Design of Power Electronic Converters
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Module: Snubber Design
Lecture 32
RC Snubber Analysis - II: Underdamped Case

Welcome to the course on design of power electron converters. So, we were discussing RC Snubber. We had started its analysis and we saw the circuit that we will be using for analysing the RC circuit. So, now in this lecture, we will be analysing the first case that is your underdamped case.

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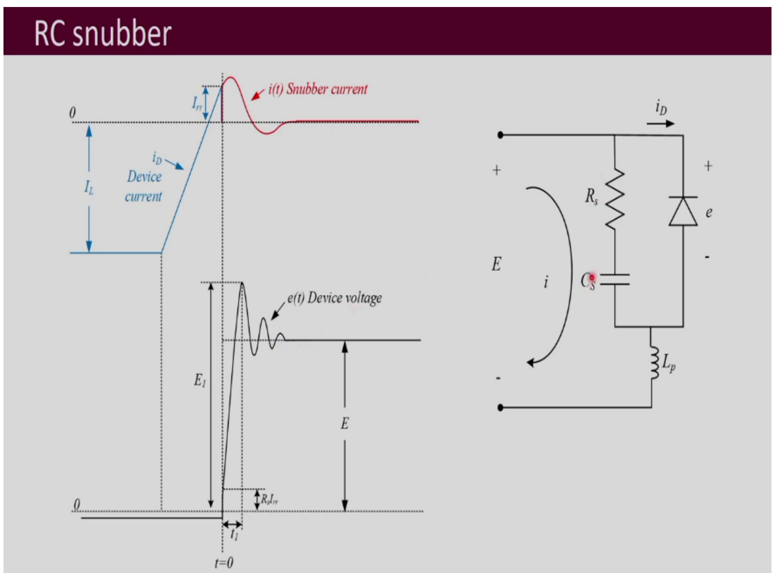

So, before going further, let me just show you the waveforms in the circuit again. So, this is the circuit that we were discussing Rs and Cs are the snubbers, i is your the current through this loop which flows through this snubber and the parasitic inductance and then you have this diode voltage e and then this is the diode current i D and these were the waveforms. So, first we are supposed to get this this snubber current i t and using that number current then we can obtain this device voltage e t and then that we will be further using to design this Rs and Cs.

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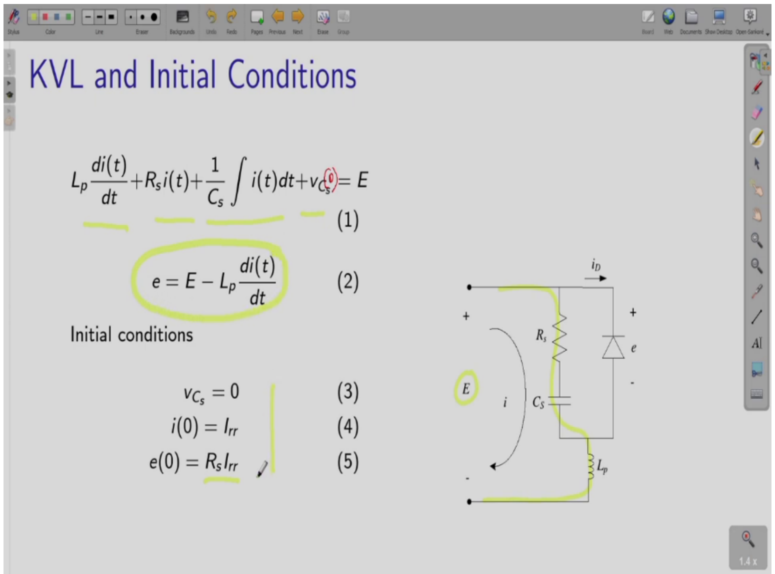

So, we had started by writing these KVL equations. This is the first equation in the second equation, please remember the second equation we will be using it again and again and these were the initial conditions.

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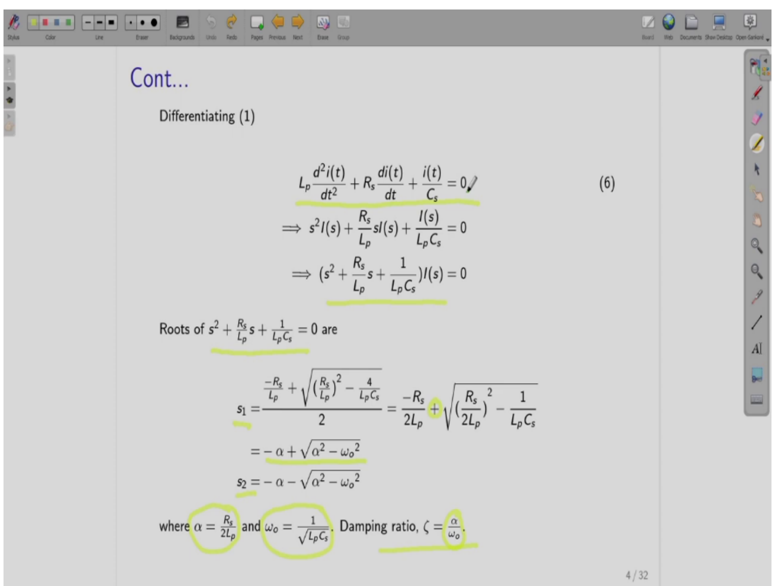

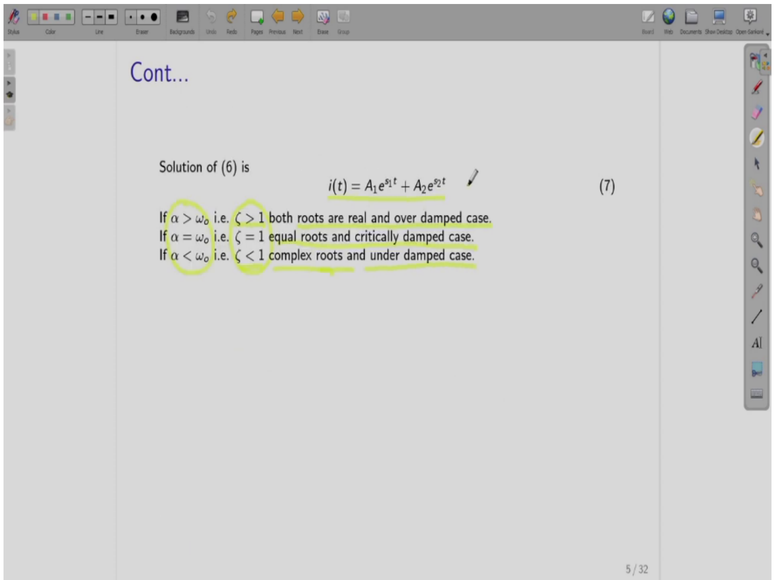

Then, we had this differential equation which we are supposed to solve and so, the solution of this differential equation is this the current this the standard solution of RLC circuit and then we had these three cases based on this value of zeta which is your over damped case critically damped and under damped case. So, we had seen till here. Now, we will take up the first case, which is your under damped case.

