INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

NPTEL

NPTEL ONLINE CERTIFICATION COURSE
An Initiative of MHRD

VLSI Design, Verification & Test

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NPTEL Phase-II
Video course on

Design Verification and Test of Digital VLSI Designs

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Module IV: Temporal Logic

Lecture II: Temporal Logic: Introduction and Basic Operators

So in last class we have introduced a logic called temporal logic by which we can capture or we can express the timing behavior. Now we are going to see in details how, what is required in temporal logic and how we can express the properties in temporal logic. So basically this is a specification language we are going to specify the properties of our system. Because in modern synapses that we have three components one is your model of the system, second one is your specification in proper the specification of the system and third is your verification method logics okay.

So basically we are going to look in the second component today, specification how to provide specification and for that we are going to use temporal logic.

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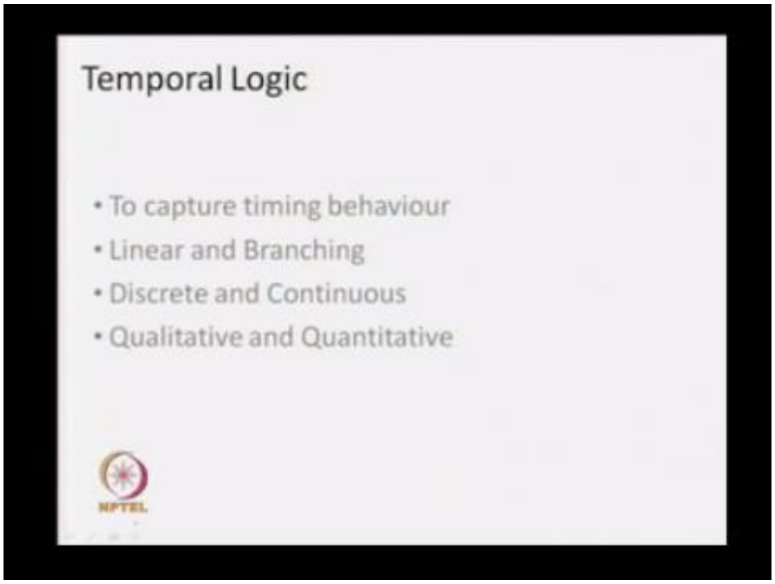

So already I have mentioned that in temporal logic basically we are going to capture the tinning behavior of the system, secondly if you look into the notion of timing where i have being two type of notion, one is your branching time and second one is linear time. Again I have mentioned about that we can capture timing in two different way one is your discrete and second one is your continuous.

Again I mentioned that how to specify the behavior again we can look into two different way one is your qualitative and second one is your quantitative. In case of qualitative we just specify the in FUTURE something is going to happen or not, or in quantitative we can specify after some unit of time something is going to happen in the system. So in this lecture basically we are going to talk about qualitative issues of temporal logic.

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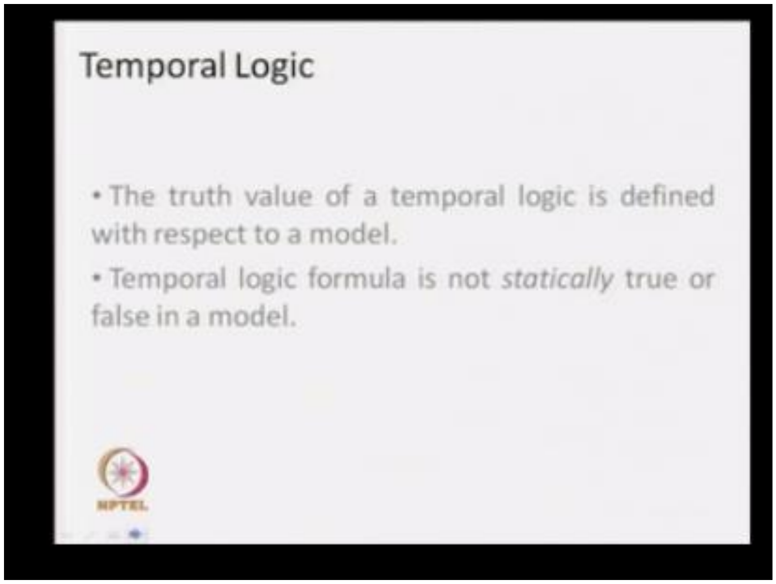

So we have already introduced about the logics we have talked about proportional logic, predicate logic or higher order logic and now we are coming to the temporal logic. So when you are going to look in the truth values of the temporal logic formula, basically the truth values of temporal logic formula is defined for a model or on logic truth with respect to a model we should have a model and it is in this particular model we are going to define the meaning of those particular temporal logic formulas.

So temporal logic formulas is not statically true or false in a model, in some of the state it may be true or in some of the state it may be false. So it is not statically true or statically false like the ado logic called proportional logic and predicate logic so it is a system dynamic behaviors in some state it will be true or in some state it will be false.

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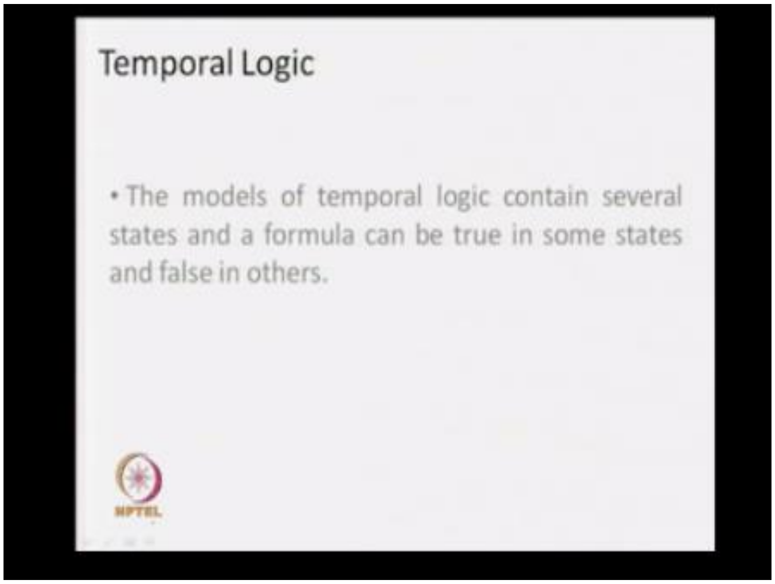

So for that what records we will need the models, the models of temporal logic contain several states and the formula can be true in some state and false in others. Now what we can represent in temporal logics, so we can expect the statements like "I am always happy", so it says about the state of my mind " I am happy, I am always happy". On the other hand it says that I will eventually be happy that means sometimes I will be happy or it says that I will be happy until I do something wrong. Si I will be happy until I do something wrong not here oh, so a countdown.

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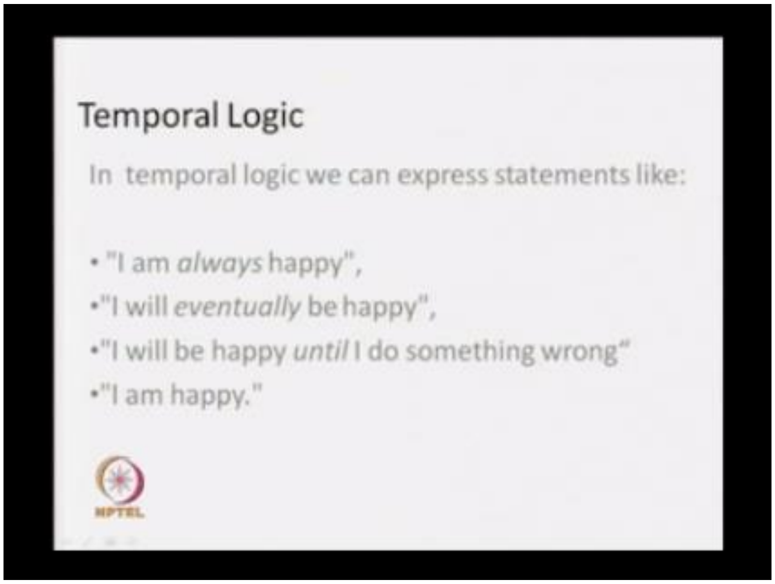

So in temporal logic we can express statements like " I am always happy" which says that I will be happy in all the times or I will eventually be happy so in FUTURE I will be happy or I will be happy until I do something wrong or I can say that I am happy. So these are the way things that we can expect now we will say this is the simple example I am giving, but we will what we can do.

