Course Name- Nanophotonics, Plasmonics and Metamaterials

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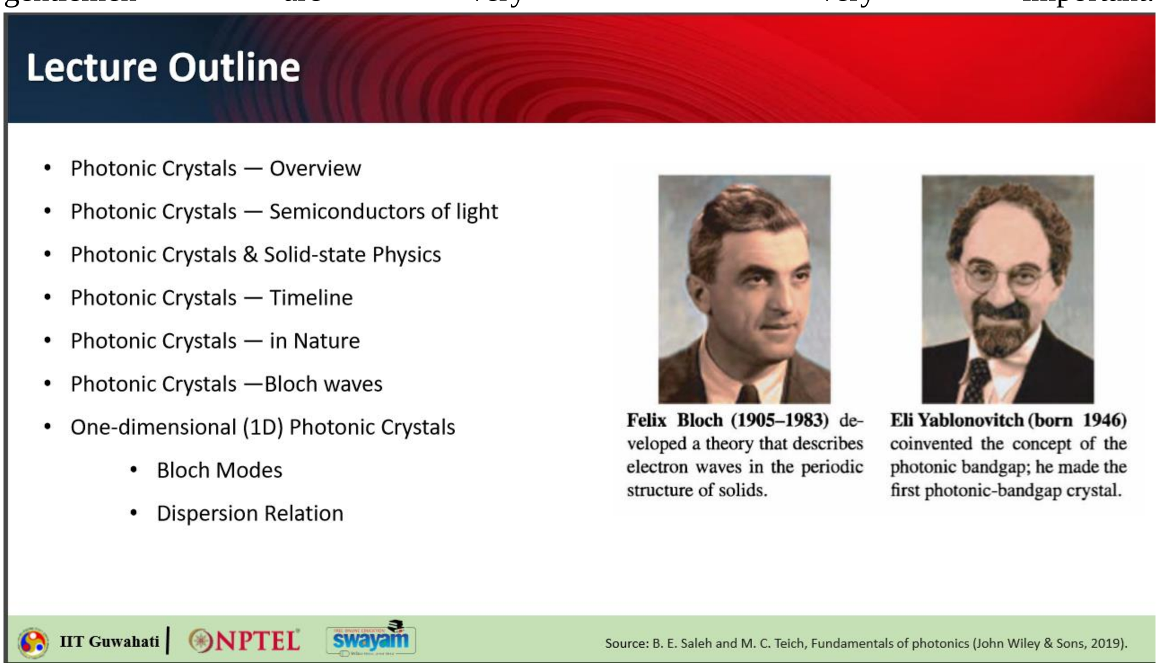
Week-04

Lecture -11

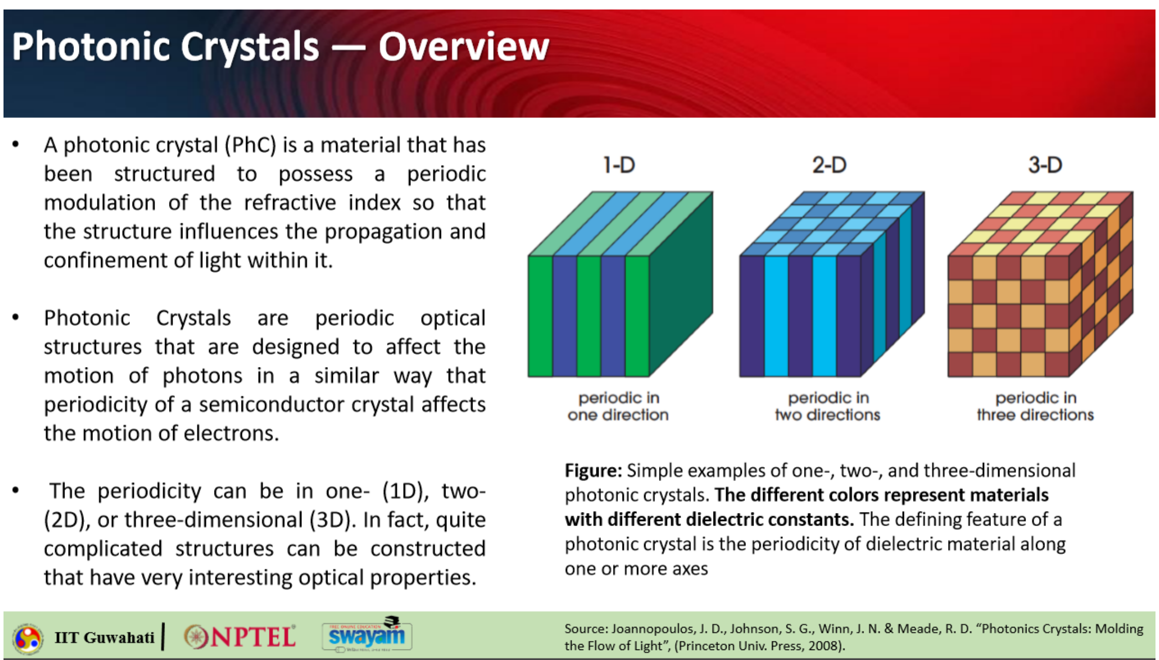
Hello students, welcome to lecture 11 of the online course on Nanophotonics, Plasmonics and Metamaterials. Today we will be covering the topic of 1D photonic crystals. So first, let us start the discussion of photonic crystals by giving a brief overview. Then we will look into the analogy of semiconductors and then we will see that photonic crystals are like semiconductors in optical domain. And we will study photonic crystals and solid state physics, how they are correlated, the timeline of photonic crystal, how photonic crystals are found in nature, then the Bloch waves to analyze photonic crystals and then we will go into the details of 1D photonic crystals by studying their different Bloch modes and dispersion relation. Now when we discuss the topic of photonic crystals, two particular gentlemen are very very important.



One is Bloch, another is Yablonovitch. So, Felix Bloch, he has developed a theory that describes the electron waves in the periodic structure of solids. So, that same theory can also be hired and brought into the optical domain and you will be able to explain how photons are behaving in a or light wave is basically behaving in a periodic crystal. And this is the picture of Eli Yablonovitch.

So, he co-invented the concept of photonic band gap. Just like semiconductors have band gap, photonic crystals also have band gap. So, he is the one to invent or co-invent this particular band gap, photonic band gap concept and he made the first photonic band gap crystal to demonstrate that. So, a photonic crystal, when we discuss about photonic crystal, we have to understand that this is basically a material, man-made material that has been structured to possess a periodic modulation of refractive index. So, that the structure can influence the propagation and confinement of light within it.

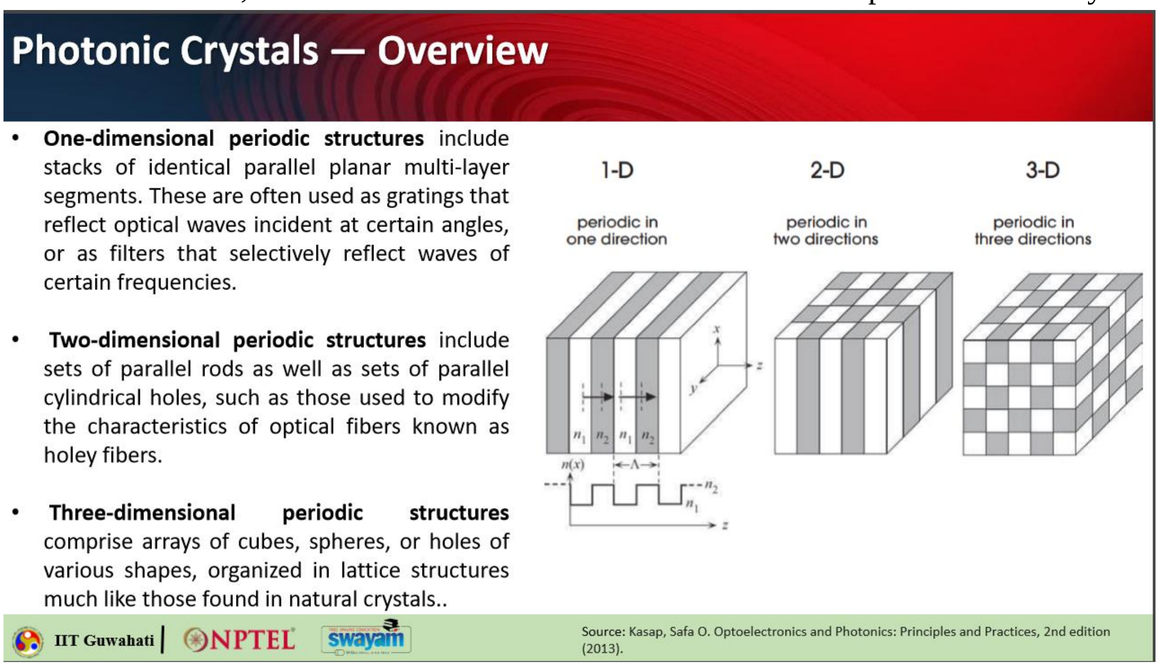
So, periodic modulation of refractive index. So, photonic crystals are nothing but periodic optical structure that are designed to affect the motion of photons in a similar way the periodicity of semiconductor crystals affect the motion of electrons. So, there is a direct analogy you can draw between semiconductor electrons and photonic crystal and photons. So, the periodicity when we are saying the periodicity can be in three dimension, one dimension or two dimension. So that way you can have 1D photonic crystal, 2D photonic crystal and 3D photonic crystal and each of them will have very unique and interesting optical properties.



So, here you can see you know different colors they represent different materials. So, you have material 1 and material 2 you can take slabs and repeat them and repeat them. So, that way it is like the periodicity is in one dimension. So, we call this as 1D photonic crystal. So, two materials are involved here.