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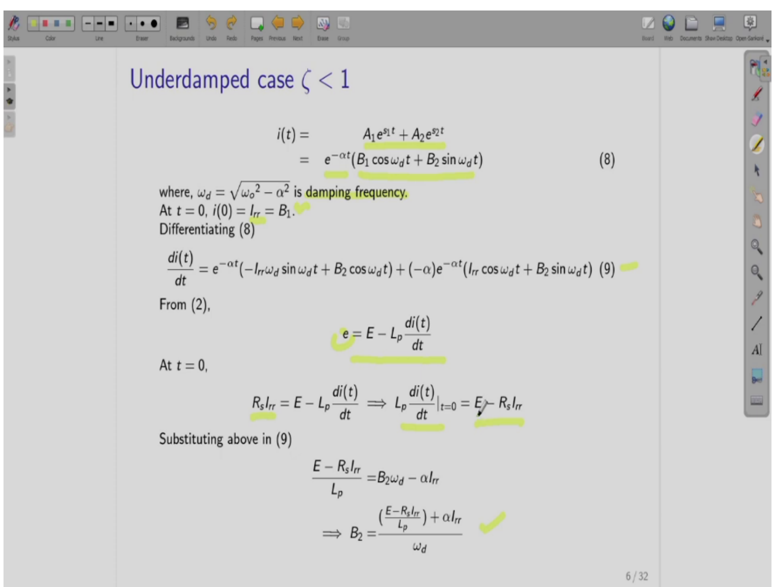

So, underdamped case zeta less than 1. So, this also you might have done in your circuit course. So, what you have is your underdamped case, this can be written as

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right) \quad (8)$$

And what is omega d? Omega d is the damping frequency which is

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

And then, we have to obtain these solutions for B1 and B2 because we have to replace this by using the initial conditions.

So, initial conditions at t = 0 you know that i(0) = Irr reverse recovery current and if you substitute here this goes out, so, B_1 becomes equal to Irr and then if a differentiate this di t by dt, you will get

$$\frac{di}{dt} = e^{-\alpha t}\left(-I_{rr}\omega_d \sin \omega_d t + B_2\omega_d \cos \omega_d t\right) + (-\alpha)e^{-\alpha t}\left(I_{rr} \cos \omega_d t + B_2 \sin \omega_d t\right) \quad (9)$$

From (2), $$e = E - L_p \frac{di(t)}{dt}$$

So, you apply the initial conditions over here. t =0 So, then you know this $e = R_S I_{rr}$

$$R_S I_{rr} = E - L_p \frac{di(t)}{dt} \Rightarrow L_p \left.\frac{di(t)}{dt}\right|_{t=0} = E - R_S I_{rr}$$

Substitute $\frac{di(t)}{dt}$ from above in (9) for t = 0, we get

$$\frac{E - R_s I_{rr}}{L_p} = B_2 \omega_d - \alpha I_{rr}$$

$$\Rightarrow B_2 = \frac{\left(\frac{E - R_s I_{rr}}{L_p}\right) + \alpha I_{rr}}{\omega_d}$$

So, we have obtained both the constants B1 and B2 by using the initial condition.

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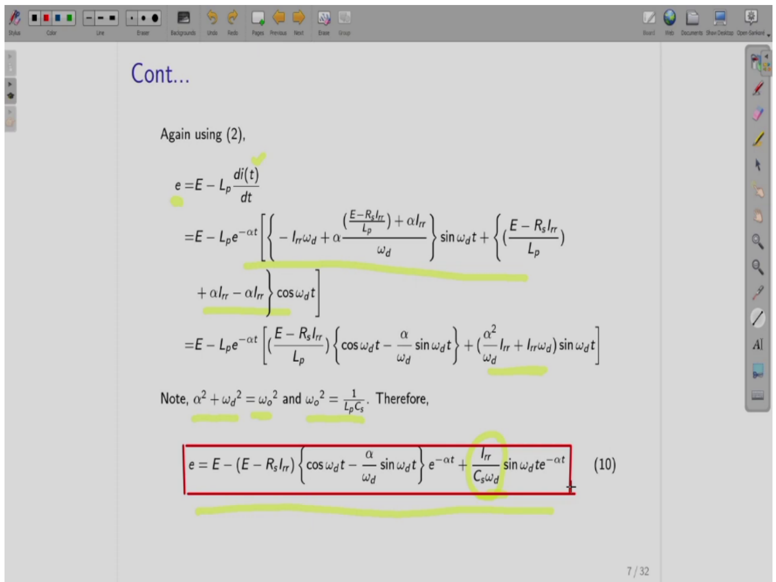

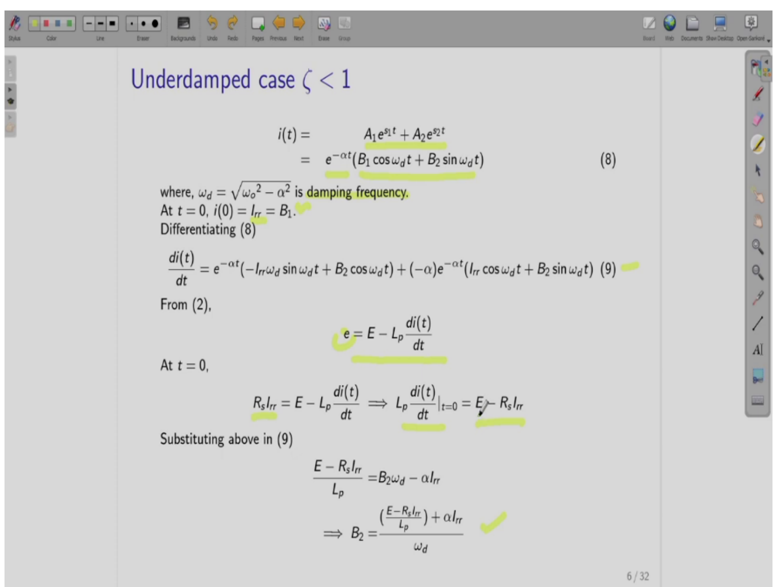

So, then from there once you have obtained it, what you can do is you can find out your e because ultimately our objective is to find out e using this it. So, di t by dt you substitute this B1 and B2 and this is what you will be then getting this is a little long expression that you will be getting and you rearrange it when you rearrange you can rearrange it like as shown in screenshot.

