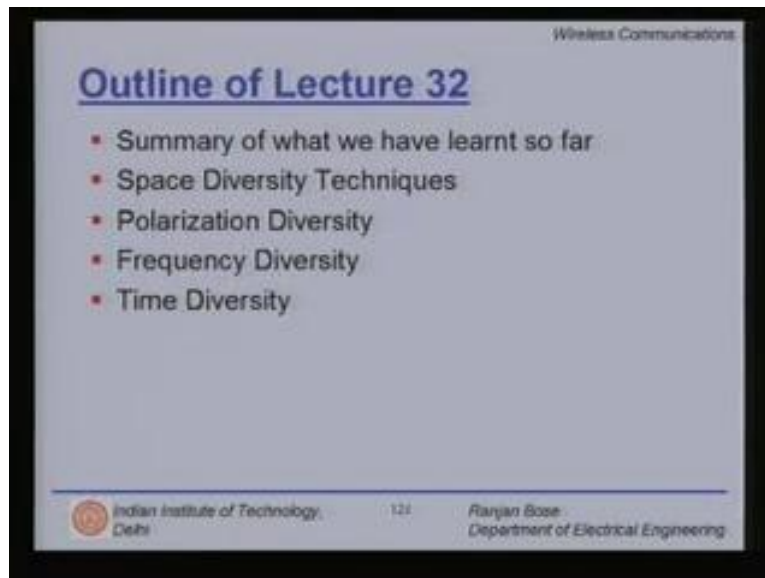


**Wireless Communications**  
**Dr. Ranjan Bose**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**  
**Lecture No. # 32**

**Equalization and Diversity Techniques for Wireless Communications (Continued)**

Welcome to the next lecture on wireless communications. Today we will deal with diversity techniques for wireless communications. First let us look at the brief outline for today's talk.

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We will summarize by learning what we have read so far, followed by space diversity techniques. Then we will look at polarization diversity followed by frequency diversity and then time diversity. Today we will touch upon most of these cases but definitely after recap what we have done so far. First a recapitulation; we have already studied different kinds of space diversity techniques and these have been the selection diversity, the scanning diversity, the maximal ratio combining method and the equal gain combining methods.

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## Recapitulation

- Diversity Techniques
  - Space Diversity
    - Selection Diversity
    - Scanning Diversity
    - Maximum Ratio Combining
    - Equal Gain Combining

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Just to go over space diversity, in totality if you remember from last lecture space diversity can be broadly divided into selection, scanning, maximal ratio and equal gain combining. We will briefly go over the results of each one of them and then move over to polarization diversity, frequency diversity and then finally time diversity.

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## Space Diversity

```
graph TD; SD[Space Diversity] --- SD1[Selection Diversity]; SD --- SD2[Scanning Diversity]; SD --- SD3[Maximal Ratio Combining]; SD --- SD4[Equal Gain Combining];
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## Selection Diversity

Principle:

Selecting the **best signal** among all the signals received from different branches at the receiving end.

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Ranjan Bose  
Department of Electrical Engineering

What is selection diversity? We know the basic principle of selection diversity is to select the best signal amongst all the signals received from different branches at the receiver end. Please note in diversity techniques we use more than one antenna and what we expect is that if the signal is fading in one of the received antennas, if it is uncorrelated fading in the other one we will more slightly get a stronger signal in the second antenna, if there M antenna then we can choose one of the best signals amongst the n antennas. The selection diversity requires you to choose the best signal from amongst the M different antenna elements.

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## Derivation of Selection Diversity Improvement

- Consider  $M$  independent Rayleigh fading channels available at receiver

Assumptions:

- Each branch has the same average SNR
- Instantaneous SNR for each branch =  $\gamma_i$

Transmitter

Voice/data

1 2 M

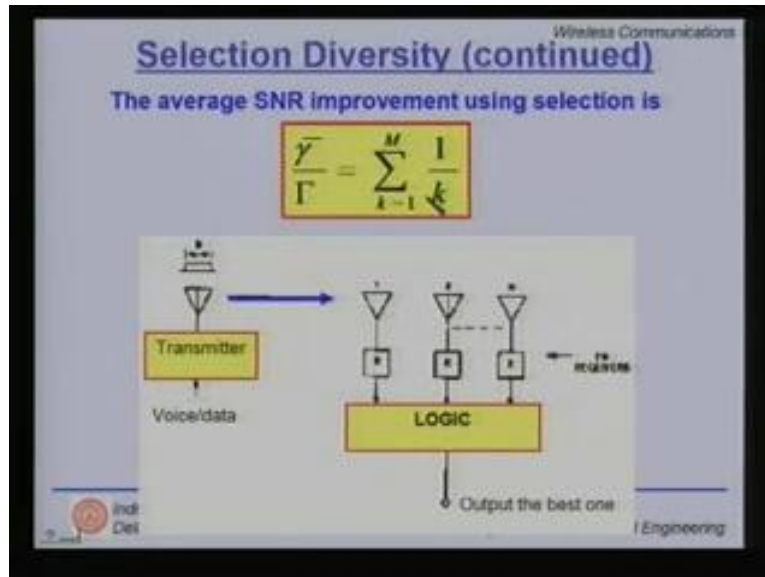
LOGIC

Output the best one

Engineering

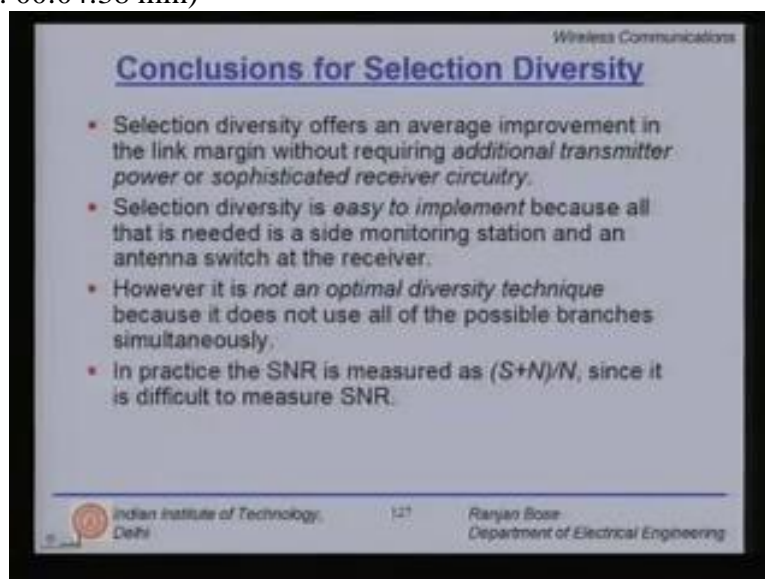
Here is a graphical representation for the selection diversity and suppose we have 1, 2 and so forth till M antenna and the transmitter transmits either a voice or a data digital format, here is the logic which outputs the best of the signals.

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What we learnt last time is that if you use selection diversity the average SNR improvement is given by gamma bar divided by capital gamma is equal to summation from k is equal to 1 through M, 1 over k. What we learn from this expression is thus as we increase k from 1 to 2 you get an improvement, you go from 2 to 3 you definitely get another improvement though a smaller one because it grows as 1 over k and then as we increase you keep on getting improvements but the incremental improvement is minimal.

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What do we learn from selection diversity? Selection diversity offers an average improvement in the link margin without requiring additional transmitter power or any sophisticated receiver circuitry. The best point for diversity techniques is the simplicity and there is no additional computational requirements in general. Selection diversity is easy to implement because all that is needed is a side monitoring station and an antenna switch at the receiver which will help pick up the most strongest signal. However it is not an optimal diversity technique simply because we throwing away the signals of all the other antennas except that which is strongest. So clearly there should be some technique which we can use to combine the signals however small from different antenna elements.

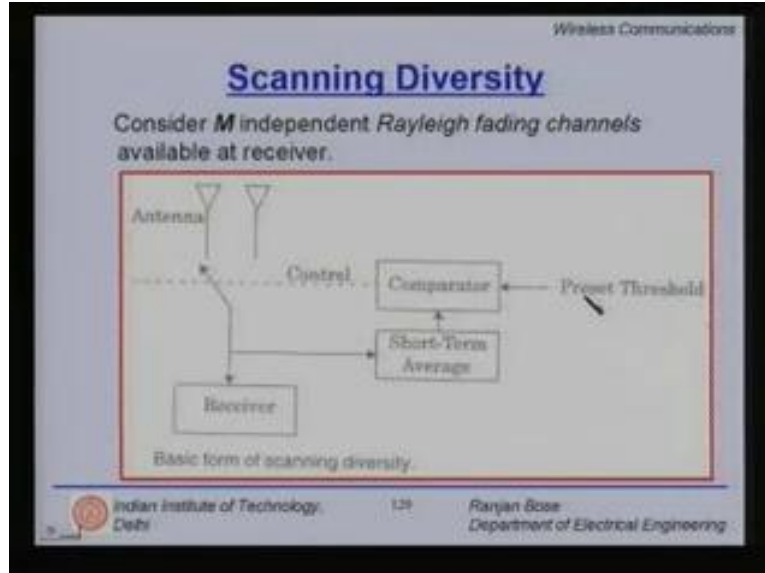
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Let's talk about the second spatial diversity technique which is called the feedback or the scanning diversity. What is the basic principle? It scans all the signals in a fixed sequence until one with the SNR more than a predetermined threshold is identified. So we need to set a threshold before we start the operation and then we go into the scanning mode.

