Course Name- Nanophotonics, Plasmonics and Metamaterials

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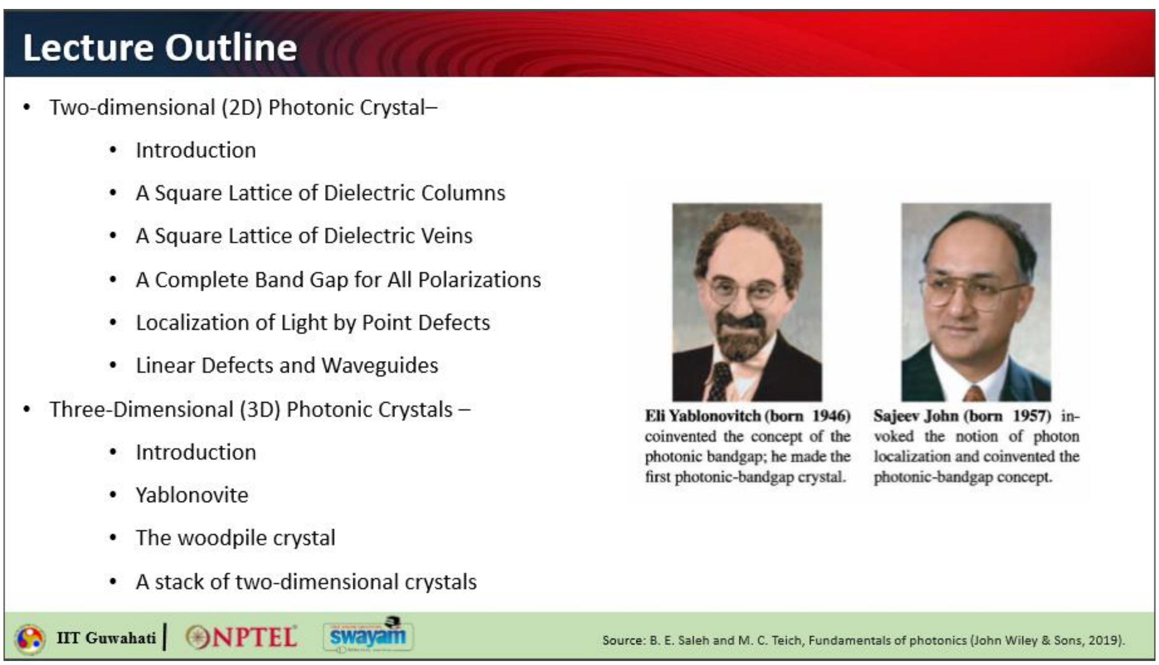
Week-05

Lecture -14

Hello students, welcome to lecture 14 of the online course on Nanophotonics, Plasmonics and Metamaterial. Today's lecture will be on 2D and 3D photonic crystals. So here is the lecture outline. So today we will first look into 2D photonic crystals. We will introduce this amazing engineering wonders. Then we will discuss about a square lattice of dielectric columns as an example of 2D photonic crystal. We will also discuss a square lattice of dielectric veins, a complete band gap for all polarization, how we are able to generate those. We will look into localization of light by point defects, linear defects and waveguiding. We will also discuss about 3D photonic crystals. We will introduce them. Then we will look into Yablonovitch kind of crystals. We will look into Woodpile crystal. So, these are different examples of 3D photonic crystals. And we will also look into a stack of 2 dimensional crystals. So that will also become a 3D photonic crystal.



So, for this wonderful engineering material, photonic crystals we are highly indebted to these two scientists Yablonovitch and Sajeev John. So, they are basically the co-inventor of photonic band gap. And he was the one Yablonovitch, so this is the crystal named after him. So, he was the first one to made a photonic band gap crystal to demonstrate this concept of photonic band gaps. So, let us introduce 2 dimensional photonic crystals.