And further from there what you see is that $\left(\frac{\alpha^2}{\omega_d} I_{rr} + I_{rr}\omega_d\right)$ can be reduced to $\frac{I_{rr}}{C_s\omega_d}$, because your

$\alpha^2 + \omega_d^2 = \omega_o^2$ and $\omega_o^2 = \frac{1}{L_p C_S}$

. So, applying that fact it reduces to this.

$$e = E - (E - R_s I_{rr})\left\{\cos\omega_d t - \frac{\alpha}{\omega_d}\sin\omega_d t\right\}e^{-\alpha t} + \frac{I_{rr}}{C_s\omega_d}\sin\omega_d t e^{-\alpha t} \qquad (10)$$

this is an important expression that we will be using later on as well. This is the expression of e the voltage across the device.

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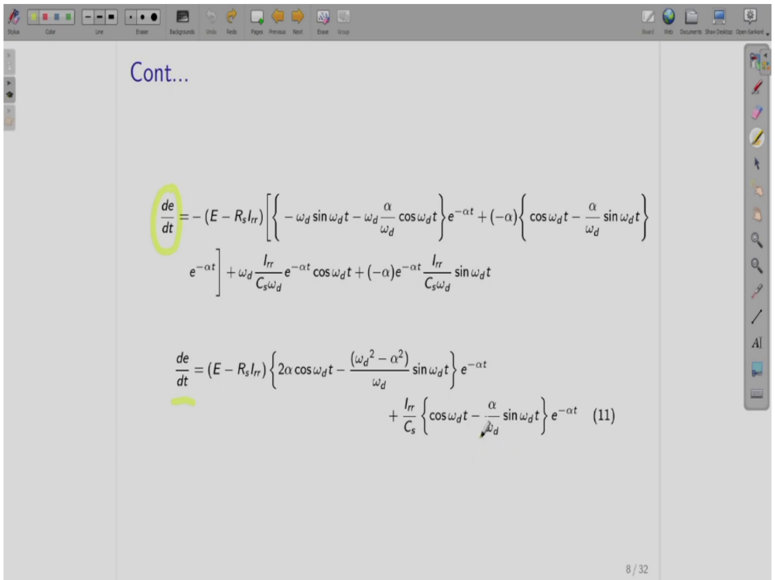

Next is that you have you can differentiate this voltage de by dt. So, whatever expression we obtained for e, you differentiate it, so, you can differentiate it, it is a little longer expression again you are getting you rearrange it. So, after rearranging this is what you are going to obtain for de by dt.

Now, what we want is that, we have to equate (11) shown in screenshot to 0 because if this de by dt if we equate to 0, so, then we can obtain the peak value the maxima of e and that is what we want to find out and that is what we want to reduce also. So, before obtaining that de by dt and equating it to 0 and finding out the peak, we would like to see what is the nature of this de by dt this rate of change. And to do that, we can actually simplify this expression because this is a little longer expression.

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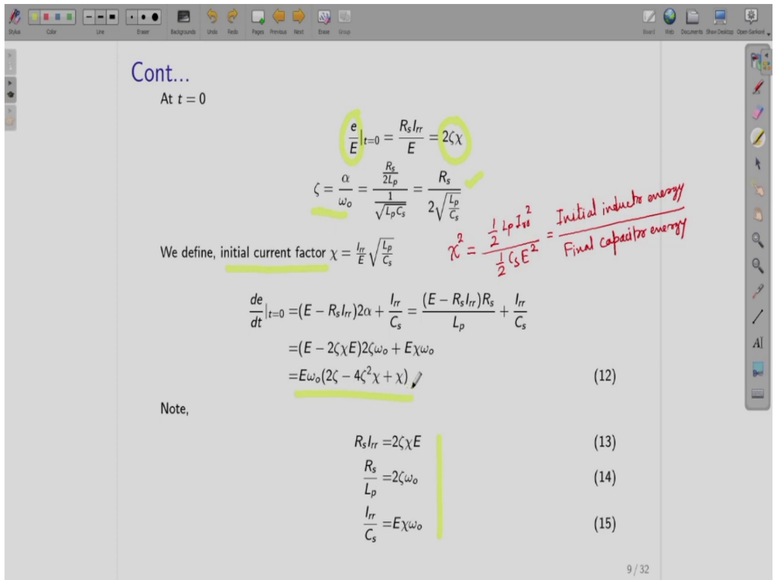

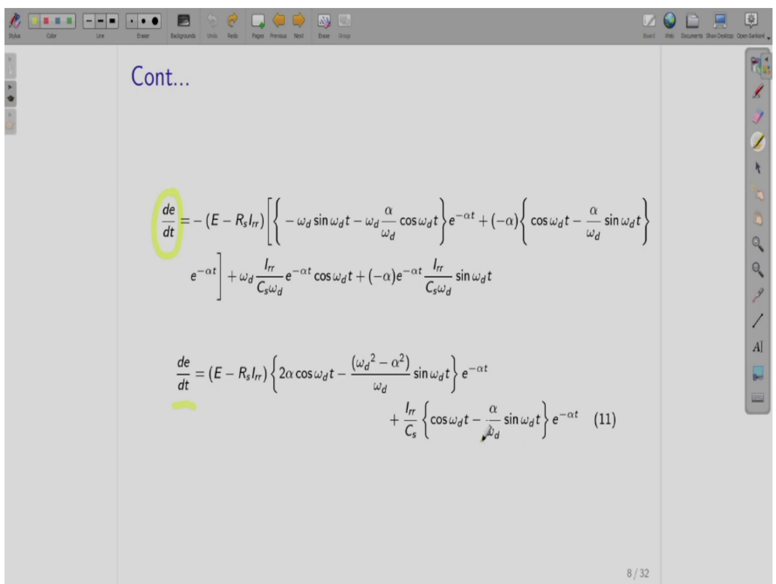

So, to simplify it, first we see that at t equal to 0 this ratio e by capital E this can be written as Rs Irr by E (shown in screenshot) and which can be further written is 2 zeta into chi. Now, what is this chi? Chi is defined as the initial current factor which is equal to

$$\chi = \frac{I_{rr}}{E}\sqrt{\frac{L_p}{C_s}}$$

So, from where is this coming up.

So, this is actually your $\chi^2 = \dfrac{\frac{1}{2}L_p I_{rr}^2}{\frac{1}{2}C_s E^2} = \dfrac{initial\ inductor\ energy}{Final\ capacitor\ energy}$. So, chi will be equal to Irr by E root over of Lp by Cs, this is called as the initial current factor. So, this initial current factor and this damping ratio zeta this we have already defined is alpha by omega 0 which . So, this is what it is going to be Rs by 2 root over of Lp by Cs (see screenshot).

So, when you multiply these ζ and χ then becomes equal to Rs Irr by E. So, e by capital E at equal to 0 becomes equal to 2 zeta chi. Then at this d by dt at t equal to 0 at initial condition what it is that we want to find out in terms of that zeta and chi. So, for that you substitute t equal to 0 here (see equation 10), then what you obtain over here is this E minus Rs Irr 2 alpha plus Irr by Cs.