So, n_1, n_2, n_1, n_2 and so on. It is not shown here. So, usually it is a periodic structure and we are just showing a small portion of it. Similarly, if we think of the periodicity in two dimension that is you think of you know columns of two different material. So, this dark light, dark light usually the dark material represents a larger refractive index lighter material represents a lower refractive index that is typically the analogy, but not always the case.

But let us assume that that helps us in understanding. So, there is the periodicity is now along say X and Y both. So, this kind of crystal can be called 2D periodic crystal. The third one will have periodicity in three dimension that means you are basically modulating the refractive index high, low, high, low, high, low and so on in all X, Y and also along Z direction. So, this becomes a 3D photonic crystal.



So, what we are gaining out of this gaining out of this you will see that basically with the periodic modulation of the refractive index you can control how light will travel in this particular medium and how you will be able to you know confine or propagate light through this particular medium. So, let us look into the details of 1D periodic crystal or periodic structure. So, here you can see that these actually includes stacks of identical, parallel, planar, multilayer segments. So, these are these two will form a period and we will now repeat this period. So, here how many periods are shown three periods are shown.

So, refractive index you can number them as n_1, n_2, n_1, n_2 and so on. So, these are often used as grating. So, when you see 1D periodic array is nothing but a grating. So, what does grating typically do? It reflects light at certain angles or you can say you can use this as a filter that can selectively reflect light waves at certain frequency. There is one very important filter called break grating filter based on this photonic crystal concept that is used in optical communication.

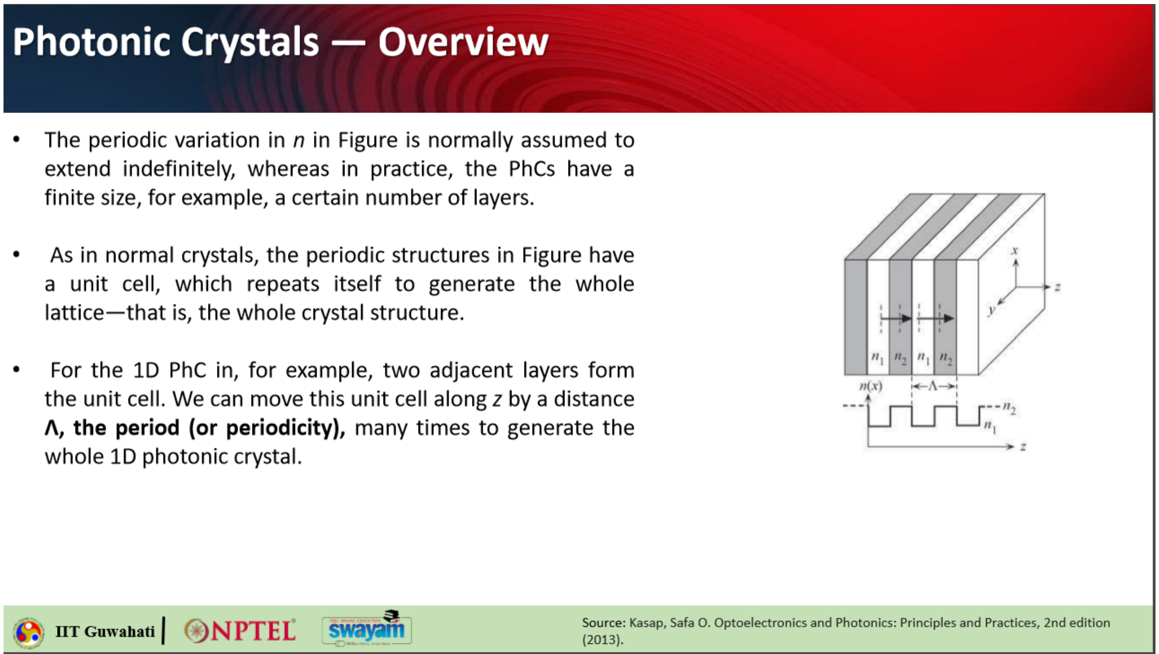
So, here also you can see along the Z so this is the Z direction that is the grating vector or along which the periodicity is lying. So, if you take that direction Z and if you plot the refractive index profile and x. So, you will see that you are basically getting low, high, low, high and so on. So, this is one period and you are repeating this structure over many periods. So, that will give you the 1D periodic grating or you can say 1D photonic crystal.

Now, if you see the 2D as I have already discussed these are nothing but sets of you know parallel rods. So, they can be have the you can have rods or you can take a solid material and drill holes in a you know linear shape or linear fashion that will also if you do that in only one axis say along Z. You are drilling holes along a particular solid material you will get a 1D photonic crystal. Now, if you take a slab and drill holes along X as well as Y you will get a 2D photonic crystal. So, you can either have the periodicity by different materials put together or you can actually take a solid slab and drill holes.

So, holes will be made of air. So, you have that material then air then material then air and so on. So, that way you are also able to create a periodic alteration of the refractive index along this material. So, one such example of having holes like parallel cylindrical holes is to is used to modify the characteristics of optical fibers which are called also holey fibers. So, there are holes in those fibers.

So, these are like photonic crystal fibers. So, you can actually have this kind of periodic holes in them. And 3D periodic crystal again you can either have 3D array of cubes, spheres or holes of different shapes. And the important thing is that they should be organized in lattice structures much like they are found in natural crystals. So, this is how 1D, 2D and 3D photonic crystals will look like.

So, let us look into this 1D photonic crystal in more details. So, as you see that there is a periodic variation of refractive index. So, n is varying along the length of this crystal. Now, as in normal crystals the periodic structure have a unit cell. So, here what is the unit cell? This n_1 and n_2 the whole thing these two together gives you the periodic cell.



So, you can define what is the period that is capital lambda and you have to repeat this period over the length. So, that you can get a 1D photonic crystal. Now, optical waves when it will encounter with this particular periodic variation of refractive index they will do something. So, optical waves they themselves are inherently periodic and when they interact with periodic media they do it in a unique way particularly when the periodicity of those material are of the order of the wavelength of light. So, if you recall our lecture from the metamaterial introduction I have shown you that when lambda is equivalent to a, a is the lattice period.

In that case wavelength of the light is able to see each of this scatterers individually. So, the way it interacts with the crystal is completely different than when it sees the crystal as a homogenized medium. So, this is where you know things become interesting and light matter interaction here also becomes very interesting. So, we will see that spectral bands will emerge in which light waves cannot propagate through this medium without severe attenuation. Means if you take this particular crystal and shine light you will see that at certain frequency band there is very severe attenuation and there is no transmission of that particular light.

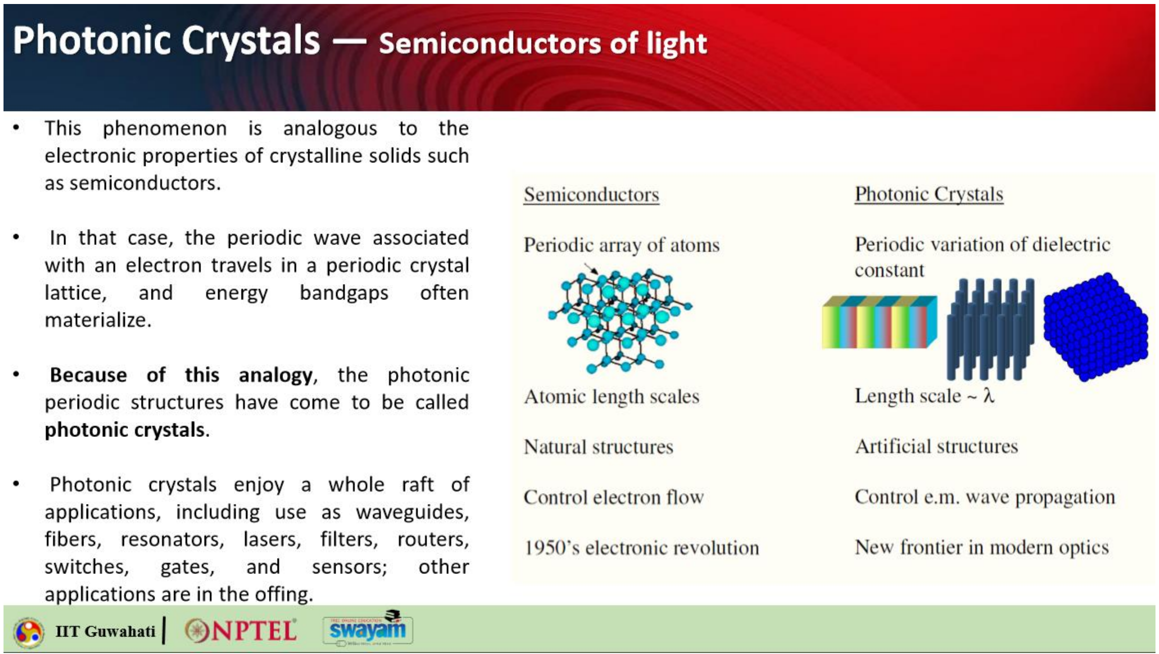
Photonic Crystals — Semiconductors of light

- Optical waves, which are inherently periodic, interact with periodic media in a unique way, particularly when the scale of the periodicity is of the same order as that of the wavelength.
- For example, spectral bands emerge in which light waves cannot propagate through the medium without severe attenuation.
- Waves with frequencies lying within these forbidden bands, called **photonic bandgaps**, behave in a manner akin to total internal reflection, but are applicable for all directions.
- The dissolution of the transmitted wave is a result of destructive interference among the waves scattered by elements of the periodic structure in the forward direction.
- Remarkably, this effect extends over finite spectral bands, rather than occurring for just single frequencies.