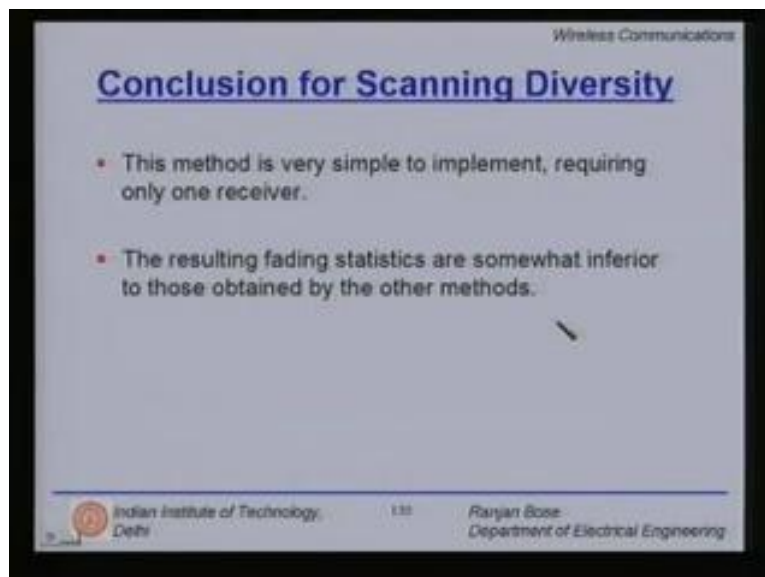
Consider  $M$  independent Rayleigh fading channels available at the receiver, so here we have the block diagram for the scanning diversity, we have the different antenna elements goes to the receiver, finds the short term average, comparator with a preset threshold and then keep on jumping from one antenna element to the other in a predefined fashion until your threshold is reached and then you stop. So you do not have to go through the entire cycle to locate the maximum as long as your threshold is satisfied, you are in business.

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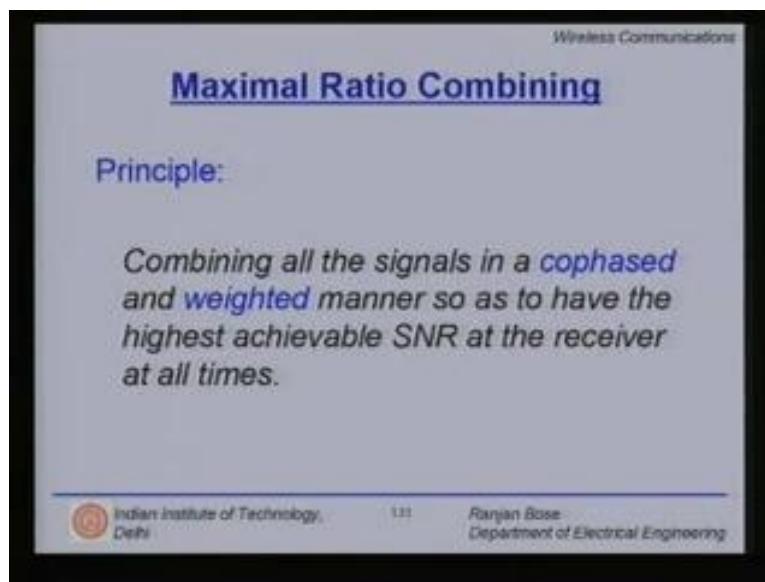
Conclusions for scanning diversity this method is really very simple requiring only one receiver. The resulting fading statistics are somewhat inferior to those obtained by other methods questions. [Conversation between Student and Professor – Not audible ((00:07:15 min))] The question being asked is what do we mean by one receiver? So if you go back to the diagram, here we definitely have different antenna elements which is hoping on a cross but this is a general property of the different special diversity schemes where we have one receiver finally which should be connected to one of the antenna elements. The logic which connects that single receiver to various antenna elements is set here, otherwise if you are doing some kind of other combining technique then you will have to have different receivers and then you combine the received output.

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Here you do not have to process it before, you first choose the larger signal and then you use only a single receiver. Is scanning a special case of selective technique? Yes it can be look at as a special case where as you reduce your effort, in the sense that you are not trying to reach the maximum signal but one of the signals whichever reaches your threshold. Please note hoping itself requires some effort and energy so we save on that. The resulting fading statistics are somewhat inferior to those obtained by other methods. Later on today we will see how different in terms of improvement in performance, really are these different techniques for diversity. Is it a big improvement? Definitely maximal ratio combining will give you a better performance because it is optimal but how much better? We look at an example to look tell us how much is the improvement.

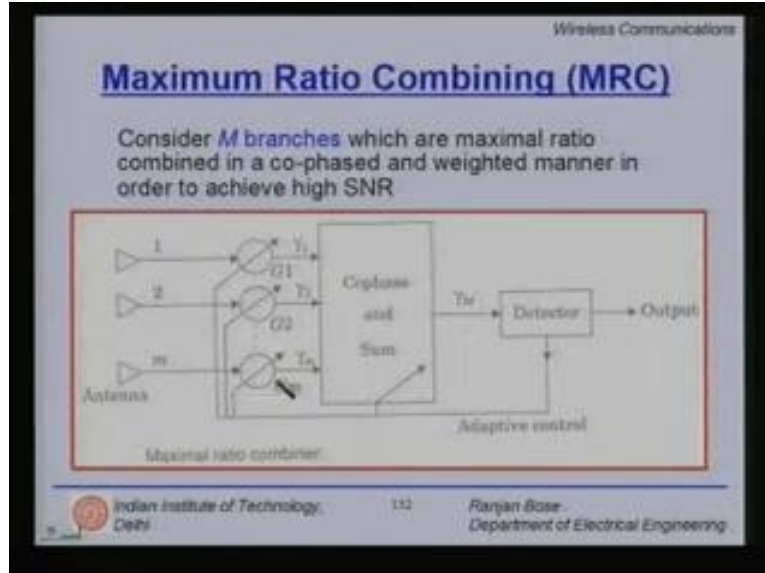
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Let us look at the third technique which is also the more popular technique which is the maximal ratio combining method. What is the basic principle? You combine all the signals in a cophased and weighted manner so as to have the highest achievable SNR at the receiver at all times. In this sense it is optimal. So please note the two terms cophased and weighted.



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Again consider  $M$  branches which are maximal ratio combined in a cophased manner, so if you have these  $M$  branches here in the antenna then you have a method of adaptive control wherein you can choose the weights and the phases. So that you can weight it, cophase it and then pass it to the detector.

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This slide, titled 'Maximum Ratio Combining (MRC)', provides the mean SNR for MRC and discusses control algorithms. It states: 'The mean SNR for MRC is' followed by the equation 
$$\bar{\gamma}_M = \sum_{i=1}^M \bar{\gamma}_i = \sum_{i=1}^M \Gamma_i = M\Gamma$$
 where  $\Gamma$  is the SNR for one branch. Below the equation, it notes: 'The control algorithms for setting the gains and phases for MRC are similar to those required for equalizers.' The slide footer is identical to the previous slide, identifying it as slide 112 from the Indian Institute of Technology, Delhi, by Ranjan Bose.

So what we learnt last time is that if you use MRC, the mean SNR is obtained as  $M$  times gamma which is for one branch. As you increase the number of branches, your average SNR increases.



It's a very interesting result so here we go by increasing the number of branches in your receiver system but please note that the control algorithms for setting the gains and the phases for MRC do require computation and similar to those required for the equalizers. So this is the additional price we have to pay to do the job optimally.