So, then you again do some substitution in terms of these and what you will be finally obtaining will be

$$\left.\frac{de}{dt}\right|_{t=0} = E\omega_0(2\zeta - 4\zeta^2\chi + \chi) \tag{12}$$

And what substitutions are we doing to obtain (12)

$$R_s I_{rr} = 2\zeta\chi E \tag{13}$$

$$\frac{R_s}{L_p} = 2\zeta\omega_0 \tag{14}$$

$$\frac{I_{rr}}{C_s} = E\chi\omega_0 \tag{15}$$

Now, we have to observe this equation, now, this can be greater than 0 or it can be less than 0.

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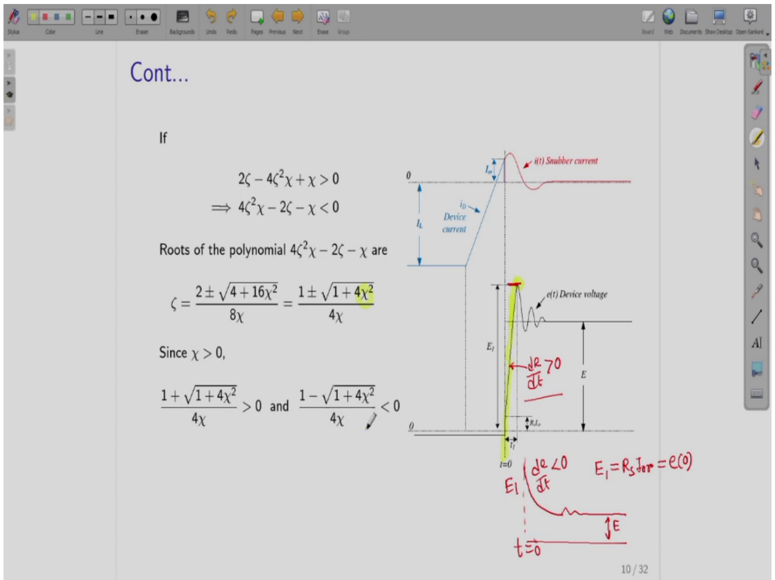

So, this one, 2 zeta the minus 4 zeta squares chi plus chi, if this is greater than 0(see the screenshot) that means, what that your this voltage will this dv by dt the rate of change of the voltage de by dt this is positive. So, at t equal to 0 whatever it was, this peak is going to be greater than that, but if it is negative that means, what at t equal to 0 whatever was your voltage over here it is if this d by dt is less than 0, so, this is going to further come down and so, this is what your finally, whatever the voltage is going to be equal to.

So, this capital E1 the peak voltage that we had considered will be equal to Rs Irr in that case because at t equal to 0 that is the voltage that is the voltage E at equal to 0. So, this is not that much of a problem because this usually is not going to be that high. But if this de by dt is greater than 0 that means, there is the this voltage is going to increase and then here it may be higher than the blocking voltage capita E.

So, and that is that the capital E1 how high it can go we want to limit it by this number design and so, we are interested more in this case when your d by dt is going to be greater than 0. So, we have to find out then what is the condition at which this can be greater than 0 this slope or the rate of change of voltage across the device is going to be positive. So, for this 2 zeta minus 4 zeta square chi plus chi to be greater than 0 this is what it implies and this is quadratic polynomial.

So, here it is going to be having 2 roots, this quadratic equation will be having 2 roots these are the 2 roots and we know that this chi is going to be greater than 0 because this is the ratio of 2 energy's says no question of it being negative. So, this is greater than 0 that means, what from here you can easily see that this is greater than 1. So, this 1 is greater than 0 this first one the plus 1 and the minus 1 is going to be less than 0.

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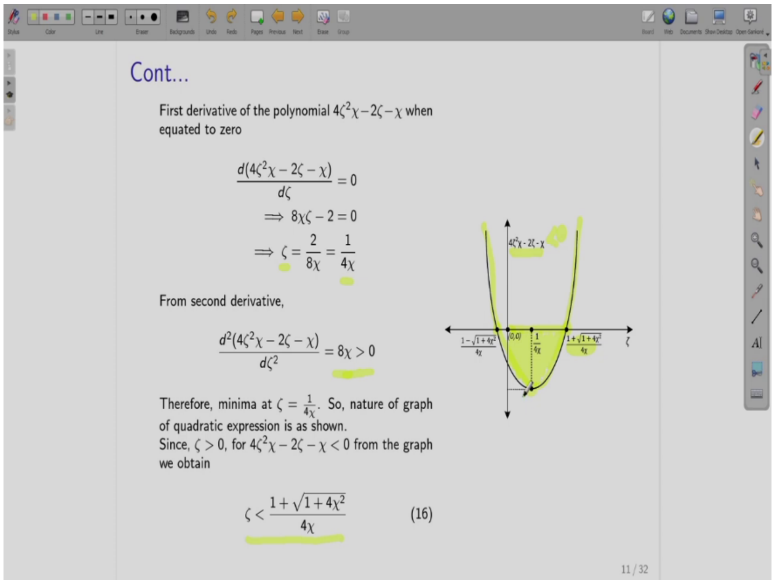

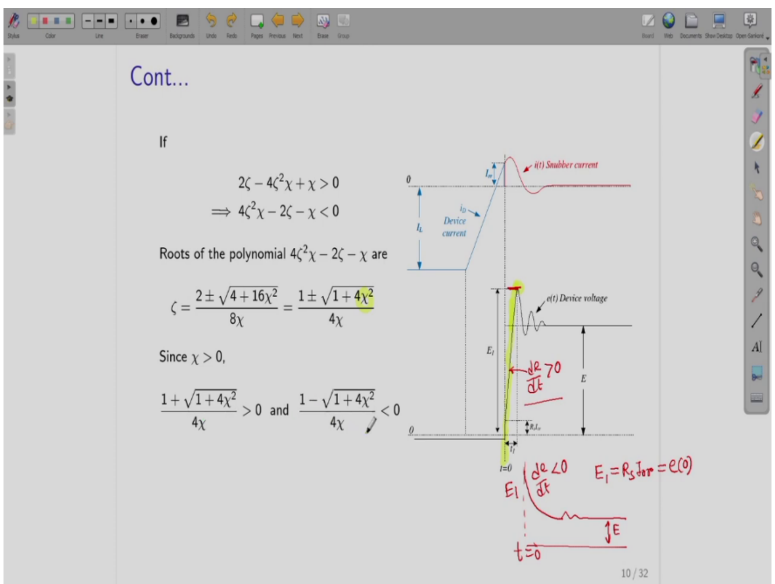

Further we can differentiate it find out the first derivative of it equated to 0. So, that will either give us the maxima or the minima. So, that gives you this zeta as equal to 1 by 4 chi and then you can find out the second derivative. So, second derivative is 8 chi, you know chi is positive so, this is greater than 0 so, that means this is minima.