It means whatever light is being incident is coming back and that happens only over a certain frequency band and we can call those frequencies lying in that forbidden band as photonic band gap. And they behave in a very similar way like you know total internal reflection, but you know total internal reflection also there is no transmission everything gets reflected back. But you know total internal reflection happens say at a certain angle, but here you have to make sure that over those frequency band or band of wavelengths you can say the light may incident on the crystal at any direction, but it will have the same effect. It is not at all permitted to enter inside the material. So, there will be no transmission of that light beyond that material or through that material you will get all reflection only.

So, that is the concept of photonic band gap. And this comes the dissolution of the transmitted wave is basically a result of destructive interference among the waves scattered by all the elements of this periodic media in the forward direction. So, you can actually consider this periodic lattice let it be 1D, 2D or 3D the concept remains same. So, one light is falling on them they are scattering waves in the forward direction. So, those waves those are the basically transmitted waves and when they destructively interfere with each other they are they all cancel out ok.

And this is the case when we say that there is no propagation allowed through this crystal it means those frequencies are lying within the photonic band gap. So, this effect extends over finite spectral band as I told rather than occurring over just a single frequency and this is where it becomes different to the normal you know total internal reflection. So, now let us look into photonic crystals as if they are basically semiconductors of light. So, this phenomena is already seen in semiconductor crystals ok. So, if you look into the electronic properties of crystalline solids such as semiconductors, you will find similar kind of you know features where you have energy band gap right.



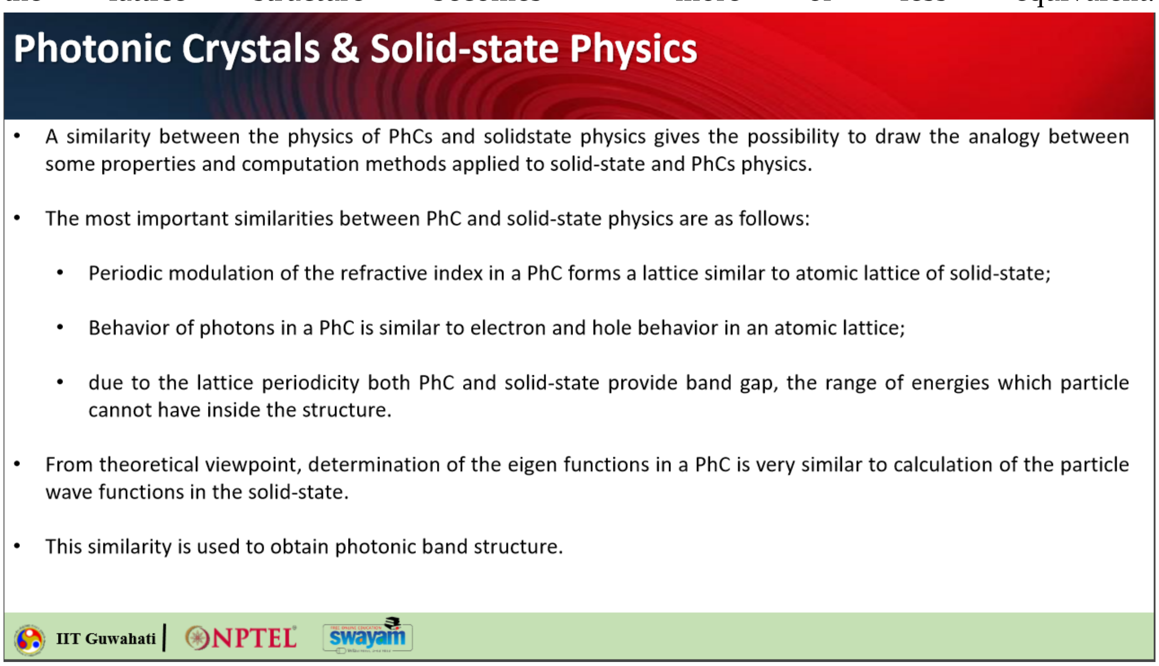
All of you must have studied this in your school days that there are metal conductors there are insulators and then there are semiconductors. and semiconductor there is a band gap it means that particular energy is not supported in that particular semiconductor crystal. So, in that case a periodic wave associated with an electron travels in a periodic crystal lattice and the energy band gaps often materialize. So, because of this analogy the photonic periodic structures can also be called as photonic crystals just like semiconductor crystals you can call them as photonic crystals. So, 1D, 2D, 3D photonic structures can be called as 1D, 2D or 3D photonic crystals.

So, here is an exact analogy as you can see on the screen. Semiconductors are nothing but periodic array of atoms. Here the periodicity is in the atomic length scale. These are natural structures whereas photonic crystals are artificial structures and if you compare the length scale here also the length scale is comparable to the wavelength of incident light. And here you have periodic array of atoms.

Here you basically have periodic variation of dielectric constant. So, this is 1D, this is 2D you see these are like dielectric rods. Rod, air, rod, air, rod, air this is how we actually can achieve this particular 2D photonic crystal and this is 3D photonic crystal. There are different ways of making 3D photonic crystal we will come to that in the next lecture. So, these are the direct analogy or you know comparison between semiconductors and photonic crystals.

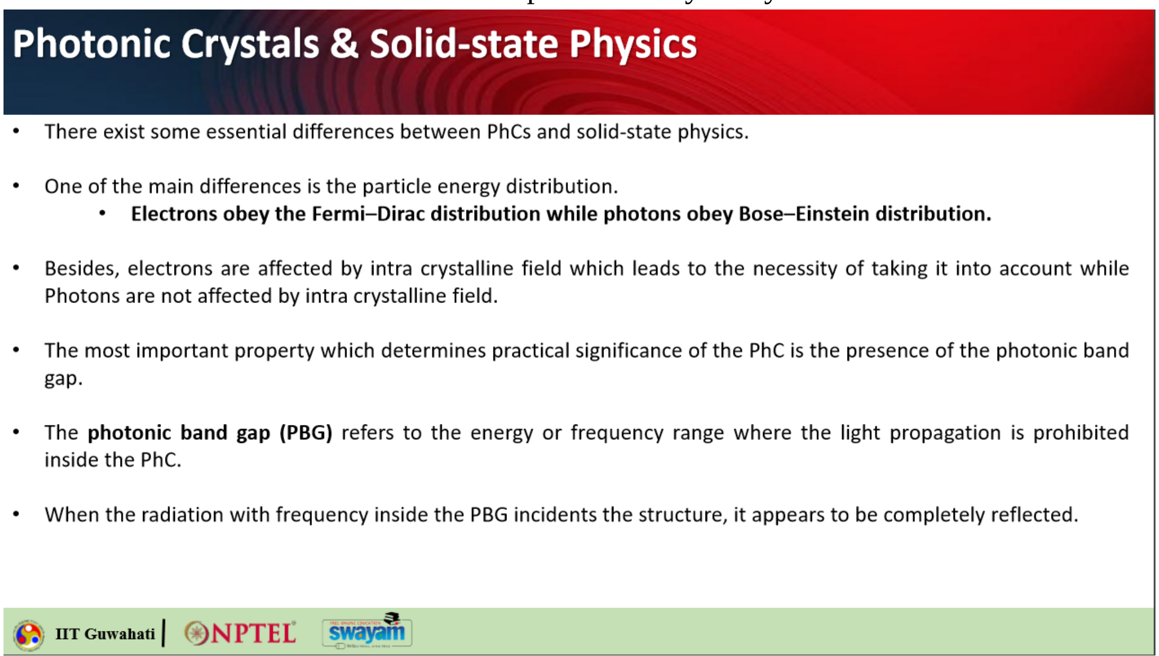
Now semiconductors allow you to control the electron flow whereas photonic crystals allow you to control the flow of light. And in 1950 it has actually revolutionized electronics by bringing in semiconductors as you can as you know that we have all these industries now electronics industry based on semiconductors. Now photonic crystals is relatively new and they are the new frontiers in modern optics. So, photonic crystals also enjoy a whole draft of application including use of say waveguides, filters, fibers, resonators, lasers, routers, switches, gates and sensors and other applications also. So, this is why photonic crystal itself is a very very interesting topic to study because you can actually make a lot of kind of lot of practical devices using photonic crystals.

One of these would be like waveguiding like we will see there are different types of 1D, 2D waveguides you can make and you can make cavities, you can make fibers. So, lot of applications, filters as I already mentioned. So, the similarity between the physics of photonic crystals and solid state physics has given us an possibility to draw the analogy between some properties and computational methods being applied to both solid state physics and photonic crystal physics. The most important similarities between photonic crystal and solid state physics are the following that periodic modulation of refractive index in case of photonic still forms a lattice similar to the atomic lattice of solid state. So, the lattice structure becomes more or less equivalent.



Then the behavior of photons in photonic crystal is similar or analogous to electrons and hold behavior in atomic lattice. And due to this lattice periodicity both photonic crystal and solid state they provide band gap and the range of energies which are not basically supported by that particular structure. And you can do different type of you know band gap engineering and explore different possibilities of using these materials for different applications. From theoretical point of view determination of the eigenfunctions in a photonic crystal is very similar to the way of calculating the particle wave functions in the solid state. So this similarity is also used to obtain photonic band structure.

So there you have electronic band structure here you have photonic band structure. But there are some important differences between photonic crystals and solid state physics. The main difference is that you know the particle energy distribution is different in both cases. The electrons they obey Fermi Dirac distributions I believe all of you know about this that is been taught in basic electromagnetic theory as well. So electrons they obey the Fermi Dirac distribution whereas the photons they obey Bose Einstein distribution.



And besides electrons are affected by the intra crystalline fields which leads to the necessity of taking into account while photons are not affected by this intra crystalline fields. And the most important property rather which determines the practical significance of photonic crystals is basically the existence of photonic band gap. And as I mentioned photonic band gap is nothing but the frequency range or wavelength range or you can say the energy range where light propagation is prohibited inside the photonic crystal. It means when such a you know when a radiation within such frequency band will fall on the photonic band gap crystal or you can say photonic crystal it will be completely reflected.

