indicates delay during which the power of the received signal is above a certain value. It is the square root of the second central moment of the power delay profile and sigma subscripts tau is given by under root tau square bar minus tau bar squared which is the second central moment of the power delay profile, tau square bar is calculated as fallows.

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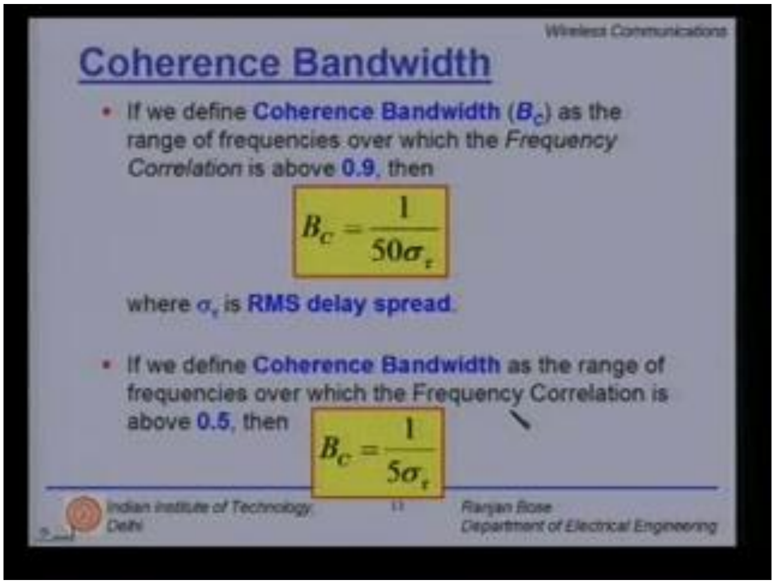

What is coherence bandwidth? We define coherence bandwidth as the range of frequency over which the frequency correlation is above 0.9. This is one of the possible definitions of coherence bandwidth. In this case B_c coherence bandwidth is given by 1 over 50 $\text{sigma}_{\text{tau}}$. On the other hand if you want to have a definition wherein we define coherence bandwidth as the range of frequencies over which frequency correlation is above 0.5 in that case coherence bandwidth is 1 over 5 $\text{sigma}_{\text{tau}}$. So $\text{sigma}_{\text{tau}}$ comes directly from measurement data and then by this basic formulae I can find out the coherence bandwidth, compared to the signal bandwidth if your coherence bandwidth is larger or smaller you decide whether the channel is frequency selective or frequency flat.

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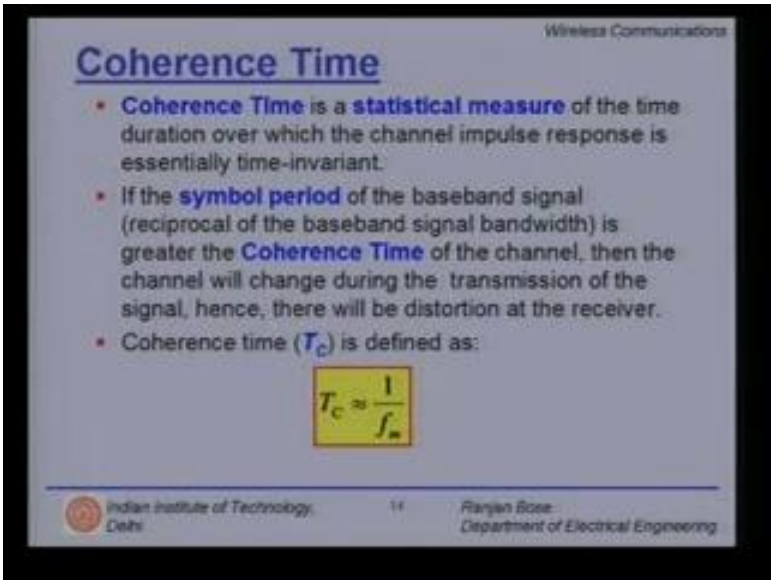

Now the other dimension is coherence time. Coherence time is a statistical measure of the time duration over which the channel impulses response is essentially time invariant. It tells us how fast the channel is changing in time. If the symbol period of the base band signal is greater than the coherence time of the channel then the channel will change during the transmission of the signal. Hence there will be distortion at the receiver. How do we define coherence time T_c? T_c is approximately given by 1 over f_m where f_m is the maximum Doppler spread. So coherence time will tell us whether your channel is fast fading or slow fading and thereby help us figure out whether to use equalization diversity or coding or a combination.

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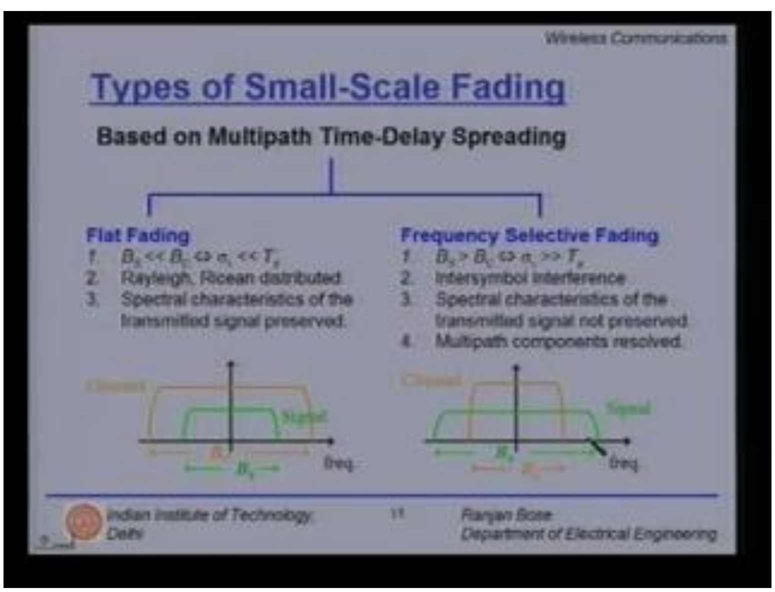

So let's see the types of small scale fading based on multipath time delay spreading we have either a flat fading or a frequency selective fading, the conditions are pretty simple B_s much less than B_c. B_c is the coherent bandwidth, B_s is the bandwidth of the signal. If the bandwidth of the signal is much smaller than the coherence bandwidth we encounter flat fading. On the other hand if B_s is greater than coherence bandwidth then we expect to see some kind of a frequency distortion. We also will see intersymbol interference and thereby there will be a need for equalization. So here in the orange is a channel bandwidth, coherence bandwidth of the channel and the green is a signal bandwidth. In the first case clearly in the frequency domain you can see that your signal frequency spectrum will not get distorted whereas here your signal is much broader in the case of frequency selective fading than the channel coherence bandwidth and you will encounter a lot of inter symbol interference.