So, from that what you obtain is that you have these 2 the roots were this expression is going to be 0 and it has the minima in between which is your 1 by 4 chi. So, this will be the nature of the quadratic expression. And so, what we are interested in is where we saw that this term is less than 0. This term less than 0 is when actually this upper 1 becomes greater than 0 and so your d by dt is greater than 0.

So, that means what this is the portion where we are interested in that means your this is the value of zeta which that is between 0 to when it is less than this term. So, that is what then we find out that

$\zeta < \frac{1+\sqrt{1+4\chi^2}}{4\chi}$ and that is where your d by dt is going to be greater than 0.

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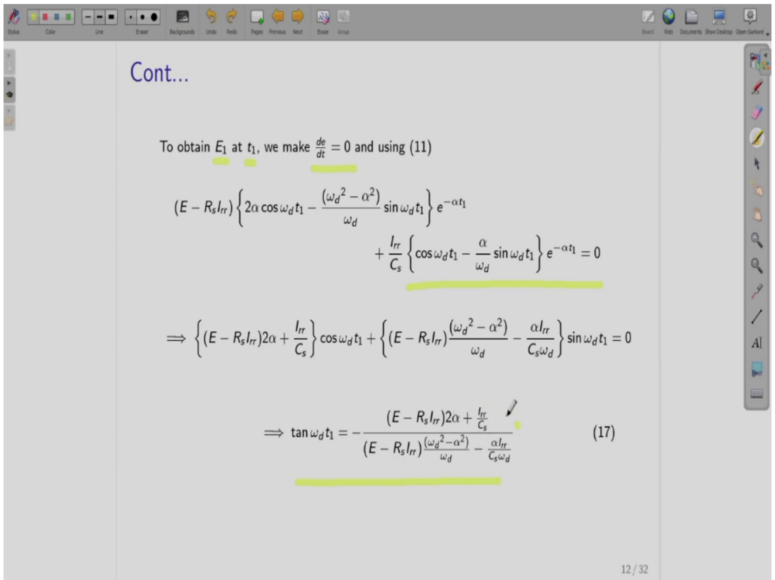

Now, we want to find out the peak of E the device voltage and we had denoted by E1 and we also said that that that occurs at time t1. So, we equate this d by dt to 0. So, you write down the expression again equated to 0 rearrange it and when you do the rearrangement, you can write this as like this tan omega d t1 and which is equal to this expression (17) shown in above screenshot.

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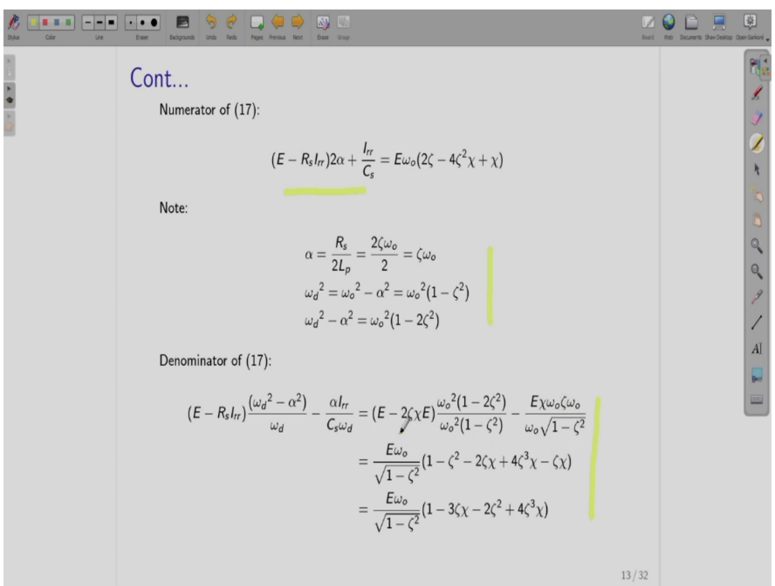

Now, further what we want to do is that we want to express it in terms of chi and zeta. So, this is the numerator of your tan omega d t1. So, you solve it to do these substitutions in terms of zeta omega 0 and chi and the denominator also the same thing you do it you substitute everything in terms of zeta omega 0 and chi.

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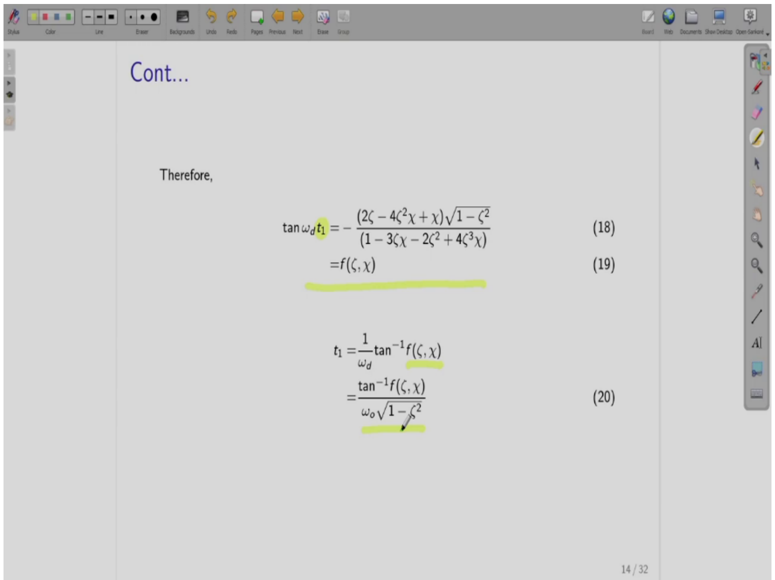

So, then this is what you will be obtaining

$$\tan\omega_d t_1 = -\frac{(2\zeta - 4\zeta^2\chi + \chi)\sqrt{1-\zeta^2}}{(1-3\zeta\chi - 2\zeta^2 + 4\zeta^3\chi)} \tag{18}$$

$$= f(\zeta,\chi) \tag{19}$$

. So, what we see that this is a function of the zeta and chi. So, that means, what we achieved is by doing this is that this omega d t1 because we are interested in this time t1 this time t1 we have obtained as a function of zeta tan chi although this is a little involved function complicated function, but we have obtained it. And then from there your t1 will be 1 by omega t tan inverse of f zeta and chi the function of that. And omega t you know that this is the damping frequency this can be written as omega 0 root over of 1 minus zeta square.