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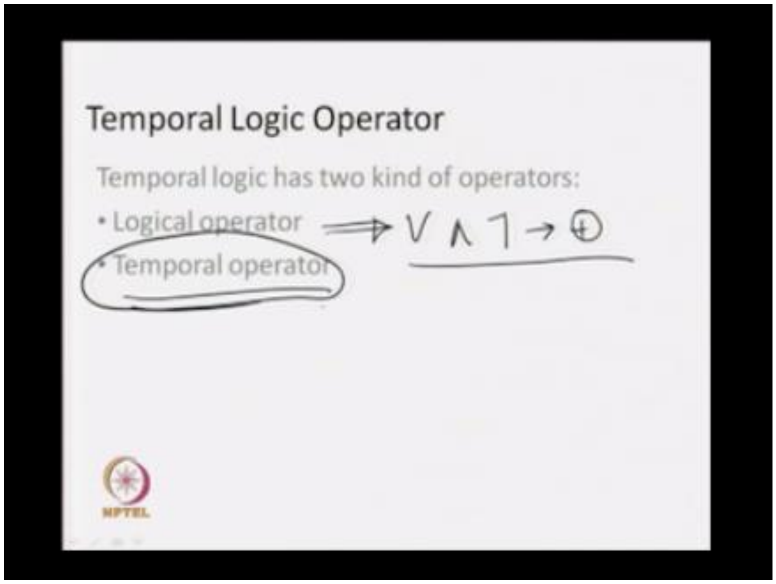

Now temporal logics are having two kind of operators one is your logical operator and second one is your temporal operator. Already we have talked about that logical operator where we are having that disjunction, conjunction, negation or say implication or EXOT like that all those logical operator that we have in other logics can be used here in temporal logic also, along with that we are having another kind of operator which is known as temporal operator.

So this is the basic things that we are having now we are going to see what are the temporal operators that we have in temporal logics. So we know the meaning of other operators logical operators, the meaning is similar to the, similar with respect to the other logic like here proportional logic or in predicate logic. So we will see what are the temporal operators that we have in temporal logic.

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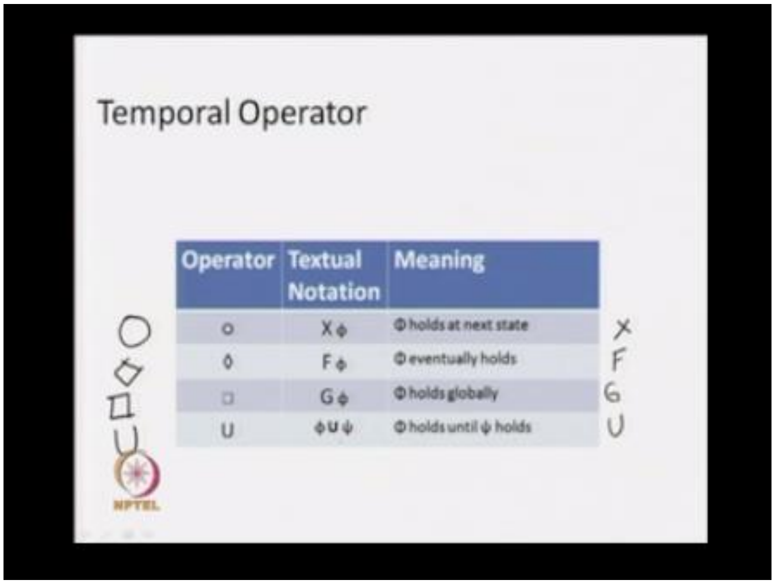

So basically here we are going to have four different type of operators we said these are temporal operators and these are basically related to or going to talk about the timing behavior of the system. So one is your NEXT operator, so this NEXT operator is basically represented by south pole, second one is your eventually operator it is represented by diamond, third one is your GLOBAL or always which is represented by box, and another operator we are having that on in which is represented by U.

In this case your NEXT state, FUTURE and your GLOBAL these are your unary operator it is going to take one particular formula, but UNTIL is a binary operator it takes two formulas, two formulas then it says talk about a meaning with respect to this formula. So in your classical notation you use these particular symbol south pole, diamond, box and UNTIL, but in textual notation this NEXT operator is represented by X.

Eventually the operator or FUTURE operator is represented by F, GLOBAL our always operator is represented by G and this UNTIL operator is represented by U. So in case of X if we write that means in NEXT state ϕ holds, if I say that Fϕ it says a dot formula ϕ holes eventually that means in FUTURE since some FUTURE state these particular formula ϕ will hold. So Gϕ which is globally ϕ it says that ϕ is always true in the system.

That means in all state ϕ is true and ϕ UNTIL size, so it is a binary operator we are having two formulas ϕ and φ it says that ϕ remains true until φ holds, that means ϕ remains true until φ holds in that particular we say that ϕ until φ holds in a particular state. So basically we are going to look for this particular four temporal operators and we will see now how we are going to represent them and how we are going to define the meaning of these operator.

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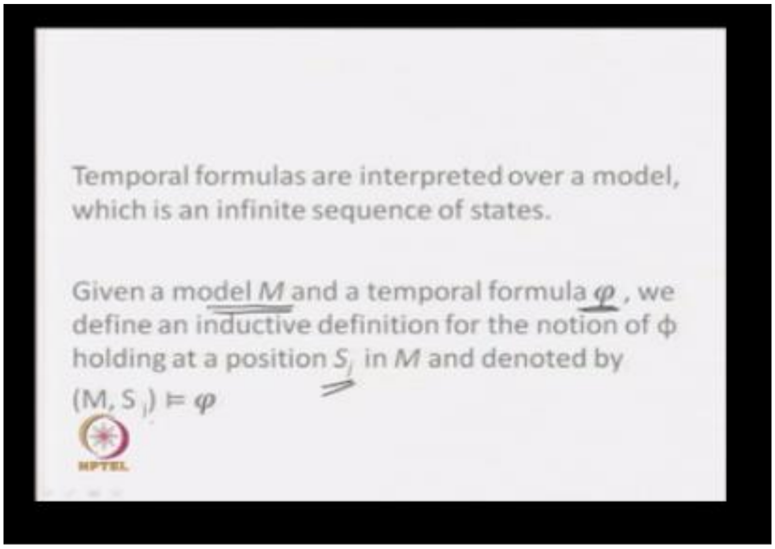

Now already I have mentioned that temporal formulas or meaning of temporal formulas interpreted over a model, which is an infinite sequence of states. So in a system we are having number of states and basically we are having an infinite sequence of state if we talk about reactive system, so we are going to define the meaning of those particular temporal formula over the states of a model.

So how formally what we can define, we can say that define a model M, so we are going to talk one model M and a temporal formula ϕ, so we are going to talk one temporal formula ϕ, so we take these two component one model and one temporal formula ϕ we define a inductive definition of the notion of ϕ holding at a position Sj so in the system we are going to look for notion Sj in M and it is denoted by M, Sj models φ.

It says that in a model M in a state Sj the formula ϕ holds and we say that M, Sj model is φ so this is notion that truth values of a temporal formula is basically defined in a model and in a state we are going to say it holds or not, but they are having some other notion also in the lecture I an going to elaborate those things.

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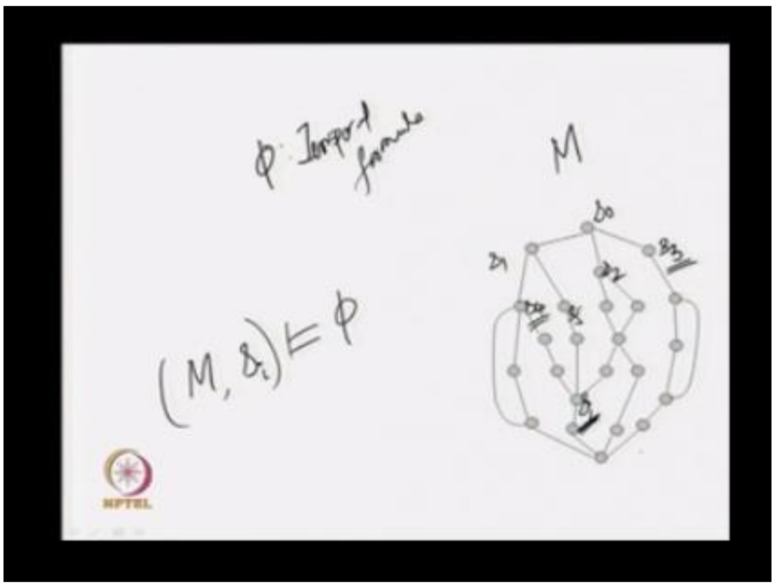

Now in this particular I say that this is a state transition model, so we can have different state called S0, S1, S2, S3, S4, S5 like that we can name the state of those particular model and if you consider about some particular step say Si so this is the model M and we are basically concerned about this particular state Si so we will say that this model M in this particular state i, it models the temporal formula ϕ if ϕ is a temporal formula.

So we are having a temporal formula and we want to check the truth values of this particular temporal formula, so we have to check with respect to a model M and we are going to define in a particular state Si, so in this particular state we will say that M Si models ϕ it means in state Si model ϕ holds. So this is basically internal meaning of the truth values of our temporal formula. So in this particular case I am saying that ϕ holds in this particular state.

But it does not mean that ϕ holds in the entire system it may be false in this particular S tree or it may be false in S port, so that is why I already mentioned that the truth values of the temporal formula basically defines about a state in this particular model. In some state it will be true or in some state it will be false.