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### Equal Gain Combining (EGC)

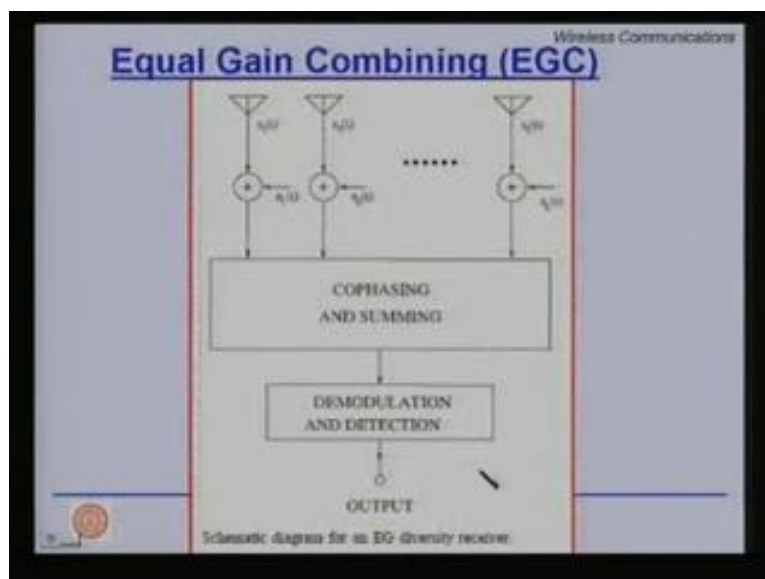
Principle:

*Combining all the signals in a co-phased manner with unity weights for all signal levels so as to have the highest achievable SNR at the receiver at all times.*

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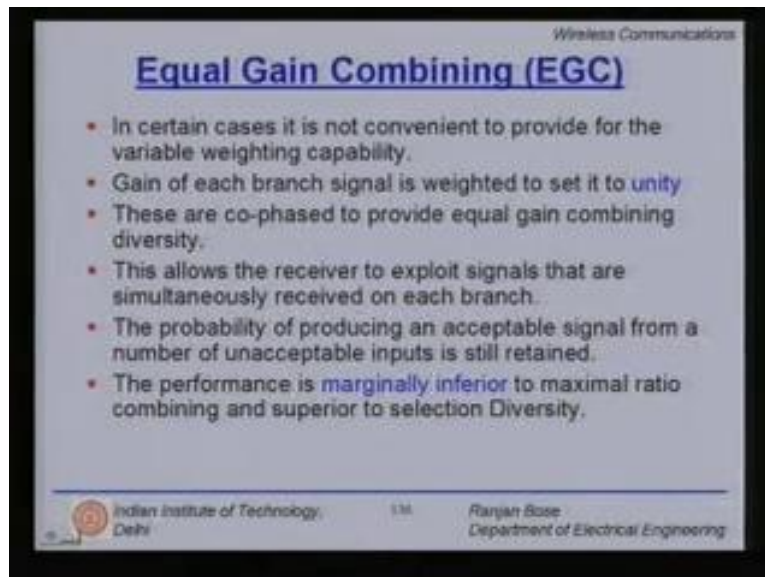
The fourth technique is a rather simple one; it is called the equal gain combining. What is the principle for this space diversity technique? We combine all the signals in a cophased manner with unity gained for all the signal levels. So that we have the highest achievable SNR at the receiver at all times, so here again we have to do a cophased addition but with unity weights.

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This is the block diagram for equal gain combining or EGC these are the M antenna elements or branches, we have the phase term then we have cophasing and summing up and then demodulation and detection to be with the desired output.

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Now let us talk about the advantages of EGC, in certain cases it is not convenient to provide for the variable weighting capability, so EGC is preferred. Gain of each branch is weighted to set to one. These are cophased to provide equal gain combining diversity, this allows the receiver to exploit the signals that are simultaneously received on each branch and the probability of producing an acceptable signal from a number of unacceptable input inputs is still retained. The performance is marginally inferior to the maximal ratio combining and definitely superior to the selection diversity. So equal gain combining sit somewhere between the selection diversity and the MRC. [Conversation between Student and Professor – Not audible (00:13:00)]. Can you say that these combining techniques are just similar to the multipath fading, in that we are just heading all the multipath components and create the final one?

Question being asked is, is it possible to equate this combining techniques as if that multipaths are coming in, we are trying to combine the multipaths. So answer is no, here clearly we are trying to overcome the effects of fading and the only thing we are trying to do is to look at uncorrelated fading. So multipaths are arriving at each antenna elements differently and then they are adding up vectorially at each antenna element differently. So there is no effort to separate the multipaths.

Now in some cases the multipath would combine to give a deep fade where as if the antenna spacing is correct we will most likely be out of fade in the second branch or the third branch and diversity techniques, spatial diversity techniques specifically looks at this property. We are not trying to separate the multipath complex. However if you go to the rake receiver kind of operation there you actually look at separating the different multipath complex. We'll talk about the time diversity technique and the rake receiver in the later part of this lecture.

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## Carrier-to-Noise & Carrier-to-Interference Ratio

### Example of a Two-branch Diversity System


- CNR  $\gamma$  is proportional to the square of a Rayleigh distributed signal envelope
- PDF of  $\gamma$ , where  $\gamma \geq 0$  is given by 
$$p(\gamma) = \frac{1}{\Gamma} e^{-\gamma/\Gamma}$$

where  $\Gamma$  denotes the average value of  $\gamma$

- CIR  $\lambda$  is given by the ratio of squared signal envelope to squared interference envelope, which are mutually Rayleigh distributed.
- PDF of  $\lambda$ , where  $\lambda \geq 0$  is given by

$$p(\lambda) = \frac{\Lambda}{(\lambda + \Lambda)^2}$$

where  $\Lambda$  denotes the average value of  $\lambda$

 Indian Institute of Technology, Delhi 117 Rajyan Bose  
Department of Electrical Engineering

Now let us look at an example and here we would like to focus on a two branch diversity system. As we have seen there is some kind of a rule of diminishing returns for selection diversity when you go from 2 branches to 3 branches and so forth. Let us look at a simple case of two branch diversity, here we are going to focus on two things carrier to noise ratio and carrier to interference ratio. So here we would try to present certain simulation results for both the C to N and C to I ratios. Now CNR or carrier to noise ratio<sub>gamma</sub> is proportional to the square of a Rayleigh distributed signal envelope we know that. The PDF of gamma where gamma is greater than or equal to zero is given by p gamma equal to 1 over capital gamma e raise to power minus gamma over capital gamma where gamma denotes the average value of this small gamma.

However the CIR or carrier to interference ratio lambda is given by the ratio of squared signal envelope to the squared interference envelope which are mutually Rayleigh distributed. What is the PDF of lambda? PDF of lambda when lambda greater than zero is given by p lambda is equal to capital lambda divided by lambda plus capital lambda whole squared. Here in this example we will look at C to N ratio and C to I ratios here capital lambda denotes the average value of small lambda.

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### For Two-branch Diversity System

Let  $\gamma_1$  be the instantaneous CNR of each diversity branch and  $\gamma$  be the instantaneous CNR of the combined branch, the three combining methods are described by

$\gamma = \gamma_1 + \gamma_2$  for Maximal Ratio Combining

$\sqrt{\gamma} = \sqrt{\frac{\gamma_1}{2}} + \sqrt{\frac{\gamma_2}{2}}$  for Equal Gain Combining

$\gamma = \max(\gamma_1, \gamma_2)$  and  $\gamma = \min(\gamma_1, \gamma_2)$  for Selection/switching

pdf's of  $\gamma$  for the three combining methods are expressed by:

$p(\gamma) = \frac{\gamma}{\Gamma^2} e^{-\gamma/\Gamma} = \frac{\gamma}{\Gamma^2}$  for Maximal Ratio Combining

$p(\gamma) = \frac{4}{3} \frac{\gamma}{\Gamma^2}$  for Equal Gain Combining

$p(\gamma) = \frac{d}{d\gamma} [(1 - e^{-\gamma/\Gamma})^2] = 2 \frac{\gamma}{\Gamma^2}$  for Selection/switching

where  $\gamma = \Gamma$  and  $\gamma$  can be obtained using the following integral:

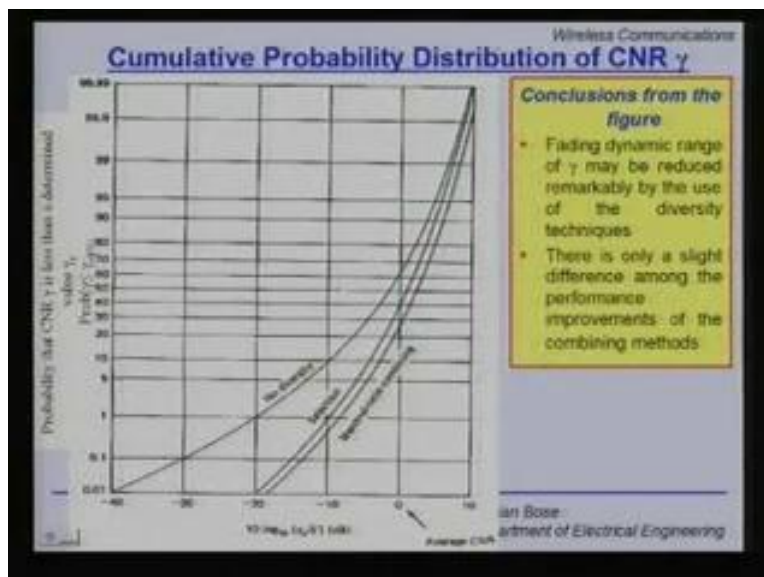
$$\text{Prob}\{\gamma \leq \gamma_s\} = \int_0^{\gamma_s} p(\gamma) d\gamma$$

Continued results are shown in next slide

Indian Institute of Technology, Delhi 111 Ranjay Bose  
Department of Electrical Engineering

Now for a two branch diversity system let  $\gamma_1$  be the instantaneous CNR or carrier to noise ratio for each diversity branch and  $\gamma$  be the instantaneous CNR of the combined branch. Now if we put it perspective, the three combining methods which are the maximal ratio combining, the equal gain combining and the selection diversity method. Then we can write the gammas for the 3 case as follows. The pdf's of gamma for the three combining methods can now be expressed approximately by these three equations. Now when gamma is much less than capital gamma, gamma can be obtained using the following integral, probability that gamma is less than gamma<sub>s</sub> is integrated zero to gamma<sub>s</sub> p gamma d gamma. Now based on this we will plot some of the results.

