Wireless Communications
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Lecture No. # 31
Equalization and Diversity Techniques for Wireless Communications (Continued)

Welcome to the next lecture on wireless communications. Today we will look at diversity techniques; the outline for today's talk is as follows.

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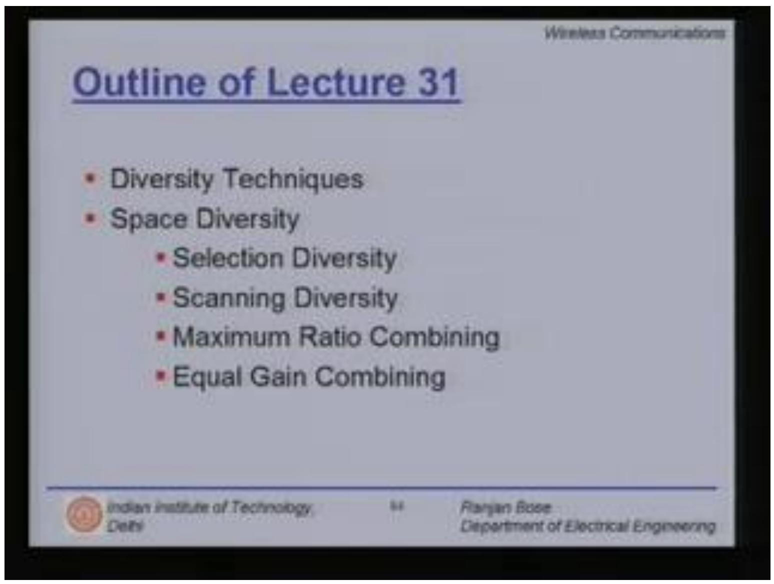

First we will look at the different kinds of diversity techniques which will form an alternative to equalization techniques for fading channels. We will learn the diversity techniques are indeed an inexpensive as well as effective way to overcome the effects of fading. Specifically we'll look at space diversity today and there are 4 important kinds of space diversity techniques. The selection diversity, the scanning diversity, the maximum ratio combining method and the equal gain combining method; of these the last two are more popular. So we will have a look at these and then look at the probability of error expressions once you use these kinds of diversity techniques. So this is the agenda for today's talk, of course we will begin by summarizing what we have learnt so far.

So let us recap. In the previous lectures we talked about the brief survey of equalization techniques followed by the study of linear equalizers. Then we moved over to the domain of nonlinear equalizers. We looked at the decision feedback equalizer or DFE followed by the maximum likelihood symbol detection and then the maximum likelihood sequence estimation. So these techniques were studied under the head of non-liner equalizers.

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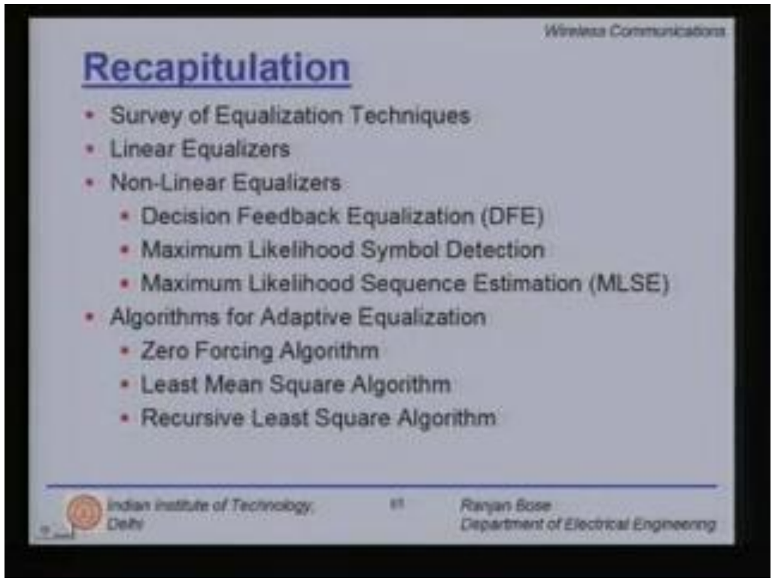

We then observed that there were several techniques and algorithms used to obtain the weights of the equalizers, these are the zero forcing algorithm, the least mean square algorithm and the recursive least square algorithms which is the faster of the lot. LMS algorithm is well known but it takes its own time to converge. We also made an observation that all of these algorithms could be used equally well for the linear case as well as the nonlinear equalizers. We however studied their cases for the case of linear equalizers. So this is in brief what we have learnt in the previous lectures.

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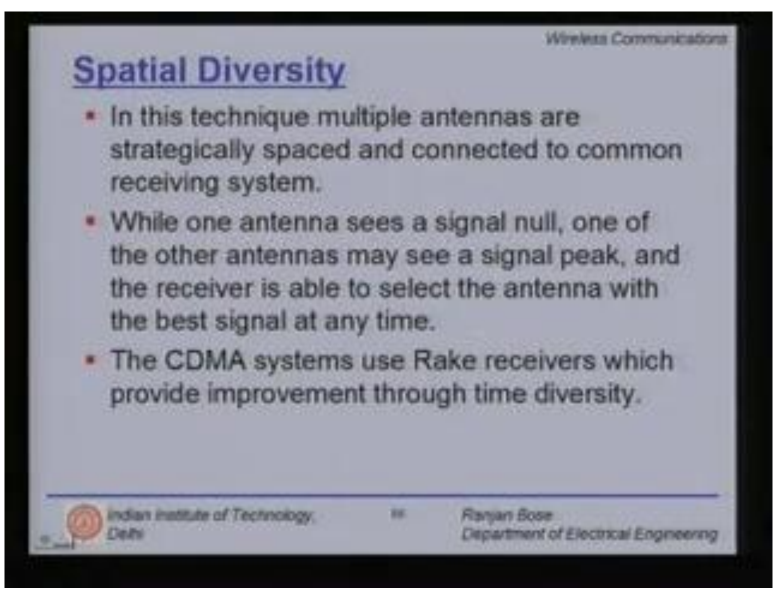

Now let us focus are vision today on spatial diversity and we will definitely look at different kinds of spatial diversity in detail, we also make an observation here that there are various kinds of diversities of which we will only focus on spatial diversity today. What is basically spatial diversity? In this technique multiple antennas are strategically spaced and connected to a common receiving system. Please note here we are talking from the perspective of receiver alone. Most of today's discussion will center around diversity at the receiving end, however it is also possible to have transmit diversity but receiving diversity is a much more simpler and hence inexpensive way to carry out the diversity scheme.

The basic philosophy is as follows for spatial diversity; while one antenna sees a signal null that is it is sitting on a fade, one of the other antennas may see a signal peak provided the signals are not fading coherently. So the receiver is able to select the antenna with the best signal at any time. This is the basic philosophy of spatial diversity. The CDMA systems use rake receivers which provide improvement through time diversity.

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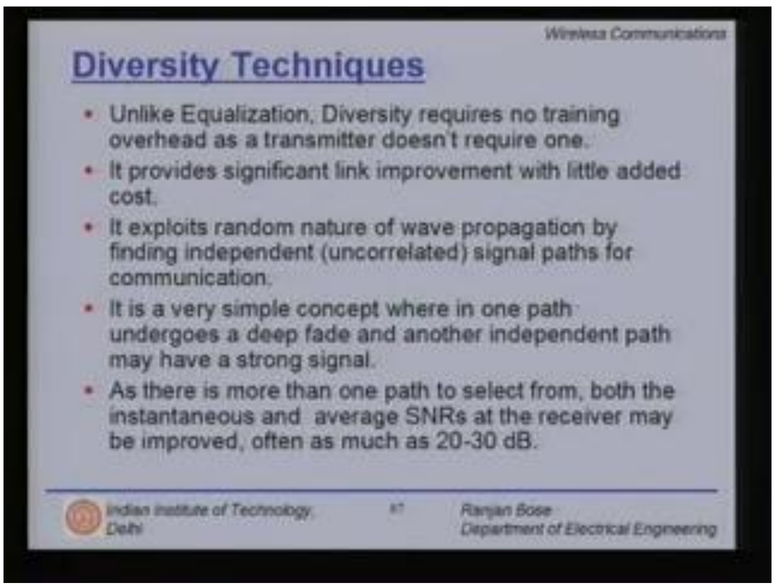

Now just let us revisit and refresh our memories about diversity techniques. Unlike equalization that we have learnt in previous classes diversity requires no training overhead as the transmitter doesn't require one. So there is no time lost in training and tracking then diversity provides a significant link improvement with very little added cost. All you need is invest once in multiple antennas and some receiver logic. Diversity techniques exploit random nature of wave propagation by finding independent or most likely uncorrelated signal paths for communication.

It is a very simple concept wherein one path undergoes a deep fade and another independent path may have a strong signal. Now in practice it is really difficult to find independent fading components but you can have uncorrelated or less correlated components, in those case also diversity techniques will work; of course we must find a smart technique to combine the signals received from different antennas.

As there is more than one path to select from both the instantaneous and average SNR's at the receiver may be improved. Today we will look at the average SNR analysis for two different kinds of diversity techniques focused on spatial diversity. The improvement may be as good as 20 to 30 dB. In previous lectures we have observed that fades can put you down by 20 to 30 dB hence spatial diversity can easily overcome the effects of deep fades.