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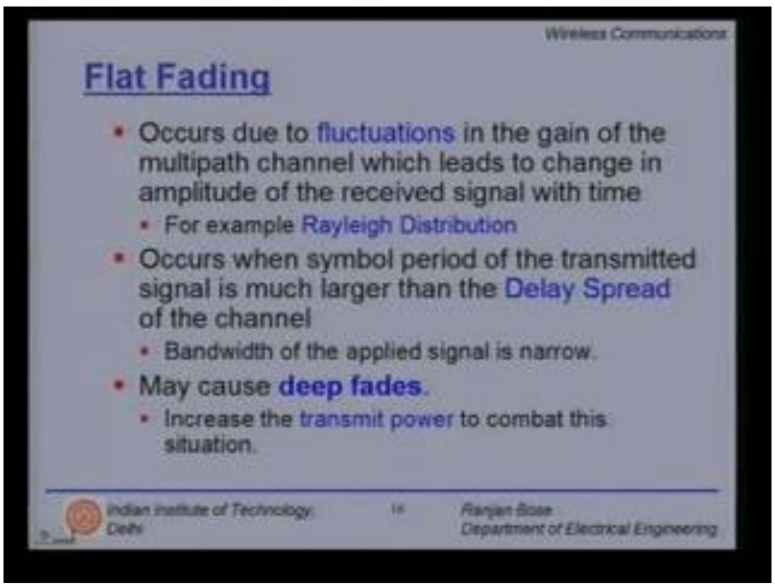

Flat fading occurs due to fluctuations in the gain of the multipath channel which leads to change in the amplitude of the received signal with time. For example Rayleigh fading, occurs when symbol period of the transmitted signal is much larger than the delay spread of the channel. May cause deep fades and increase in the transmit power to combat the situations may help but what we saw last time is that the requirement in the fade margin that you have to put in is ten's of dB's. It is not a very good way to overcome, the easier cheaper more effective solution is to use some kind of a diversity.

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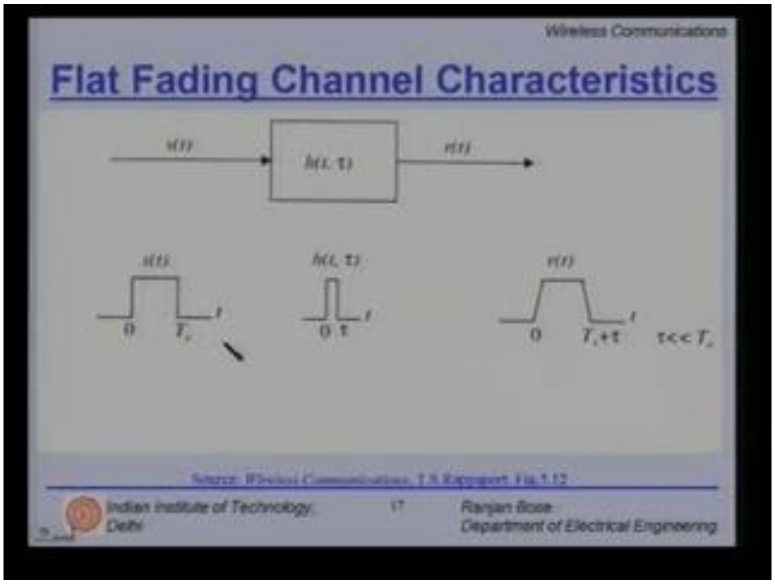

So flat fading channel characteristic, you have the signal this is your fading channel and this is the received signal, here s (t) in time domain looks like this and as we have seen we have a flat fading scenario. So h(t) is a narrower impulse response here and if you convolve these two you get r(t); in the frequency domain you have s (f), h (f) and multiplication gives you r (f) this is the frequency flat channel characteristic from the time and the frequency domain.

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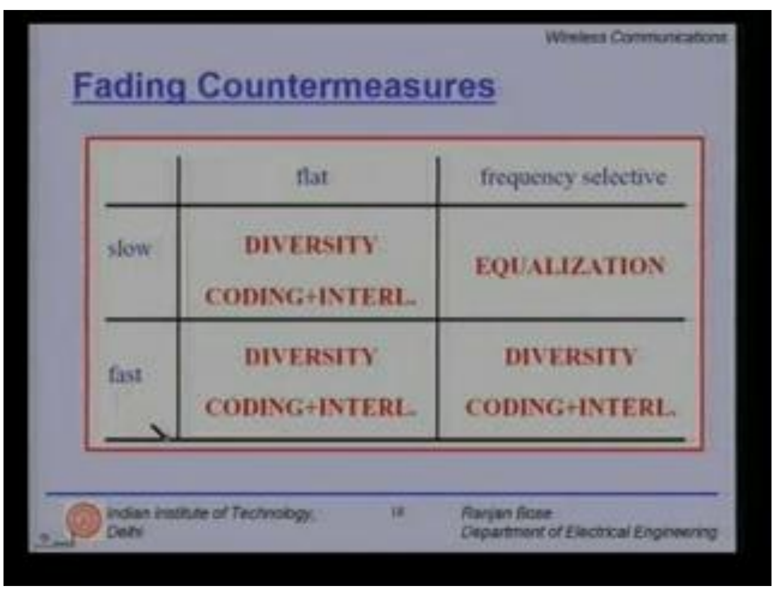

Now let us see how we fill in our matrix. So here we have on this axis frequency flat fading and frequency selective fading. These two depend on the coherence bandwidth of the channel. On this axis we have slow fading and fast fading this depends on the coherence time of the channel. Now if you look at the first location in this grid which is frequency flat slow fading channel there we can use diversity. We have seen the diversity works well for flat fading. Similarly even if it is flat but fast fading you still have the choice of diversity technique. On top of that you can also use coding and inter leaving. So inter leaving is a technique which we studied in previous classes where it causes a little bit of delay in the signal we have a memory where in read in row wise read out column wise, at the receiver read in column wise and read out row wise.

It is used to spread out the burst errors so both diversity techniques and coding plus inter leaving techniques will work for flat fading scenario. Clearly equalization will not be effective here, equalization becomes effective in a frequency selective environment. Equalization has been designed to overcome the distortions put in by the channel, is an inverse filter and if you have frequency selective but fast fading then I may not have the time to effectively track the channel and therefore I might have to resort to diversity and coding and inter leaving. So this is a handy reckoner which helps us figure out which of the three techniques or a tandem of them could be used for flat slow, flat fast, frequencies selective slow and frequency selective fast fading channels.

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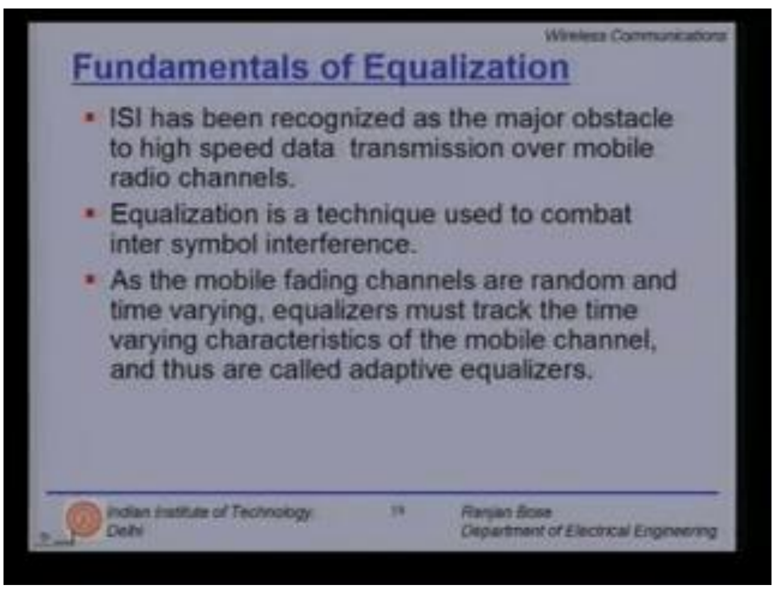

Now let us look carefully at equalization. We start off with fundamentals of equalization. We have learnt so far that equalization is used to eliminate or reduce the effects of inter symbol interference. ISI has been recognized as a major obstacle to high speed data transmission. Earlier in the lectures we have done a basic calculation for different kind of fading channels and how fast you can send data without equalization and we saw that the rates of pretty decimal. So if you have to go in for high speed data transmission, we have no choice but to use some kind of equalization technique.