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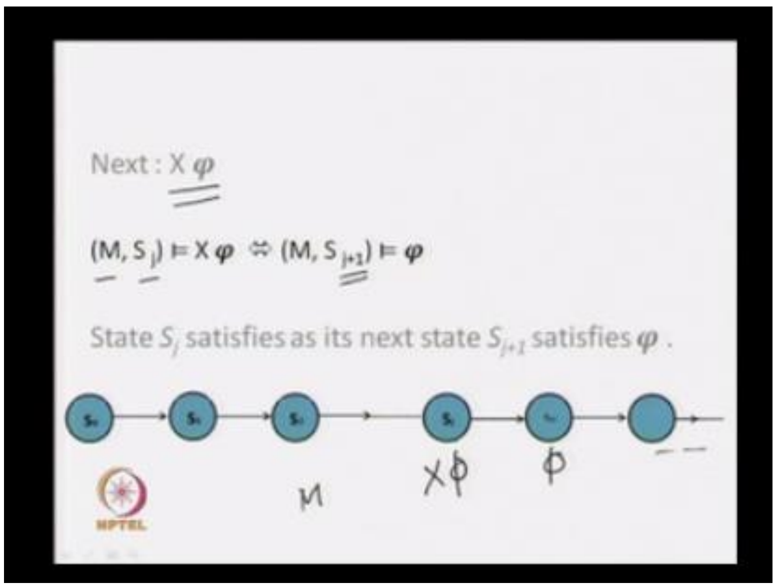

Now we are going to see the operator by operator how, what is the meaning of these things, so first we are going to look into the NEXT operator. So basically we have already mentioned that in textual notation NEXT will be represented by X, so this is the operator NEXT Xφ that means the temporal formula Xφ it says that NEXT φ.

So what is the internal meaning of how we are going to define the meaning of this particular formula, so in a model M in state Sj model Xφ provided in a very next state Xj+1 φ hold. So if in a particular state φ holds then what will happen in the previous state we are going to set the next φ holds. So basically if I am going to look into this particular model it is going from say state S0, S1, S2 like that Sj, Sj+1 and for that.

So what will happen if I level this particular Sj+1 with ϕ it means that ϕ holds in this particular state Sj+1 of this particular model M. So by meaning of these things we are going to say that where this particular formula NEXT ϕ holds, then we will find that state Sj models next φ, because when you come to this particular state Sj during the transition time then we will find that from Sj wherever I am going in this next state ϕ holds so we say that Xφ holds in this particular state Sj.

So this is the meaning of this particular NEXT operator. So Xφ holds or Xφ through in a particular state provided NEXT state φ holds. So this is the meaning of your Xφ and we are going to define the meaning within a model.

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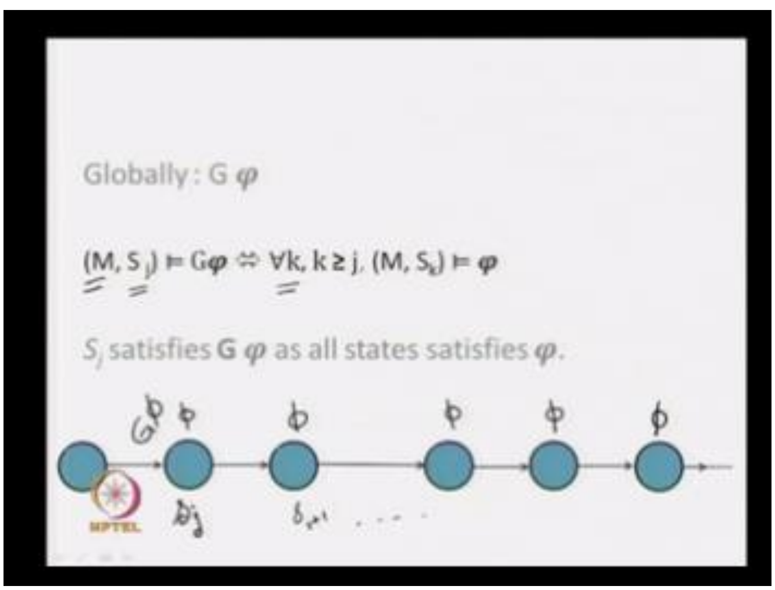

Now next operator we are going to look for eventuality or FUTURE Fφ, now how we are going to define, now you see the meaning of these things we can say that in a model M, in state Sj a φ holds so this is the temporal formula Fφ so in FUTURE φ holds basically it says that we are going to get some state K in the model where K is greater than j if we are going to look in a particular state j and it says that model M and in state K that φ holds.

Now in this particular model say this is my model I am going to look for one particular state say Sj so this is the state we are looking into it, in the slide we are going to get some state say Sk and say Sk is leveled with this particular φ, that means it says that in Sk φ holds. So when we are going to say that Fφ holds in a particular state here φ is also another temporal formula, so we are using a temporal operator φ where we are going to say in which that Fφ holds.

So if in FUTURE we are going to get a state where φ holds then we can say that in that particular state Fφ holds. So we can mark this particular state with Fφ, so what it says if I'm in state Sj in this transition line I am going to get some state in FUTURE where φ holds, so we can say that Fφ holds in this particular Sj. So that is why I'm saying that there exist some K where K is greater than j in my timeline where M Sk model is φ.

So if we are going to get such type of state in my model, then we can say that in my model in the state or the state that I am looking into the U set up f5 holds in that particular state that means in further we are going to get state where 5 holds so f5 holds in sz. Now next operator is your globally G Ø so weather globally Ø holds are not so again similar way we are going to define model M we are look a particular state Sj and we say that M Sj model Gv provided to we are going to get some k for all k is greater than equal to j M Sk Ø that means in all state where all state Sk where k is greater that j if it models Ø then we say that G Ø or that Sj model G Ø globally Ø holds.

So basically if I look into this particular state Sj now we are having this particular state Sj + 1 like that we are having so if all the states if we have model Ø it says that k greater than Z so Ø must be true in this particular state also now we are infinite sequence so basically we have to look for all the state that whatever coming in further so it is basically reason what an infinite system but we will come down in a state that everything can be represented by finite state model so your number of state will be finite but they will repetitive in measure.

So we are going to get an infinite sequence so in that particular case from a particular state Sj if all other state in further that Ø holds then we can say that GØ holds in this particular state so basically now I can level this particular state with GØ so we will say that Ø holds in Sj because in all the states that can be reachable for that particular Sj that Ø holds so this is the meaning of globally operators so globally Ø it says the globally Ø holds in a particular if ψ holds all the states in this particular system. So this is the meaning of G Ø.

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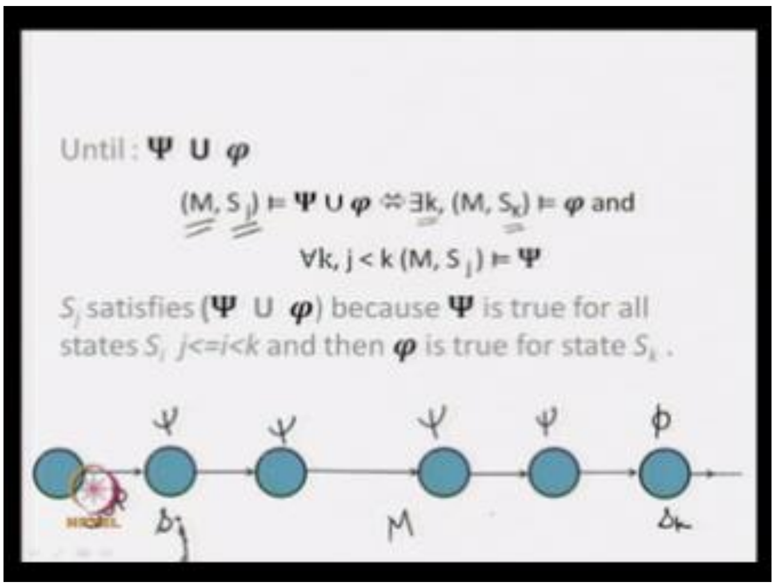

And the next operator is your until we will say that already I have mentioned that until is a binary operator having two operant 1 is your ψ and second one is Ø so it is ψ until Ø now what we are going to say that again we are going to take a model M we are going to take one particular state Sj we are going to see that weather in state Sj or model M ψ until Ø holds are not it will be holds if we are going to get some k there exits some k we are going to get a state k Sk so in the model M or state Sk ψ holds are what they are.

And for all other k where distancing see out j is less that k that M Si model ψ okay so in this particular case we are going to say that particular state holds ψ until Ø so basically we can say that I am looking into particular state say Sk it models this particular Ø so in this particular state

Ø holds in your state Sk now we are looking similar states say Sj now we have see whether Sj holds ψ until Ø holds in Sj or not it says that in further we are going to get some Sk where Ø holds and in all the state in between them where coming from this particular Sj to that particular Sk ψ must be true.

So there must be level by this particular ψ so in that particular case we say that Sj model ψ until Ø that means ψ remains to until Ø becomes true so in Sk Ø is true and all the state that is going from all the states that we are having in between from Sj to Sk for ψ must be true so in that particular case we say that Sj models ψ until Ø so this is the internal meaning of a until operator so in a distance model M we are mark this particular state Sj by the formula ψ until Ø provided if say this Ø full fill this particular behavior that ψ remains to until Ø becomes true.

So these are sun meaning of your until operator and we are defining a meaning of this until operator in a model so basically.