So as you understand a 2 dimensional photonic crystal is basically periodic along 2 dimensions or 2 axis and it is homogeneous in the third axis. So, in this particular diagram this is the orientation as you can see. So, this is basically a square lattice of dielectric columns. So, you can think of you know chalks or pencils like that, and these are arranged in a square lattice. So how we characterize them like the radius of each column is r and the period is a.

And you can see this particular red box it shows the unit cell, okay. So, it is basically a square lattice, right. So, if you look into the dimensions so x and y are the dimensions where the periodicity lies and along the z axis it can be infinitely tall, okay. Now for certain values of column spacing that is a you can actually see that this crystal can actually gain a photonic band gap in the xy plane. So, when I say band gap in xy plane it means it will not allow any light propagation through this crystal along xy plane for certain frequency band and that will be known as the photonic band gap for this crystal.

Inside this gap no extended states are permitted. So, if light propagation is not permitted through this crystal at those particular frequency band what will happen to light? They will simply get reflected, okay. So, the incident light gets reflected. And unlike multilayer film this two-dimensional photonic crystal can prevent light from propagating in any direction within the plane. So that is something very interesting that here when we say about photonic band gap it means that the frequency of incident light it can actually fall in any direction from any direction and it will have the same reflection effect, okay.

It will not be able to you know penetrate into this crystal and propagate through it. So that is the difference between multilayer films. In multilayer films you are able to say restrict the light propagation in one or two directions but here in photonic crystal it will be from any direction light can fall on the crystal and the propagation will be prohibited if the frequency lies within the photonic band gap, okay. So, we can use the symmetries of the crystal to characterize its electromagnetic modes because the system is homogeneous in z. So, this is the z direction, okay, along the length of the dielectric cylinders, okay.

Two-dimensional (2D) Photonic Crystal– Introduction

- We can use the symmetries of the crystal to characterize its electro-magnetic modes.
- Because the system is homogeneous in the *z* direction, we know that the modes must be oscillatory in that direction, with no restrictions on the wave vector k_z.
- In addition, the system has discrete translational symmetry in the *xy* plane. Specifically, ε(r) = ε(r + R), as long as R is any linear combination of the primitive lattice vectors $a\hat{\mathbf{x}}$ and $a\hat{\mathbf{y}}$.
- By applying Bloch's theorem, we can focus our attention on the values of **k**· that are in the Brillouin zone.

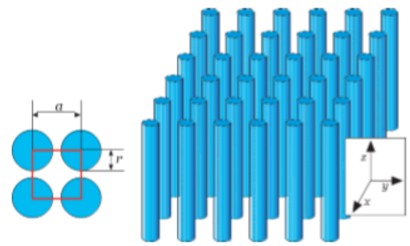


Figure: A 2D photonic crystal with lattice constant *a* (periodic along *x* and *y*). The left inset shows the square lattice from above, with **unit cell** in red frame.