Even GSM provides for equalization though the standard doesn't specify a particular type and the manufacturers are free to use some of their proprietary equalization techniques. Equalization as we know is used to combat inter symbol interference, as the mobile fading channels are random as well as time varying equalizers must track the time varying characteristics of the mobile channel and thus are called adoptive equalizers. This is also an interesting concept which is called blind equalization wherein it doesn't need a training sequence. It uses some property of the signal may be constant envelop or a constant amplitude. So as to guess the distortion being put in by the channel and undo those effects but the more common once are adaptive equalizers which work on the concept of a training sequence.

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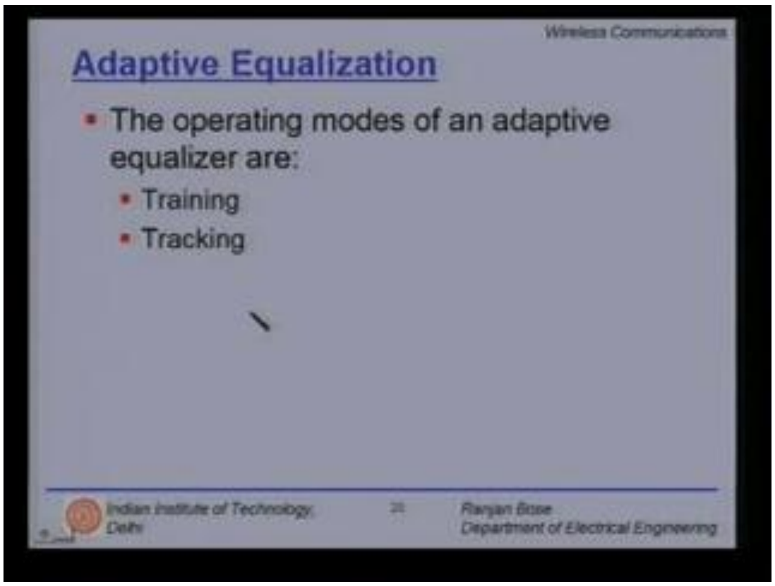

So let us look at adaptive equalizers. The two operating modes of an adaptive equalizers are training mode and tracking mode. The training mode proceeds the tracking mode, in the training mode we have to send a known sequence or a PN sequence in order to understand how the channel is, it's basically a method to measure the frequency response and then fix a weights accordingly in order to overcome the effects of channel and then is a tracking mode. If a channel changes fast then you have to resort to training mode again and then when you are happy with the estimate of the channel then you go back into the tracking mode and you keep switching between the training and the tracking mode.

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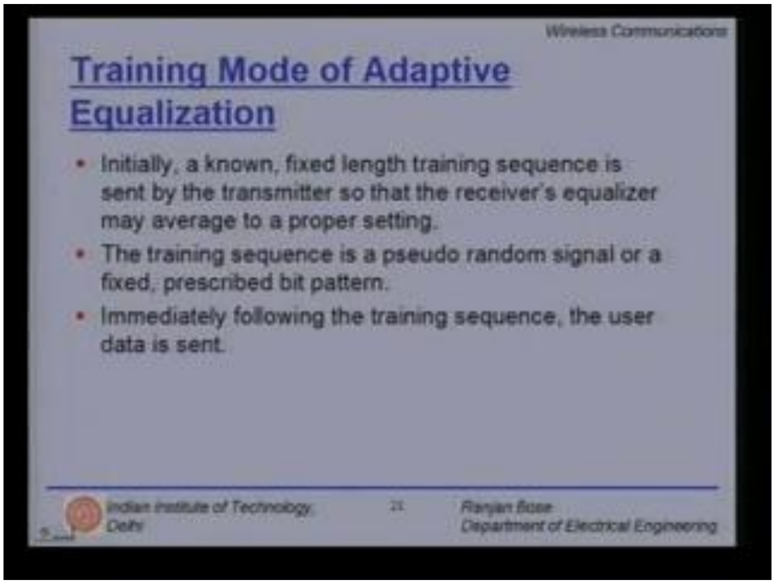

Let us talk about the training mode of adaptive equalization. What do we do in a training mode? Initially a known fixed length training sequence is sent by the transmitter so that the receiver's equalizer may average to a proper setting. The training sequence is usually a pseudo random sequence or a fixed known prescribed bit pattern usually put in the standard. So you know what you are sending. Immediately following the training sequence the user data is sent.

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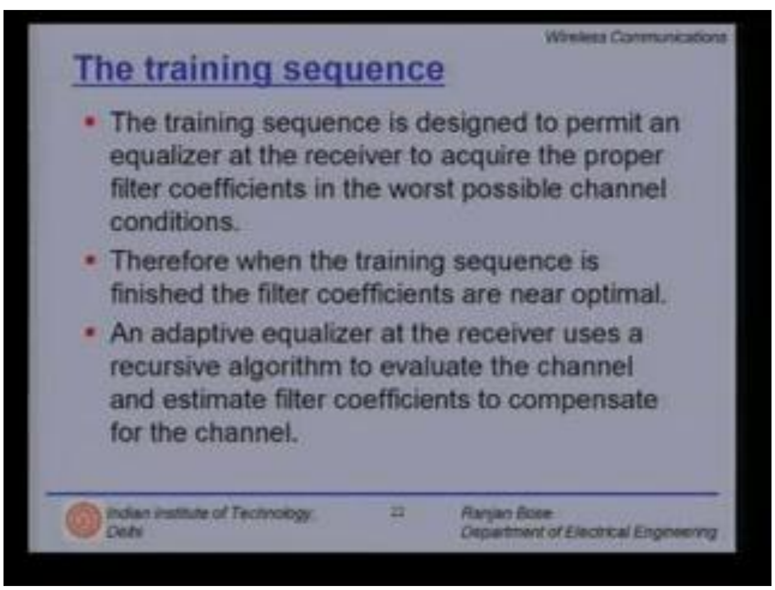

Now let us talk about the training sequence. The training sequence in an adaptive equalizer is designed to permit and equalizer at the receiver to acquire the proper filter coefficients in the worst possible channel conditions.

So it says a lot of things here, it talks about filter coefficients. So we will see in later slides that your equalizer could be like a finite impulse response FIR filter which has filter coefficients or it can look like a tab delay line with bits so we have to figure out what the weights should be but please note that the training sequence is kept in mind so that we can account for the worst possible channel conditions. What could be the bad channel scenarios? Maximum delay, deepest fades, maximum inter symbol interference. These are some bad conditions and when we design the training sequence we must keep in mind the worst case scenario, we should able to account for these worst case scenarios as well.

Therefore when training sequence is finished, the filter coefficients are near optimal. An adaptive equalizer at the receiver uses a recursive algorithms to evaluate the channel and estimate filter coefficients to compensate for the channel. So it has to be some kind of a recursive method which finally converges to the exact optimal value or close to the optimal value.