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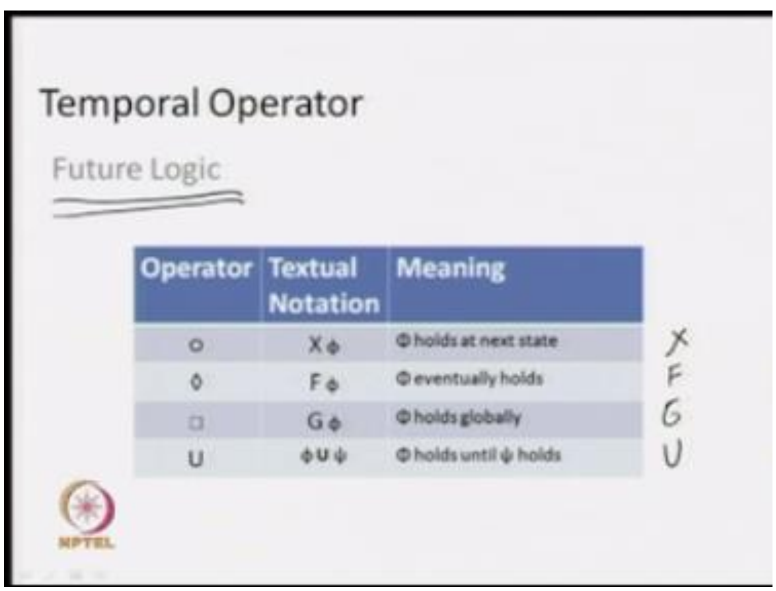

Now we have see this particular 4 operator 1 is next state operator next one is your eventuality operator third one is your globally operator and 4th is your until operator so these are the 4 temporal operators that we have and in classical notation we just represent x by your next state by your circle then the x is for eventuality box is for globally and U is for until so if you looking into that your saying that if we are in particular state whether something is going to happen in further.

Weather in next state or in all the state in the further so basically these particular 4 operators that I have introduced basically forms the further logic it is a further temporal logic we are going to reason about the further behavior of the system I am in particular state say the system is coming to a particular state now from that particular state how system is going to behave in further so if we are going to rezone about this particular behavior then we are going to look for this particular further temporal logic so this further temporal since we are talking about the further temporal logic whether we are having something called pass temporal logic yes indeed we have so we say that what are the motion of the pass temporal logic.

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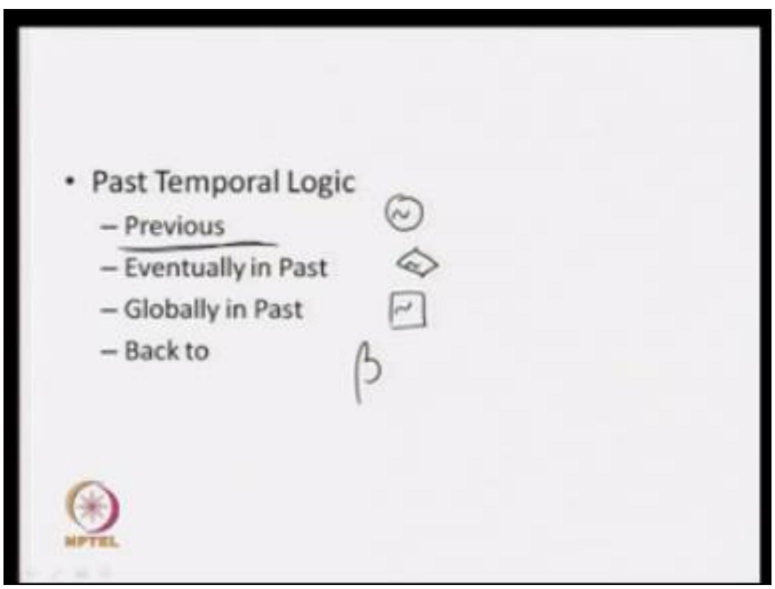

So in case of your pass temporal logic like a further temporal logic here also we are going to define 4 temporal operators which are correspondence to those particular 4 further temporal logical so first is your pervious we are going to talk about the pervious state and this is notational we are going to represent it by say for further we are having this particular charcoal so in case of previous that charcoal in between we are going to use that the symbol ~ it say that this is the pervious operator.

That means it is a first temporal logic we are going to look for the pervious state so eventually passed so basically in further we are going to have this particular diamond okay in the inside diamond we are going to write this particular ~ then basically it is going to talk about past or it is going to represent this eventuality in pas eventually in passed operator so similar we are having globally passed or globally in pass so again it is the same symbol we are going to use the box is used for your globalist operator.

So box in that inside this box we are going to but this ~ inside it and we are going to set up this is the globally in passed operator and back to it is again corresponds to the until operator so back to is a occurred by this particular symbol so now we can listen about the pas behavior of the system so in that particular case what happens you have to look for the pass temporal logic so again your further temporal logic we are going to have this particular 4 operator pervious eventually in pas globally in pas and back to.

Now in that see that like your further temporal logic now we are look for the meaning of your pass temporal logic so that pervious operator.

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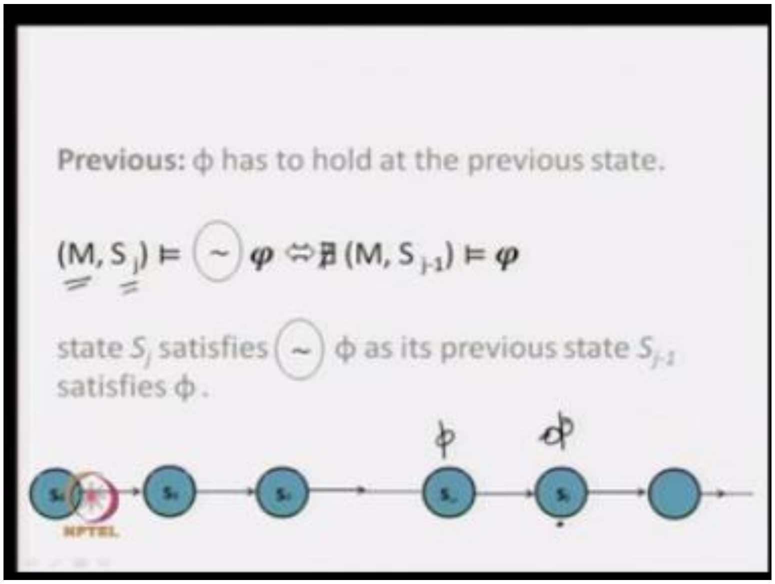

We are going to look for a particular model M and we are look for state Sj now we can say that state in Sj or model M with the formula pervious Ø force provided we are going to get a state this a wrong symbol it is reword so if M Sj – model Ø then we can say that pervious is 2 in that particular state Sj so in when we are going to say that in the particular case weather we are going to look for a particular state M, Sj+ 1 say this look Ø holds over here then we can say that in the particular state Sj your previous Ø holds.

Okay so this is the way that we are going to say that in if pervious state that Ø holds and you can say that in that particular state I your previous Ø holds at them so this is the meaning of the pervious state so if we are in particular state whether if in pervious state something happens then we can say that pervious Ø holds in that particular state now second one is your eventually in pass.

So this is similar to further so in case of further we are saying that if we are in particular state in further something is going to happen or not in case of a eventually past from particular state we are going to say that weather something happen in the past or not so basically if I say a look into this particular model M and say this the state Sj now wither these particular state Sj models some eventually in past Ø holds or not.

Then in this time line we are going to get look for some state and say that this particular state say Ø holds in this particular state Sk and where k is your less than your j then we will say that eventually in past holds in this particular state Sj so now we can say that this particular state model with eventuality Ø so say that meaning is corresponds to your further logic so in if you are in a particular state we are going to see in further something holds are not but in case of past we are going to set something happen in a past or not.

So this is a the way we are going to have the meaning so we are going to define a meaning of those particular first temporal logic operator with respect to the model also so similarly next will be our globally in past that means weather something happens globally or not.

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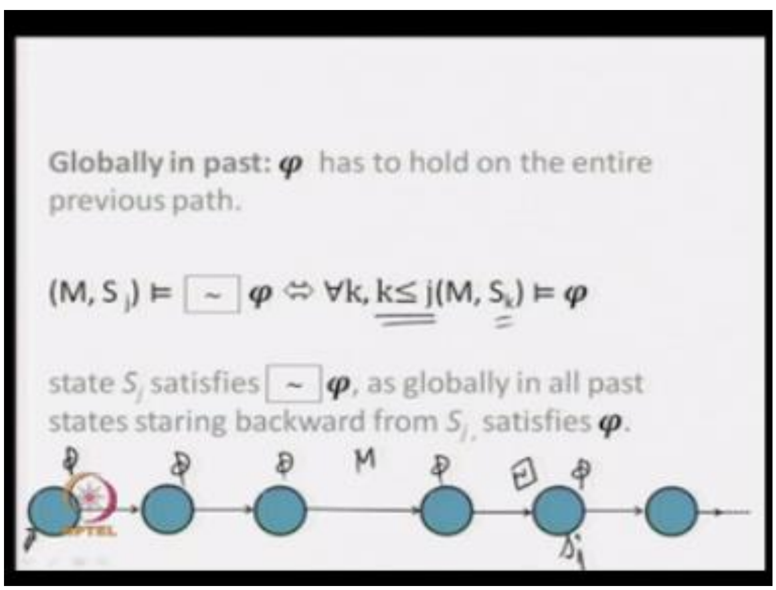

So meaning is something similar to all globally operator in further but here we are going to talk about similar ways say this is model M so we are look for one particular state say Sj whether in this particular state Sj it is globally Ø holds or not so in that particular case generally in past we are going to have something but again we will restrict it to some other initial state we will go up that particular initial state on it because if you talk about a slight in further we are having infinite sequence state.