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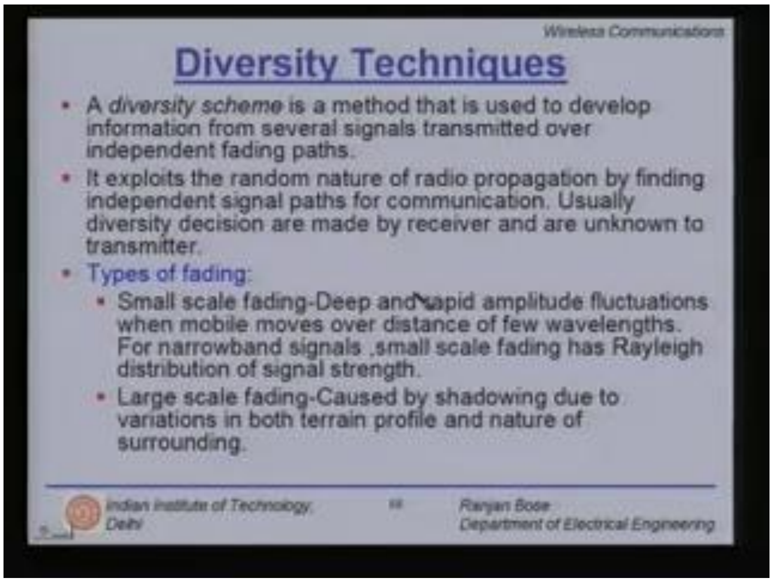

Again revisiting concept of diversity, a diversity scheme is a method that is used to develop information from different signals transmitted over independent fading paths. Now you could either have it in space or time or angle or polarization or frequency or code. It exploits the random nature of radio propagation by finding independent signal paths for communications. Now please note that earlier we have observed the two broad classes of fading, one is the small scale fading and the other is of course the large scale fading. In small scale fading deep and rapid amplitude fluctuations occur when the mobile moves over a distance of a few wavelengths only.

As opposed to large scale fading caused by shadowing due to variation both in terrain profile and the nature of surroundings. Now if we are trying to device diversity schemes to overcome the effects of fading, we must understand what kinds of fading effects are actually being caused and how to overcome them. If we device a method to overcome small scale fading then clearly the spacing between the antenna elements should be of the order of lambda by 2 or lambda, we do not need very large spacing. However if you are trying to overcome large scale fading it's a different matter altogether.

We have to increase the antenna spacing. Now there are physical limits to how much you can space the antenna elements. For example clearly if I have a handset I cannot have too many antennas spaced by lambda by 2 if I am working on a 1.8 gigahertz range or 900 megahertz range. So we must understand what kind of fading we are trying to counter at and device a special diversity scheme accordingly.

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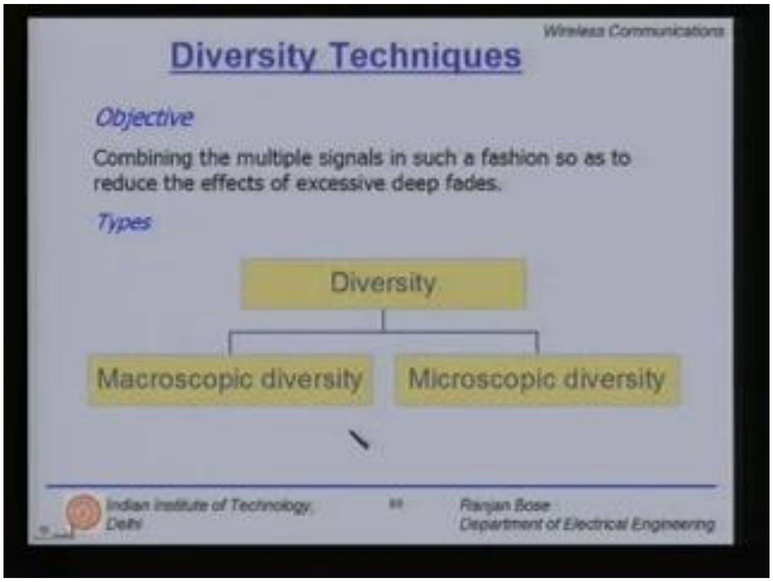

Now let's have a brief classification of diversity schemes. So we already know this objective combining multiple signals in such a fashion so as to reduce the effects of excessive deep fades. Types: you can have macroscopic diversity for large scale fading and microscopic diversity for small scale fading. Both this kinds of diversity techniques are used today.

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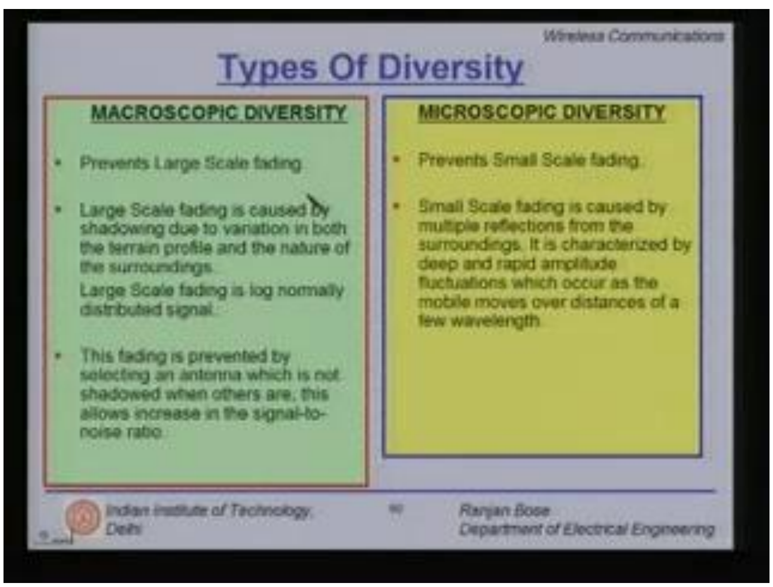

So let us now look into greater detail about the types of diversity. First the macroscopic diversity, it prevents large scale fading and shadowing effects. Large scale fading as we know is caused by shadowing due to variations in both the terrain profile and the nature of surroundings.

So if I move from the dense urban environment to a hilly terrain or a wooded terrain then my macroscopic diversity techniques would change. We also have seen that the large scale fading is modeled log normally. This fading is prevented by selecting an antenna which is not shadowed when others are very simple philosophy of fading and diversity. So the objective is to ensure that not all of the antennas are sitting in the shadowed region. This allows increase in the signal to noise ratio. Clearly if in the macroscopic diversity requirements if the shadow region is large then most likely my antennas have to be placed on different base stations. So I have a separation over several kilometers between different antennas, it’s a possibility.

Now let us look at the other aspect which is the microscopic diversity. The objective is to prevent small scale fading, in fact small scale fading is the more serious of the threat because it causes deep fades in the case of Rayleigh fading. Small scale fading as we know is caused by multiple reflections from the surroundings. It is characterized by deep as well as rapid amplitude fluctuations which occur as the mobile moves over a distance of a few wavelengths only. Now how do we prevent this fading by diversity? This fading is prevented by selecting an antenna which gives a strong signal that mitigates this small signal fading effect. So the multiple antenna elements must be spaced of the order of wavelength apart only to overcome the microscopic fading effects. Based on an understanding of the different kinds of fading I can have different kinds of diversity both spatial in nature.

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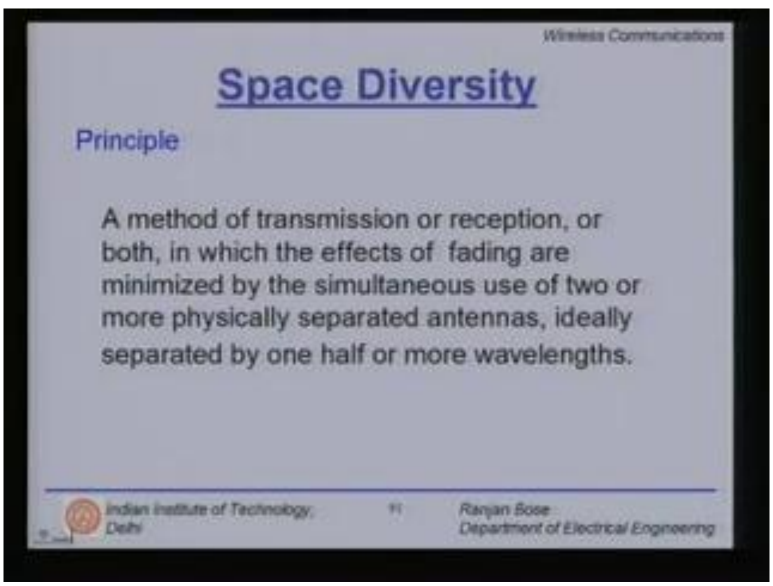

So let us now go deeper into space diversity also known as spatial diversity. What is the principle behind spatial diversity? It’s a method of transmission or reception or both in which the effects of fading are minimized by the simultaneous use of two or more physically separated antennas, ideally separated by one half or more wavelengths. So there are lot of things being said in this one sentence, first of all please note it is a method both for transmission and reception. So spatial diversity though more popular at the receiver can also be employed at the transmitting end, you can have a transmit diversity and receive diversity. Clearly it is a question of creating independent uncorrelated fading paths.

I can also have a combination of transmit and received diversity and the separation depends on several things, wavelength is one of them so which frequency you're operating at will also determine how much spacing you can have between the antenna elements. On top of that another thing that effects this separation of antennas is how cluttered is your environment. If you are in a highly dense cluttered environment with lot of reflections then this lambda by 2 separation works well. However if you move into a less cluttered environment then you need to separate your antennas more and more.