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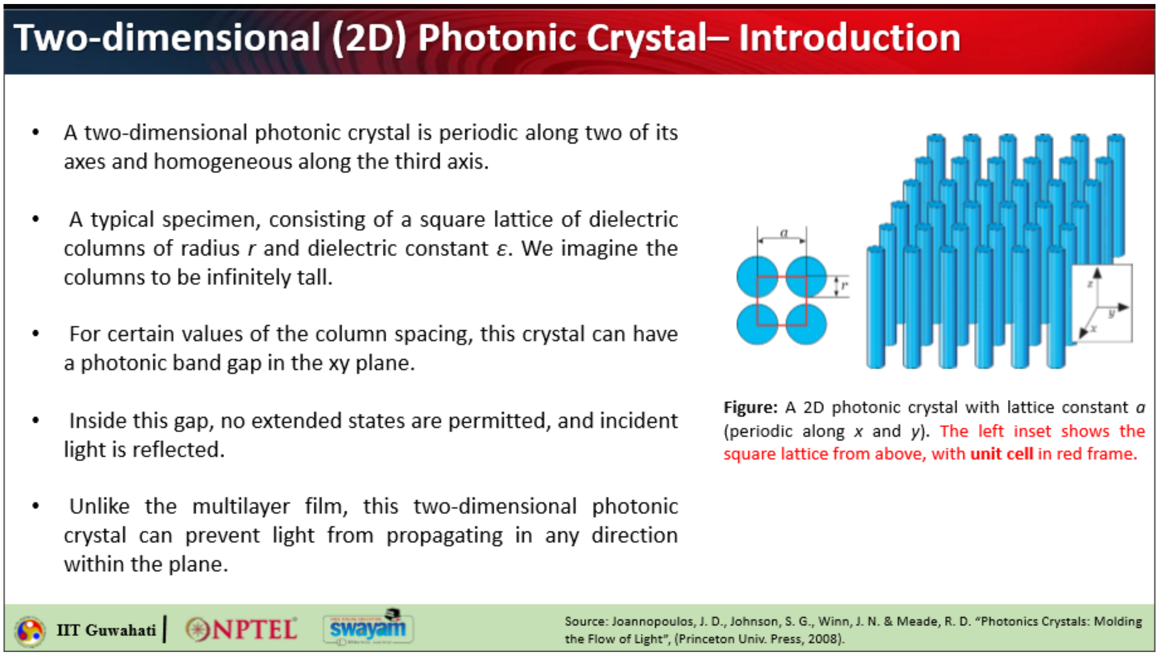
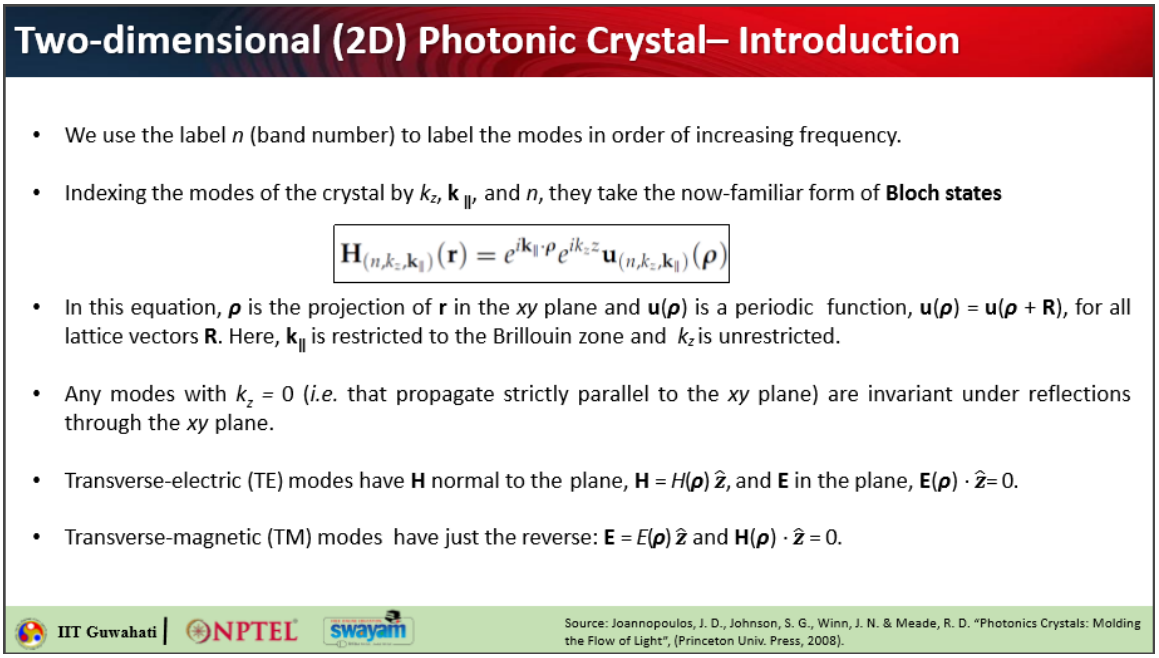
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

We must we know that the modes must be oscillatory in this particular direction as there is no restriction on the wave vector that lies in this z direction that is k_z. So k_z is a wave vector which has got no limitation or restriction. So, the modes must be oscillatory in this direction. However, the system if you look into the system from xy plane it has got basically the discrete translational symmetry which we discussed in the previous lecture. So, in that case what you can write that $\varepsilon(r)$ will be simply $\varepsilon[r+R]$.