So that means it may happens infinitely in you past also but always you can say that we are going to listen about from a particular state that is a stating of my model starting of my systems so we will go up that particular point so we are going to look for all k such that k is less than j and say all those particular Sk this Ø holds that means if say that from Sk all those particular states Ø all say that I am staring from this particular state say that this is the I am look for this particular behavior only then in all those particular state from Sj if Ø holds then we can say that globally in past Ø holds.

So we can say that we can mark this particular state with globally in past operator Ø so this is the meaning of past operators so we are going for a particular state and from that particular state we will see all the state in a past and if that particular formula holds in all that past state as then we can say that globally in past Ø holds in that particular state.

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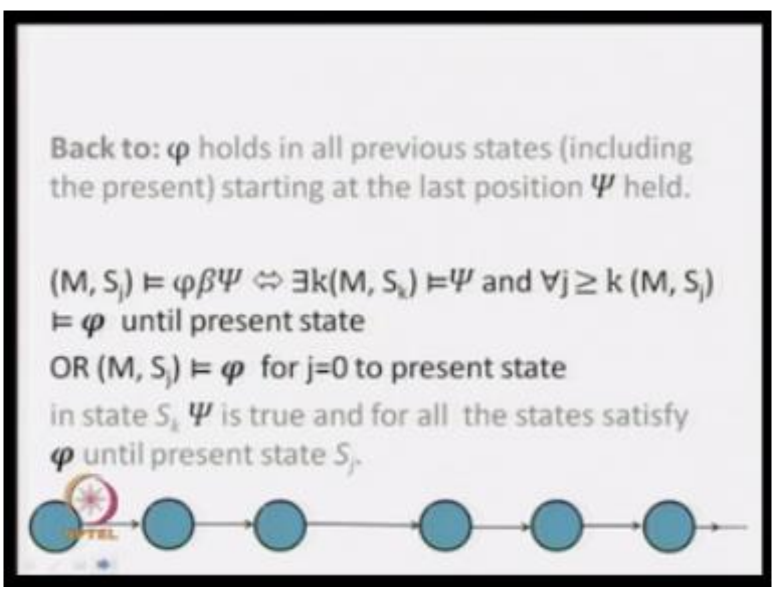

Similar the last operator in this particular series is your back to Ø so it say that back to Ø we will say that it holds in particular the Ø in pervious state including this present starting at a last positions ψ holds that means again since in past we may have infinite state but we will retracts to some initial state say that we are going to look from this particular state and we all say going to look for a particular state say Sj now we are going to set a weather in this particular Sj back to Ø holds on or not so similar say in further something holds or say until operator Ø until ψ.

So here Ø back to ψ that means we are going to get some state in this particular model in past that ψ holds and all the states between Sj and ψ that Ø holds so if we are going to get this particular model say in some states ψ holds in the past and from that particular state to the state that of my concern say Sj in all the state in between these two states Ø holds including Sj because your say j is greater than equal to k then what we can say that in that state Sj this particular back to holds that means Ø back to ψ holds in this particular state so you can mark this particular state with Ø back to ψ.

So you just say that now we are trying to capture the behavior of timing system so when we are in a particular state we are coming to particular configuration form that either we cab rejoin about the past behavior of the system or you can design about a future behavior of the system. In case of your past behavior of system we are going to use the past temple logic, so we are having this particular pore past temporal logic operator previous eventually in past globally in past and back to.

If you are going to rejoin about a future behavior then we are going to look for the future temporal operator which is your next step eventually, globally and anti-operator so these are the four operator basic operator we are going to discuss about that we have seen how we are going to define the meaning of those particular four operator and we have seen that to define a meaning of those particular temporal operator.

We are going to define with respect to a model, so this model will come from a system, now just look for some example in this particular scenario.

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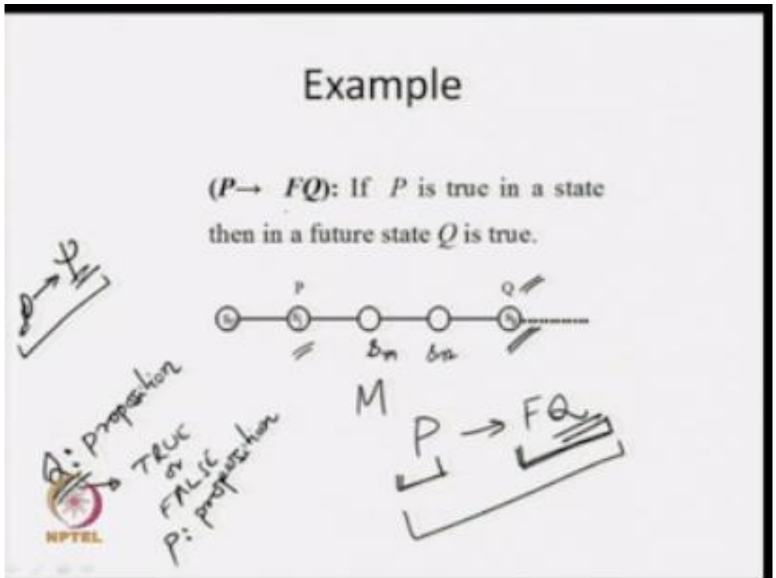

Now we are saying that P → F(Q) so this is the formula so now if I am giving a formula so here P →F(Q) now what does it means so here Q is a proposition basically we have to see that PU is a proposition that means it is the atomic proposition basically we have to see that PU is a proposition that means it is the atomic proposition that truth values of your Q may be either true or false.

So all the atomic proposition will be treated as a temporal formula because we have to see what are the temporal formula so all atomic proposition will be treated as a some part of formula, so with temporal operator we can again construct a temporal operator so here in this particular case I can say that Q is a temporal formula, now FQ that means whether in future Q holds or not so in this particular case I will say that these FQ also temporal operator.

Similarly P again it is a atomic proposition okay, so that means here I am going to walk with two atomic proposition on one is Q and second one is P, since P is an atomic proposition it can two fellow either true or false so that atomic proposition will be treated as another temporal formula so this is another temporal formula, so in the pure gas say P → FQ that means P → a this is the implication of what that we have in your logic and we say that this can be used in our temporal logic also.

So one temporal operator implies another temporal formula so this whole things will be treated as a temporal formula that means if may be something like that part implies side so here Ø is the temporal formula ψ is a temporal formula so Ø→ ψ is a temporal formula so P → FQ will be the temporal formula where F is the temporal operator few steps, now we are going to say that P → FQ if P is true in a step than in futures step Q is true.

So this is the mean now, to how to going to get a meaning of this particular formula always we are going to define it with respect to a model, now consider this particular model m so you are getting a step SK where Q is true so this particular state SK Q is true and another step we are having say SJ 12 P is true, now in this particular model say we are going to have this particular step as 0, SJ, SK and to some model.

I can say that some SM and SM now this formula P → FQ why it is true, so if you look into it I can have come into this particular notion if you look into the future behavior it says that if you look into okay I will go back to this particular definition I again.

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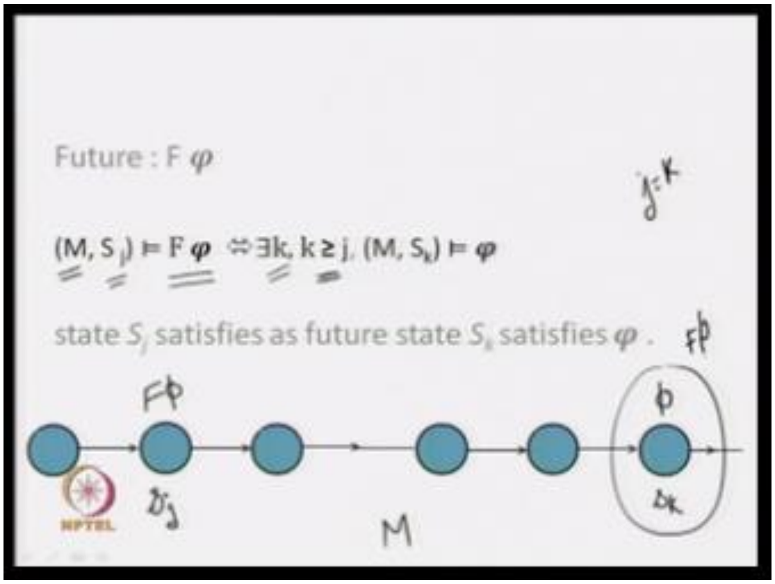

We will say you just see that where I am having this particular things. So here use select if a state SJ I am going to say that in step SJ, F Ø holds we are saying that there exist some K where K > = J and M, S_k models Ø you just see this particular symbol where greater that equal to that means it may happened at what will happen in case of when J = K because there is that means if v holds in a particular step itself.

According to this definition what I can say that, F Ø holds in this particular step also because this k = J so we will come to the particular point that one again this notion is been some significant so that means again we see that by looking into this definition we say that if I holds in a particular step we will say that a Y holds in that particular step. So now come back to this example.