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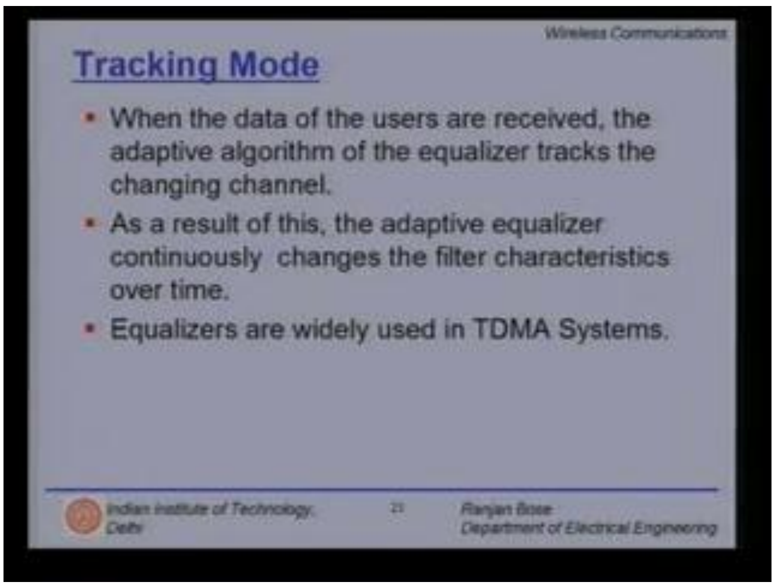

Now we move over to the tracking mode, what happens once you have been able to fix your filter coefficients? When the data of the users are received, the adaptive algorithms of the equalizer tracks the changing channel. Therefore we must have a notion of an error and it has to be possibly a feedback equalizer but that’s not the only kind of equalizer possible. So as a result of this the adaptive equalizer continuously changes the filter characteristics over time. So whenever you do an analysis of the filter coefficients you should have a time index associated with the weights of the filter. The weights of the filter changes continuously with time in the tracking mode. We never fix it and forget it because of the time varying nature of the wireless communication systems equalizers are widely used in TDMA systems.

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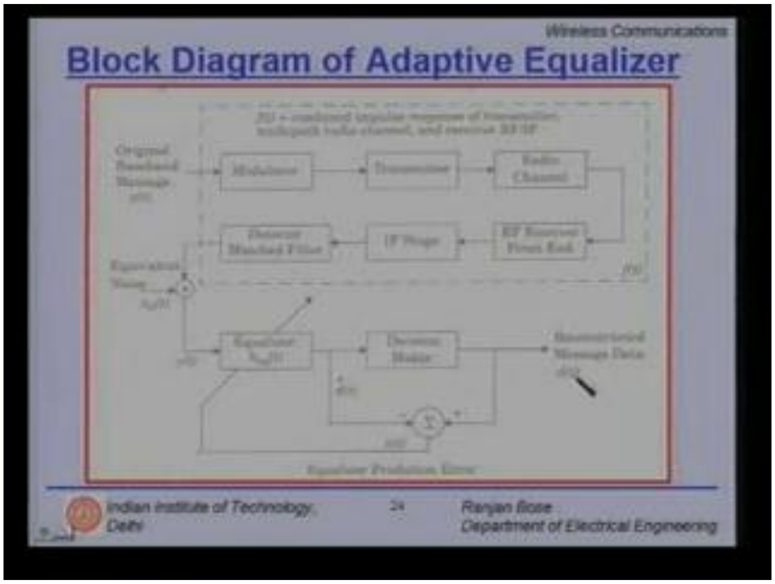

Let us now look at the block diagram of an adaptive equalizer. Let us say we have the original baseband message x of t, it passes through a modulator then through the transmitter over to the radio channel. Here noise and fading effects get added leading to a very distorted received signal. Here it goes through the RF receiver front end, passes through the IF stage and then detector in match filter. So your equalizer usually works either at the IF stage or at the baseband. So here is your equalizer, decision maker and error which kind of tracks and then finally you have the reconstructed message data d (t). In the training section this d (t) and x (t) should be equal because you know what your expected d (t) should be.

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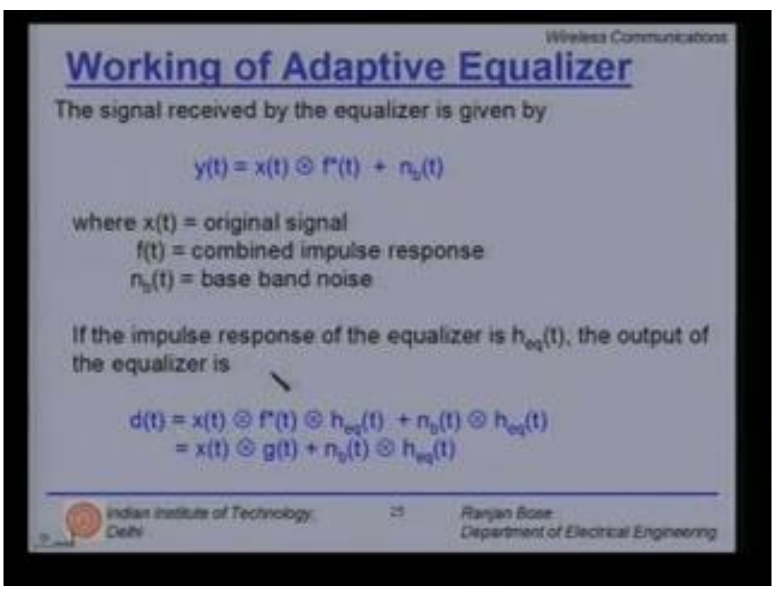

Let us now look at the working of an adaptive equalizer. The signal received by the equalizer is simply given by y (t), what is it, x (t) convolved with $f^*(t)$ plus $n_b(t)$. If you go back here f (t) is the combined impulse response of the transmitter multipath radio channel and the reviver RFIF here is the f (t). X (t) is what you sent so if you convolve this and then add your noise $n_b(t)$ you get y (t). This is the input to the equalizer as shown here y (t) equalizer input. In $n_b(t)$ the subscript b stands for the baseband noise, if the impulse response of the equalizer is $h_{eq}(t)$, the output of the equalizer is x (t) convolve with $f^*(t)$ convolved with h_{eq} (t) plus n_bt convolve with h_{eq} (t) you convolve y (t) with your h equalizer which can be written as x(t) convolve with another term g(t) plus $n_b(t)$ convolve with $h_{eq}(t)$, g(t) is $f^*(t)$ convolved with $h_{eq}(t)$. So $f^*(t)$ convolve with $h_{eq}(t)$ gives you g (t).

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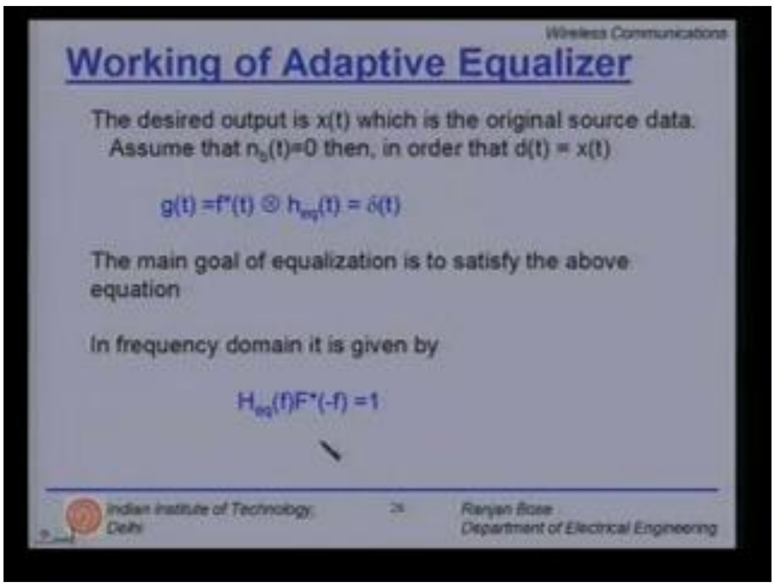

The desired output is x (t) which is the original source data. Assume that $n_b(t)$ zero for the time being in order that dt is equal to x(t). What is d (t)? If you go back here d (t) is the reconstructed message data and x (t) is the original baseband message. If you want these two to be the same in that case g (t) which I have already defined as $f^*(t)$ convolve with $h_{eq}(t)$ should be a delta function, delta (t) only then you will have d(t) same as x(t) in the options of a baseband noise. So the main goal of the equalizer is to satisfy this above equation. In frequency domain this same thing is given by $h_{eq}(f)$ F^* (-f) =1. The time domain and frequency domain implementation are the two basic equations that have to be satisfied for any equalizer to work effectively.