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So let us first talk about the cumulative probability distribution of carrier to noise ratio gamma. On the x axis we have log gamma over capital gamma in db and so this is the average SNR comes to zero where gamma is equal to the capital gamma. On to the y axis we have the probability that the  $CNR_{\text{gamma}}$  is less than a determined value  $\text{gamma}_s$ . So this  $\text{gamma}_s$  is what is reflected in the x axis,  $10 \log$  to the base 10  $\text{gamma}_s$  over capital gamma.

Now here please note when there is no diversity, we have a plot like this. Whereas if you use a selection diversity or MRC you see an improvement on this axis. So what is important to note is that there is a big jump when you go from no diversity to diversity but there is not much of a jump, when you jump from one kind of spatial diversity to another kind of spatial diversity. So what do we conclude from this figure? The fading dynamic range of gamma may be reduced remarkably by the use of diversity techniques and please note here we are talking about only 2 branch diversity.

However there is only a slight difference among the performance improvements of different combining methods. So if it is worth the time and energy you would like to spend on MRC do it otherwise just a selection diversity will also give you substantial improvement.

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### For Two-branch Diversity System

The pdfs of  $\lambda$  for the three combining methods are expressed as

$p(\lambda) = \frac{d}{d\lambda} \left\{ \left( \frac{\lambda}{\lambda + \Lambda} \right)^2 \right\}$	for Maximal Ratio Combining
$p(\lambda) = \frac{d}{d\lambda} \left\{ \left( \frac{\lambda}{\lambda + \frac{\sqrt{2}}{2} \Lambda} \right)^2 \right\}$	for Equal-gain Combining
$p(\lambda) = \frac{d}{d\lambda} \left\{ \frac{2\Lambda}{(\lambda + \Lambda)^2} - \frac{2\Lambda}{(2\lambda + \Lambda)^2} \right\}$	for Selection/switiching

$\lambda$  can be obtained using the following integral

$$\text{Prob}[\lambda < \lambda_s] = \int_0^{\lambda_s} p(\lambda) d\lambda$$

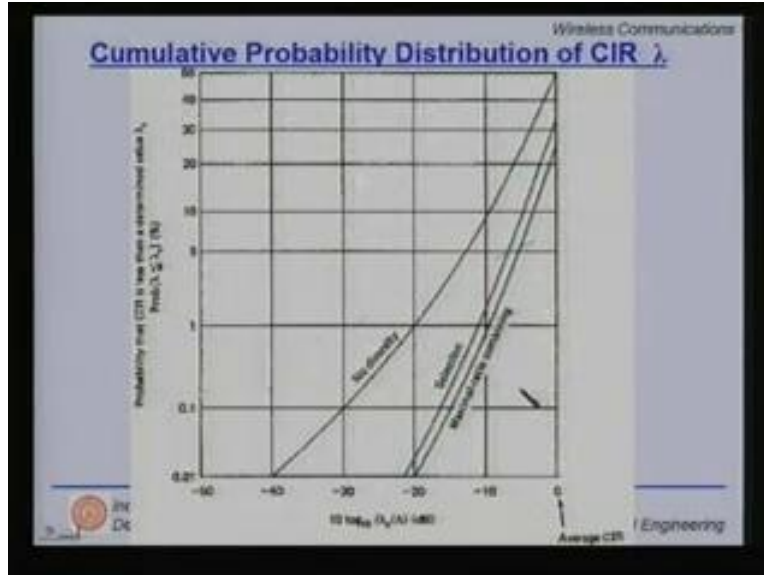
Computed results are shown in next slide

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Delhi
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Ranyan Bose  
Department of Electrical Engineering

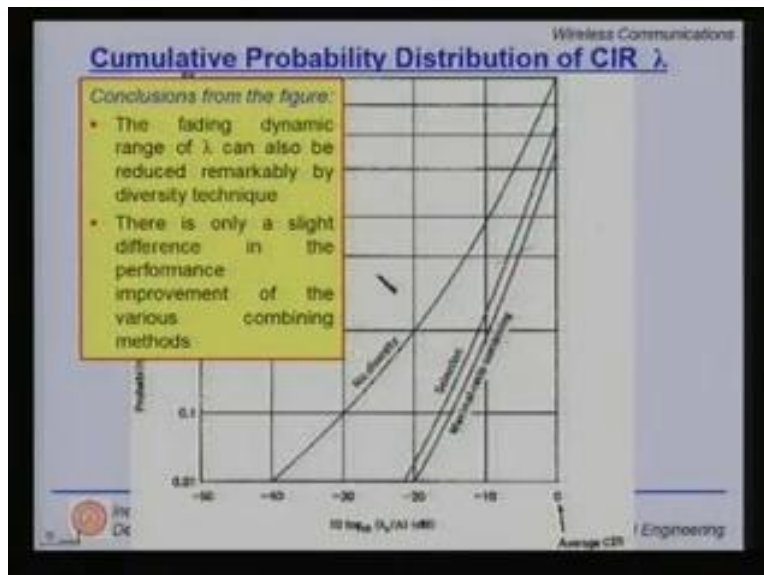
Now for the two branch diversity system only we have the three different kinds of pdfs for maximal ratio, for equal gain and for selection and switching. Then based on these 3 pdfs the computer results are shown as follows.

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Note here we are talking about the cumulative probability distribution of carrier to interference ratio CIR. Again the curves are similar, on the y axis we have the probability that the CIR is less than a pdf determined value  $\lambda_s$ , on the x axis we have  $10 \log$  to the base 10  $\lambda_s$  over capital  $\lambda$  db. Again zero denotes the average CIR because it is the log scale and zero pertains to the fact that  $\lambda_s$  is equal to capital  $\lambda$ . Again when you move from no diversity to some diversity this is a big improvement but amongst the diversities there is hardly any improvement.

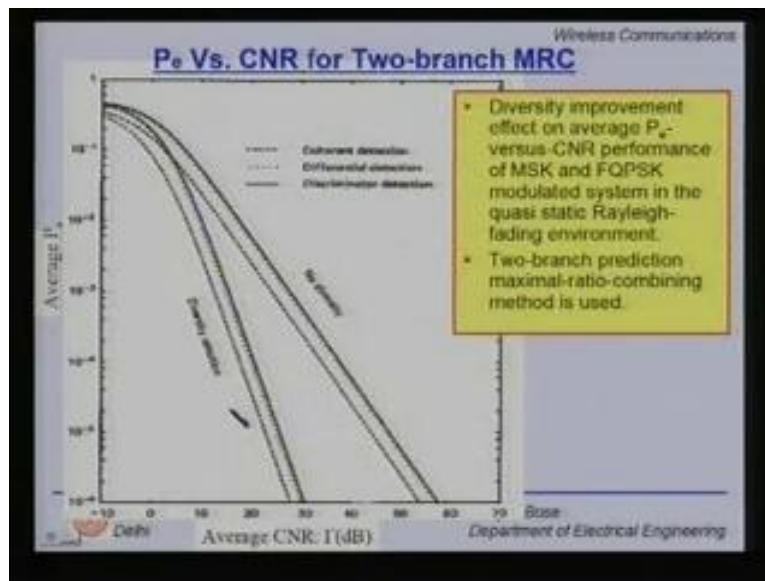
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So conclusions from this figure, the fading dynamic range of lambda can also be reduced remarkably by diversity techniques. So the point we are trying to make is diversity has been found useful not only against noise in a fading environment but also interference specially co channel interference. There is only a slight difference in the performance improvement of the various combining methods similar conclusions whether you talk about CNR or CIR.