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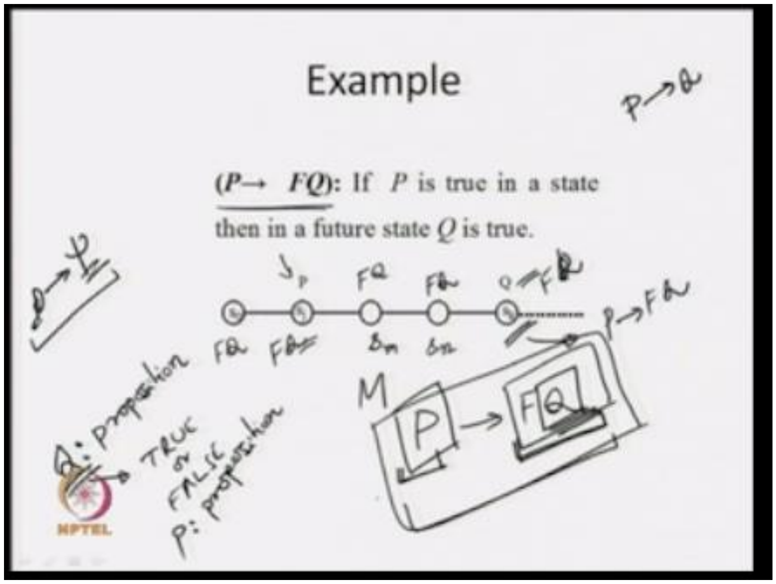

So since S part is particular definition if Ø holds in this particular step then we can say that F Ø holds in this particular step, now when I come to this particular steps and from here in a future step Ø holds so I can say that F Ø Oh! Sorry F Q here I am using this particular symbol Qs, FQ holds in this particular step also, when I come to this particular step SM from here also I am going to get a state in future where Q holds so I can say that FQ holds in this particular step also, when I come to this particularly state SJ that say the form SJ we are going to get a state in future when Q holds so I can say that FQ holds here also.

When I come back to SO we will find a from S0 in the time line I can go down and In future I am going to get a state SK where Q holds so similarly I can say that FQ holds in this step also, now we know the meaning of these things know that logical operator say P → Q say if this is the implication version that means EP than Q.

If P is true then Q must be true but if P is false you are silence so if P is false then whatever momentum below Q we are going to say P → Q is to over there, so here if you look into this particular model so here Fq holds so P is not true over here but if P is false and Q is true then also we are going to say that P → Q is true so basically you can say that in SK your P → FQ is true so in a similar reasoning I can say that in SN and SM and S0 that P → FQ is true.

Now what about SJ, P → FQ again it will be true in this particular step because now P is true if P is true then Q is true then we are going to say that P → Q is true in that particular state, so basically in this particular state SJ also P → FQ is true, that means in this particular model we have seen that in all the steps P → FQs is true, so basically if step dependent we have to look for each and every step.

And when we are going to talk about a particular formula so like that P → FQ now we have to look for the truth values of exam component now here we are having components one temporal formula Q we are forming another temporal formula FQ that one temporal formula P that means we should know the truth values of this particular temporal formula in each and every step then only we can talk about a truth values of this particular whole temporal formula, okay. So this is the way we are going to look into it.

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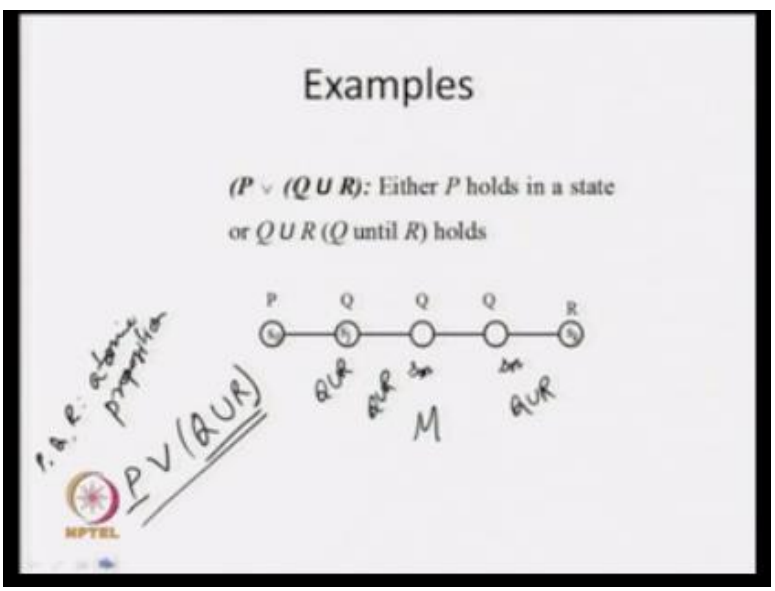

Now another example is see that if that saying that P or XQ now in this particular case now again similarly we are having P and Q as our atomic proposition, so like that what we can say now, P or XQ so is the atomic proposition so P is the temporal formula, Q is a atomic proposition so Q is also temporal formula X is a temporal operator next step so XQ is a temporal formula so these two temporal formula is connected by this OR operators.

So this is also temporal formula, now we are going to look for the truth values of this particular temporal formula, now in this particular case say either P is true or XQ is true now in this model if I am having this things so the similar way can say that P is true over here and Q is true over here, so in this particular step what I can say that XQ is true over here, what about this particular step?

Because XQ is not true because Q is not true in the next step, so XQ is not true so here Q is true so in this particular state I can say that XQ is also true over here, now since this is your P or XQ now we will see that in all the step either P is true or XQ is true, okay. So here I am going to say that P or XQ is true in all the step now if I simply sends this operator from P and XQ then what will happen, we are not having any step where both are true so in this particular case I say that these P and XQ is false in all the step, now I slightly modify the your distinct leveling of this particular state in all the states.

I say that in S0 also P holds okay, now in this particular case you say that in S0 you will find that both P is true and next Q is also true, that means you can say that if this is my model M so in this model M in state S0 that P and XQ so user see that in that example what we are getting we are getting only one state S0 where P and XQ is true but in all other state P and XQ is false but if you talk about P or XQ we will find that in all the states P or XQ is true.

So this is the width now we have to prove for the truth values of each and every temporal formula in the each and every states and then only we can talk about a truth values of a particular temporal formula in a step, so it is state dependence, in some state it will be true or some state it will be false, now come to this particular step again we are having now here we are using P until a P or Q until that.

That means we are having PQR as your atomic proposition, okay we are starting with their atomic proposition and we are saying that P or Q until R so Q and R are temporal formula so Q until R is a temporal formula P is a temporal formula so P or Q until R is a temporal formula, so now what we are saying that either P holds in a state or U until holds now if you look into this particular model M.

Then you will find that in step R Sk this R is true and when you become back to this particular step I will just level it as say SM and SN so in this particular SN we will find that Q until R is true when you come through that SM we will find that U until R is true if you come back to this particular step SJ you will find that Q until R is true when you come to S0 you will not find that Q until R is true over it.

Because Q is not true in that particular point, now when we are going to look for P or Q until R you will find that in this three step SJ, SM, and SN, Q until R is true so P or Q until R will be true in this particular three step, when you come to this particular S0 since P is true we will find that P or Q until R is true, when you come to this particular SK then we will find that neither P is true nor Q until R is true over here.

So we will not we will find that P or Q until R is false in this particular step Sk so this is the width that we are going to look for the truth values, so we are just looking into some examples through get the meaning of those particular temporal operator.

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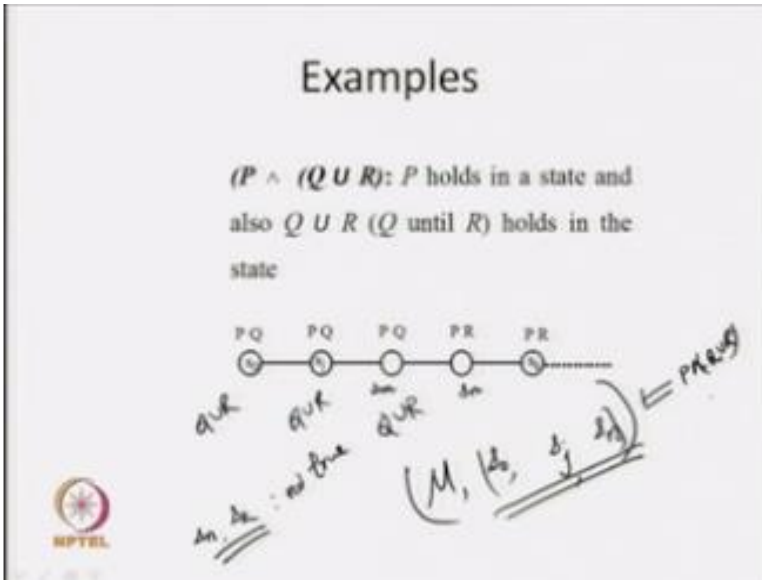

So now thing you can look into it when I can talk about P and Q until R, so this is similar in earlier case we are having P or Q until R now we are talking about P and Q until R so P and Q until R so similarly we can inspect this particular thing Q until R so here we are going to find out now said at I am because I am going to look into it, if you look into that P and R is true over here so what is the states of Q until R.