What is capital R? It is any linear combination of the primitive lattice vectors. So, in this case the primitive lattice vectors can be $a\hat{x}$ and $a\hat{y}$ because it is a square lattice, okay. So, any combination linear combination of this primitive lattice vectors will see repetition of the dielectric property. So, what is the property here? It will be same here, here, here, here and so on. If you actually look into this gap that is air filled, so if you look into the same after this distance r, so you will see there is a repetition in the property as well.



Now by applying Bloch's theorem we can focus our attention on the values of k that are within the Brillouin zone because from the last lecture we have understood that if we study the properties of the Bloch modes within the Brillouin zone or even better if we study it within only the irreducible Brillouin zone we will be able to know the entire characteristics of the crystal, right. So, let us now use the level n, n is the band number to level the modes in order of the increasing frequency, okay. So, indexing the modes of the crystal by k_z, $k_{\parallel}$, so k_z you understand, the k_z are basically those modes which are lying along the z direction. $k_{\parallel}$ will be the modes lying in either x or y that is the parallel plane and you can use this level n, right. And when you put them they take the familiar form of the Bloch states.



So, you can write $\mathbf{H}_{(n,k_z,\mathbf{k}_{\parallel})}(\mathbf{r})$ that can be written as $e^{i\mathbf{k}_{\parallel}\cdot\rho} e^{ik_z z}\mathbf{u}_{(n,k_z,\mathbf{k}_{\parallel})}(\rho)$. So, what is ρ here? This is basically the projection of r in XY plane. So ρ is basically the distance, so if you take its projection on the xy plane, so r is any vector, okay. If you take the projection of r on the xy plane you will get ρ. So, $u(\rho)$ is basically a periodic function.

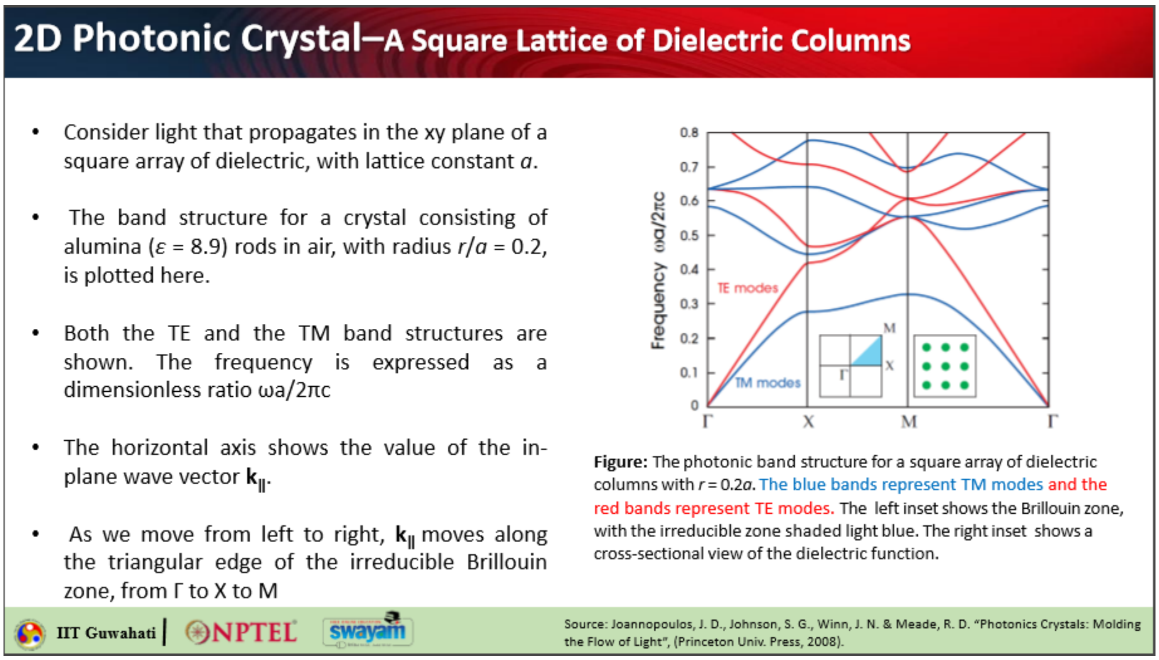
So you can write $\mathbf{u}(\boldsymbol{\rho})$ basically equals to $\mathbf{u}(\boldsymbol{\rho} + \boldsymbol{R})$, okay. And that is true for all lattice vectors capital R, okay. And as you understand this R is basically any combination of the primitive lattice vectors a x cap plus y cap, okay. And what is $k_{\parallel}$? That is restricted to the Brillouin zone, okay because that is where the periodicity comes into picture. So, we will only study for the Brillouin zone.