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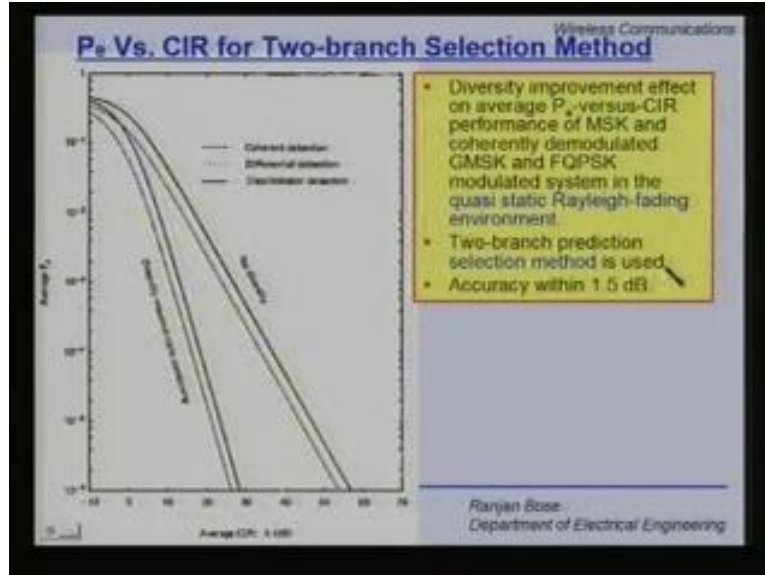


Now let us look at the probability of error versus CNR for the two branch MRC. So we have specifically fixed maximal ratio combining as the chosen space diversity technique. We are only using two branches and we would like to see the improvement in probability of error as we increase the carrier to noise ratio. So on the x axis we have the average CNR, on the y axis we have the average probability of error. Now this curve pertains to no diversity whereas this one pertains to diversity with two branches. The first thing that we observe is in the probability of error curves; diversity techniques increase the slope of the curve. In fact how fast the slope increases will have something to do with the diversity order. So when you say this technique gives me diversity gain, we talk about how much is the improvement in the slope with respect to no diversity.

So the diversity improvement effect on average probability of error versus CNR performance of MSK and FQPSK modulated schemes in the Quasi static Rayleigh fading environments are plotted, please note these are Quasi static Rayleigh fading environment and we have put for simulation certain kinds of MSK or frequency shift keying. Now the two branch prediction maximal ratio combining method is used in these curves. The point to be noted if there is a big improvement in the probability of error performance.

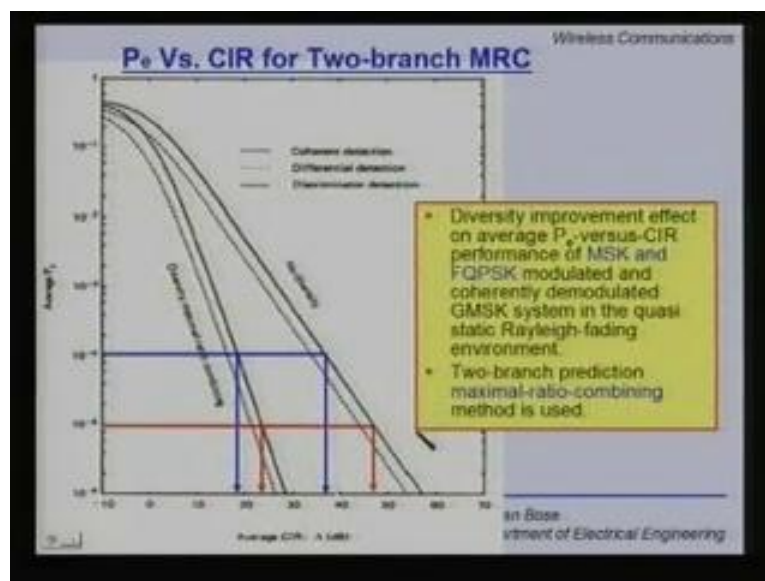


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Now let us talk about the two branch selection method, again probability of error versus CIR. So in this case we are talking about the interference, carrier to interference ratio; on the y axis we have average probability of error, on the x axis average CIR. Again the top curve represents no diversity whereas this is with two branch diversity using the selection method. Again there is a big improvement in performance. Here again we are using MSK kind of modulation scheme over a Quasi static Rayleigh fading environment, again two branch prediction method is used.

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Let us now look at the probability of error versus CIR interference carrier to interference ratio for two branch MRC maximal ratio combining. Here again we see that with respect to no diversity you have a substantial improvement. How much? Let us take this case, let us say we are working at 10 raise to power minus 4 average probability of error. So when you go from the case of no diversity wherein you require close to 38 dB C to IR you require only about 18 dB here. If you now put another restriction that you are working now not a 10 raise to power minus 4 but 10 raise to power minus 5, for no diversity case you have to go as high as 47 dB C to I ratio that is extremely high but if you are just using two branch maximal ratio combining, you are done with only close to 23 dB.

Now let us also understand the implication of the slope of the curve. Suppose I want to have one order of magnitude improvement in performance. Suppose my competitors are offering better performance, bit error rate performance then I would say I would like to move from 10 raise to power minus 4 to 10 raise to power minus 5, if I do so I have to increase my C to I ratio close to 10 dB whereas if I am using a diversity technique which is two branch MRC, I just have to improve it by about 5 dB. So that's where my gain comes in, that is the importance of the slope of these two curve.

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### Conclusions for Two-branch Diversity System

- The figures indicate that in order to improve the average  $P_e$  10 times:
  - It is required to increase the average CIR by about 5 dB when two-branch diversity improvement is available.
  - However, an increase of 10 dB is required in the case of no-diversity reception.

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So let us conclude from these two branch diversity system example. the figures indicate that in order to improve an average  $P_e$  to 10 times or one order of improvement, it is required to increase the average CIR by about 5 dB only when a two branch diversity improvement technique is used. However an increase of 10 dB is required if you have no diversity, so clearly diversity is the way to go.

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## Polarization Diversity

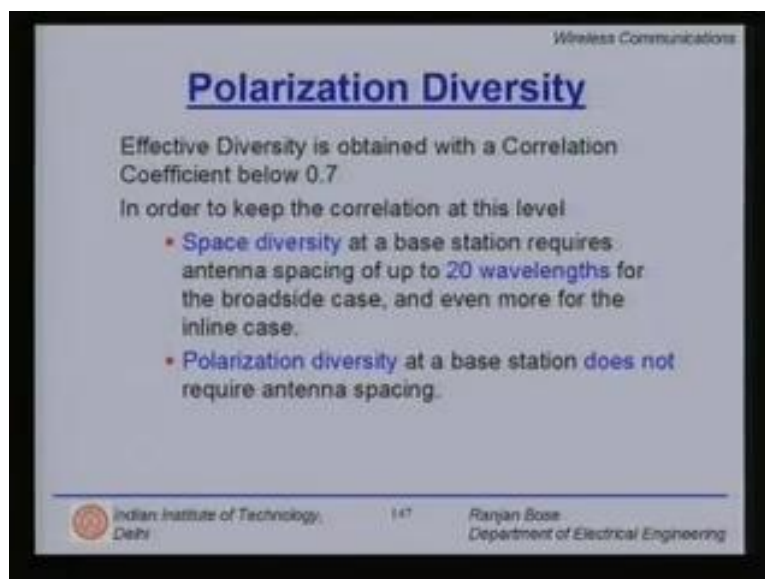
Principle:

*Polarization diversity relies on the decorrelation of the two receive ports to achieve diversity gain. The two receiver ports must remain cross-polarized.*

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Now let us switch gears and move into something different than the space diversity technique which is the polarization diversity. What is polarization diversity? The basic principle is as follows. Polarization diversity relies on the decorrelation of two receive ports to achieve diversity gain. The two receiver ports must remain cross polarized. So what was observed is that when you send a vertically polarized waves from the base station to the receiver and then the horizontally polarized waves, these two tend to be uncorrelated and we have to use this fact that we can use diversity which is based on polarization.

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## Polarization Diversity

Effective Diversity is obtained with a Correlation Coefficient below 0.7

In order to keep the correlation at this level

- Space diversity at a base station requires antenna spacing of up to 20 wavelengths for the broadside case, and even more for the inline case.
- Polarization diversity at a base station does not require antenna spacing.

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The effective diversity is obtained when a correlation coefficient is below 0.7. In order to keep the correlation at this level, space diversity at a base station requires antenna spacing up to twenty wavelengths. We learnt last time that you have to increase the spacing between antenna elements in a less cluttered environment which is true of the base station which is situated high up on top of a tower or on top of a tall building. Whereas you do not have to separate antenna elements by 20 wavelengths but just a few wavelengths or even  $\lambda$  by 2 for a fairly cluttered environment that the mobile station moves in. Now space diversity diploid at the base station really requires a lot of separation. In many cases it may be impractical. At the same time polarization diversity at a base station does not require any antenna spacing. It requires just two elements, one is vertical and the other is horizontally polarized or left hand circular polarized or right hand circular polarized.