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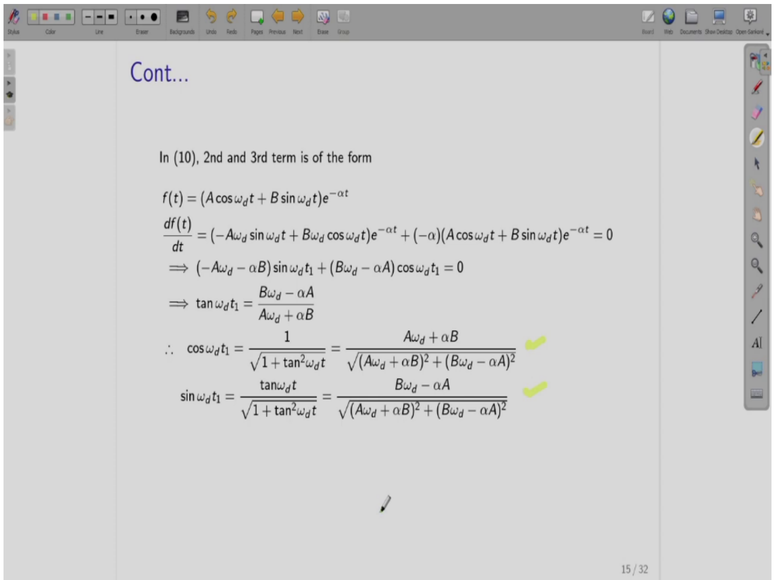

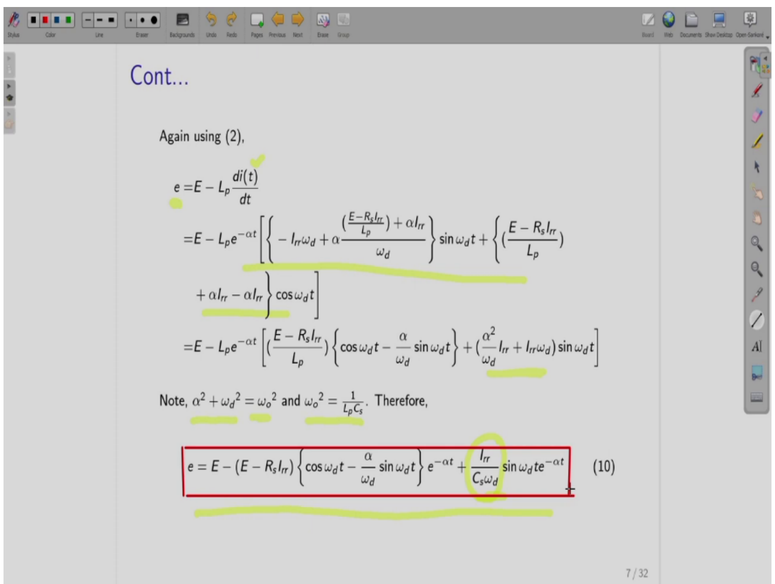

Now, what we are interested in is that, we want to find out the expression for capital E1 the peak voltage. We found out the time and t1 at which it occurs, but more than that, we want to find out the peak voltage. So, to do that, let us see this part of the derivation. So, this expression of e that we had obtained this expression can be written as a sum of for cosine and a sine the second and the third term. You can directly solve it also, but that may be more tedious. So, to simplify it, I have shown the derivation in this form. So, that expression of e the second and the third term can be written as A cos omega dt plus B sine omega dt e power of minus alpha t, you can write it like that.

And then you differentiate it df t by dt. So, this is what we will be obtaining and we are interested in the peak. So, let us equate it to 0. So, once you have equated it to 0, you can obtain this tan omega d t1 which is B omega d minus alpha A by A omega d plus alpha B. Then further you know that your cos omega d t1 can be written as 1 by root over of 1 plus tan omega square omega d t1. So, from there this is what you will be obtaining cos omega d t1 and then similarly, you can obtain sine omega d t1 this is what you will be getting(see above screenshot).

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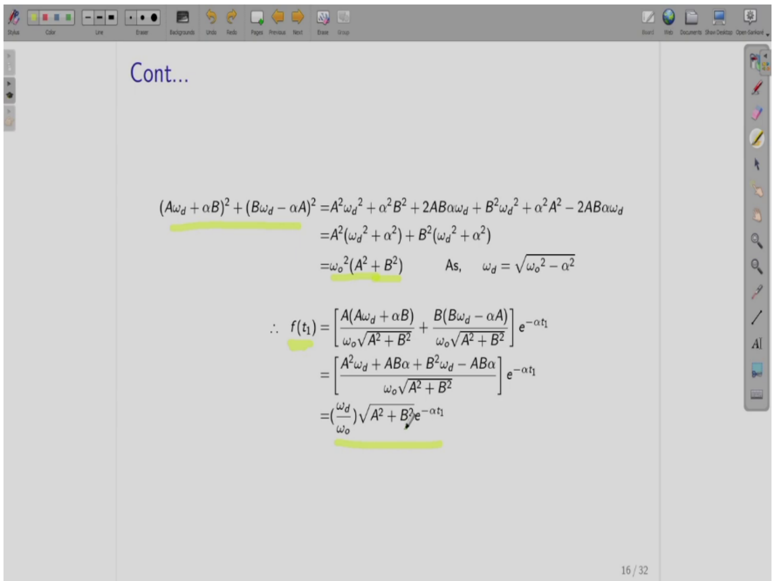

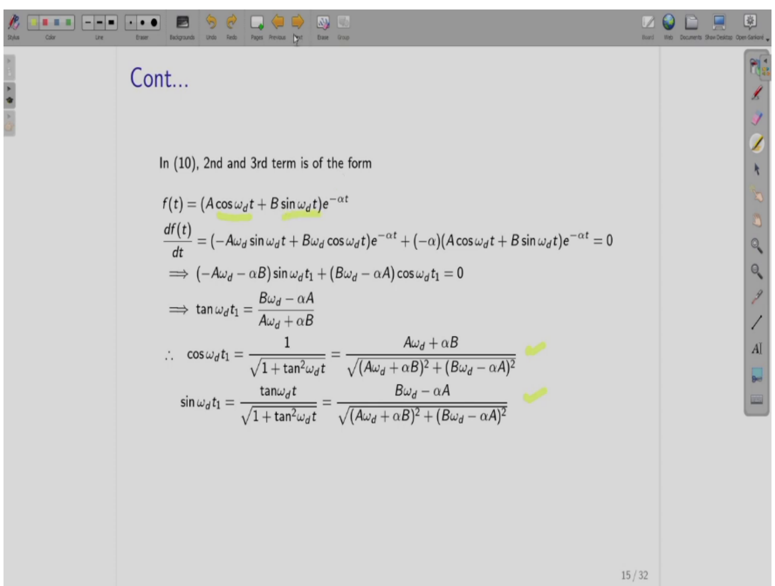

Now, we solve the denominator (see above screenshot). So, the denominator is this, you open it up and you try to reduce it. So, when you reduce it this is what you will be obtaining omega 0 square is A square plus B square and that you substitute in the original expression. So, original expression means this was your f t1 we substitute for cos omega dt and sine omega dt 1.

So, when you do that, this is what you will be obtaining and you try to reduce it. This is finally what you will be getting

$$f(t_1) = \left(\frac{\omega_d}{\omega_o}\right)\sqrt{A^2 + B^2}\, e^{-\alpha t_1}$$

. So, this is what we see at time t1 this is what your second and third term will reduce down to. So, we have to square out square the coefficient of cosine and squared the coefficient of sine part.