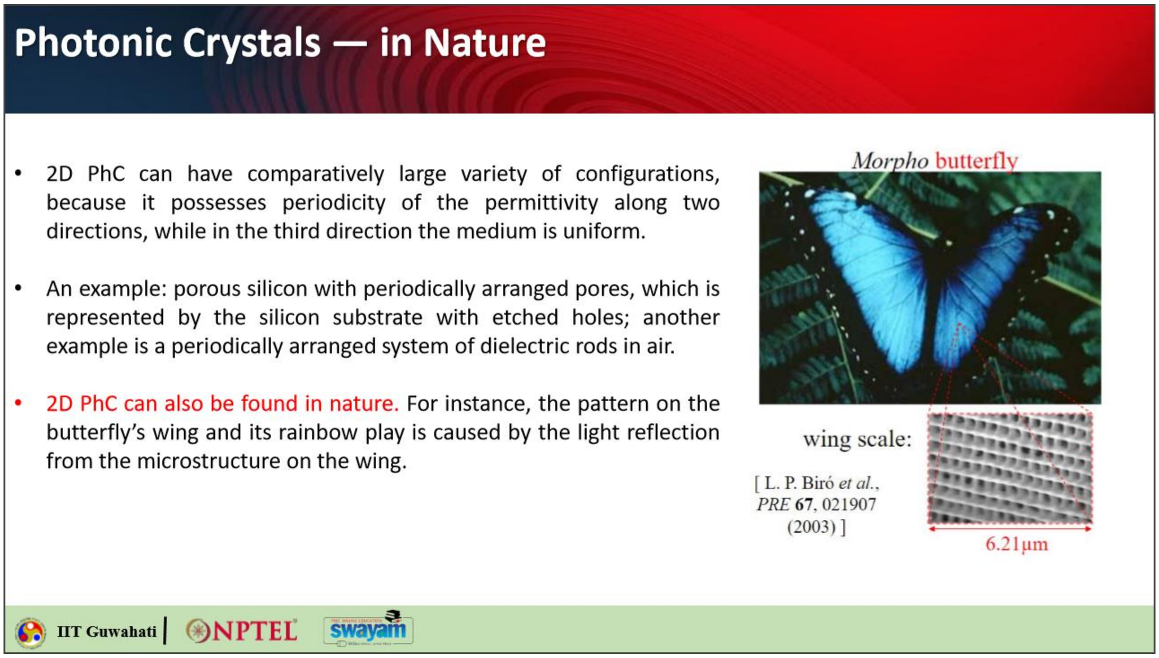
Photonic Crystals — Timeline

Photonic Crystals Major Time Steps

- 1987: Prediction of photonic crystals
    - *S. John, Phys. Rev. Lett. 58,2486 (1987), "Strong **localization of photons** in certain dielectric superlattices"*
    - *E. Yablonovitch, Phys. Rev. Lett. 58 2059 (1987), "**Inhibited spontaneous emission** in solid state physics and electronics"*
- 1990: Computational demonstration of photonic crystal
    - *K. M. Ho, C. T Chan, and C. M. Soukoulis, Phys. Rev. Lett. 65, 3152 (1990)*
- 1991: Experimental demonstration of **microwave** photonic crystals
    - *E. Yablonovitch, T. J. Mitter, K. M. Leung, Phys. Rev. Lett. 67, 2295 (1991)*
- 1995: "Large" scale 2D photonic crystals in **Visible**
    - *U. Gruning, V. Lehman, C.M. Englehardt, Appl. Phys. Lett. 66 (1995)*
- 1998: S.Y. Lin, Sandia National laboratories, N.M., designed a 3D photonic crystal operating at infrared wavelengths
- 1998: Philip St. J. Russell, University of Bath, England, demonstrated photonic band-gap fibers.

So here is a brief timeline of photonic crystals. So first prediction of photonic crystals was made in 18 sorry 1987 as you can see it is relatively new field which were mentioned in these two papers research papers. In 1990 the computational demonstration of photonic crystal was done. Computational demonstration means you were able to compute the photonic band structures and you can identify that there are photonic band gap possible where light can fall on the crystal from any direction and it will get reflected. So there is a possibility of photonic band gap and that is what was computation is shown there. And then in 1991 experimental demonstration of microwave photonic crystals were done by Yevlonovitch.

This is where his contribution comes into picture. So he was the first one to predict it as you can see and then in 1991 he was able to demonstrate this concept. So he was able to do it using a microwave range analogy for photonic crystals. In 1995 large scale 2D photonic crystals in visible range was made by this group and in 1998 3D photonic crystals operating at infrared wavelengths were fabricated or designed in this particular laboratory. And in 1998 also in University of Bath England they demonstrated photonic band gap fibers. You can actually make optical communication fibers using photonic band gap. So we look into all of these different applications and the fundamentals how things are working using photonic crystals in this two three lectures which are dedicated for photonic crystals. Now when you take a 2D photonic crystal they can have a comparatively large variety of configurations because in 1D photonic crystal it is pretty much you know high low high low high low and that is how you are basically varying the refractive index. There is nothing much variation you can bring in. So typically the Bragg grating that you have seen those are the examples of 1D photonic crystal or you can as I mentioned you can take a solid slab and you can drill holes in a linear fashion.



That also becomes a 1D photonic crystal. But when you come to 2D photonic crystals they have comparatively large wide variety because it possesses periodicity of the permittivity along two dimensions. So you have more room to play. Well the third dimension the depth dimension is maintained uniform. There is no variation in refractive index in that particular direction. So, we can take an example say porous silicon with periodically arranged pores, which is represented by you know silicon substrate with etched holes.

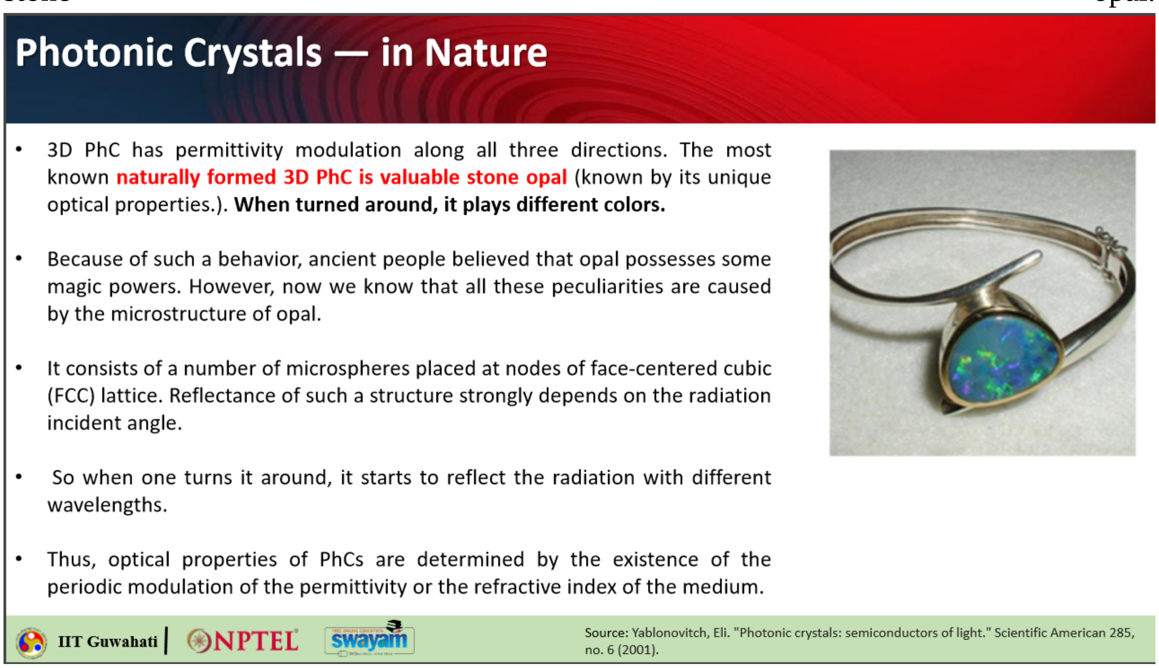
So you take a silicon substrate a 2D or a 3D structure like this and then you drill holes you make a 2D array of holes drilled into it or etched into it. So that becomes a you know that becomes a 2D periodic crystal. Another example is to have periodically arranged system of dielectric rods in air. So I believe all of you have seen chalks in your school days or even in colleges. So if you take the bundle of chalk that is like you know a periodic array of chalk.

These are like dielectric rods you can think of and they are actually in surrounded by air. So chalk air chalk air and so on and that happens in you know both X and Y direction. So just have them a little spaced. Usually the bundle of chalk will be all packed together. So if you allow some spacing between the two chalks you will have chalk air chalk air and so on and that will happen in both the direction.

So this is also another type of structure that is having a 2D variation of refractive index. So there are two types of structures possible as you have seen here. One is you take a solid silicon slab which is a 3D material and then you drill holes in a 2D array of holes. So this structure is also a 2D periodic crystal and that array of or bunch of chalks as I told you that is also another 2D periodic crystal.

Now this 2D periodic crystals are also found in nature. So if you take the you know morpho butterflies wing under microscope you will see that they actually have this kind of 2D periodic lattice. You see they are also having some whole kind of structure in X and say Y direction. So it is a 2D periodic variation so you are having 2D photonic crystal. And based on that what is happening why this color looks blue. So when white light falls on this this is basically a 2D photonic crystal which has got a band gap that lies in the blue frequency range.