In fact a mobile which is normally close to the ground level and of course usually present in a dense reflection environment doesn't require antenna separations by more than lambda by 2 but if we want to have receive diversity at the base station where there are hardly any reflectors around, very few you need to separate them by ten's of wavelengths. So it's not just the wavelength that it decides how far the antenna elements must be separated from each other. This wave length is one parameter as well as the clutter around the receiver system.

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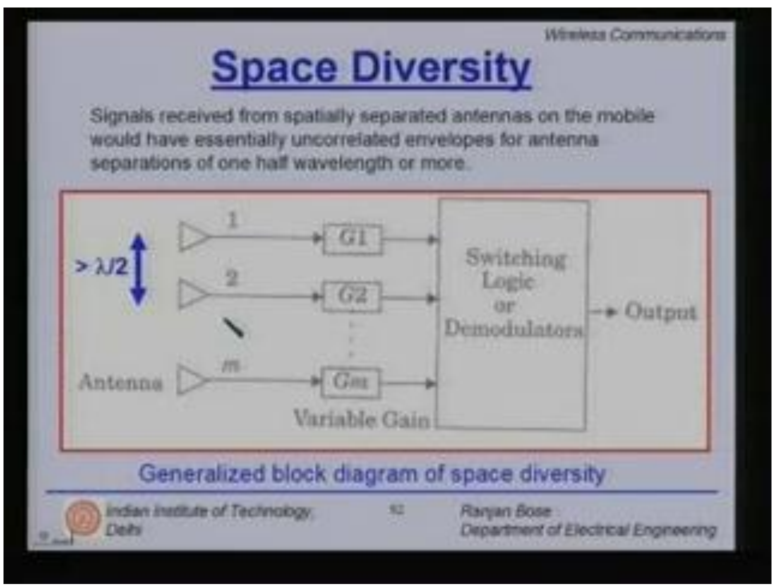

Now let us look at the generalized block diagram of space diversity. So signals received from spatially separated antennas on the mobile would essentially have uncorrelated envelopes for antenna separations of one half wavelength or more, that's the assumption. So what you have is antenna element 1, 2 so and so forth till m and then you have some kind of a variable gain and if you are just choosing one of them then one of the gains will be one and the others will be zero or an appropriately scaled value and then there's a logic which one to choose and how and then there is an output. This is a generic structure, it can have several of the possible kinds of spatial diversity included in this diagram. Now what is important here is the separation because the whole thing works provided the signal in element one is fairly uncorrelated to signal in element two and so and so forth till antenna element m. So this should be greater than lambda by 2 provided you are in a cluttered environment.

Question:
Sir in the cluttered environment is there any restrictions in the number of antennas ((00:17:08 min)) to this system. The question being asked is in a cluttered environment is there any restrictions on the number of elements antennas whether 1, 2, 4, 8 or 100, how many antennas can we have? So let us look at it like this. Clearly if you are having more and more antennas your chances of getting a better and better performance will improve, by how much we will talk about it analytically soon. Clearly if you have more diversity you will have an improvement in performance but as we will go along we will see that there is a rule of diminishing returns.

So when you go from 1 to 2 the improvement is drastic, when you go from 2 to 3 yes there will be an improvement but not so much, from 3 to 4 of course there will be an improvement but marginally more and so and so forth. So beyond a certain number of antenna elements your improvement will not match up to the money you put in to put more number of antennas, so there will be a tradeoff.

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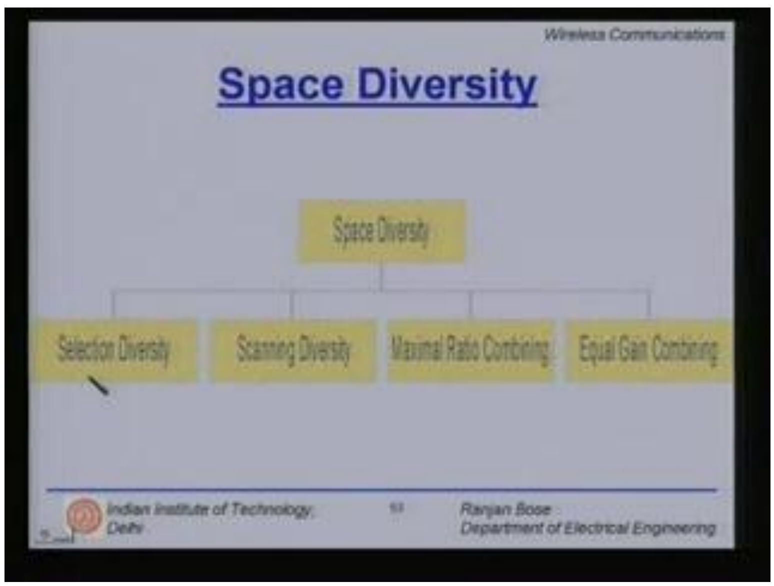

Now let us classify the different kinds of space diversities that are possible and are usually employed in wireless communication systems. So the first one is the selection diversity and then another kind of space diversity is scanning diversity. Then the third kind is the maximal ratio combining is one of the popular methods of space diversity and then there is the equal gain combining. So the four kinds of spatial diversities that we'll study today are selection, scanning, MRC or maximum ratio combining and equal gain combining; of these maximum ratio combing is the optimal way.

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So first let us look at selection diversity. What is it, why do we use it? The basic principle of selection diversity is as follows; you select the best signal among all the signals received from different branches at the receiving end period. So the objective is to choose the maximum of the received signal values in the receiver systems with multiple antennas.

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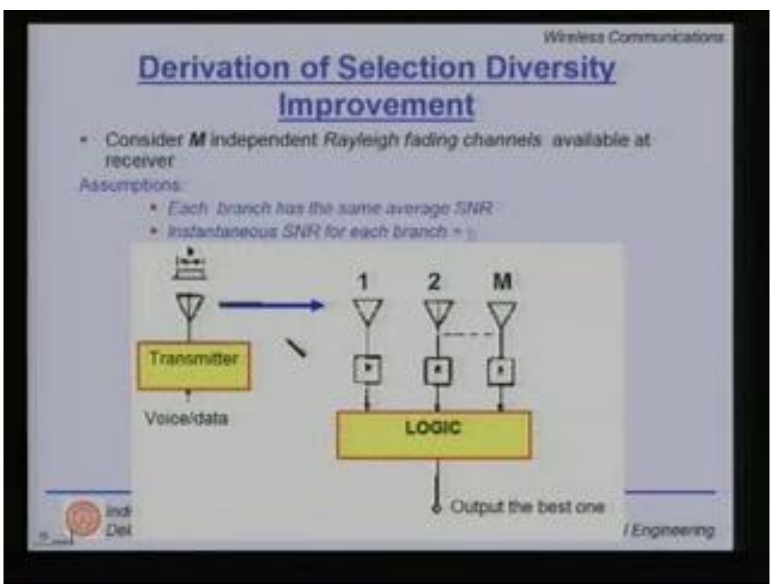

Here is the basic block diagram, so let us consider M independent Rayleigh fading channels available at the receiver, so I have ensured my separation of antenna elements is such that they are fairly independent so I have 1, 2 and so and so forth till M then there’s a transmitter.

So voice or data is being sent through the transmitter and I have M independently fading paths. Here we have a logic which picks up the maximum signal strength from any one of them. So it only chooses one of the branches whichever branch is offering you the maximum received signal path. Assumptions: each branch has the same average SNR, we are making these assumptions right now to do some basic mathematics to predict what will be the effective SNR if you have M receive antennas or what is the effective probability of error if you have M independent fading paths.

So each branch has been assumed to have the same average SNR, it's not a bad assumption because they're approximately the same distance from the transmitter. Instantaneous SNR for each branch is denoted by gamma$_i$. So it will be wise to have another parameter gamma bar which will be the threshold which is required to work at it. So we'll have to find out can we meet a certain threshold requirement even though the average SNR's in each of the branches is lower.

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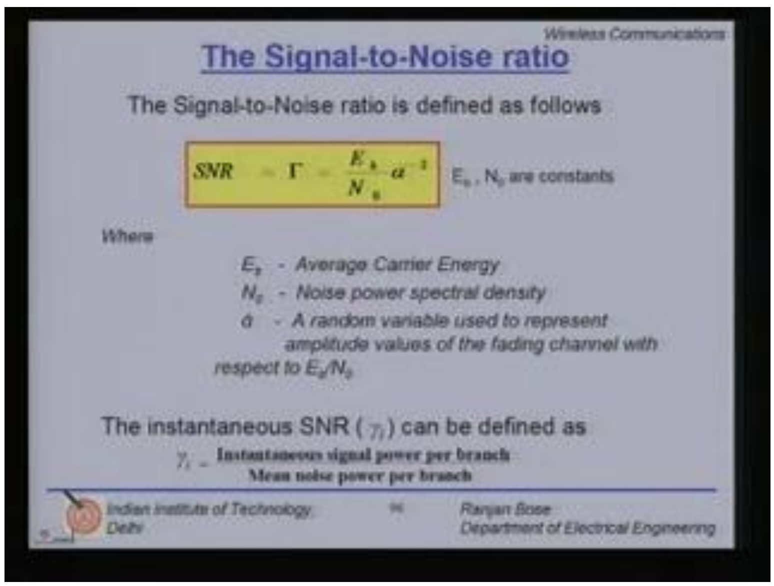

Let's talk about the signal to noise ratio aspect. The SNR is defined as follows; we define it by capital Gamma is E_b over N_0 alpha bar squared where alpha is the fading multiplicative constant here E_b and N_0 are considered constants. E_b is the energy per bit, N_0 is the noise power. Here alpha is a random variable used to represent the amplitude value of the fading channel with respect to E_b over N_0. If you go by this definition then the instantaneous SNR gamma$_i$ can be defined as instantaneous signal power per branch over mean noise power per branch. So this is how we defined our gamma here. So we need to derive our expressions for probability of error based on this value of gamma and gamma$_i$.