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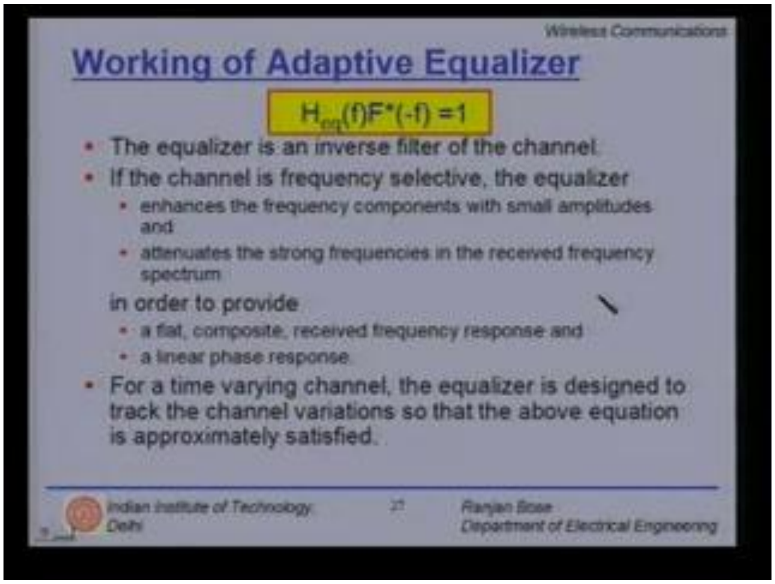

So we start from the frequency domain $H_{eq}(f)$ times $F^*(-f)$ =1. What does it take? It implies simply that the equalizer is kind of inverse filter of the channel. If the channel is frequency selective, the equalizer enhances the frequency components with small amplitudes working as an inverse filter and attenuates the strong frequencies in the received frequency spectrum. So as to obtain a flat, composite, received frequency response and on top of that a linear phase response. So that is the basic philosophy of working of an adaptive equalizer. It works as an inverse filter by enhancing the frequency components of small amplitude, attenuating the frequency components with larger amplitudes, in the same time keeping in mind that a linear phase response is obtained. For a time varying channel, the equalizer is designed to track the channel variations so that the above equation is approximately satisfied.

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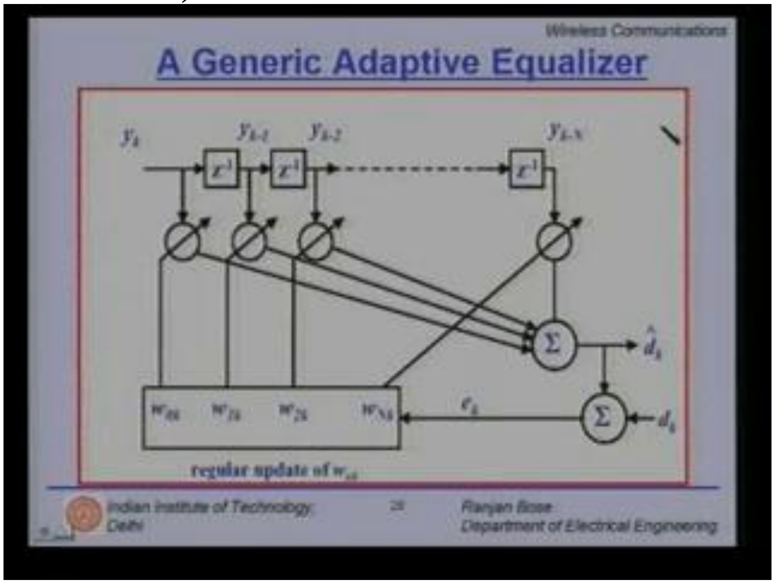

Now let us look at a generic adaptive equalizer. How does it look like, why does it work the way it is designed to work? Suppose your input is y_k it passes through something that looks like a tapped delay line so delay 1, delay 2 so and so forth. So there are end points and then there are n+1 taps. Here each of this tapped value is multiplied with the weight, given by this weight vector w_0 k, w_1 k so and so forth up to w_nk. So they are n+ 1 weights. Please note besides this w_0, w_1 and so and so forth up k denoting the time index has been put. We have observed earlier that these weights must change with time then we have the estimate d_k hat, we have the d_k there is an error and this error is used to update the weights. How we do it? There are various techniques to do it we'll talk about those techniques. So this is the generic block diagram of an adaptive equalizer.

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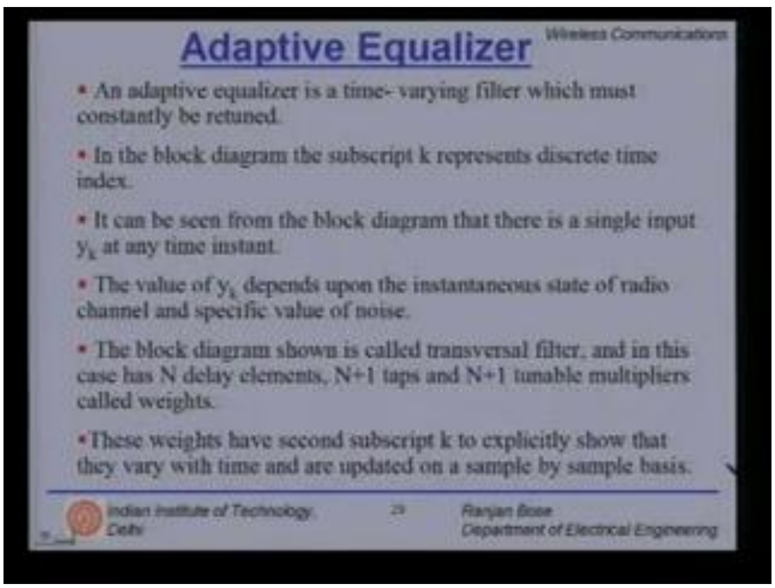

So what is an adaptive equalizer in principle? An adaptive equalizer is a time varying filter which must constantly be retuned. In the block diagram the substitute k represents the discrete time index, here k is the discrete time index. It can be seen from the block diagram that there is a single input y_k at any time instant here. The value of y_k depends on the instantaneous state of the radio channel and the specific value of the noise. The block diagram shown is called the transversal filter and in this case it has n delay elements, N+1taps and hence N+1 tunable multipliers which we will called weights. So here is my weight vector. These weights have a second subscript k as pointed out earlier to explicitly show that they vary with time and are updated on a sample by sample basis or in some cases for the whole block. So either you work sample by sample or you work with the whole block.