However k_z is unrestricted. Now any modes with k_z equals 0 it means they will not be propagating along the z direction, it means they are strictly propagating along the xy plane, okay. So, they are invariant under reflections through the xy plane. So, they will not change. Invariant means they will be same under reflection, okay.

So now let us actually look into the TE and TM modes separately. Now transverse electric modes, so transverse electric I believe everybody knows what is TE and TM mode. So here the electric field is perpendicular to the propagating direction, okay. So TE modes they have H normal to the plane, okay. So, we can actually take H equals $H(\rho)\hat{\mathbf{z}}$, okay.

And E is in the plane, okay. So, you can write that $E(\rho).\hat{\mathbf{z}}$ is 0, okay. And for TM modes you just have the reverse of it. So, you can write that E equals transverse magnetic means there is no magnetic field in the propagation direction. So magnetic field is basically transverse.

So you can write $H(\rho).\hat{\boldsymbol{z}}$ is 0 and E equals $E(\rho)\hat{\boldsymbol{z}}$. So, with that let us consider light that propagates in the xy plane of a square array of dielectric with lattice constant a. Now let us take some particular specific example. Say we are talking about this cylinders made of alumina, okay. So, alumina rods are considered what is the permittivity of alumina? So, epsilon equals 8.9. So, this epsilon is basically the relative permittivity. We are just simply using epsilon here, okay. And the radius is normalized to the lattice period and that is 0.2, okay. So, when you take this you can actually plot this particular band diagram.



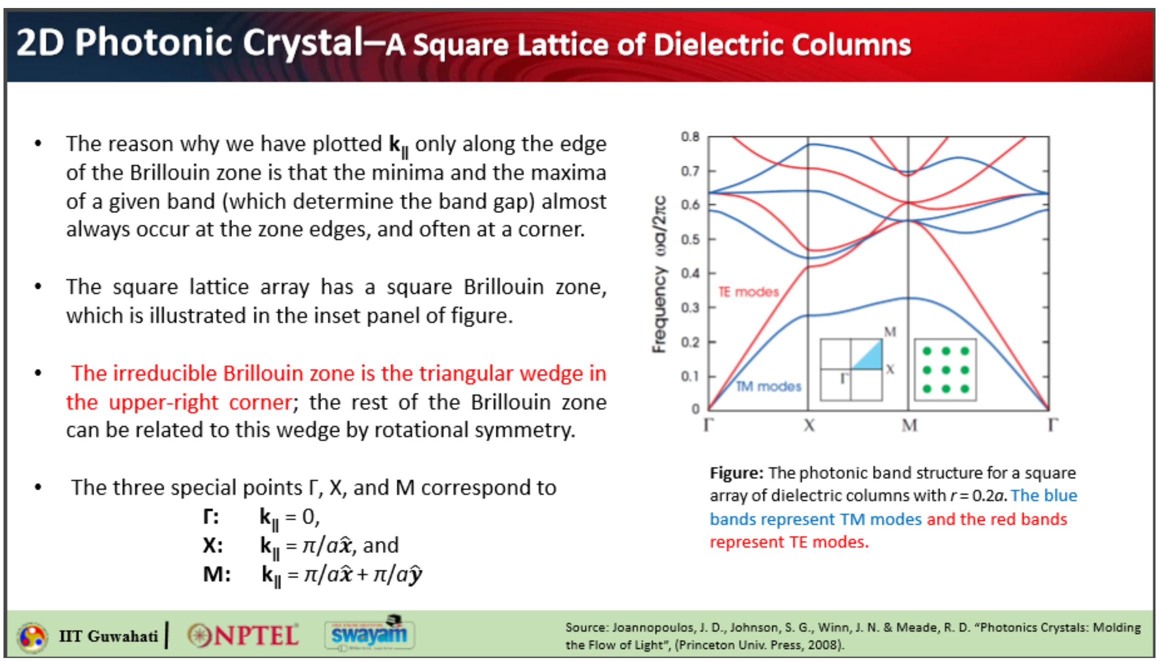
Now what are these points? Γ, X, M, Γ. As you can see here, so this is the square lattice. This is how the rods are placed, okay. And if you think about the blue zone, it will also be of a square shape. But in that you can actually find out that this blue shaded triangle is basically the irreducible Brillouin zone.