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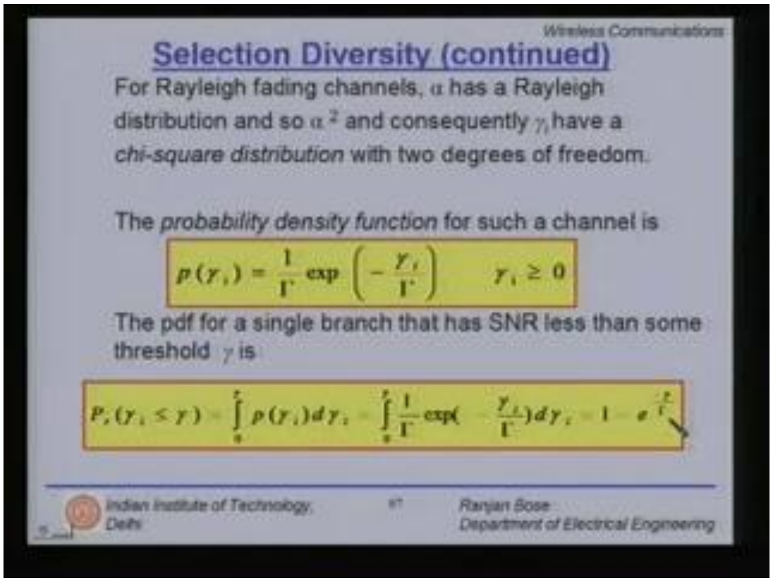

So continuing with selection diversity which tells us to pick the maximum of the branches. For Rayleigh fading channels clearly alpha has a Rayleigh distribution and so alpha squared and consequently gamma_i have a chi squared distribution. So if alpha has Rayleigh distribution, it is well known that alpha squared will have a chi squared distribution with two degrees of freedom.
The probability density function for such a channel we know is this one and a pdf for a single branch that is in SNR less than some threshold gamma. So we have started defining a threshold gamma here is given by this following. Probability that gamma_i is less than gamma is given by the pdf of gamma_i integrated from zero to gamma because gamma is greater than or equal to zero and if you integrate this expression you get 1- e raise to power minus gamma over capital gamma. So this is actually probability that gamma_i is less than a threshold value gamma.

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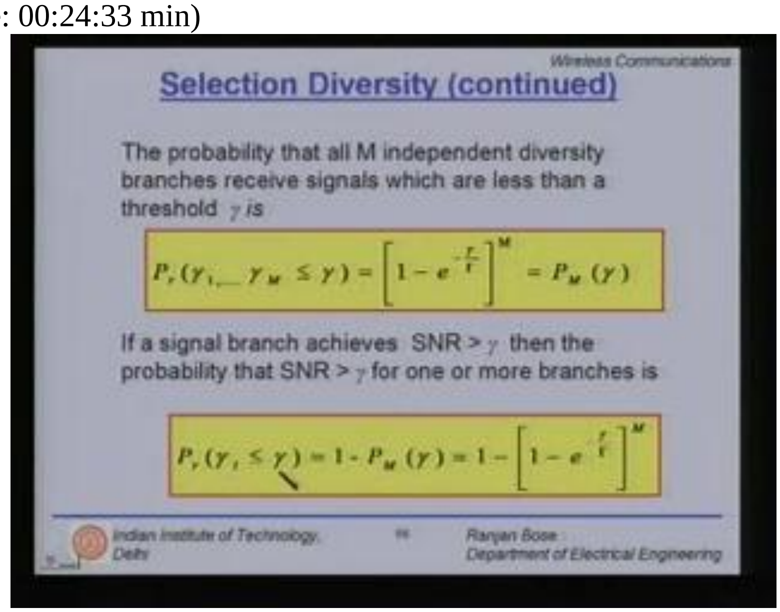

The probability that all M independent diversity branches receive signal which are less than threshold gamma. We are trying work around it, we have to find out when will our diversity scheme help us, for that we find out the worst case scenario when it will not help us. It will not help us when all the independently fading branches have the signal strength below this required signal strength threshold gamma and then one minus that is the probability that it'll fade. So if you consider the probability that all the M independently faded diversity branches receive signals below gamma is gamma$_1$ less than gamma, gamma$_2$ less than gamma and so and so forth since the independent events we have a multiplication here. So it is just simply raise to power M is equal to P_M gamma, this is kind of a cumulative distribution function, all of them are less than gamma.

Now if a signal branch achieves SNR greater than gamma then the probability that SNR is greater than gamma for one or more branches is simply given by one minus this quantity. Here all of the branches are less than gamma so the probability that one or more branches have the signal strength greater than gamma SNR is equal to one minus, here bracket open 1 - e raise to power gamma over capital gamma with the negative sign raise to power M. So this quantitatively gives us a very good feel as how increase in M will improve your probability of error or at least the required average SNR question. [Conversation between Student and Professor – Not audible ((00:26:38 min))] In this expression, yeah. P_r gamma greater than here lets go over it. So the probability that gamma$_i$ is less than equal to gamma, it should be the other way around, thank you.

So at least one of them is greater than one is gamma is equal to one minus this expression. So there is a type of graphical error here is 1 –(1- e raise to power minus gamma over capital gamma e raise to power M. So this tells us the probability that at least one of the branches exceeds the SNR greater than gamma.

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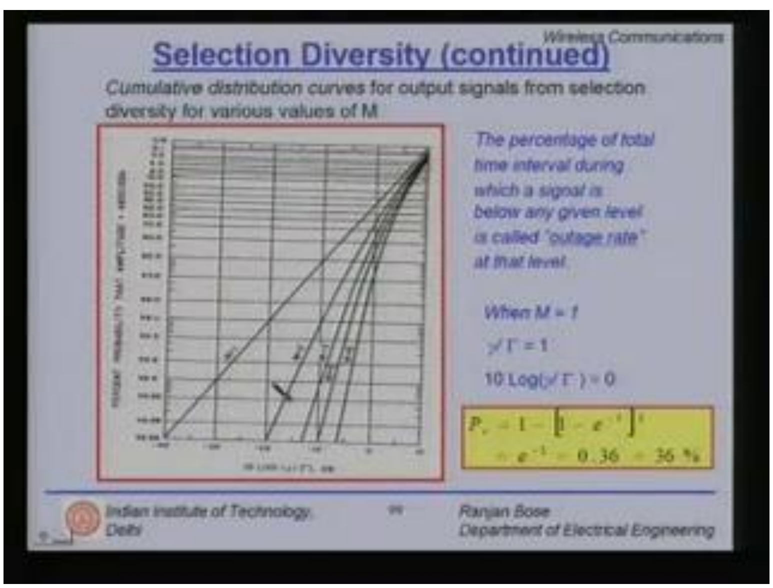

So what we have so far achieved is a cumulative distribution curve for the output signals from selection diversity for various values of M. The percentage of total time interval during which a signal is below any given level is called the outage range of that level. So it's the outage rate and then there is an associated outage probability, when M is equal to one then gamma over gamma capital is one, so 10 log gamma over gamma capital is zero so in that case P_r the probability that at least one of the branches will have threshold greater than the gamma is one minus bracket open 1 - e raise to power - 1 nothing but e raise to power - 1 is equal to 0.36 or 36%. So this is the case when a single branch is being considered, it's pretty bad so here we have the curve for the outage probability and 10 log gamma over capital gamma lets expand this.

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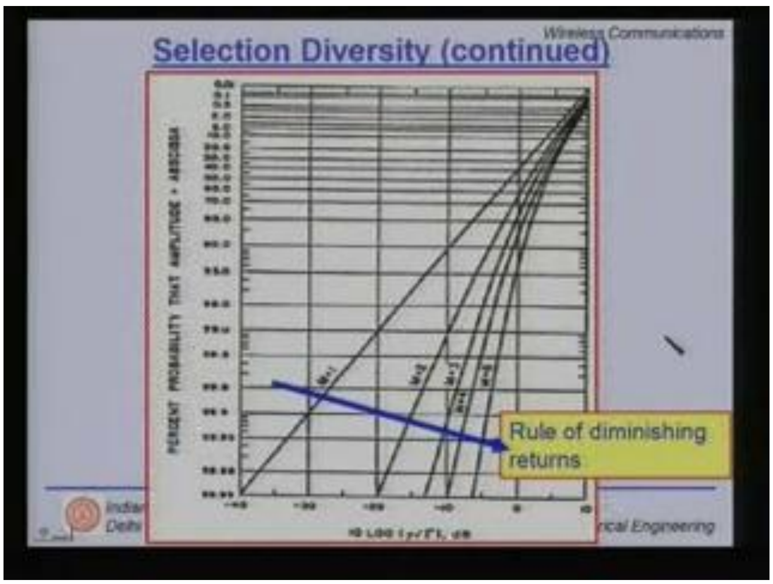

So on the x axis we have 10 log gamma variable divided by capital gamma in dB, on the y axis we have the percentage probability that the amplitude exceeds abscissa. So you have studied from 99.99 and then it goes up to 0.1. Here you have dB so - 40 increasing up to – 30, - 20 and so and so forth. The first curve is for M is equal to one and then you have a diversity of 2, 3 and so and so forth. So first thing to observe is the rule of diminishing returns, if you just follow one of the horizontal lines which is the percentage probability that the amplitude is greater than abscissa then if you have M is equal to one you have to have this - 30 dB as the value here.