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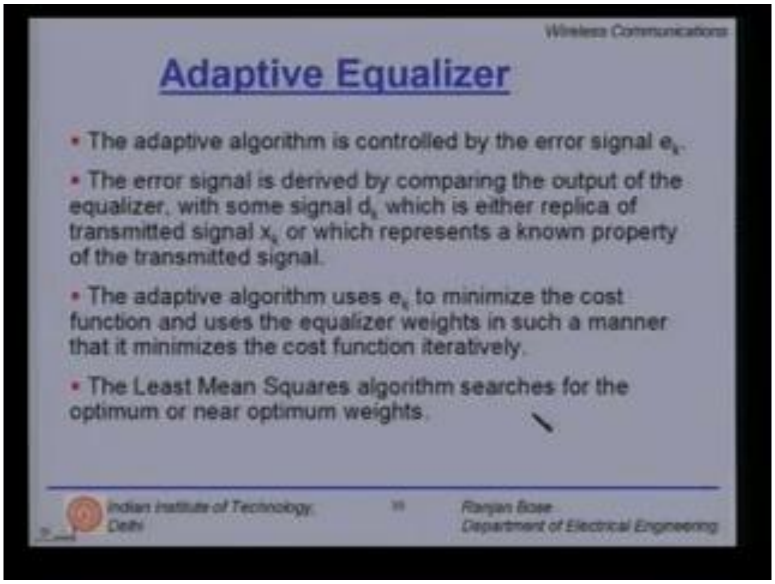

The adaptive algorithm is controlled by the error signal e_k. The error signal is derived by comparing the output of the equalizer with some signal d_k which is either replica of the transmitted signal x_k or which represents a known property of the transmitted signal. So either we have the luxury of knowing what x_k is or with some property of the transmitted signal. The adaptive algorithm uses e_k to minimize the cost function and uses the equalizer weights in such a manner that it minimizes the cost function iteratively, this is important. How many iteration it will it take to converge those depend on the particular choice of the algorithm. The least mean square or LMS algorithm searches for the optimum or near optimum weights, this is one of the popular methods.

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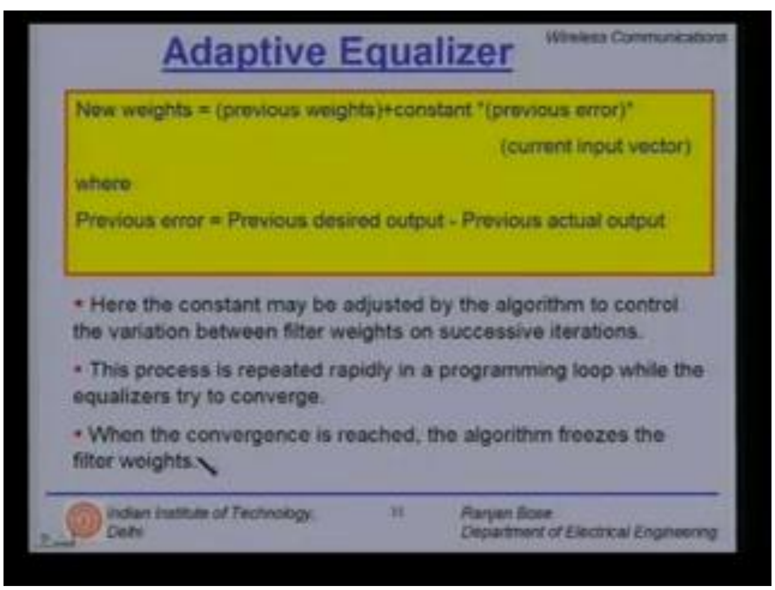

Now let's see how the weights are updated. New weights is previous weight plus a constant times previous error times current input vector where previous error is previous desired output minus previous actual output. So very simple scheme for updating the weights, the constant shown here may be adjusted by the algorithm to control the variation between filter weights on successive iteration. This process is repeated rapidly in a programming loop while the equalizer tries to converge. When the convergence is reached, the algorithm freezes the filter weights. This is the basic way how the adaptive equalizer works, how the weights are updated. There is a recursive algorithm trying to freeze the filter weights.

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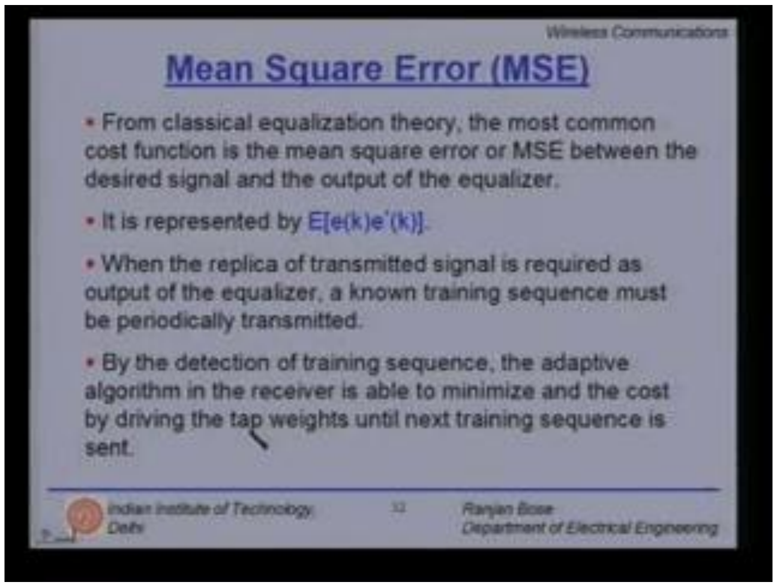

Let us talk about the mean square error, from classical equalization theory the most common cost function is the mean square error or MSE between the desired signal and the output of the equalizer. The mean square error is represented by the expected value e_k times e^*k. When the replica of the transmitted signal is required as output of the equalizer, a known training sequence must be periodically transmitted, we know this already. By the detection of the training sequence the adaptive algorithm in the receiver is able to minimize and the cost function by driving the tap weights until the next training sequence is sent. When is the time to change the training sequence or go to the training mode? It depends on how fast the channel is changing.

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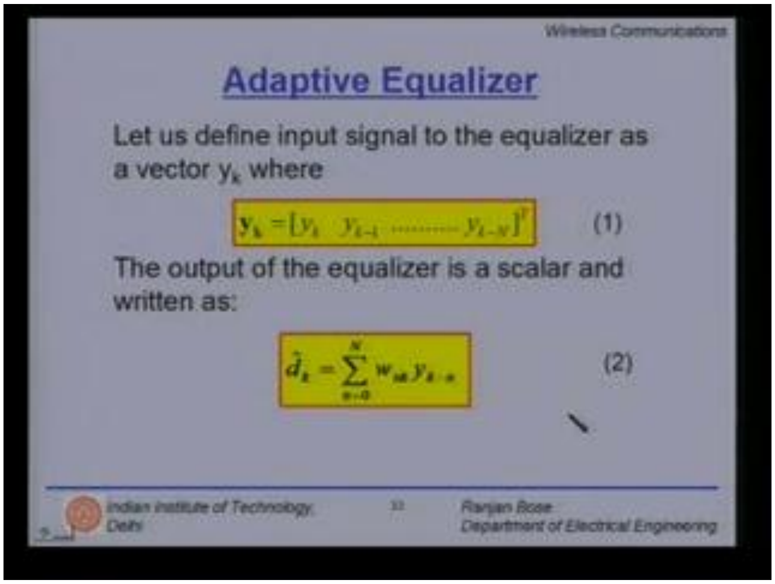

Let us now quickly look at the basic mathematics involved with an adaptive equalizer. Let us define the input signal to the equalizer as a vector y_k where y_k in bold phase is y_k y_{k-1} up to y_{k-n} transpose say it's a column vector. The output of the equalizer is a scalar and is written simplify by d_k hat is equal to summation so n=0 to N so there are n+1 terms w_nk the weights multiplied with the appropriate delayed version of the input signal comes on the basic block diagram. The weight vector can be written as fallows w_0k w_1k and so and so forth up to w_Nk transpose. Using the previous two equations we have d_k that is nothing but y_k transpose time w_k is equal to the w_k transpose y_k.