It means the blue color is not allowed to pass through it. So what will happen to blue color it will be all reflected. So you can look into it from but the blue color will actually come back to your eyes and that is why it looks blue. There is 3D photonic crystal also found in nature. So in this case the periodic variation of refractive index happens in all the three dimensions X, Y and Z. And the most commonly known natural 3D photonic crystal is the stone opal.

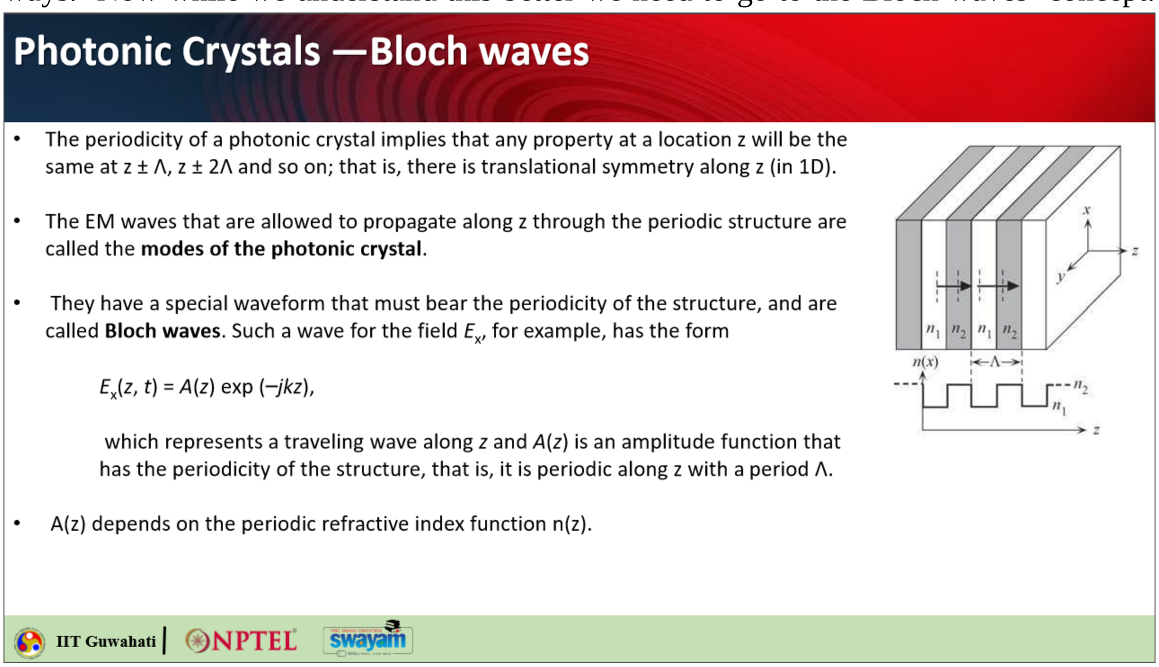


So when you take the opal gemstone and turn it in different direction you will see that it plays different colors. It looks different. Different different colors are reflected out of it.

And because of this such you know amazing or strange behavior you can say ancient people used to believe that opal possesses some kind of magical powers. But later on they could understand when when the scientists put this opal into a microscope and they could see the microstructure of the opal they figured out that you know this consists of a number of microspheres which are placed at the nodes of FCC lattice.

FCC is face centered cubic lattice. And because of those they are able to reflect light at different angles. Or you can say you know the reflectance of the structure is very strong dependent on the incident angle. So when someone turns it around it starts to reflect the radiation with a different wavelength. So that is the speciality of this particular photonic crystal.

So these are 3D photonic crystals. So as discussed the optical properties of photonic crystals are determined by the existence of periodic modulation of permittivity or refractive index in the of the medium. So these are natural objects.So that also gives us the opportunity and flexibility to make this kind of modulation in our laboratory and manipulate light in a different way. So that is how photonic crystal gained so much of popularity that it gives you the ability to engineer light matter interaction in many possible ways. Now while we understand this better we need to go to the Bloch waves concept.

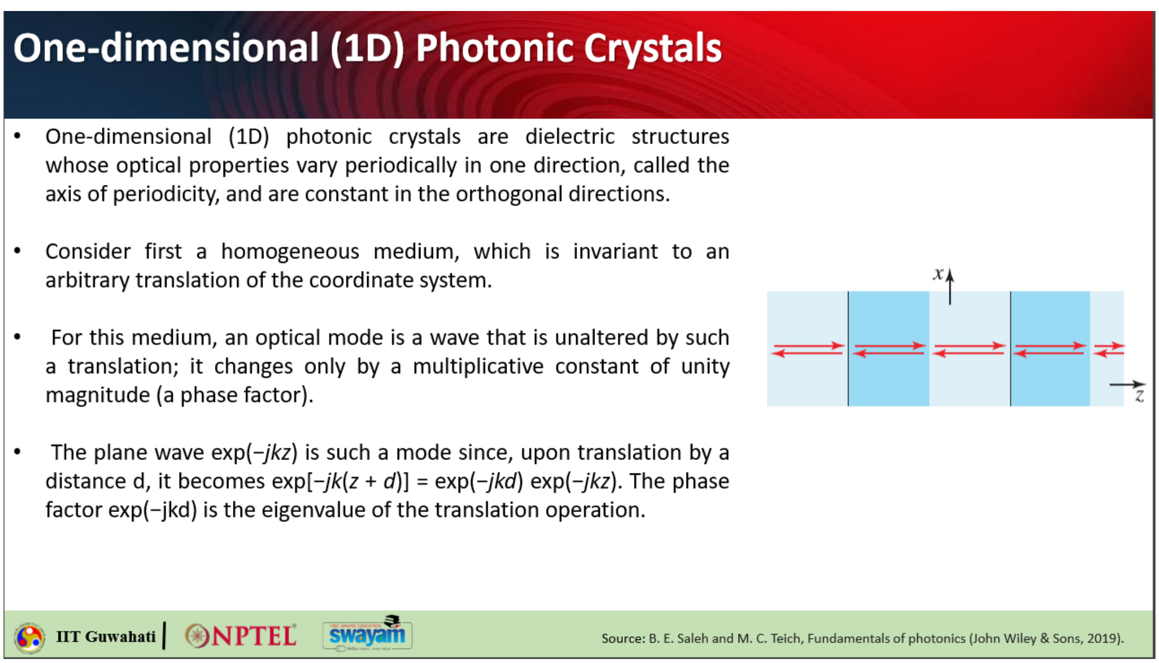


So let us assume this particular structure here which shows the 1D photonic crystal which we have seen in our previous slides. Here the periodicity of the photonic crystal implies that the property at any location z will also be repeated. How? They will be same at $(Z \pm \Lambda)$ that is the period $(Z \pm 2\Lambda)$ and so on. That is there is a translational symmetry along Z means this way it is 1D. So there is a translational symmetry because the same feature is repeating after lambda (Λ).

Now EM waves that are allowed to propagate along Z through this periodic structure are called the modes of the photonic crystal. Now they have a special waveform that must bear the periodicity of the structure and these are called the Bloch waves. So this is what happens like when a plane wave will propagate through this periodic medium there that plane wave will also have this kind of periodicity of the medium and that we will try to see here mathematically. So such a wave for the field E_x for example has the form $E_x(z,t)$. So E_x is what electric field along X direction, Z is the propagation direction, t is the time dependence.

So that will have *A(z)*. What is *A(z)*? That is basically the amplitude function of this wave that has the periodicity of the structure. So the amplitude will be like this high low high low depend and what is the variation in the amplitude that is from the periodicity of the structure that is lambda. What is exponential minus j omega d? This is how it oscillate and propagate along the Z direction with a wave factor of *k*. So I believe it is clear. So a plane wave when it interacts and it propagates through a periodic medium the amplitude function picks up the periodicity from the medium.

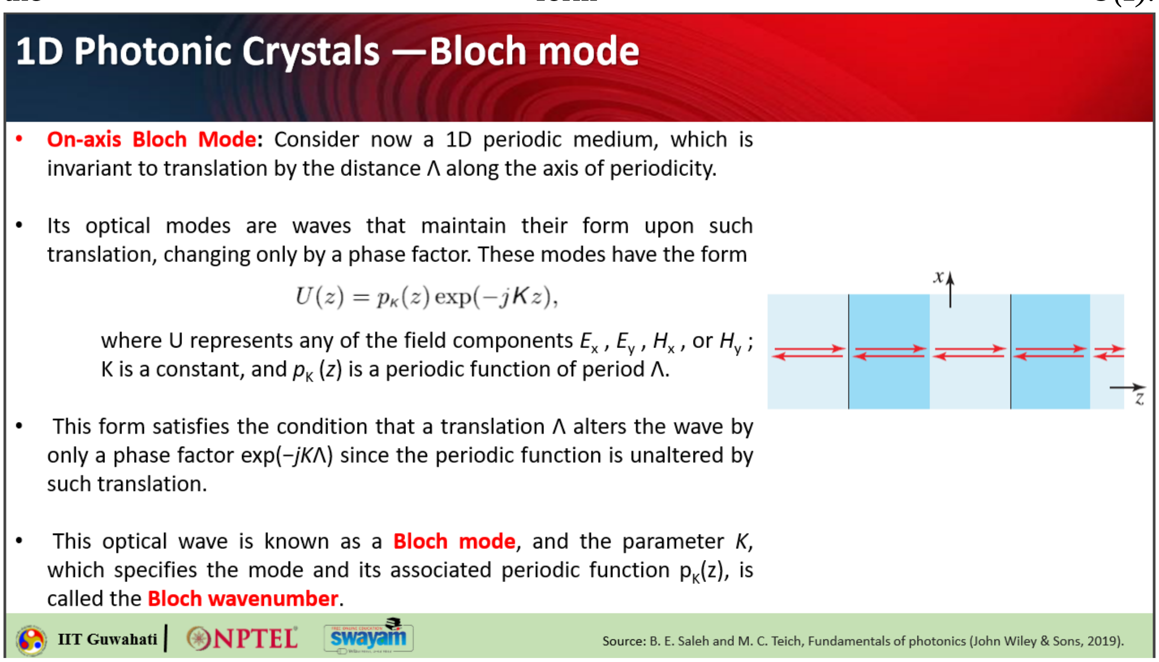
So whatever is the periodicity here that you can see lambda your amplitude of the plane wave will also get modulated with that periodicity. And we have seen that *A(z)* depends on the periodic refractive index function that is *n(z)*. So when you look into 1D photonic crystals they are basically dielectric structures whose optical properties vary periodically in one dimension. So here we can define the axis of periodicity.