Q until R is false over here I am going to talk about Q until R so if you look here the Q until R is false over here, if you look into this particular step Q is true until R is true so you can say that Q until R is true over here when I come to this particular point again you will find that Q is true Q is true un till R is true so I will say that Q until R is term over here when you come back top this particular state again you will find that Q is true q u is true qu is true until r is true so q u r is true over here also.

So again as such I am going to give them ms Mn Sn to this particular two step not we have to said that [p and q u and r so in that particular case both p and q angtela Mal b true so here we are going to get that Ns 0 both are true sj both re true when we coming to th particular case in your sn both p and q until r is true so in find that in this three step p and q until r I true but when we are coming to step sn and your sk you will find that p is true in both the cases but q until r is not true in this two particular step.

So will said that sn and sk so this formula is not true basically it is false okay what in this three step I can sad that it models this particular formula p and q until r so in the model m if I am going to take this particular state of step we will find that in this three state p and q until r is true.

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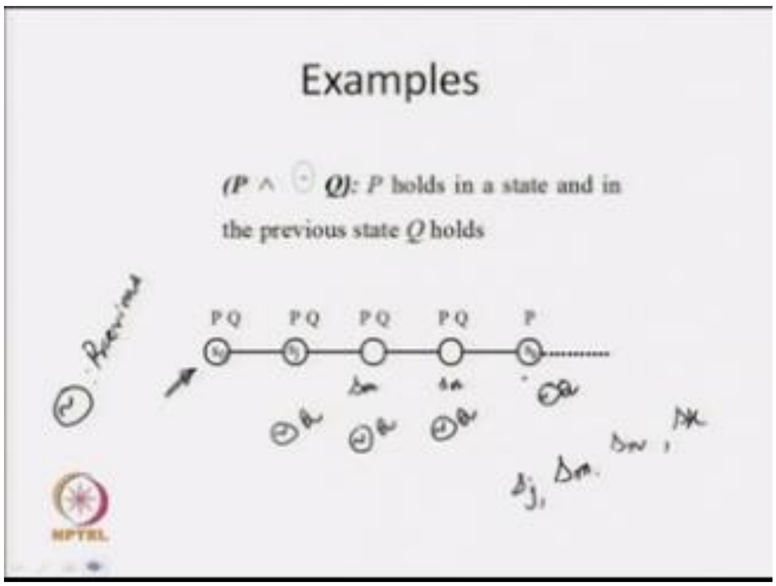

Now this is another formula that I am talking so we are going to have this circle and inside that I am having ~ that means this is the previous operator means this is an example of past tempo logic so in that particular case you said that p and previous q so we have to see whether p is true and the previous that q is true so if you look at this particular state this is the simple model that we are having so you aid that here if I am in this particular step say sk in find that in the previous step q is true so I can say that previous q is true in this particular step.

One I am in this particular step say again I am marking it as you say sm and n so when I am in your sn you will find that in previous step your q is true so I will said that previous q is true in this particular case similarly when you are come down to this particular step in sm again you will find the previous q is true similarly n I will find previous q is true. So we are talking about p and previous case true so we are going to get that and you just say since this is the spackling I am change that in past who cannot go beyond certain punch so we will said that we are having some start step and we are going to starting form s0.

So here that previous q is not problem as we are saying that we know the we have form this particular point we do not the behavior of the system for this particular step so here we cannot talk about that whether that previous q is true or not so, here I am not going to say p and previous case true but in this particular forth state Sj Sm Sn and your Sk that p and previous q I true okay so this is the way I am going to look for the true fellows about tempo logic formula.

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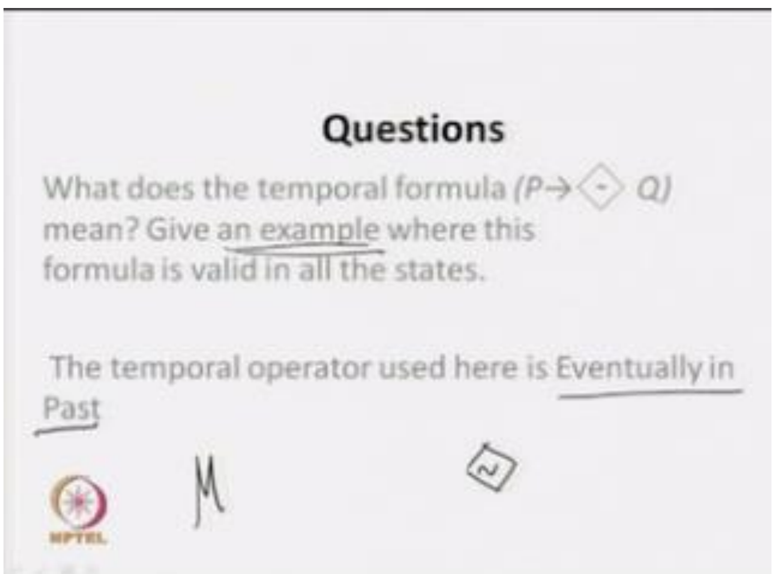

Now I will give you some question let say whether what we are going to do here say that what does the tempo logic formula p in + the tilde inside the box q mean give an example where this formula is valid in all the states. Now say I am here in this question what I am giving I am giving a temporary formula p and I am writing a symbol this is basically diamond and inside the diamond I am having the tilde and q.

So what this particular symbol means I think you will very well identify this symbol means basically eventually in past okay so diamond is future if I write tilde in diamond then we say that this is eventually in past so we are going to said that going to look for this particular thing eventually in past q holds or not if p in + eventually q so it says that what does this temporary formula p implies.

Eventually in past q mean op that means we are going to said that if these two whether eventually in future past q was true or not so given an example where this formula is valid in all the steps now if we talk about such have the question given example that means we have to come up with the model and we have to said that in this model we have to look for a true fellows of this particular formula in this model and we have to see whether this formula is true or not. Now we have to come with the example where this particular formula is true in all the steps.

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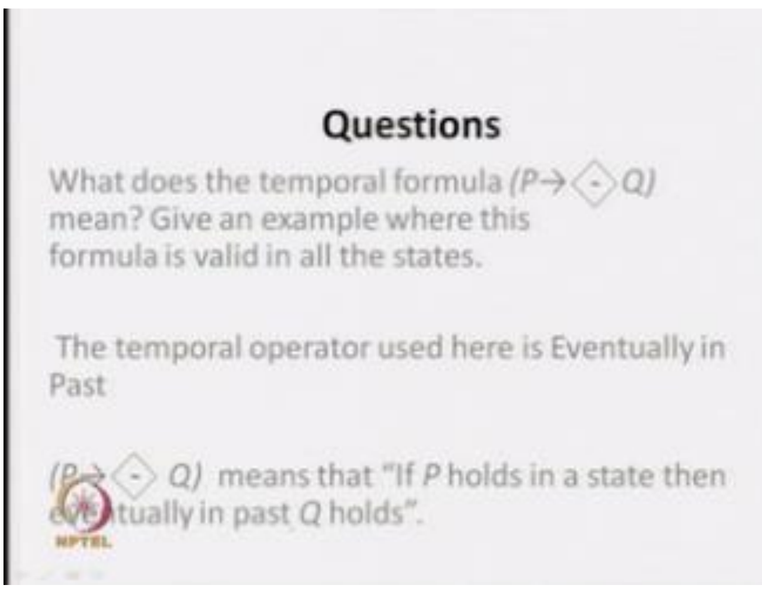

So now this is basically what it say that p implies tilde inside diamond q means if p holds in a step[then eventually in past q holds so if q holds in a step that eventually in past w holds.

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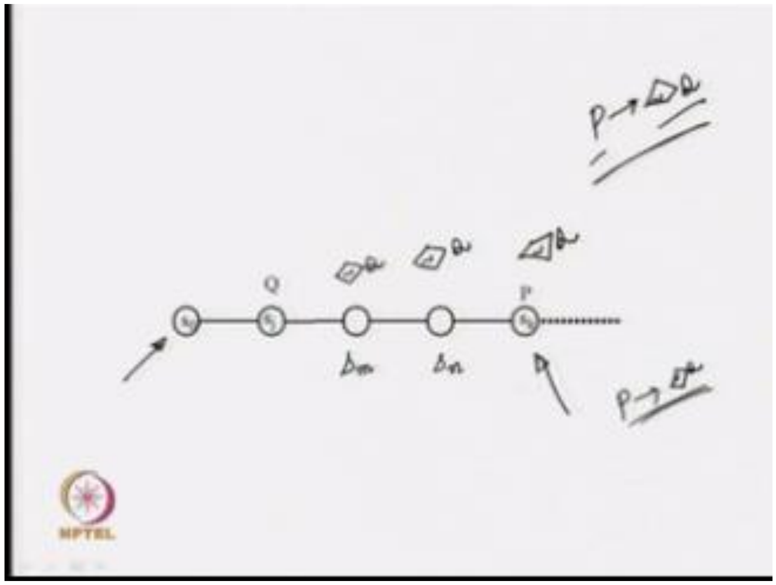

So it is a very simple model that I have come up with this things so we have talking a step sk and beyond that in case of we are going to have said that just think at this sk I the current step and I am going to look in to it that I 0 is a starting point of my system and this as a what this thing I am going to give the name of this few things sm and sn and what is my formula it says p implies eventually in past q means.