And you can mark the key points of that irreducible Brillouin zone as gamma X and M, okay. And bend diagram is basically plotted when you traverse through this Brillouin zone boundary. So, you can start with gamma, you move to X, then from X you move to M, then from M you move back to gamma. So, these are the key points gamma x m gamma plotted here, okay. So, after that you will also do the calculation for, these are basically the k parallel, okay.

And this is the frequency which is normalized. So $\omega a/2\pi c$ is basically normalized frequency, okay. So, depending on your size, okay, you can find out what is the exact frequency. Size in the sense, the lattice constant. You can find out what is the exact frequency where the band gap is lying.

So here the blue bands, they correspond to TM modes and the red bands correspond to TE mode, okay. So, as I mentioned the frequency axis, this is basically the omega k diagram, the dispersion relation. But the omega or the frequency is basically expressed in a dimensionless form which is given as $\omega a/2\pi c$. And this axis is nothing but the in-plane wave vector that is $k_{\parallel}$. Now as you move from left to right, so $k_{\parallel}$ moves from, you know, the triangular edge of the irreducible Brillouin zone.

So it will move from Γ to X to M to Γ as I have already discussed. Now the reason why we have plotted k parallel only along the edge of the Brillouin zone is that the minima and the maxima of a given band that actually determines your band gap, they almost always occur in the, you know, zone edges. So, there is no important information, okay, in the middle point of this particular Brillouin zone, okay, region. So that is why we are always going across the periphery instead of going inside. You can understand that if you start calculating all these points inside, okay, this will actually take a 3D shape, okay, and the computation time for the band structure will be really really high, okay.