And these variations they are constant in other orthogonal direction. So along the plane of this paper or inside like this or this way or along X it is same. So this variation only takes place along the Z direction. Now let us first consider a homogeneous medium which is invariant to any arbitrary translation of the coordinate system. And for this medium an optical mode is nothing but a wave that is also unaltered by a translation. So it changes only by a multiplicative constant of unity magnitude or a phase factor.

So let us show how it looks like. So if you take a plane wave $exp(-jkz)$ so this has got a fixed amplitude. So, this one is such a mode since upon translation by distance d it will only become $exp(-jkz + d)$. So that is the distance it has propagated. There is no variation in amplitude we can take that as fixed. So we said that you have a multiplicative constant here so you can see it is you can actually split this exponential into two parts.

So it is $exp(-jkd). exp(-jkz)$ and in this case this term $exp(-jkd)$ is a phase factor and that turns out to be the eigenvalue of this translation operation. So when you consider a 1D periodic medium which is invariant to the translation by distance capital lambda along the axis of periodicity invariant you understand that you know after one period the property is again repeating so the property is remaining same. So lambda then next lambda it is similar. So that is how you are able to define this you know on axis Bloch modes. So its optical modes are basically waves that maintain their form upon such translation changing only by a phase factor depending on the distance they are travelling and these modes have the form *U(z)*.



Now U can be anything E_x, E_y or H_x, H_y so U is a generic representation $p_\kappa(z)$, $p_\kappa(z)$ is nothing but a periodic function pk is a periodic function which is having a period of capital lambda that is what we are seeing here that there is you know periodic variation of refractive index and the period is capital lambda. So this term comes here because this will be the amplitude variation also for the bloch mode of the wave that is traveling and times $\exp(-jkz)$. So this is the you know propagation of the wave. Now this form satisfies the condition that a translation lambda alters the wave by only a phase factor that is $\exp(-jk\Lambda)$ since the periodic function is unaltered by such translation. So you can take you know z equals lambda and you will see that the wave is actually repeating itself or the conditions you can see that they are repeating.

So this kind of optical wave is called bloch mode and the parameter k that satisfies you know the mode and its associated periodic function which is $p_\kappa(z)$ is also called the bloch wave number. So bloch wave is nothing but the wave that is propagating in a periodic medium. So here also we are saying the same thing the Bloch mode is thus a plane wave e to the power minus jkz with propagation constant of k modulated by a periodic function $p_\kappa(z)$. So that is very important the amplitude of the Bloch wave will get the same pattern of the periodicity of that particular medium.

So we can say that it is getting modulated by the periodic function $p_\kappa(z)$. And this is this has the character of a standing wave. So you know how standing waves form so there is a traveling wave when it goes into any crystal or something it will get reflected. So when these two waves will you know combine they form the standing waves. So you can see here this is how the standing wave will form the blue one is the traveling wave.

So you can see the periodic variation of the refractive index. So these are like the periods or you can consider from here to here is a period or you can say from here to here is the period same thing. And this is way the mode is propagating. So if you take a reflected version and you add this up you will get this kind of you know dashed line pattern which shows you the standing wave pattern. Now since the periodic function of period given as capital lambda can be expanded into its Fourier series as a superposition of the harmonic components of the form $\exp(-jmgz)$ where m is nothing but $0, \pm 1, \pm 2$, and so on.

So what will be g that will be the inverse of this period so $2\pi/\Lambda$. So you can actually draw the spatial spectrum of the bloch mode in this form. So this is spatial frequency. So you have K, K+g, K-g you will have K+2g, K-2g, and so on. So we are able to convert this from you know time domain to spatial frequency domain. So it allows it follows that you know the Bloch mode is basically a superposition of plane waves of multiple spatial frequencies that can be given as K+mg, m can be $0, \pm 1, \pm 2$, and so on.

So this is how the bloch mode is comprised of. It is not only a single plane wave it is basically a plane wave or you can say it is a superposition of plane waves with multiple spatial frequencies. Now the fundamental spatial frequency will be this one corresponding to the bloch wave number k. So that will have the strongest contribution also as you can see from this spectrum. So the fundamental spatial frequency is g small g of the periodic structure and its harmonics which are mg, m is $0, \pm 1, \pm 2,$ and so on.

1D Photonic Crystals —Bloch mode

- The Bloch mode is thus a plane wave $\exp(-jKz)$ with propagation constant K, modulated by a periodic function $p_K(z)$, which has the character of a standing wave, as illustrated by its real part displayed in Fig. (a).
- Since a periodic function of period Λ can be expanded in a Fourier series as a superposition of harmonic functions of the form $\exp(-jmgz)$, $m = 0, \pm1, \pm2, \ldots$, with

$$g = 2\pi/\Lambda$$

- it follows that the Bloch wave is a superposition of plane waves of multiple spatial frequencies $K + mg$.

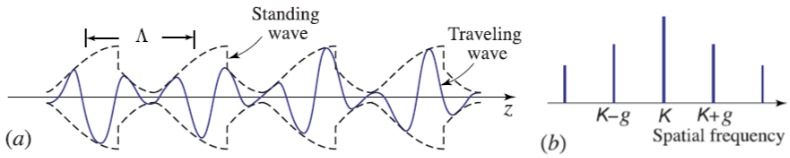


Figure: (a) The Bloch mode. (b) Spatial spectrum of the Bloch mode

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

1D Photonic Crystals —Bloch mode

- The fundamental spatial frequency g of the periodic structure and its harmonics mg, added to the **Bloch wavenumber** K, constitute the spatial spectrum of the Bloch wave, as shown in Fig. (b).
- The spatial frequency shift introduced by the periodic medium is analogous to the temporal frequency (Doppler) shift introduced by reflection from a moving object

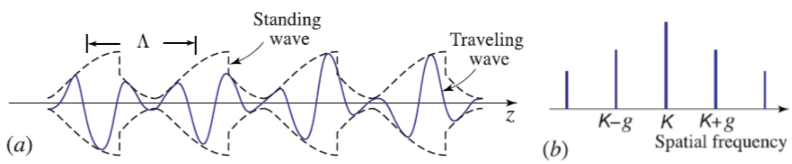


Figure: (a) The Bloch mode. (b) Spatial spectrum of the Bloch mode

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

Added to the Bloch wave number capital K constitute the spatial spectrum of the bloch wave. So this is the spatial spectrum of the bloch wave as we have discussed. The spatial frequency shift that you see here there is a shift in the frequencies by the periodic medium is analogous to the temporal frequency Doppler shift that is introduced by reflection from a moving object. So it is this kind of analogy you have also seen in Doppler shifts. So let us take two modes with Bloch wave numbers one is k another is k prime which is nothing but, K+g and let us assume that these two modes are equivalent since they correspond to the same phase factors. So you can actually write down so $\exp(-jK'\Lambda)$ will be same as $\exp(-jK\Lambda)\exp(-j2\pi)$ because from here to here it is 2 pi and then also here it is another 2pi.

1D Photonic Crystals —Bloch mode

- Two modes with Bloch wavenumbers K and $K' = K + g$ are equivalent since they correspond to the same phase factor, $\exp(-jK'\Lambda) = \exp(-jK\Lambda)\exp(-j2\pi) = \exp(-jK\Lambda)$.
- This is also evident since the factor $\exp(-jgz)$ is itself periodic & can be lumped with the periodic function $p_K(z)$.
- Therefore, for a complete specification of all modes, we need only consider values of K in a spatial-frequency interval of width $g = 2\pi/\Lambda$.
- The interval $[-g/2, g/2] = [-\pi/\Lambda, \pi/\Lambda]$, known as the first **Brillouin zone**, is a commonly used construct.