So basically we are going to look for formula p in implies eventually in past q holds now we have to look for this particular formula first say since q is set of atomic proportion so eventually ion past this also temporary of a top p is a atomic proportion so it is a temporary formula so even we are looking for a temporary formula. So eventually in past q holds so if you look in to sk if you go in the past direction then you find that how we are getting step that sj where q hold.

So I can said that eventually in pat q hold ion this particular step so similarly in sm in sn also eventually in past q holds okay now in sj I cannot say this to because we do not know because q was not true in s0 and again s0 we cannot rejoin about this thing because we have just single this is the starting part beyond that what is happen we did not about this things. So this is formula we say now p in plus eventually in past q hold.

So if p holds in a step then eventually in past in q must told so you have to look in to the sk this formula is holds over here okay so this is the step so similarly but p is false we do not have any many over here so we have not require to look for the foot paddles because it false in plus because we are not implication if p is false that formula is true in this particular step, so in you sj sn and sn this formula hold. So in all those particular step we in plus eventually in past q holds.

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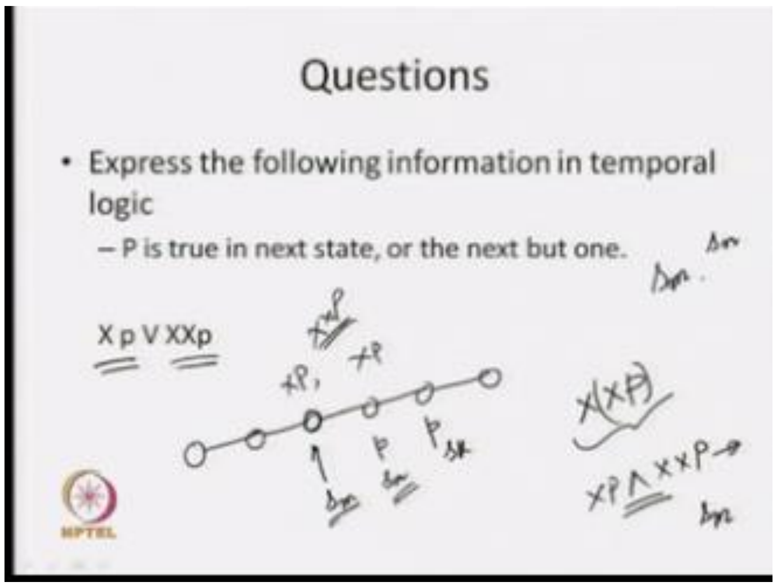

Now look for another one I m saying that x was the following information in tempo logic so basically we are going to design bout it we are having some behavior now whether we can express this thing with tempo logic or not we are saying that express he following information in tempo logic p is true in next step or the next what one here you just think that p is the atomic proportion.

So we are talking about one atomic proposition p or since atomic proposition are temporary formulas so it can replace by any temporary formula fires so that means p is true in next step or in next but one so now what I can in the express in this tempo logic let us see we can say write the formula for this particular step one xp we are going to say that in next step p holds and xx p so you say the next top next step p holds so what is say that p is a atomic proposition so it is a temporary formula xp next step p holds.

So we said that this is also I am not a temporary formula either I am writing this particular next step what are x how x it is so again so th9i thing will also temporary formula you are saying that p is true in next that on next part one sop we said that x xp is basically next part one and xp is your next step. So now in this particular case now what will happen now we are going to look in right now you can how it is true but is it is basic thing and again we have to look for one models sy if I am going to have some model like that say this is the model saying this particular case that p holds.

And in p holds then what will happen if you come to this particular state will find that xp holds over here in this state also xp holds over here if I come to this sy I can said that this is your Sn Sm and Sk if we come back to a Sn then what will happen sn is the next step p holds and then next step[and next but the next that also p holds so here xxxp also holds in this particular step.

So I am having this r combination so you can say that xp or xxp hold in this particular step similarly seeing I am having this xp or xxp so in sn also in this particular formula also what happens I can say that the sn and sm this particular formula xp or xxp holds, now if I replace this formula why say xp and xxp that means in next step p holds are n next but next step p hold now see since I am having this particular conjoin and who person then you will find that base particular formula holds only in that step sm it I not true in sn because in sn xp holds. But xp not holds so that I why I am getting all the m where this xp[an xxp holds.

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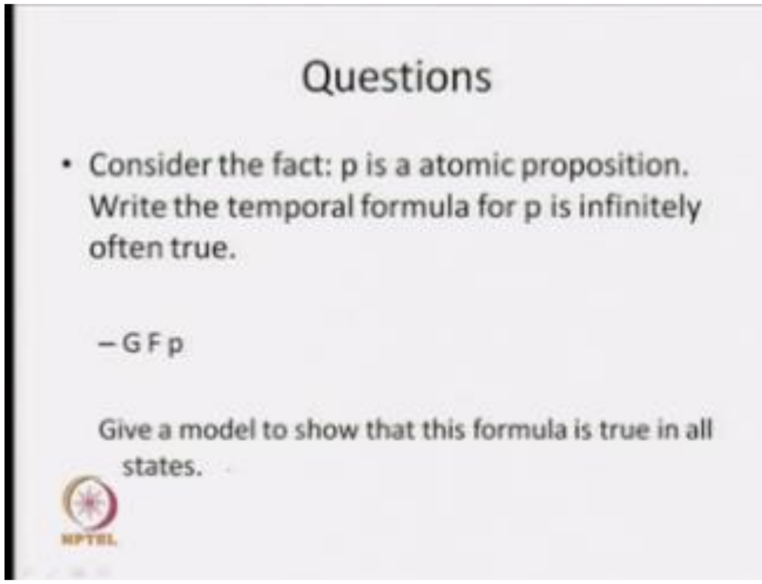

Now whenever question you just see that you can think about with p is an atomic proposition now I am saying that write the temporary formula for p is infinitely often true I am talking about is infinitely often true that one does it means so it I infinitely often it is coming that means it is true and after some time again it will be true so in temporal logic we can write this behavior in this particular concept so globally GFp.

So infinitely Fp so infinitely often that means globally it should come how globally it should come in future p hold so we are saying that p is infinitely often for it not like that Gp if I said that globally p I true it is different and infinitely often it is true that means it is not always true but infinitely often it is true.

This coming true after some instance of time so that is why we are saying that globally have p hold so it is infinitely often it is true, now can we give model for this particular step where it is true in order step because globally infinitely often it should true. So it is we can construct it because.

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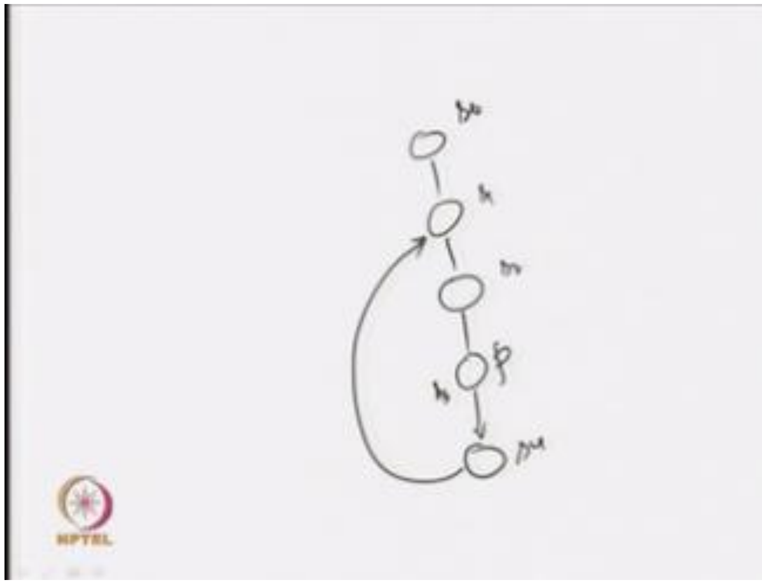

If you know the behaviors say I am just going to give one example say I am having a step machine something like that and said that in this particular step say p is true now say these are the step say s0 s1 s2 s3 s4 so I am coming up the very small models so globally infinitely often this dept we must to infinitely what about I can make a step like that now this things is this execution trace s1 s2s3s4 is repeating.

So in this particular case you said that where about you are in future that we are going to get a step when p is true that means p i not globally true but infinitely often it is true so tht I why we are saying that globally a p is true when I said that p is infinitely often it is true. So this is now in this class what we have seen we have seen what are the temporal of parts that we are having we are having two type of operator logic one is your past tempo logic and another one is your future tempo logic in both the cases we are seen four operators that basic operators one is your next eventually globally in until.

And we have seen in how to defined them meaning of those particular tempo operator we are going top defined the meaning with respect to model sop we have seen what is the meaning of those particular operators and give you some clear idea about those particular tempo operator I have given some example and along with that we have discussion also so with this today I am going to find up so in next class we are going to look for a particular tempo logic.

Now we are having with this particular for temporal logic operator we have having suffered kind of temporal logic so we are going to look for particular class and we will discuss about that particular class and you know lecture we are going to use that particular class okay.

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