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## Polarization Diversity

- At the base station, space diversity is considerably less practical than at the mobile because the narrow angle of incident fields requires large antenna spacing.
- The comparatively high cost of using space diversity at the base station prompts the consideration of using orthogonal polarization.
- Polarization diversity provides two diversity branches and allows the antenna elements to be considered.

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At the base station space diversity is considerably less practical than at the mobile because the narrow angle of incident fields require large angle antenna spacing. Also the comparatively high cost of using space diversity at the base station prompts consideration of using orthogonal polarization. Please note the signal coming from the mobile to the base station is a weaker signal and that's where we need to employ the diversity techniques. Polarization diversity provides two diversity branches and allows the antenna element to be considered.

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## Polarization Diversity

- In the early days of cellular radio, all subscriber units were mounted in vehicles or used vertical whip antennas.
- Today, however, over half of the subscriber units are portable.
- This means that most subscribers are no longer using vertical polarization due to hand-tilting when the portable cellular phone is used.
- This recent phenomenon has sparked interest in polarization diversity at the base station.



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Now let's take a step backwards, in early days of cellular radio all subscriber units were mounted either in vehicles or used with a vertical whip antenna. So you were dealing with vertically polarized waves. Today however over half the subscriber units are just portable; this means that most subscribers are no longer using vertical polarization due to hand tilting when the portable cellular phone is used. This recent phenomenon has sparked interest in polarization diversity at the base station. So earlier round these were vehicle mounted and the antenna elements were vertical most of the time so made sense to use only one kind of polarization. Now my antenna can have any kind of orientation and hence I will receive both vertically and horizontally polarized waves at the base station and then I can use polarization diversity.

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## Theoretical Model for Polarization Diversity

- Assuming that a signal is transmitted from a mobile station with vertical (or horizontal) polarization, and is received by a polarization diversity antenna with two branches at the base station.
- The measured horizontal and vertical polarization paths between a mobile and a base station are uncorrelated.
- The decorrelation for the signal in each polarization is caused by multiple reflections in the channel between the mobile and base station antenna.
- This results in signals of different amplitudes and phase reflections.
- In reality there is some dependence of the received polarization on the transmitted polarization.

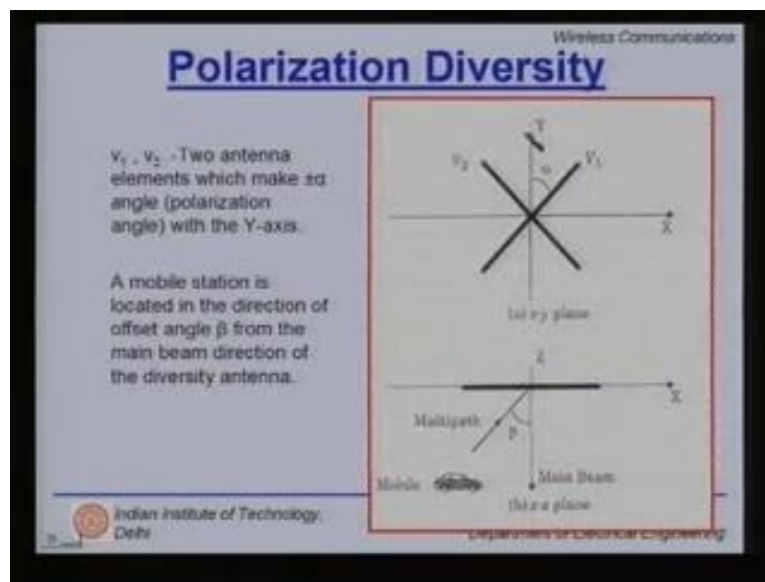
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Now let us look at briefly the theoretical model for polarization diversity. Assuming that a signal is transmitted from a mobile station with vertical or horizontal polarization and is received by a polarization diversity antenna with two branches at the base station. So this is the scenario that we are going to develop a model form. So we have two branches at the base station pertaining to either the vertical polarized, horizontal polarized or 45 degree of vertical, 45 degrees of horizontal. The measured horizontal and vertical polarization paths between a mobile and a base station are uncorrelated so this is the assumption you make. The decorrelation for the signal in each polarization is caused by multiple reflections in the channel between the mobile and the base station antenna. This results in signals of different amplitude and phase reflections. In reality there is some dependence of the received polarization on the transmitted polarization.

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So here we have the two antenna elements which make an angle alpha here and we are trying to derive some results based on this angle alpha and a mobile is located in the direction of the offset angle beta shown here. So this is the main b, either is the offset angle beta and this is your mobile. So the antenna elements of the base station are also at an angle alpha with respect to the y axis.




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## Polarization Diversity

- Some of the vertically polarized signals transmitted from the mobile station are converted to the horizontally polarized signal, because of multipath propagation.
- Horizontally Polarized Component  $x = r_1 \cos(\omega t + \phi_1)$
- Vertically Polarized Component  $y = r_2 \cos(\omega t + \phi_2)$
- These signals  $x$  and  $y$  are received at  $\beta = 0$
- Assuming that  $r_1$  and  $r_2$  have independent Rayleigh distribution,
- $\Phi_1$  and  $\Phi_2$  have independent, Uniform distribution.
- No cross coupling between the diversity antenna elements is assumed.

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Department of Electrical Engineering

Some of the vertically polarized signals transmitted from the mobile station are converted to the horizontally polarized signal because of multipath propagation. So each time the wave undergoes reflection, there will be some component of the vertically polarized beam coming into the horizontally polarized antenna element. So let us denote the horizontally polarized component  $x$  equal to  $r_1 \cos \omega t + \phi_1$  and similarly the vertically polarized component  $y$  equal to  $r_2 \cos \omega t + \phi_2$ . These signals  $x$  and  $y$  are received at  $\beta$  is equal to zero. What is  $\beta$ ?  $\beta$  we have seen earlier is the angle here. So let's go back to our  $x$  and  $y$  which are the horizontally and vertically polarized components. Assume that  $r_1$  and  $r_2$  have independent Rayleigh distribution, these are the two received signals,  $\phi_1$  and  $\phi_2$  have independent uniform distribution here  $\phi_1$  and  $\phi_2$  that's not a bad assumption and no cross coupling between the diversity antenna elements is assumed. So this is the model we are building up.

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## Correlation Coefficient $\rho$

The received signal values at  $v_1$  and  $v_2$  are

$$S_1 = x \sin \alpha \cos \beta + y \cos \alpha \quad S_2 = -x \sin \alpha \cos \beta + y \cos \alpha$$

Substituting  $x$  and  $y$


$$S_1 = (r_1 a \cos \phi_1 + r_2 b \cos \phi_2) \cos \omega t - (r_1 a \sin \phi_1 + r_2 b \sin \phi_2) \sin \omega t$$

$$S_2 = (-r_1 a \cos \phi_1 + r_2 b \cos \phi_2) \cos \omega t - (-r_1 a \sin \phi_1 + r_2 b \sin \phi_2) \sin \omega t$$

Where

$$a = \sin \alpha \cos \beta$$

$$b = \cos \alpha$$

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Let us first talk about the correlation coefficient rho, the received signal values at  $V_1$  and  $V_2$  are given by, this is a script  $V_1$  is equal to  $x \sin \alpha \cos \beta$  plus  $y \cos \alpha$  and  $V_2$  is equal to minus  $x \sin \alpha \cos \beta$  plus  $y \cos \alpha$  where alpha and beta are the angles defined earlier. Substituting x and y we get these two expressions for  $v_1$  and  $v_2$ . Here the two constants a and b are given by  $\sin \alpha \cos \beta$  is equal to a and  $\cos \alpha$  is equal to b. So it is possible to derive the received signal values at  $v_1$  and  $v_2$ .

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## Correlation Coefficient $\rho$

Amplitudes for  $v_1$  and  $v_2$  are

$$R_1 = \sqrt{(r_1 a \cos \phi_1 + r_2 b \cos \phi_2)^2 + (r_1 a \sin \phi_1 + r_2 b \sin \phi_2)^2}$$

$$R_2 = \sqrt{(-r_1 a \cos \phi_1 + r_2 b \cos \phi_2)^2 + (-r_1 a \sin \phi_1 + r_2 b \sin \phi_2)^2}$$

Therefore,

$$R_1 = \sqrt{r_1^2 a^2 + r_2^2 b^2 + 2r_1 r_2 ab \cos(\phi_1 + \phi_2)}$$

$$R_2 = \sqrt{r_1^2 a^2 + r_2^2 b^2 - 2r_1 r_2 ab \cos(\phi_1 + \phi_2)}$$

The correlation coefficient is defined as follows

$$\rho = \frac{R_1^2 R_2^2 - R_1^4 - R_2^4}{\sqrt{((R_1^2)^2 - R_1^4)((R_2^2)^2 - R_2^4)}}$$

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So the amplitude for the signals received at  $v_1$  and  $v_2$  is under root, the first element squared plus second squared. Similarly for  $R_2$  these are the two expressions. So if you simplify it, you get your expressions as  $R_1$  is equal to under root small  $r_1$  squared a squared plus  $r_2$  squared b squared plus  $2 r_1 r_2 ab \cos$  **zeta one** plus  $\phi_2$  and so and so forth. Now let us define the correlation coefficient as follows in terms of  $r_1$  and  $r_2$ . So this is the definition of the correlation coefficient. Clearly correlation coefficient depends on a and b which depend on the angles alpha and beta.