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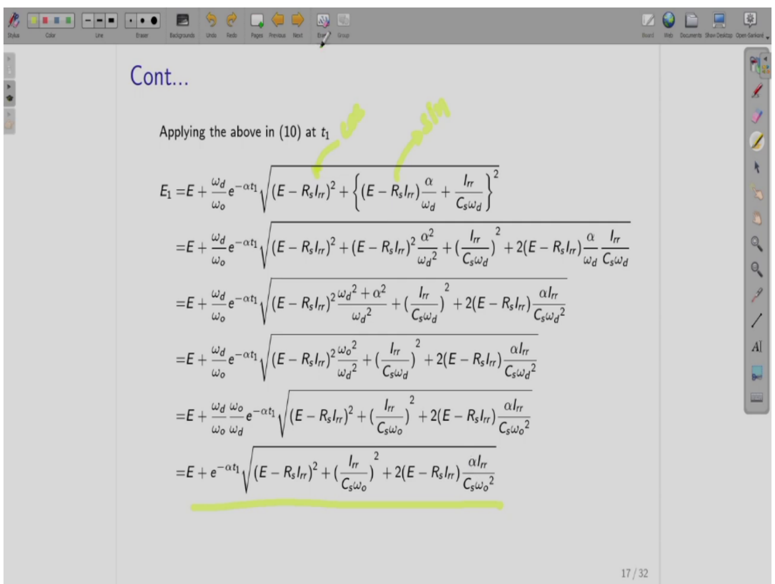

So, then that is what I had applied here (see above screenshot). So, at time t1 that is that peak, this is what it is going to be this is the cosine your coefficient and this is your sine coefficient. So, if you square it out and then take the root of it, square root of it, so, you just basically expand it and then try to reduce it. So, after reduction solving of this this is finally what you will be obtaining E plus e power of minus alpha t1 root over of this expression. And we do not like this Rs Irr and so forth, we want to express all in terms of those ratio chi and zeta.

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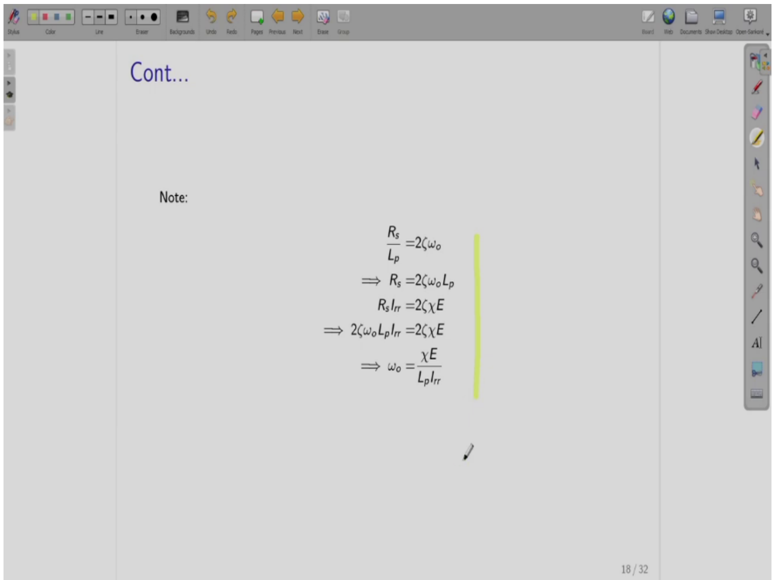

So, that is what we will be doing then we will be doing all these different substitutions.

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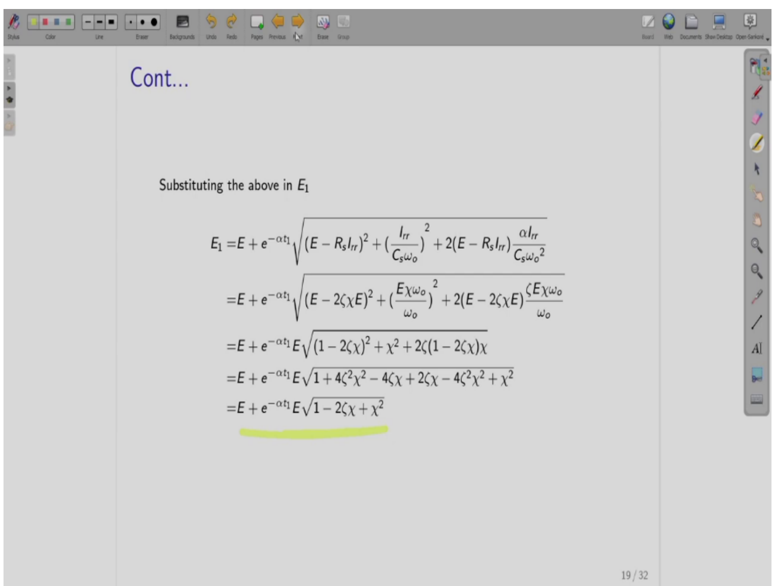

And when you substitute in this expression of E1, this is finally what you will be reducing it to. I am not going through all these things, you can do all these derivations on your own it is just simple math, you have to sit down and do it. So, this is what you finally obtain

$$E_1 = E + e^{-\alpha t_1} E\sqrt{1 - 2\zeta\chi + \chi^2}$$

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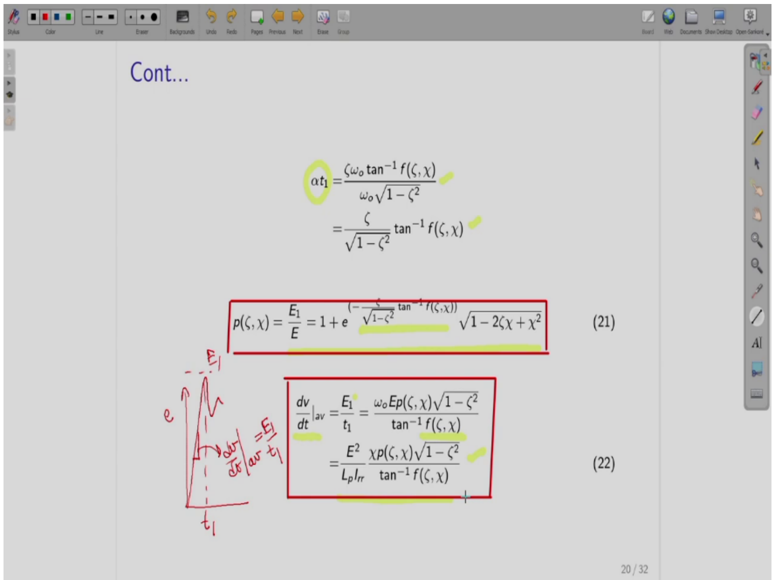

So, that is what you obtain here E1 by E as equal to this here we have also substituted for your t1, alpha t1 is this because t1 you already have. So, you can write for alpha t1 as well and so, this is what you will be getting

$$p(\zeta,\chi)=\frac{E_1}{E}=1+e^{\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\tan^{-1}f(\zeta,\chi)\right)}\sqrt{1-2\zeta\chi+\chi^2}$$

(21)

and so, that is what is written over here. Now, we are also interested in this dv by dt average. Now, what is this dv by dt average that we are talking about. Now, here this is the way your voltage rises this is your e waveform and this is your capital E1 which occurs at time t1.