When you go from M is equal to 1 to 2 there is a significant improvement, when you go from 2 to 3 ,yes there is an improvement but smaller, from 3 to 4 improvement much less, then I jump 4 to 5 it is somewhere in the middle less improvement and even less improvement. So it tells me that yes by all means go from 1 to 2 and may be up to 3 but beyond that choose wisely, only increase the diversity if it is worth it. So this is an important rule, the rule of diminishing returns observed for the diversity techniques.

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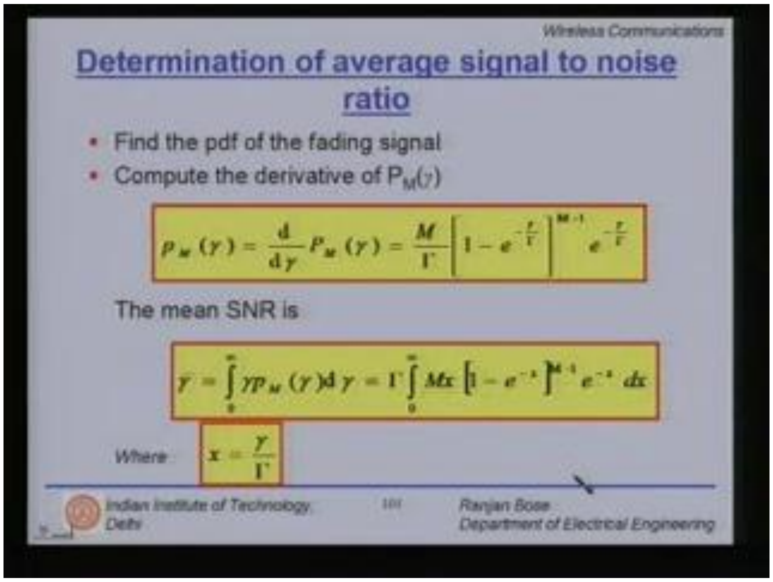

Now let’s go over the determination of average signal to noise ratio, so the job that we have to do carrying on from the mathematics developed so far is find the pdf of the fading signal. We already have the cumulative distribution function so we need to differentiate it somewhere and then compute the derivative of P_M gamma to do so. So probability density function for gamma is derivative with respect to gamma and if you do that you get this expression. The mean SNR is given by the following equation; gamma bar is equal to integrated over all gammas zero to infinity gamma P_M gamma d gamma this is the average and if you integrate it you get this following expression, read out as capital gamma zero to infinity Mx bracket open 1 - e raise to power x with a negative sign whole raise to power M - 1 times e raise to power minus x dx where x is defined as gamma over capital gamma. So this tells you the average SNR obtained when you have an M order diversity.

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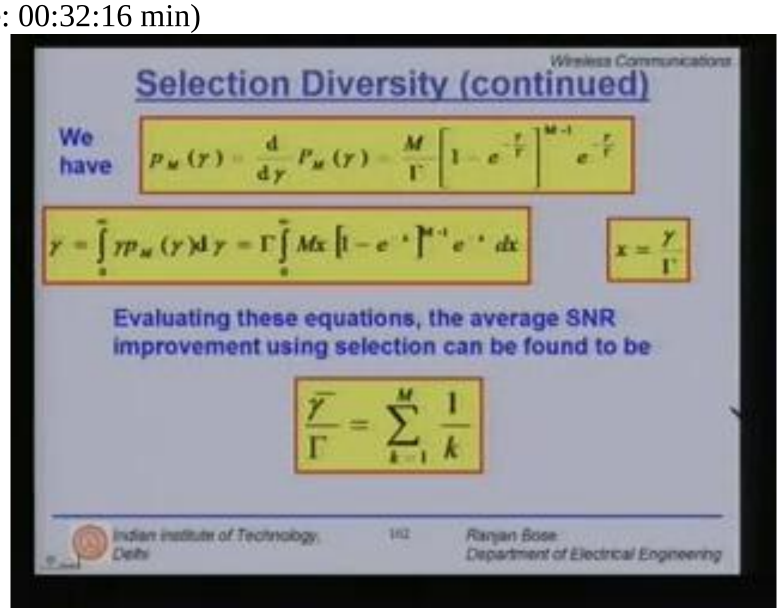

So continuing with selection diversity let's now juggle the different equations that we have obtained so far and try to derive something meaningful out of it. So far we have these 3 equations, this we have obtained just by differentiating and we have the average gamma and x is already defined as gamma over capital gamma using these, the average SNR improvement using selection. What is selection? Choose the maximum, can be found to be gamma bar over gamma is equal to summation k is equal to one through M one over k. It's a very simple relationship that comes up which is the improvement in SNR average. Expression for average SNR improvement using selection diversity.

So now you can easily pick and choose your M and see how far you want to go. If you have a larger value of M there will be more terms in the summation but very clearly you can see that how fast the diminishing return goes, it actually goes as one over that value k. So going from 1 to 2 you increase by 0.5, normalize from 2 to 3 you increase by 0.33 from 3 to 4 you improve by 0.25 and so and so forth.

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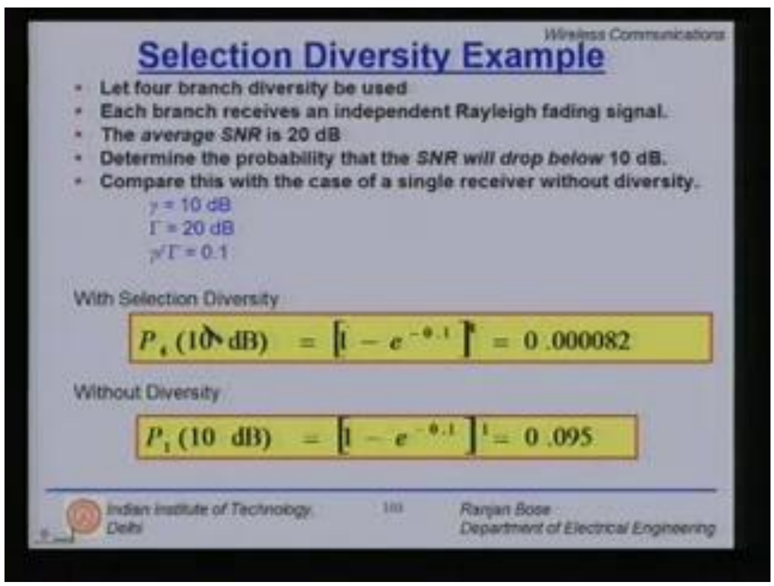

Now let us look at an example. Let us take M is equal to 4 that is four branch diversity, let us see how well it performs with respect to no diversity that is single branch. Each branch, here branch means the antenna element receives an independent Rayleigh fading signal that's our assumption because all our mathematics so far has been developed on independent fading channels. Let us say the average SNR is 20 dB so what we would like to do is find out the probability that the SNR will drop below 10 dB. This is an average dB, it's a fading channel clearly it will drop but we would like to know that the SNR will drop below 10 dB. That's my definition of a deep fade here and compare this with the case of a single receiver without diversity. So what do we have? We have this gamma which is our threshold which is 10 dB, now this capital Gamma is defined as 20 dB average SNR. So if you take it up in real terms it is gamma by capital Gamma is 0.1. 10 dB is in real life 10, 20 dB is 100 so 10 over 100 is 0.1.

Now let us say we use selection diversity that is we pick and choose one of the four branches at any time which gives the maximum signal. Then using the expression probability with 4 diversity order is given by 1 - e raise to power minus 0.1 raise to power 4 is this value 0.000082. So there is 1, 2, 3, 4 of the order of 10 raise to power minus 4. Now at the other hand if you have no diversity that is only using one antenna element this four here will be replaced by one and you have 0.095 which is unacceptable. So this is a successful receiver this is an unsuccessful receiver. What did we do? We have M is equal to 4.

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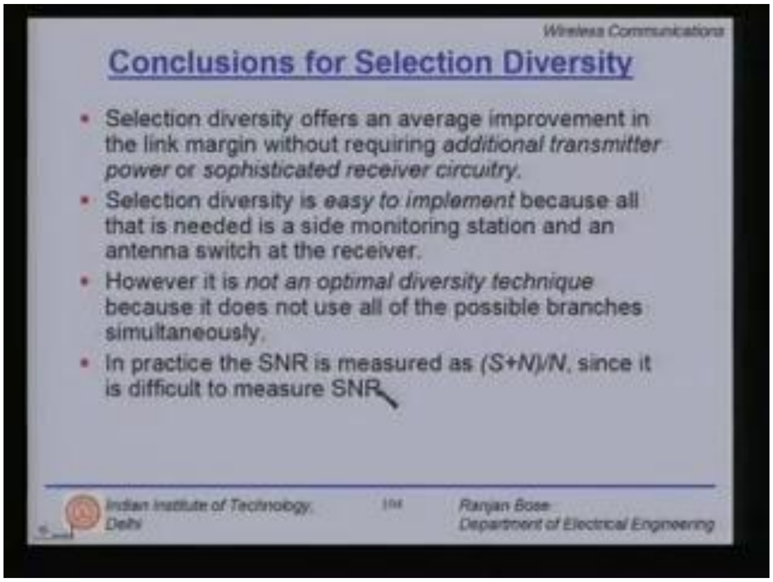

So let’s conclude only for the selection diversity part, selection diversity offers an average improvement in the link margin clearly without requiring additional transmitter power very important or any sophisticated receiver circuitry as was required for equalizers. Selection diversity is easy to implement because all that is needed is a side monitoring station and an antenna switch at the receiver, all it is doing is choose maximum there is no post processing. However intuitively also this is not an optimal diversity techniques why, because it does not use all the possible branches simultaneously. Yes, we select the maximum but may be the second branch has a little less but still it has some signal.