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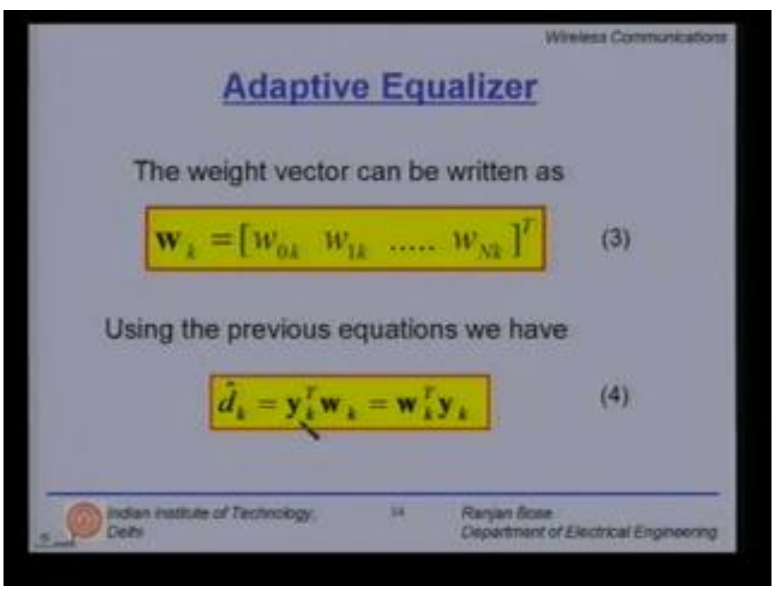

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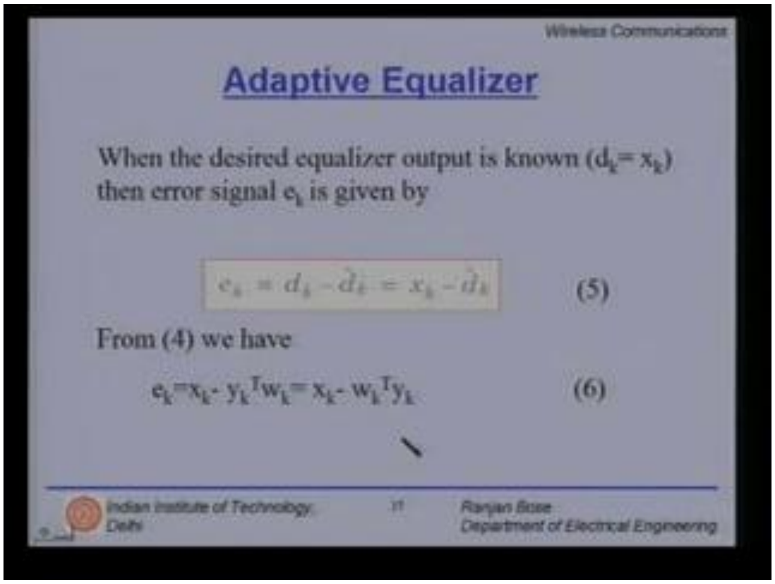

When the desired equalizer output is known in the training phase d_k is nothing but x_k then the error signal e_k is given by d_k minus d_k that the estimate of d_k but d_k is x_k so x_k minus d_k hat. The equation number 4 here we have the error e_k is equal to x_k minus y_k transpose w_k is equal to x_k minus w_k transpose y_k. Now to compute the mean square error, the mod of e_k squared at the time instant k must be taken. So equation 6 is squared to obtain e_k square and you square the right hand side 2 to obtain equation number seven and now when you take the expected value of e_k squared over k which in practice amounts to computing time average yields the following equation. Expected value x_k squared plus w_k T, we assume this is out of the expectation operator expectation over time because we expect in the weights of converged E [y_k y_k transpose].

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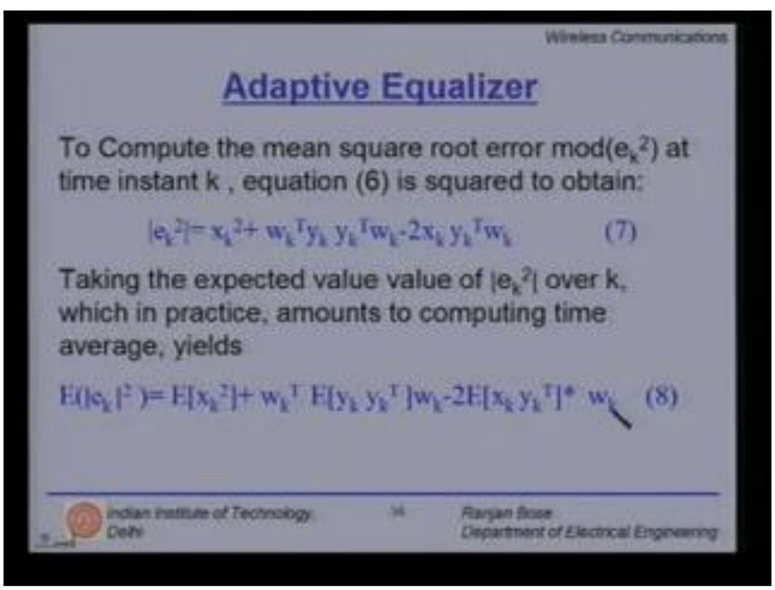

So this is what we measure so it is easily available to us, w_k minus expected value x_k y_k transpose complex conjugate w_k.

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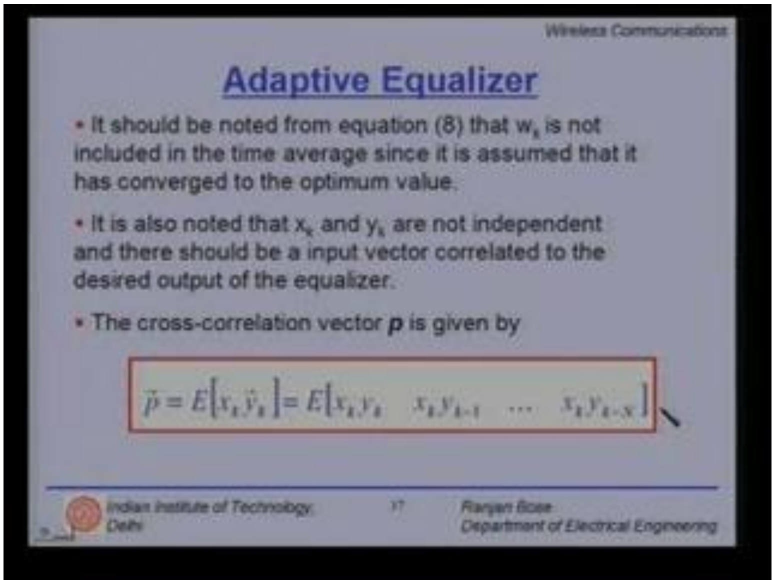

It should be noted from equation 8 that w_k is not included in the time average. Since it is assumed that it has converged to the optimal value. It is also noted that x_k and y_k are not independent, it was fairly easy to simplify the equation and there should be an input vector correlated to the desired output vector of the equalizer. So we define the cross correlation vector p as follows which is nothing but the expected value of x_k y_k given by this.