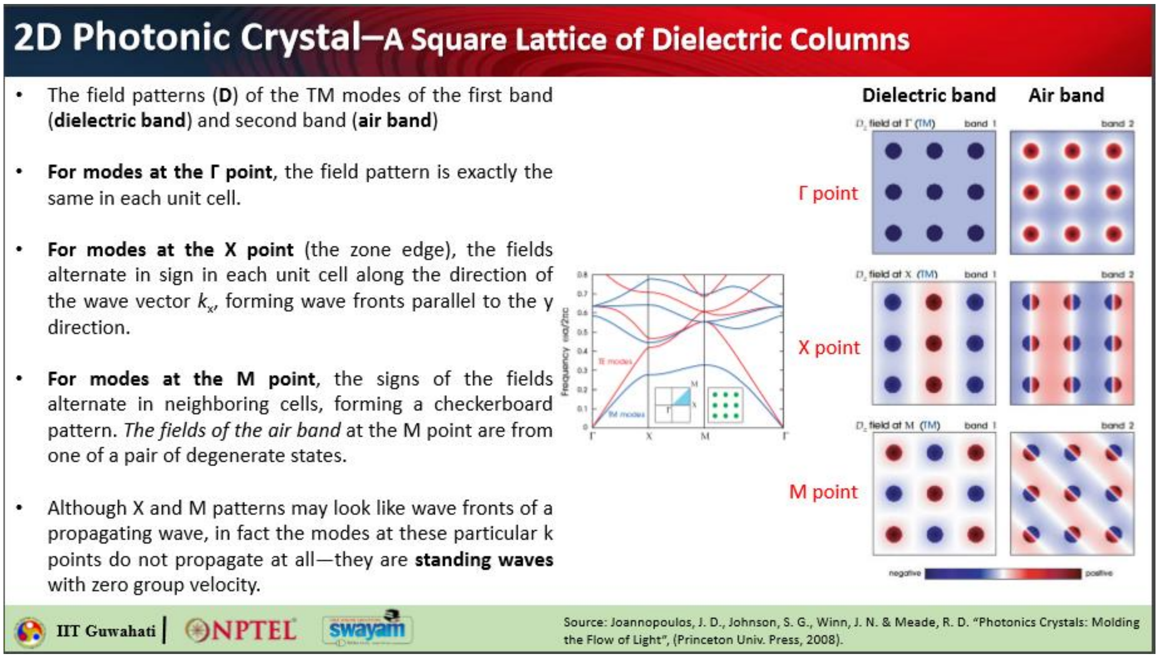


So, it is very obvious that a square lattice will have a square Brillouin zone as you have seen here, okay. But the irreducible Brillouin zone is basically a triangular wedge which is shown here, okay. And this is coming because of the rotational symmetry. So, you can rotate this by 45 degree, you can get this one and then you can use the reflection symmetry, okay, to get this box and then once you have the upper half you put a mirror here that will reflect back the bottom half and you get the entire Brillouin zone, right. We have discussed all this in the previous lecture.

Now when we take these three points gamma X and M they correspond to some particular information. What is that information? So $\mathbf{\Gamma}$ is basically telling you it is $\mathbf{k}_{\parallel} = 0$, okay, that is the center point, okay. X corresponds to $\mathbf{k}_{\parallel} = \pi/a\hat{\boldsymbol{x}}$, okay. So, and obviously this M corresponds to $\mathbf{k}_{\parallel} = \pi/a\hat{\boldsymbol{x}} + \pi/a\widehat{\boldsymbol{y}}$ and that is how you can go to this vector. So, these are basically the three important directions or that are marked by these three important points.

Now let us study the electric field and magnetic field pattern but here particularly we will be talking about the displacement field pattern D, okay, for the TM modes. Now one important thing is that here why do you see band gap? So, if you consider the blue lines which correspond to TM modes, so you see that this is the first band of TM mode and then this is the second band of TM mode, the second blue line. So, there is a gap in between, nothing, no other possible TM modes are here. So, this is a TM band gap and are you able to see a TE band gap in this structure? So TE mode, the first band is here but it crosses the band 2 of TE. So, this is red lines, only focus on the red lines, okay, and you will see that they are crossing each other.

So there is no TE band gap here, okay. So, this particular structure has only got TM band gap. Now let us study the dielectric field, okay of the TM modes of the first band which is also called as dielectric band and the second band which is called air band. Now what are these dielectric band and air band? Now if you think of photonic bands above and below the band gap, they can be distinguished by where exactly the energy of that particular mode is concentrated. So, energy of the mode can be concentrated in a high dielectric region or a low or say you can say high refractive index region that is in this case the dielectric cylinders or they can be in the air gap that is in the low dielectric region. So that is how you know you can actually say that the higher frequency band is called air band and lower frequency band is called dielectric band, okay.



So, this is band 1, so you can call this as dielectric band. These are the 3 points gamma X and M we are showing and this is band 2 for the TM mode, okay. This is also corresponding to gamma X and M points, okay. Now for modes at the gamma point, the field pattern is exactly same in each unit cell, you see there is no difference. The field pattern in each unit cell is exactly same.