So third branch may have some signal though less so we have to use them we cannot throw away any of the signal strength, specifically when we have multiple antennas. So we must use in a better scheme away to find out and use the signals received in all the other branches. In practice this SNR is measured as S + N by N since it is difficult to measure the SNR by itself alone.

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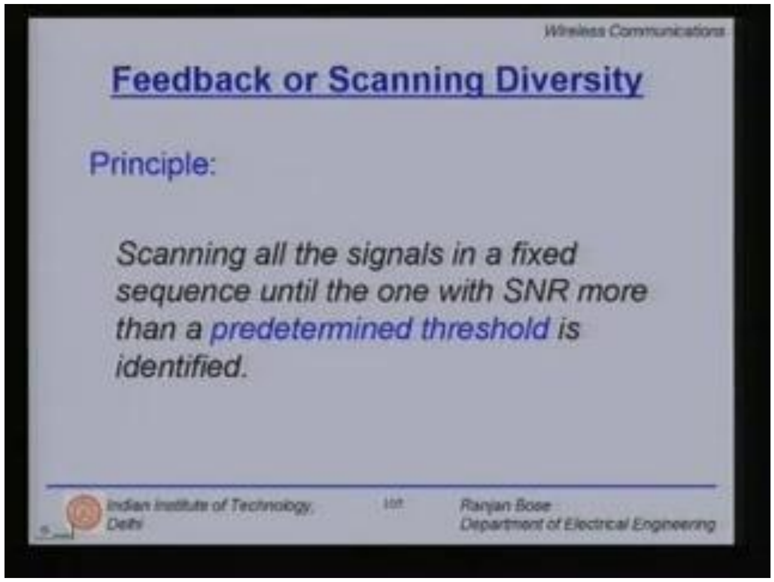

Now let us look at the second kind of spatial diversity which is called scanning diversity or sometimes also called as feedback diversity. What does it do? The basic principle is as follows; its scans all the signals in a fixed sequence until the one with SNR more than the predetermined threshold is identified, that is slightly different from the selection diversity you keep on scanning the signals in the different antenna elements in a particular predefined sequence until one with SNR more than a predetermined threshold is identified.

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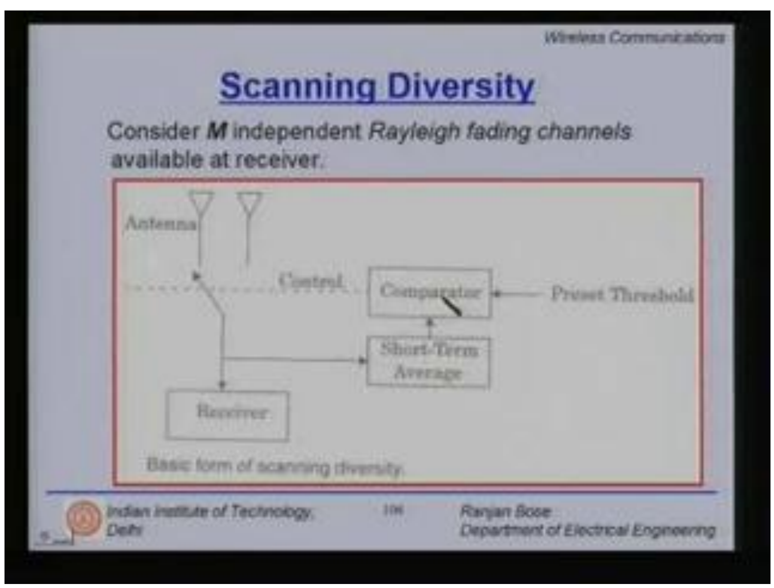

So again consider M independent Rayleigh fading channels available at the receiver pertaining to the M antenna elements. Here is the basic block diagram of scanning diversity so you have the antenna elements and then there is a receiver, it gives to a processor which does a short term average comparator based on the preset threshold and then if it is required it switches. So it keeps on scanning.

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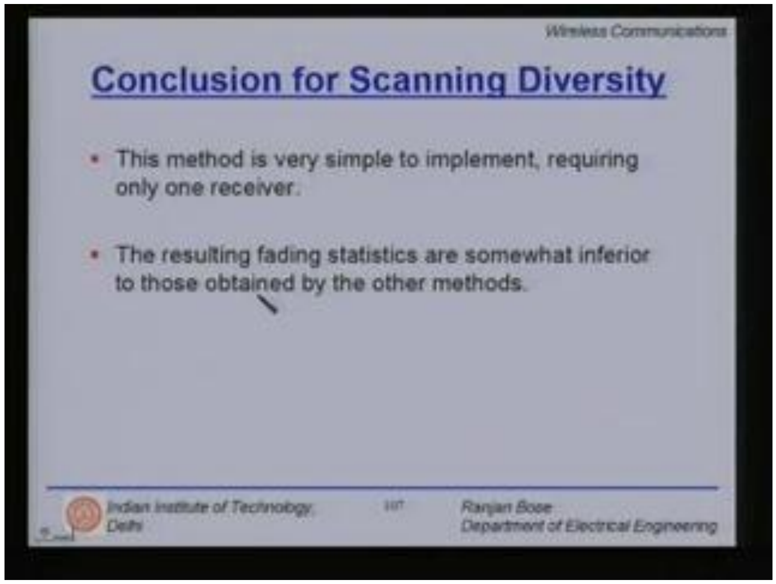

So this method is much simpler, very simple to implement requiring only one receiver. The resulting fading statistics are somewhat inferior to those obtained by other methods but the bonus is it is a very simple method.

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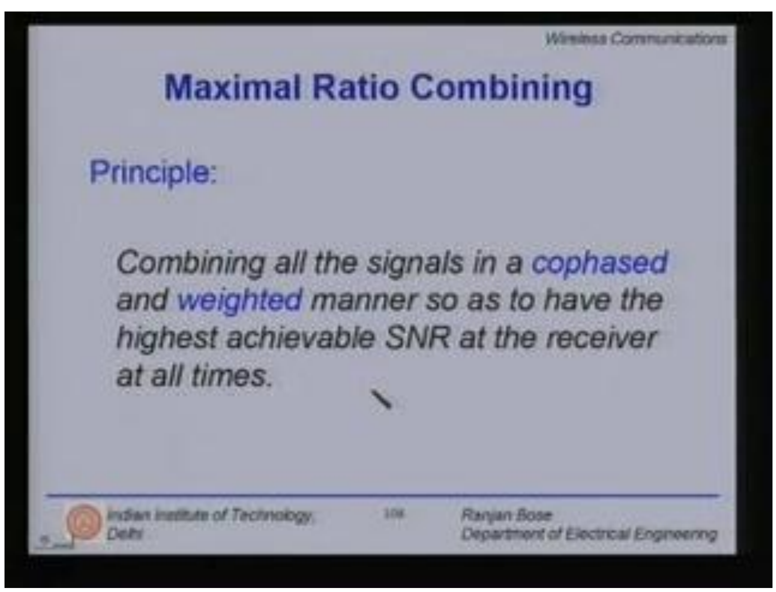

Now we come to a more complicated, more useful and almost optimal method called maximal ratio combining. What is the principle? We combine all the signals in a cophased and weighted manner so as to have the highest achievable signal to noise ratio at the receiver at all times. So we are not going to throw away any other information. In fact we pick up all of the signals, cophase it and then weight it and multiply add and then obtain so as to maximize SNR. Clearly it requires you to adjust the weights if they're complex, you have to look at the phase as well as the attenuation in that path. So you do require some kind of a processing here and updation of weights.

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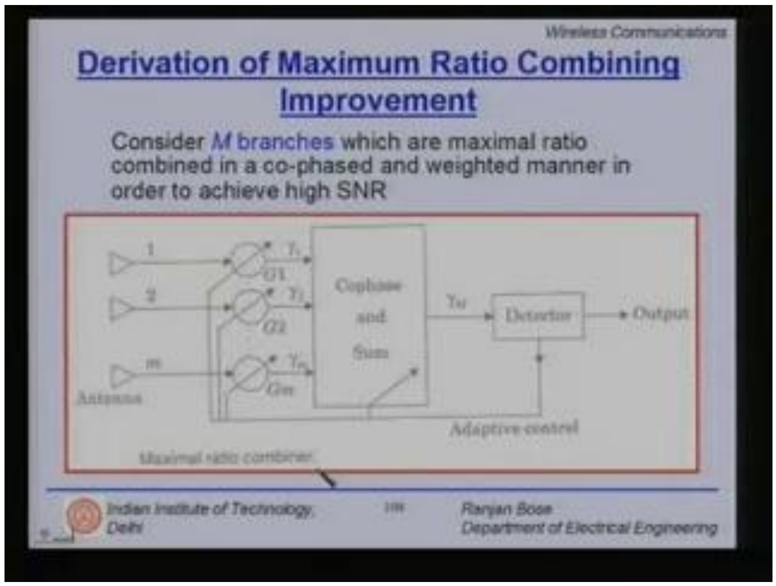

Now let us look at the derivation of maximum ratio combining improvement. How much improvement do we obtain if you are using MRC or maximal ratio combining? Again we consider M branches which are maximal ratio combined in a cophased and weighted manner in order to achieve the highest possible SNR. Here is the block diagram of MRC, again we have antenna elements 1, 2, 3, 4 up to M but look here we have put weights with phases so complex weights cophase and sum and then you have a detector you have to have adaptive control that means you have to have an algorithm to update your weights and then finally you have an output. So the output is only based once you have updated your weights. So you do have to have some kind of a processing here, this is called the maximal ratio combiner.