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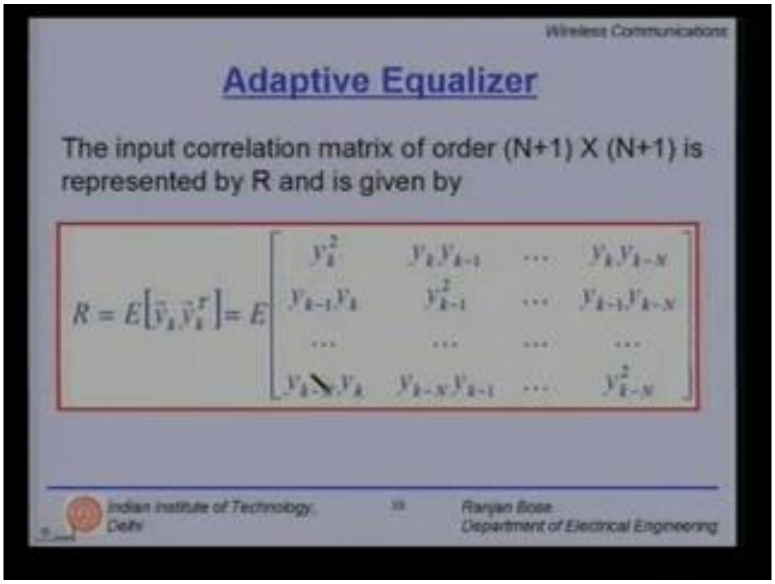

Let us define the input correlation matrix of the order N+1 by N+1 and lets represent it by R, R is nothing but the expected value of y_k y_k transpose and given as follows. Notice the principal diagonal, it is y_k squared y_{k-1} squared and so and so forth. So it is the power at the kth instance, k-1th and so on so forth up to k-Nth instance. Note there is an expectation operator outside and these are the cross correlation between the tapped delay points.

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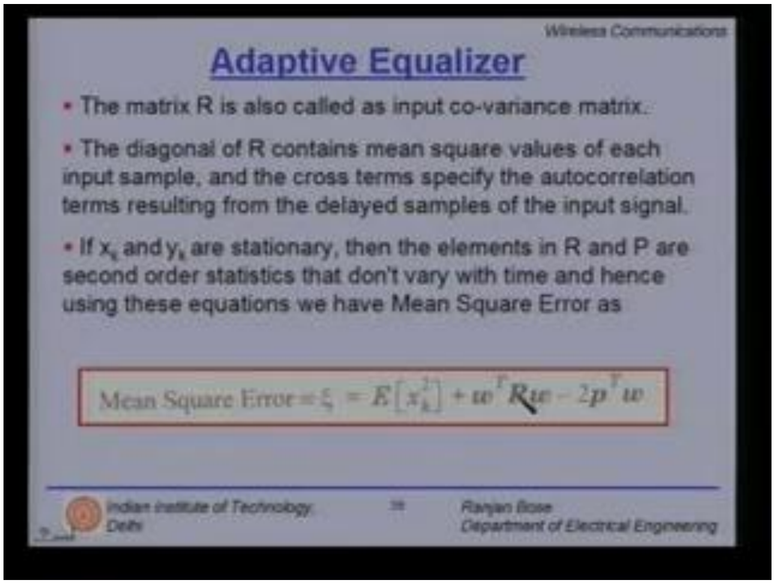

So the matrix R is also called as an input covariance matrix, the diagonal of R contains mean square values of each input sample and the cross terms specify the auto correlation terms resulting from the delayed samples of the input signals as observed before. Now if x_k and y_k are stationary then the elements R and P are second order statistics that do not vary with time and hence using these equations we have the mean square error given as follows, mean square error is equal to expected value of x_k squared plus weight vector transpose or weight vector minus 2 p transpose weight vector. This is the expression we have for mean squared error. We have to minimize the mean squared error so as to obtain the optimal weight values. So please note r can easily be computed if you know the y, y_k, y_{k-1} up to y_{k-n}.

So let us now summarize today's lecture. Today we talked about the use of equalization, diversity and coding. We saw that fading channels cause major impairments in the modulation technique performance simply because of getting into deep fades and the fade duration that we stay within the faded region. Equalization is a very simple in expensive method to overcome the effects of fading but it works only when there is a frequency selective environment. Equalization is used for inter symbol interference.

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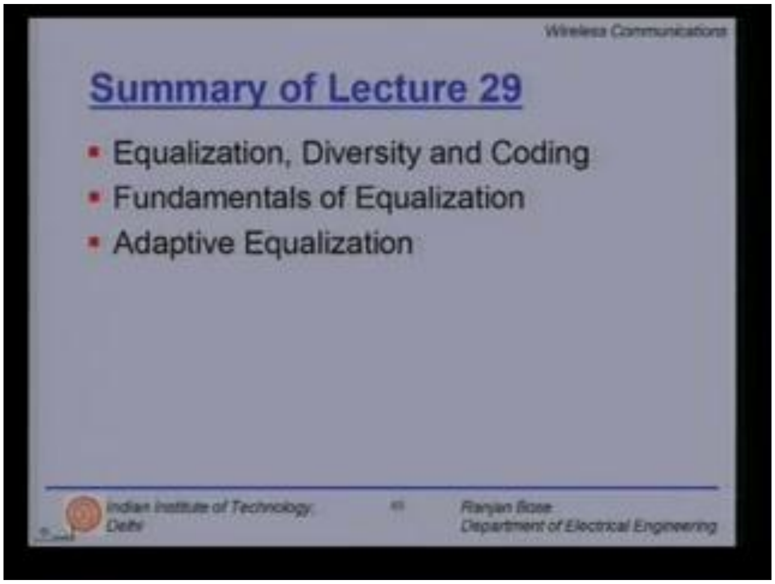

Diversity on the other hand can be used in most of the cases and we use any one kind of diversity or a combination of diversity. We saw there is a series of possibilities for example you have the space diversity, the time diversity, the frequency diversity, the polarization diversity, the angle diversity or even the code diversity. The third method is also popular to overcome the channel impairments put in by fading channels, it is called coding. Coding basically requires you to add redundant bits in a known manner in the transmitted signal so that at the receiver you can detect and probably correct the errors.

Coding is often used with inter leaving in fading environments. There is an advanced technique which combines coding with modulation called trellis coded modulation as an example or a block coded modulation wherein we use the effects of coding together with modulation because the objective of modulation is also to have a good bit error rate performance and same is the objective of channel coding. You can combine them together to use coded modulation. We then moved over to the study of the fundamentals of equalization, why it works, what is the basic structure of an equalizer?

Specifically we talked about adaptive equalization and how a weights of the equalizer may be adjusted depending upon the input signal. We saw that equalizations work in two modes, the training mode and the tracking mode. The weights however always have a time index that is the changed continuously with time, fading channels and mobile channels in particular provide you with time varying channels and hence your equalizer must be fast enough and should converge to the optimum value at the earliest. In the next few lectures we will look at more details about equalization, diversity and coding. We will conclude today’s lecture here.