What are the assumptions? Let us say that  $R_1$  squared average value is  $r_1$  squared a squared plus  $r_2$  squared b squared average plus  $R_2$  squared. Then you have this following expression because I am trying to compute the value of the correlation coefficient here so I need to compute each of these values independently. So if you do the basic mathematics in this slide, these are the steps involved in order to achieve the final expression. This one shows that  $R_1$  squared whole squared is equal to  $R_2$  squared whole squared average value.

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## Correlation Coefficient $\rho$

Assumptions

$$\langle R_1^2 \rangle = \langle r_1^2 a^2 + r_2^2 b^2 \rangle = \langle R_1^2 \rangle$$


$$\langle R_1^2 R_2^2 \rangle = \langle (r_1^2 a^2 + r_2^2 b^2 + 2r_1 r_2 ab \cos(\theta + \phi_1)) (r_1^2 a^2 + r_2^2 b^2 - 2r_1 r_2 ab \cos(\theta + \phi_1)) \rangle$$

$$\langle R_1^2 R_2^2 \rangle = \langle r_1^4 a^4 + r_2^4 b^4 - 4r_1^3 r_2 a^3 b^2 \cos^2(\theta + \phi_1) \rangle$$

$$\langle (R_1^2)^2 \rangle = \langle (r_1^2 a^2 + r_2^2 b^2 + 2r_1 r_2 ab \cos(\theta + \phi_1))^2 \rangle$$

$$= \langle r_1^4 a^4 + r_2^4 b^4 + 4r_1^3 r_2 a^3 b^2 \cos^2(\theta + \phi_1) \rangle$$

$$= \langle (R_1^2)^2 \rangle$$

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Substituting these values it is finally possible to compute the correlation coefficient rho which is given by this, if you want to simplify it you will be able to get a much easier to write expression but  $r_1$  and  $r_2$  which we have known follow Rayleigh distribution. If you define  $r_2$  squared average value over  $r_1$  squared average value equal to capital gamma then this expression rho can be simplified as follows, rho is equal to a squared minus b squared gamma over a squared plus b squared gamma whole squared. Now if you substitute the values of a and b you get the following expression. Please note that capital gamma has been defined as  $r_2$  squared average over  $r_1$  squared average.

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## Correlation Coefficient $\rho$

Substituting the values

$$\rho = \frac{\langle r_1^4 \rangle - \langle r_1^2 \rangle^2 \langle a^4 \rangle + \langle r_2^4 \rangle - \langle r_2^2 \rangle^2 \langle b^4 \rangle - 2 \langle r_1^3 \rangle \langle r_2 \rangle \langle a^3 b^2 \rangle}{\langle r_1^4 \rangle - \langle r_1^2 \rangle^2 \langle a^4 \rangle + \langle r_2^4 \rangle - \langle r_2^2 \rangle^2 \langle b^4 \rangle + 2 \langle r_1^3 \rangle \langle r_2 \rangle \langle a^3 b^2 \rangle}$$

Because  $r_1$  and  $r_2$  follow Rayleigh distribution

$$\langle r_1^4 \rangle = 4 \langle r_1^2 \rangle^2$$

$$\langle r_2^4 \rangle = 4 \langle r_2^2 \rangle^2$$

Let  $\frac{\langle r_2^2 \rangle}{\langle r_1^2 \rangle} = \Gamma$ , then  $\rho$  will be

$$\rho = \frac{a^2 - b^2 \Gamma}{a^2 + b^2 \Gamma} = \frac{\tan^2 \alpha \cos^2 \beta - 1}{\tan^2 \alpha \cos^2 \beta + 1}$$

Where  $\Gamma$  is the cross polarization discrimination of the propagation path between a mobile and a base station.

In some sense this gamma is the cross polar discrimination factor, it is telling you how much is the ability to reject the horizontally polarized wave by the vertically polarized antenna and so and so forth. So here gamma is the cross polarization discrimination of the propagation path between a mobile and a base station is also called XPD or cross polar discrimination.

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Wireless Communications

### Average Signal loss L

The average level of vertically polarized signal level is  $\langle R_0^2 \rangle$

We have seen before that  $\langle R_0^2 \rangle = \langle r_2^2 \rangle$

Hence we have  $\langle R_1^2 \rangle = \langle r_1^2 a^2 + r_2^2 b^2 \rangle = \langle R_0^2 \rangle$

The signal loss is

$$L = \frac{\langle r_1^2 \rangle}{\langle r_2^2 \rangle} a^2 + b^2$$

$$\Gamma = \frac{\langle r_1^2 \rangle}{\langle r_2^2 \rangle}$$

$$L = \frac{1}{\Gamma} a^2 + b^2$$

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Let us talk also about the average signal loss L, the average level of vertically polarized signal level is given by the average value  $R_0$  squared. We have seen before that  $R_0$  squared is equal to average value  $r_2$  squared, hence we can write the following expression. The signal loss L is given by average  $r_1$  squared over  $r_2$  squared a squared plus b squared. So this basic steps can be carried out to come up with this expression. Now we already know that gamma or the XPD cross polar discrimination is defined as  $r_1$  squared over  $r_2$  squared both average. Hence the average signal loss L comes out as one over gamma a squared plus b squared. Please note a and b both depend on the angles alpha, the angle between the two antenna elements at that base station and beta which is the angle with which the mobile is moving with respect to the main beam. So this is an expression that we'll use to calculate the average signal loss.

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### Example

$$\rho = \frac{(a^2 - b^2\Gamma)}{(a^2 + b^2\Gamma)} \left( \frac{\tan^2 \alpha \cos^2 \beta - \Gamma}{\tan^2 \alpha \cos^2 \beta + \Gamma} \right)$$
$$L = \frac{1}{\Gamma} a^2 + b^2$$

Three factors determine  $\rho$  and  $L$

- Polarization angle  $\alpha$
- Offset angle  $\beta$
- Cross polarization discriminator  $\Gamma$

When  $\Gamma = 0$   
At  $\alpha = 50^\circ$  and  $\beta = 33^\circ$ ;  $\rho = 0$

When  $\Gamma = 0 \text{ dB} = 1$   
At  $\alpha = 45^\circ$  and  $\beta = 90^\circ$

$$L = \frac{1}{1} \sin^2 45 \cos^2 90 + \cos^2 45 = -3 \text{ dB}$$

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Department of Electrical Engineering

Let us take an example to understand these equations. So far we have derived very briefly two equations, one is an expression for rho and the other is an expression for the loss factor L. Please note both of them depend on this gamma, gamma is the cross polar discrimination. Note if you substitute gamma is equal to zero then both the numerator and denominator coincide and correlation is one, perfect correlation. So the idea is for a good polarization diversity scheme gamma should be as high as possible. In real life measurement gamma is measured to be close to 20 to 25 dB. Three factors determine rho and L these are the polarization angle as you have seen, the offset angle beta and the cross polar discrimination gamma.

Now let's take two small examples when gamma is equal to zero and if you take these special angles if you substitute it here then rho will also come out to be zero. When gamma is equal to zero dB that is one, in that case if you substitute the values alpha is equal to 45 and beta is equal to 90 then the loss factor comes out to be minus 3 dB. So you can say that for zero dB cross polar discrimination and 45 degree alpha which is typical and beta to be 90 degree you will get a loss factor of minus 3 dB.

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## Frequency Diversity

**Principle:**

*The same information signal is transmitted and received simultaneously on two or more independent fading carrier frequencies.*

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Now let us move to another diversity technique which is the frequency diversity. What is the basic principle? The same information signal is transmitted and received simultaneously on two or more independent fading carrier frequencies.