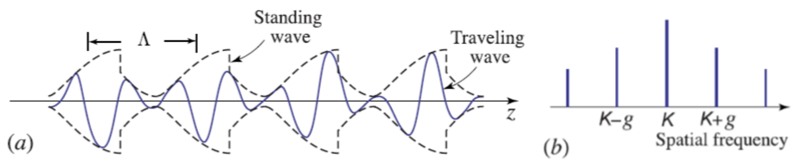


Figure: (a) The Bloch mode. (b) Spatial spectrum of the Bloch mode

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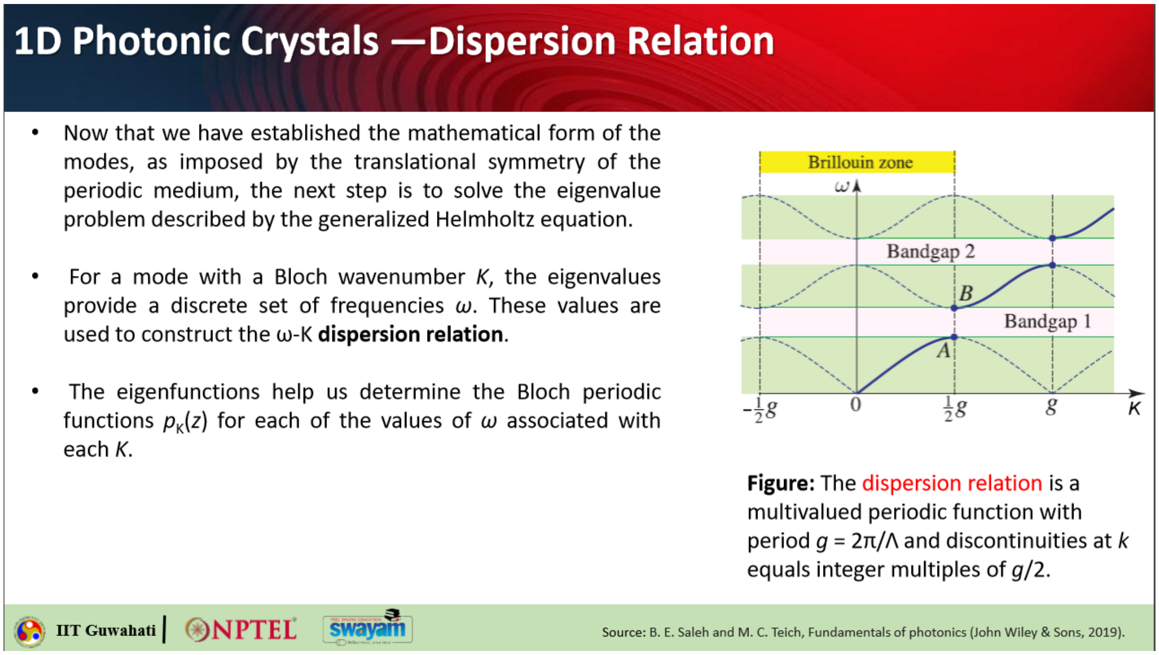
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

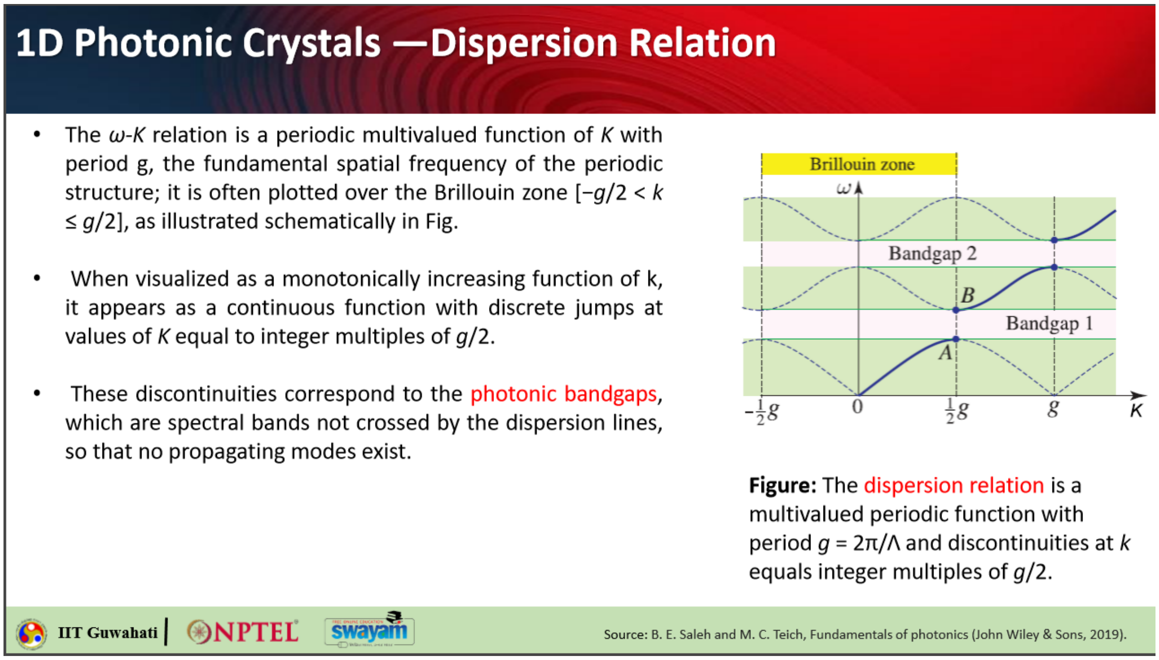
So you can add that extra phase and you will see you will land up actually to this so you can say these two are having same phase factors. It is also evident since the factor $\exp(-jgz)$ is itself periodic and it can be lumped with a periodic function which is $p_\kappa(z)$. You can say that for a overall or you say complete specification of all modes we need only to consider the values of K in a spatial frequency interval of width g equals $2\pi/\Lambda$. So if you consider only this width that will tell you about all the you know spatial frequencies because whatever is happening here is getting repeated in all other intervals which are also periodic. So if you take the interval from $[-g/2, g/2]$ this is a frequency spatial frequency interval or you can say that this is $[-\pi/\Lambda, \pi/\Lambda]$.

We can identify that as the first Brillouin zone and Brillouin zone if you remember the concepts from lattice, Brillouin zones are the you know small portion of the lattice that can be reproduced replicated to form the actual lattice. So this is a commonly used construct in kind of periodic crystals. So, now that we have established the mathematical form of the modes we know how the modes look like as imposed by the translational symmetry of the periodic medium. The next step would be to solve the eigenvalue problem described by the generalized Helmholtz equation.

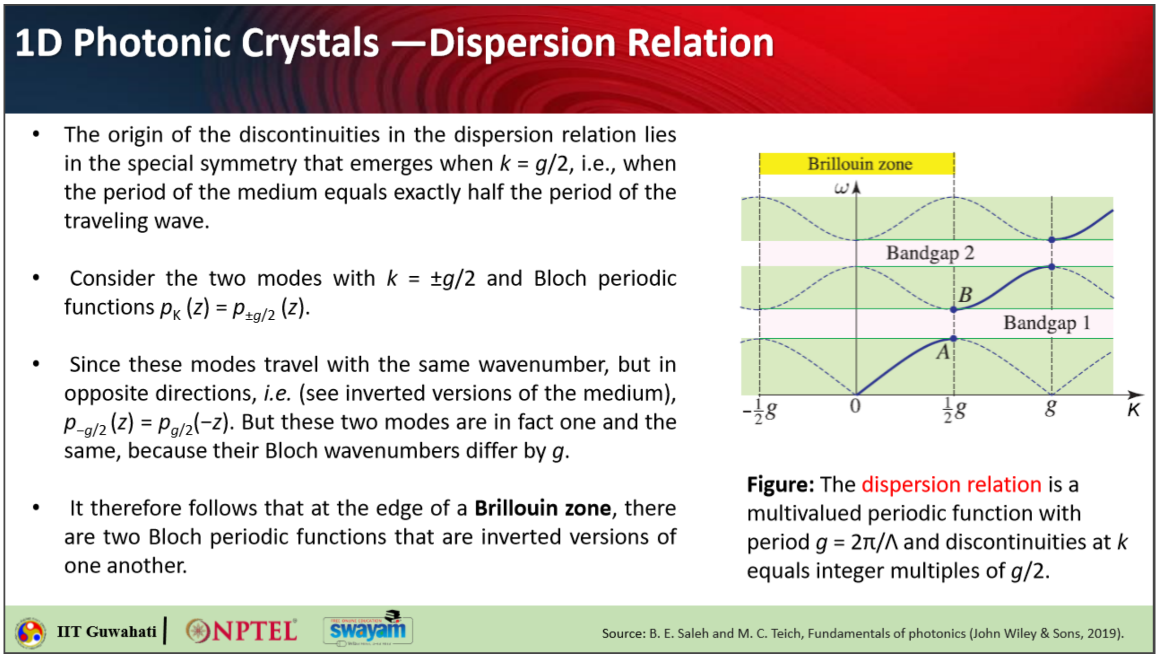


So you take Helmholtz equation dou square u plus k square u equals 0 you can be E_x, E_y, Hx, or H_z. And then you will see that you know for a mode with Bloch wave number K the eigen values provide a discrete set of frequencies omega. And these values will be used to construct the $\omega - K$ that is the dispersion relation. So if you try to plot the dispersion relation curve so this will be your ω this will be your K and you know the eigen functions will help us to determine the bloch periodic function that is $p_\kappa(z)$ for each value of ω associated with each K. And that is how you are able to calculate this particular thing. The $\omega - K$ relation or the dispersion relation in a periodic multivalued function of K with the period g in the spatial frequency the fundamental spatial frequency of the periodic structure right g is the fundamental spatial frequency of the periodic structure it is often plotted over the Brillouin zone that is why you try to plot over $[-g/2, g/2]$ right.

And when visualized as monotonically decreasing function of K so here you can see that you know one particular part is monotonically increasing this one the dark line so you can actually see the plots here. So as you start from 0 the central point and you add half g or g/2 you get this point and then there is a discrete jump ok. And again you monotonically increase you go up to another g again then there is a discrete jump and so on. So these discontinuities are nothing but the band gaps it means this particular frequencies there is no solution to the you know Helmholtz equation. So you are looking for the bloch modes which are able to propagate and you could not find any solution of bloch modes it means wave propagation there for this particular frequencies are not possible ok.