So, at this part we are basically interested in the first rise only. So, this is what we are calling it as the dv by dt average, which is your ratio of E1 by t1. This rise in the voltage also we want to limit by this snubber design we can do that that we had discussed before. So, you divide this E1 by t1 and this is the function that you are going to get.

$$\left.\frac{dv}{dt}\right|_{av} = \frac{E_1}{t_1} = \frac{\omega_0 E p(\zeta,\chi)\sqrt{1-\zeta^2}}{\tan^{-1} f(\zeta,\chi)}$$
$$= \frac{E^2}{L_p I_{rr}} \frac{\chi\, p(\zeta,\chi)\sqrt{1-\zeta^2}}{\tan^{-1} f(\zeta,\chi)}$$

(22)

So, because t1 expression we know that this is what it comes out in terms of this tan inverse of this function zeta comma chi. So, that is what we have written here this dv by dt average that is what you are going to get. So, this one (equation 21) and this expression (equation 22) these are the ones that we will be using later on for doing the snubber design.

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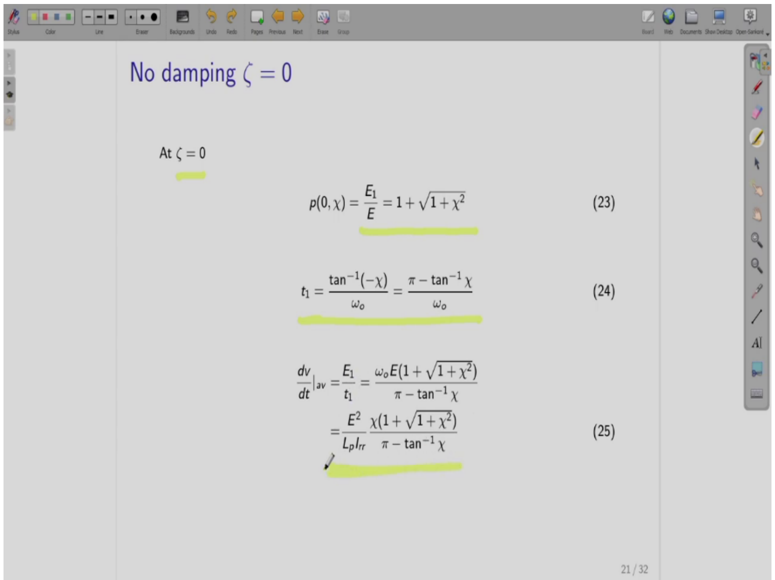

Now, we saw zeta less than 1 it could be also that your there is no damping that is zeta equal to 0 and that is a special case of this underdamped condition that we in the expressions that we obtain. So, you simply substitute zeta equal to 0 in all the above expressions that we had obtained. So, this is what you will be obtaining

$$p(0,\chi) = \frac{E_1}{E} = 1 + \sqrt{1+\chi^2} \tag{23}$$

$$t_1 = \frac{\tan^{-1}(-\chi)}{\omega_0} = \frac{\pi - \tan^{-1}\chi}{\omega_0} \tag{24}$$

$$\begin{aligned} \frac{dv}{dt}\Big|_{av} &= \frac{\omega_0 E(1+\sqrt{1+\chi^2})}{\pi - \tan^{-1}\chi} \\ &= \frac{E^2}{L_p I_{rr}} \frac{\chi(1+\sqrt{1+\chi^2})}{\pi - \tan^{-1}\chi} \end{aligned} \tag{25}$$

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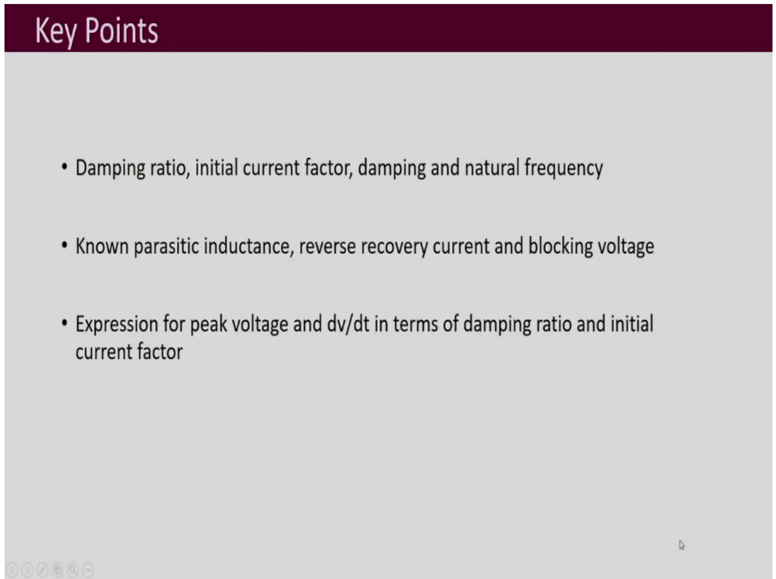

So, we did some derivations and you saw some big big expressions and equations solving out. So, do not get lost in the equations or the derivation, what we did was that that we first applied the KVL we got the equations and then we got the solution for i current for the RLC circuit, then those who are standard solutions and then you have to get the constants applying the initial condition, once you obtain that from there we found out the expression for e which is your l minus di by dt once you have the equation for i, then you can do differentiation of it obtain di by dt and then from there you can obtain the equation for the device voltage e which is what we want to limit the peak value and also the rate of change the dv by dt.

So, our objective of the derivation is to obtain the peak voltage E1 and this E1 by t1 that is the initially how the voltage changes. So, for that, we solved it and we saw what are the conditions at which the slope dv by dt can be positive and then we obtained those expressions in terms of zeta and chi your damping ratio and the initial current factor. So, that is what overall in the derivation that we had done.

And here your damping ratio initial current factor, damping frequency and natural frequency these are the important terms that you should be remembering for this derivation and further when we see this snubber design. And what is known here the parasitic inductance value Lp is assumed to be known and that means, you have some idea of what it is and the reverse recovery current Irr also you should be having some idea of it or an estimate of it and what is the blocking voltage capital E that usually we will be knowing based on for which circuit you are doing this snubber design. And as I told you, what we have finally found out is the expression of the peak voltage and the rate of change of the voltage that is your dv by dt in terms of your damping ratio and initial current factor. Thank you.