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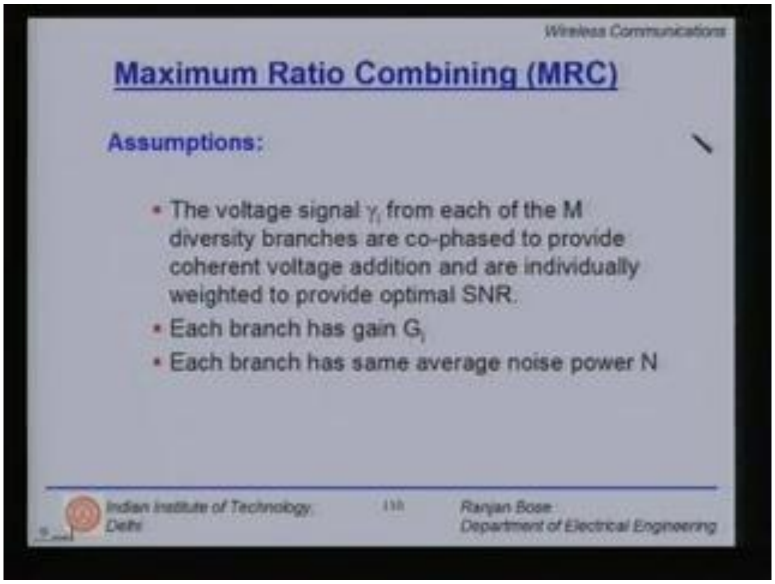

What are the assumptions? The voltage signal gamma$_i$ from each of the M diversity branches are cophased to provide coherent voltage addition very important and are individually weighted to provide optimal SNR. Each branch has gain G_i, each branch has a same average noise power N so these are the assumptions being made for MRC.

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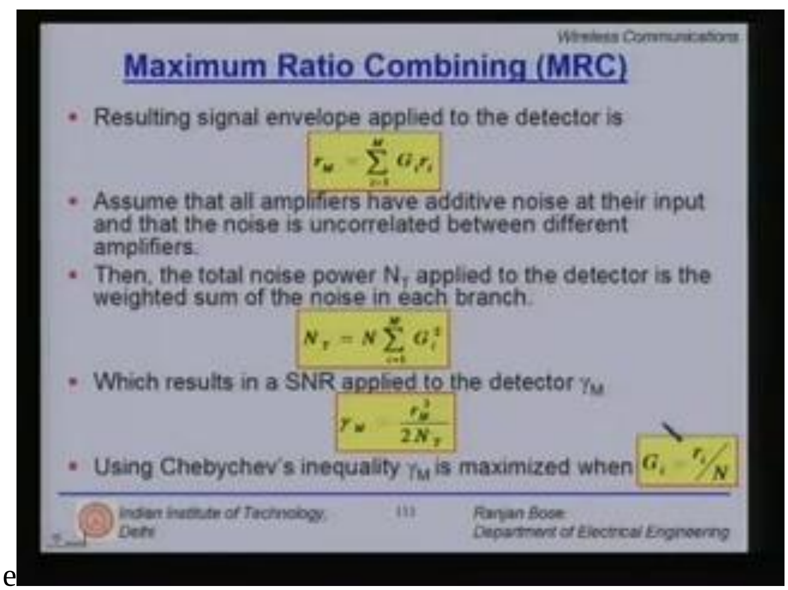

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Now let us derive certain conditions. the resulting signal envelope applied to the detector is as follows, this is resulting from M different branches summation i is equal to one through M gain times the received signal in ith branch.

Assume that all amplifiers have additive noise at their inputs and the noise is uncorrelated between different amplifiers. Under this assumption we have the total noise power N_T applied to the detector is weighted sum of the noise in each branches because in the process of weighting the received signal we are also weighting the noise. So the received noise total is N times summation G_i this N will come out of the summation sign I mean i is equal to one through M. Now this results in SNR applied to the detector gamma M equal to r_m squared where r_m is here over 2 N_T. [Conversation between Student and Professor – Not audible ((00:44:25 min))] The question being asked is if all the amplifiers are independent then the noise should also be independent that's true but please note that we have closely spaced antenna elements also. So this is a good assumption but we still have to make this assumption because the processing is being done on the same receiver so there could be a fairly correlated noise as well.

We make this assumption to ensure that things work out please note it's not a bad assumption to make as you pointed out. Now working on this gamma M which is the SNR for the M received signals we use the Chebychev's inequality and gamma M is maximized when we obtain G_i the gain in each braches equal to r_i over N. So in fact the gains are matched to the signal strength so whichever branch has a stronger signal will also have a stronger gain, whichever branch is a weaker signal will have a proportionately smaller gain, it makes sense.

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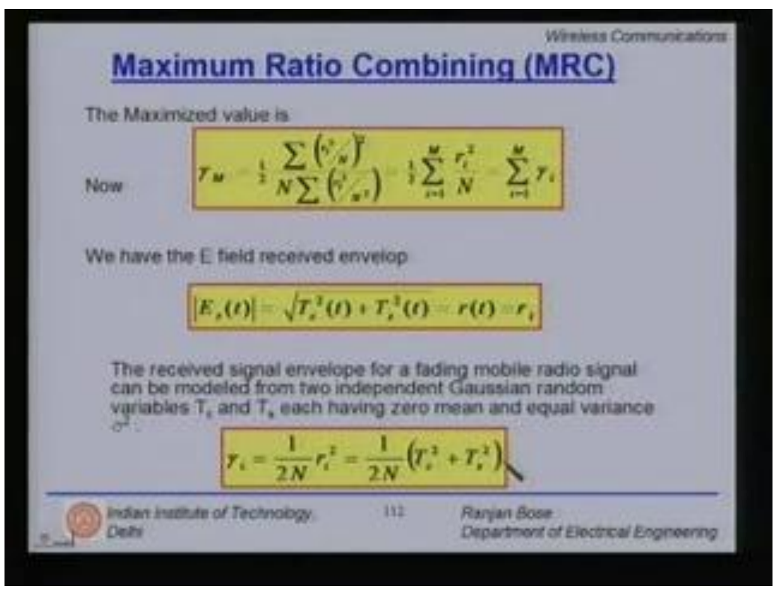

So the maximized value based on the Chebychev's inequality if you used that expression for gain is equal to gamma M equal to half summation r_i squared over N whole squared divided by N summation r_i squared over M squared. If you solve this out is equal to summation i is equal to one through M gamma$_i$. Now we have the E field received envelope which can be expressed as followed at the antenna E_Z (t) absolute value is under root T_c squared as a function of time plus T_s squared as a function of a time. What is this? The received signal envelope for a fading mobile radio signal can be modeled from two independent Gaussian random variables T_c and T_s, subscript c stands for the cosine and s stands for the sign each having zero mean and equal variance sigma squared.

If you have this under root T_c squared plus T_s squared, you have the Rayleigh faded envelope. Thus you have the $gamma_i$ equal to 1 over 2 N r_i squared and r_i can be substituted as 1 over 2 N T_c squared plus T_s squared.

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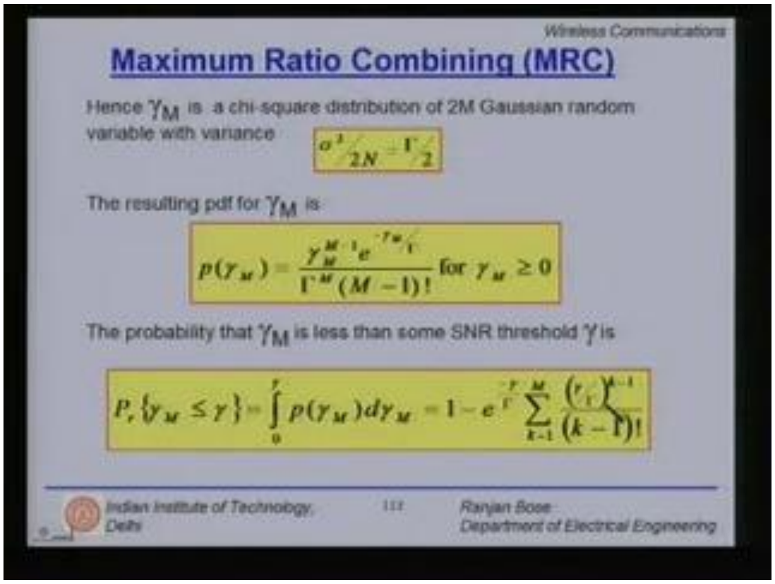

Hence what do we have, that $gamma_M$ is a chi squared distribution of 2 M Gaussian random variables with the variance sigma squared over 2 N is equal to gamma over 2 and the resulting pdf of $gamma_M$ is given by this following expression. So Jakes has done this kind of an analysis in his book and a lot of details can be found from there but this is the probability density function for $gamma_M$, M independently faded branches and using maximal ratio combining.

Now the probability that $gamma_M$ is less than some SNR threshold gamma that's how we work in most diversity scheme what is your target threshold and then we work whether it is less than or greater than it. So probability that $gamma_M$ is less than this threshold value gamma is simply obtained by integrating from 0 to gamma P $gamma_M$ d $gamma_M$ and if you do the integration you will land up with this following expression. Please note this normalize term minus gamma over capital gamma appears here also.