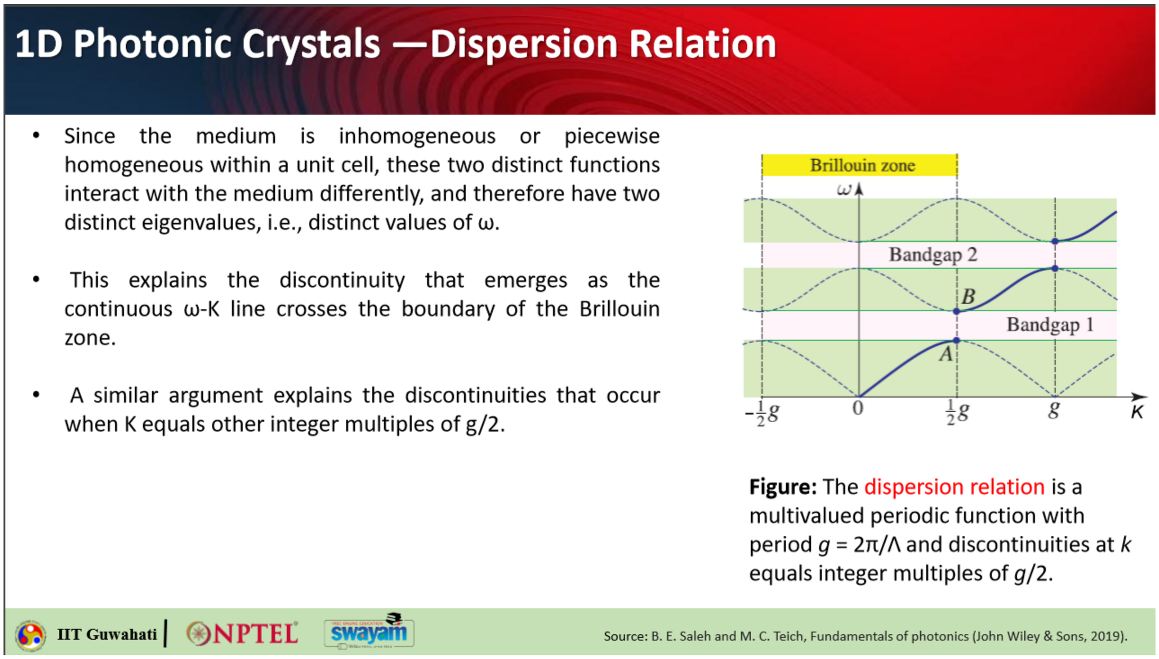


So again you have seen here also this particular wavelengths or frequencies are not allowed. So these discontinuities they correspond to photonic band gaps which are nothing but you know spectral bands and not crossed by the dispersion lines so that you know no propagation modes are existing in those. The origin of the discontinuities as you can see here there are discontinuities lies in the spatial symmetry that emerges from this relation when K equals g/2. That means when the period of the medium is exactly equal to the period of half of the travelling wave ok. So in that case you can consider the two modes with K equals plus minus g/2 ok.



So and the bloch periodic function we have seen $p_K(z)$ so you can replace K with $\pm g/2$. So in these cases since these modes travel with the same wave number because this and this they have the same wave number right but they are in opposite direction ok. So you can actually see the inverted versions of the medium they will propagate in the opposite direction this one ok. So you can write $p_{-g/2}(z)$ will be equal to $p_{g/2}(-z)$. So that is how you get you know the inverted version of the medium here.

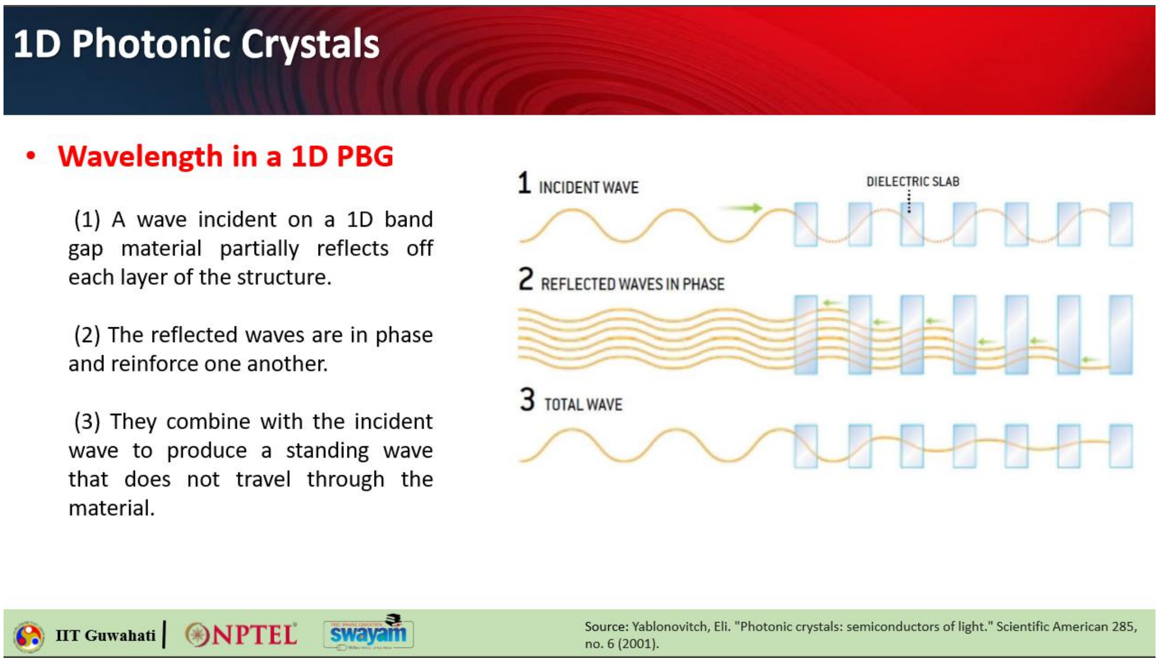
But these two modes are in fact basically one and the same just that their bloch numbers differ by g ok. Because one is g by 2 another is minus g by 2. So what is the difference between their bloch numbers is basically g. So when the modes are separated by integral multiple of g they are basically the same modes. So it means there are two different ways of modes that is possible having the same wave number in that case it is only possible if they have two different frequencies and that is why there is a discrete jump in the frequency ok. It therefore follows that at the edge of the Brillouin zone so this is the edge of the Brillouin zone there are two bloch periodic functions that are inverted versions of each other.



So these two are basically inverted version of each other fine. And the dispersion relation that you see here is basically a multivalued periodic function with period g equals $2\pi/\Lambda$ and the discontinuities at K equal to integral multiple of g by 2. And the reason of the discontinuities I have just explained here right. Now since the medium is inhomogeneous or you can say piecewise homogeneous within a unit cell. the two distinct functions they interact with the medium differently and therefore you have two different eigenvalues or distinct values of omega. That is what I was telling that it is only possible if you have these two modes having different frequencies and that explains the discontinuity that emerges from the continuous omega K line across the boundary of the Brillouin zone. So this is the Brillouin zone boundary this vertical dashed lines so whenever they will cross the Brillouin zone boundary you will find a discontinuity.

Here also they are crossing the Brillouin zone boundary we will get this discontinuity. So a similar argument explains the discontinuities that occur at K equals to any other integral multiple of g by 2 that is happening. Here it is 2 times g by 2 so this is where also you will get the discontinuity. If you proceed further you will find 3 g by 2 that case also you will find another discontinuity. Now another thing is across the central frequency as the 0 spatial frequency you see this part and this part basically they are symmetrical.

You can draw a mirror image of what is happening here to here and you will be able to construct the band diagram. So this is for forward propagating waves these are for the backward propagating waves or waves in the other opposite direction. Now let us look into the wavelength in 1D crystal. So what happens a wave incident on a 1D crystal which is nothing but a periodic variation of refractive index in one direction that can be achieved by dielectric slab air dielectric slab air so slab air slab air and so on. So this is the incident wave and you can see that you are actually having you know reflected wave from each of this structures So here this is reflecting here you this is the one reflected from this one but it is also carrying the reflection from the previous one and so on. So the reflected waves they are in phase and they reinforce with each other and in that case what happens when they are in phase they combine with the incident wave and they produce a standing wave.



So they constructively interfere and give you a standing wave and standing wave cannot travel and that is how you know this particular wave will not be able to travel through this particular material. So this is giving you the band gap. But when the material if you choose you know wavelengths which are not in the 1D photonic crystal band gap. PVG means photonic crystal photonic band gap or photonic crystal band gap. So if you choose the wavelength that is outside the band gap and that light will enter you will again get all the reflected waves but this time the reflected waves are not in phase.

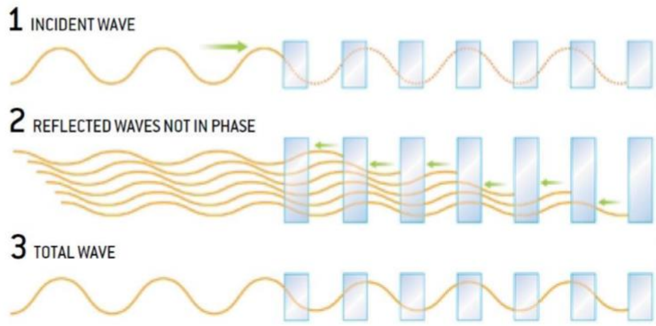
1D Photonic Crystals

- **Wavelength not in a 1D PBG**

  (1) A wavelength outside the band gap enters the 1D material.

  (2) The reflected waves are out of phase and cancel out one another.

  (3) The light propagates through the material only slightly attenuated.



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Source: Yablonovitch, Eli. "Photonic crystals: semiconductors of light." Scientific American 285, no. 6 (2001).

So they will not be able to you know form a standing wave rather what will happen the light will able to propagate through this material but with slight attenuation. So this is where transmission through this crystal will be possible when the wavelength does not lie in the band gap. So with band gap there is standing wave formation and that stops your you know propagation or transmission through this material. Outside the band gap light is able to propagate through this material but with slight attenuation.

Slight attenuation is coming because of this kind of you know interference with the reflected wave. So with that we will stop here. Thank you and we will start the discussion of dispersion relation and other details of photonic band structure in the next lecture. If you have any doubt regarding these lectures and any other previous lectures mention the lecture number and MOOC on your subject line and you can drop email to this particular email address.

Thank you.