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## Frequency Diversity

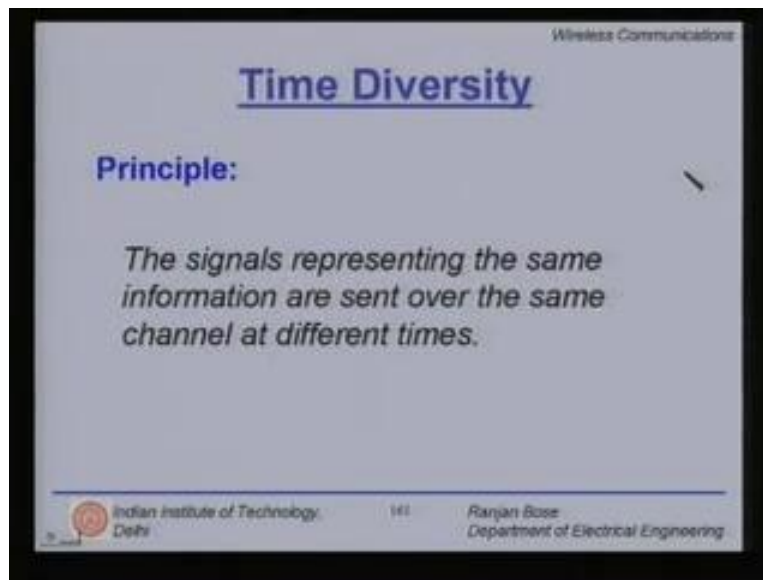
- The rationale behind this technique is that frequencies separated by more than the **coherence bandwidth** of the channel will not experience the same fade.
- The probability of simultaneous fade will be the product of the individual fading probabilities.
- This is often employed in microwave LOS links which carry several channels in a frequency division multiplex mode (FDM).
- This technique not only requires **spare bandwidth**, but also requires that there be as many receivers as there are channels used for the frequency diversity.
- However, for critical traffic, the expense may be justified.

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The rationale behind this technique is that frequencies separated by more than the coherence bandwidth of the channel will not experience the same fade. Several lectures earlier we have talked in detail about coherence bandwidth and how to determine coherence bandwidth for a particular wireless channel. The probability of simultaneous fade will be the product of the individual fading probabilities because they are fading independently.

This technique, the frequency diversity technique is often employed in microwave lines of sight links which carry several channels in a frequency division multiplex mode or FDM. This technique not only requires spare bandwidth because you are transmitting the same information in two different carriers separated by more than the coherence bandwidth but also requires that there may be as many receivers as their channels use for frequency diversity, each one tuned to the different frequency. However for critical traffic this expense may be justified, so the point being illustrated in this frequency diversity scheme slide is that frequency diversity is expensive in terms of bandwidth but if you are working in a wide band scenario anyway then this can be used.

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## Time Diversity

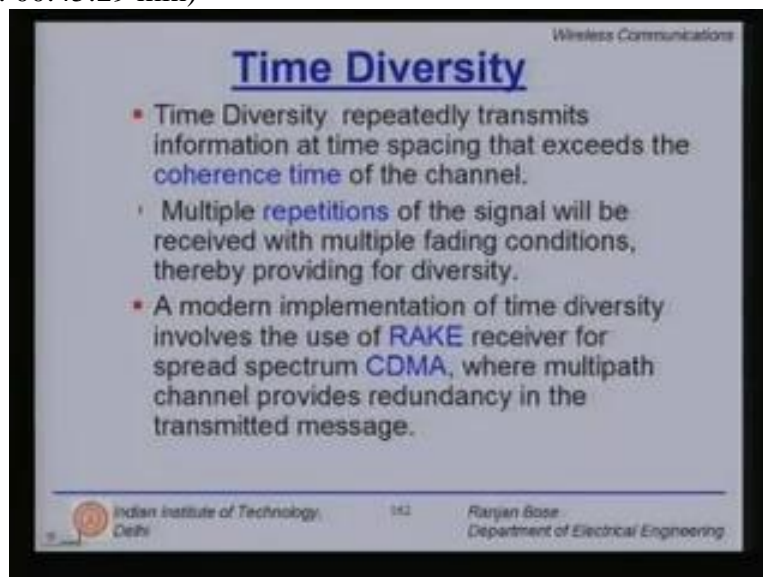
**Principle:**

*The signals representing the same information are sent over the same channel at different times.*

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Let us now talk about the fourth method which is the time diversity technique. The principle is fairly simple, the signals representing the same information are sent over the same channels at different times. Now how far apart?

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## Time Diversity

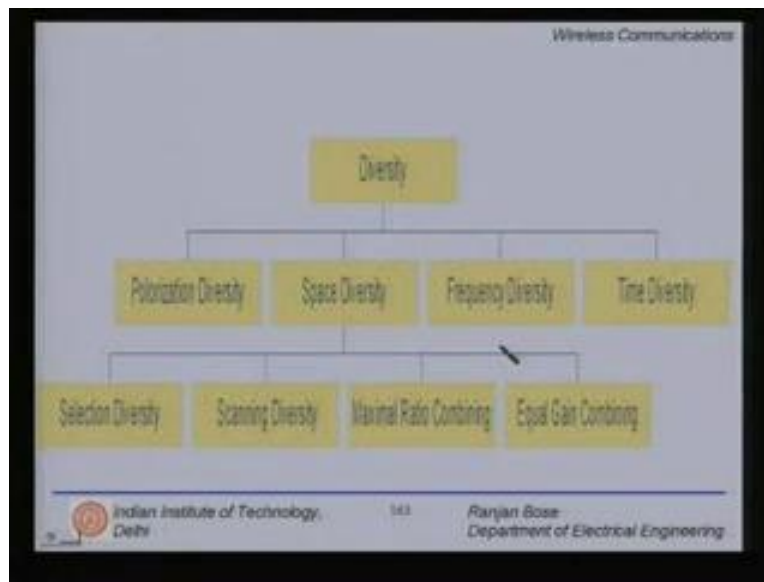
- Time Diversity repeatedly transmits information at time spacing that exceeds the coherence time of the channel.
- Multiple repetitions of the signal will be received with multiple fading conditions, thereby providing for diversity.
- A modern implementation of time diversity involves the use of RAKE receiver for spread spectrum CDMA, where multipath channel provides redundancy in the transmitted message.

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The time diversity repeatedly transmits information at time spacing's that exceed the coherence time of the channel. In earlier lectures we have seen that coherence time is related to the maximum Doppler shift of the channel. So if you know the Doppler shift that your channel introduces you can come up with the coherence time value and then you can use that value to determine how much spacing you have to put for time diversity. Multiple repetitions of the signal will be received with multiple fading conditions thereby providing the diversity. A modern implementation of time diversity involves the use of rake receiver for spread spectrum CDMA where multipath channels provide redundancy in the transmitted message.

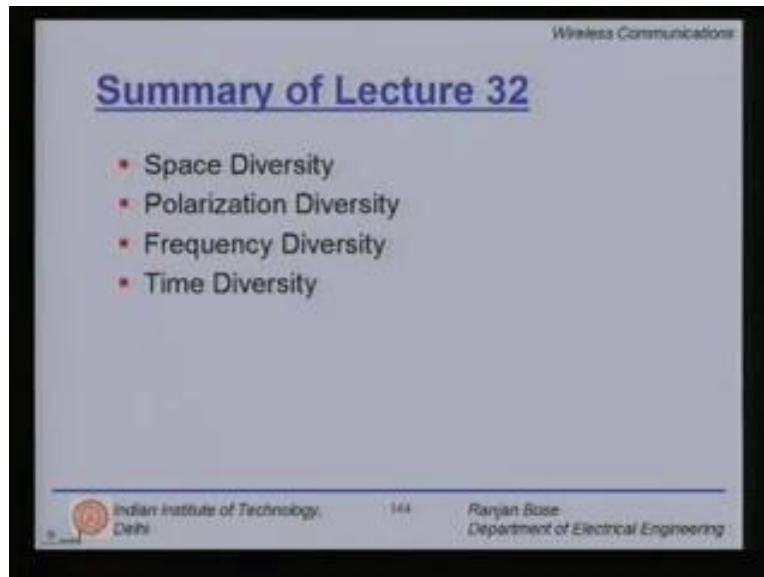
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So just let's have a bird's eye view of the various diversity techniques we have discussed so far. The important sub classes are the polarization diversity, the space diversity, the frequency diversity and the time diversity. The space diversity we have seen is one of the more popular techniques wherein we have sub classes selection diversity, scanning, maximal ratio combining and equal gain combining. So this gives us a broad perspective of the various kinds of diversity techniques used in wireless communications.



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Let us now try to summarize today's lecture. We started off with space diversity and different kinds of space diversity the selection, the scanning, the MRC and the equal gain combining methods then we talked about the polarization diversity we set up a model and we computed the  $\rho$  the correlation factor and  $L$  the system loss model. Finally we talked about the frequency diversity wherein you must transmit your signals spaced in frequency domain for more than the coherence bandwidth of the channel. Finally we talked about time diversity where multiple copies of the signal are transmitted separated in time and the separation should be at least equal to the coherence time of the channel. So this brings us to conclusion of different kinds of diversity techniques. We will conclude our lecture here.