So determination of the average signal to noise ratio. Equation that we have seen just before is the probability distribution function for maximal ratio combining, hence the mean SNR give a $gamma_M$ bar is given by again i is equal to 1 through M $gamma_f$ bar is equal to the following expression is a very simple M times gamma. That's an excellent result, with M I am growing linearly in terms of my average SNR. If I increase my M I increase my average SNR.

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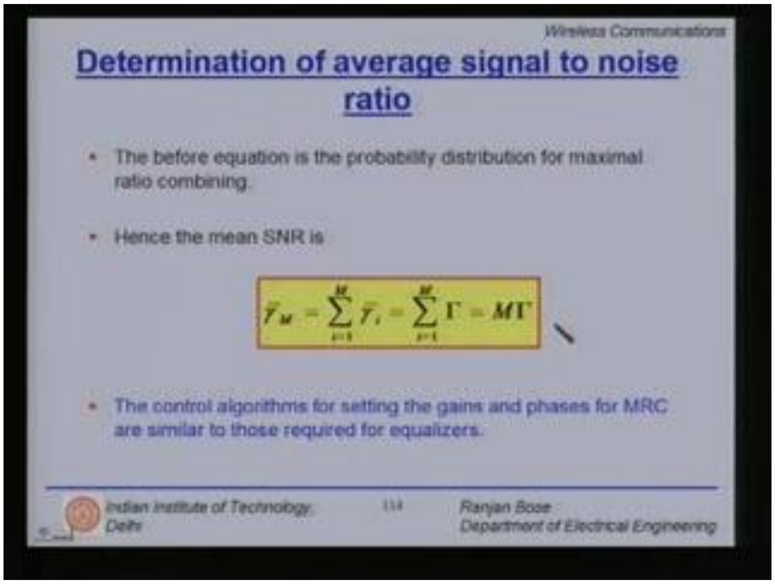

Now the flip side is that the control algorithms for adjusting the weights and phases and the gains also have to be set properly. The control algorithms are setting the gains and phases for MRC are similar to those required for equalizers. So you need to have some time to converge to the right weights and your performance is as good as the knowledge of the channel but if you do know your channel well then your average SNR grows as M times the SNR in a single branch. We come to the fourth technique which is the equal gain combining which is slightly sub optimal but a lot easier than MRC. What is the principle? Combine all the signals in a cophased manner but with unity gain for all signal levels so as to have the highest achievable SNR at the receiver at all times, so cophased addition but unity weights.

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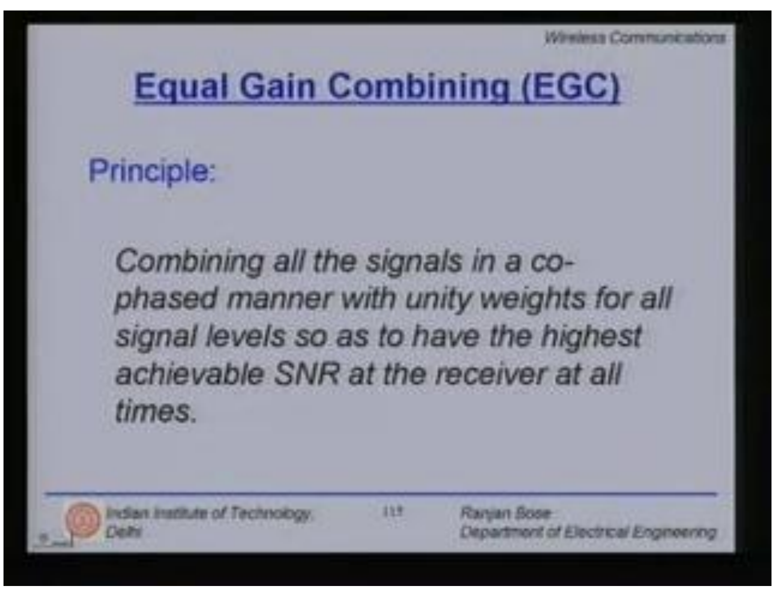

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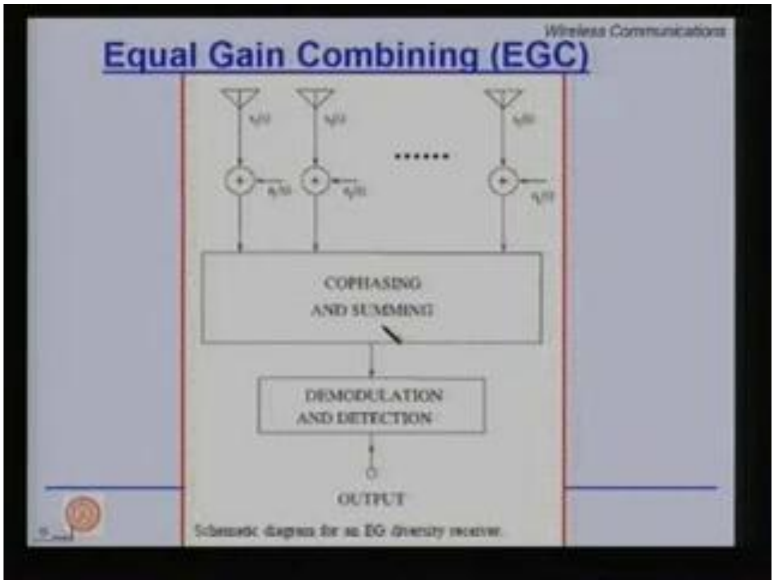

So here is the equal gain combining block diagram you have M receive antennas, you just cophase and sum demodulation and detection and out goes your output, a fairly simple implementation.

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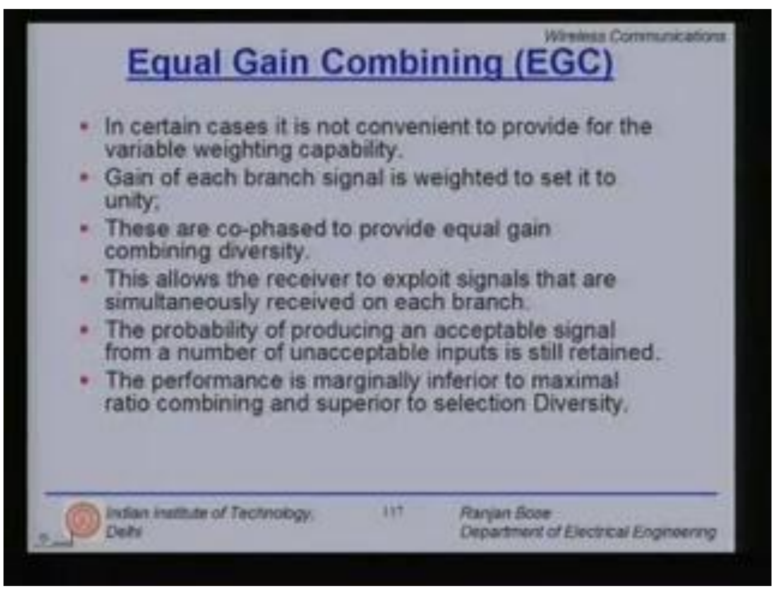

What are the salient features? In certain cases it is not convenient to provide for variable weighting capability it’s too complex, it takes too much time to converge we do not know the channel well and may various other factors.

Gains of each branch signal is weighted to set it to unity, these are then cophased this is important if you add them just like that you are in trouble. These are cophased to provide equal gain combining diversity, this allows the receiver to exploits signals that are simultaneously received on each of the branches. The probability of producing an acceptable signal from a number of unacceptable inputs is still retained and you still combining them. The performance of equal gain combining EGC is marginally inferior to the maximal ratio combining MRC but definitely superior to the selection diversity were only pick one of them.

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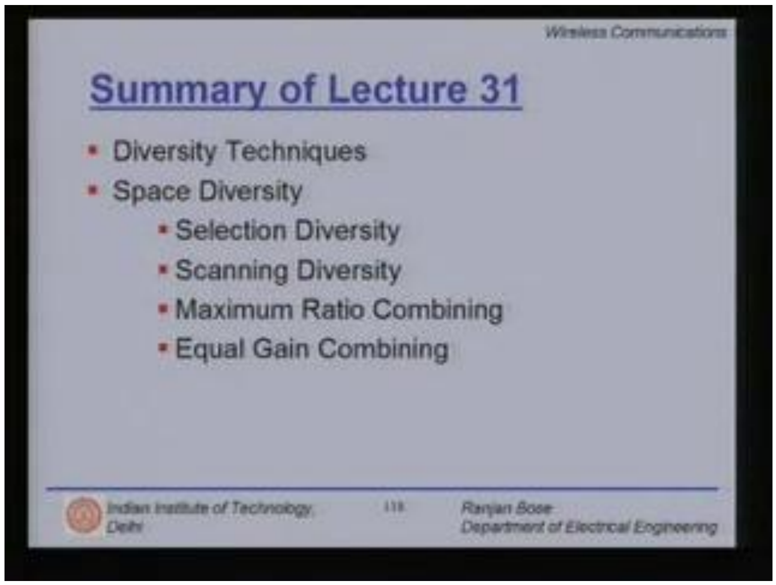

So let us summarize today's lecture. We started off with the need for diversity techniques to overcome deep fades, we then looked at space diversity and the different kinds of space diversity. Specifically we discussed selection diversity followed by the scanning diversity then we spent some more time discussing MRC or maximum ratio combining and then finally we talked about the equal gain combining. We will conclude our lectures here.