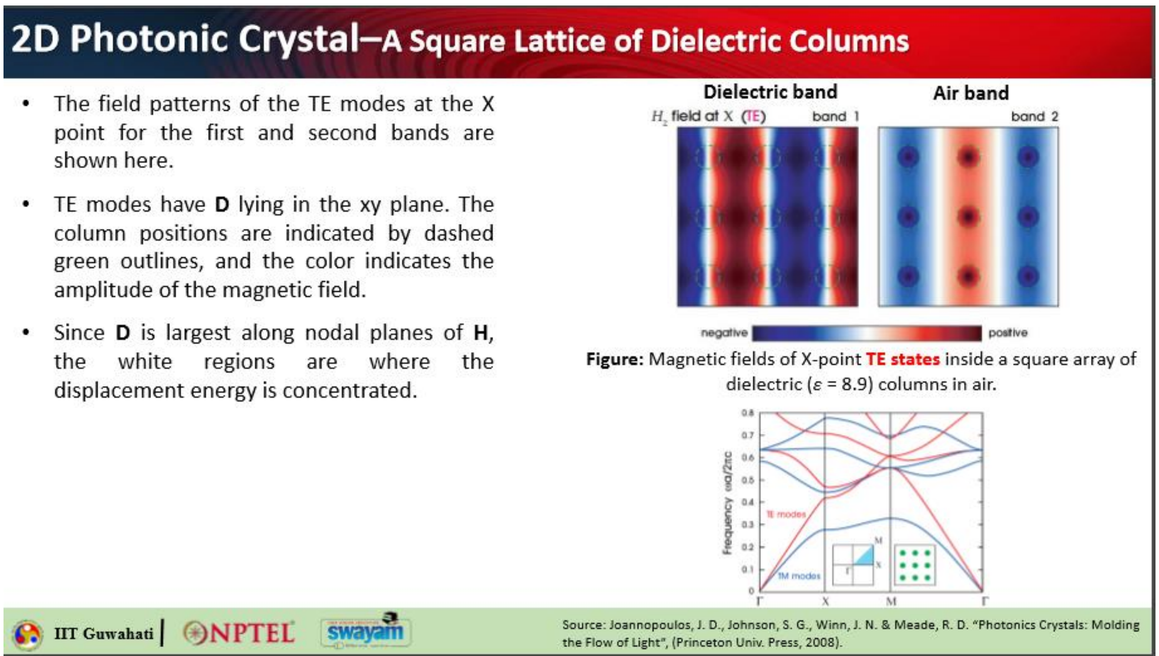
However, when you move to the X point, okay that is you have moved from the center to the band edge, okay or you can say zone edge. So, you can see that the fields are basically alternating in sign. So, this is the blue one is negative, this one is red, so it is positive, then again blue and the same thing happens here blue, red, blue, red and so on, okay. So, this is happening along you know for the unit cell along the direction of the wave vector K X. So, forming wave fronts which are parallel to the Y direction.

So you can actually see the wave fronts in the Y direction. So, this is these are basically this is X, this is Y and up which is outside the plane of this screen is basically the Z direction. So, you can actually see they are forming some wave fronts parallel to the Y direction, okay. And for the M mode, okay the signs of the field alternate in each unit cell along both X and Y direction.

So that is where it is a corner point basically. And the fields in the air band here you can see that they are basically a pair of degenerate states. So here you can see you can get degenerate states, degenerate states and so on. So, this is the difference between the displacement field pattern. So, as you can see this we are only plotting the Z component here dZ, okay for the TM modes, okay and at the three different important points we are able to see the patterns. Now although X and M patterns may look like wave fronts of a propagating wave but they are actually not wave fronts, okay.

In fact these modes at this particular K points they do not propagate at all. They are basically standing waves with zero velocity, okay. So that is something you should keep in mind. Now if you look into the field pattern of the TE modes which are the red ones, okay. So, the TE modes you will see let us look into the X point first.

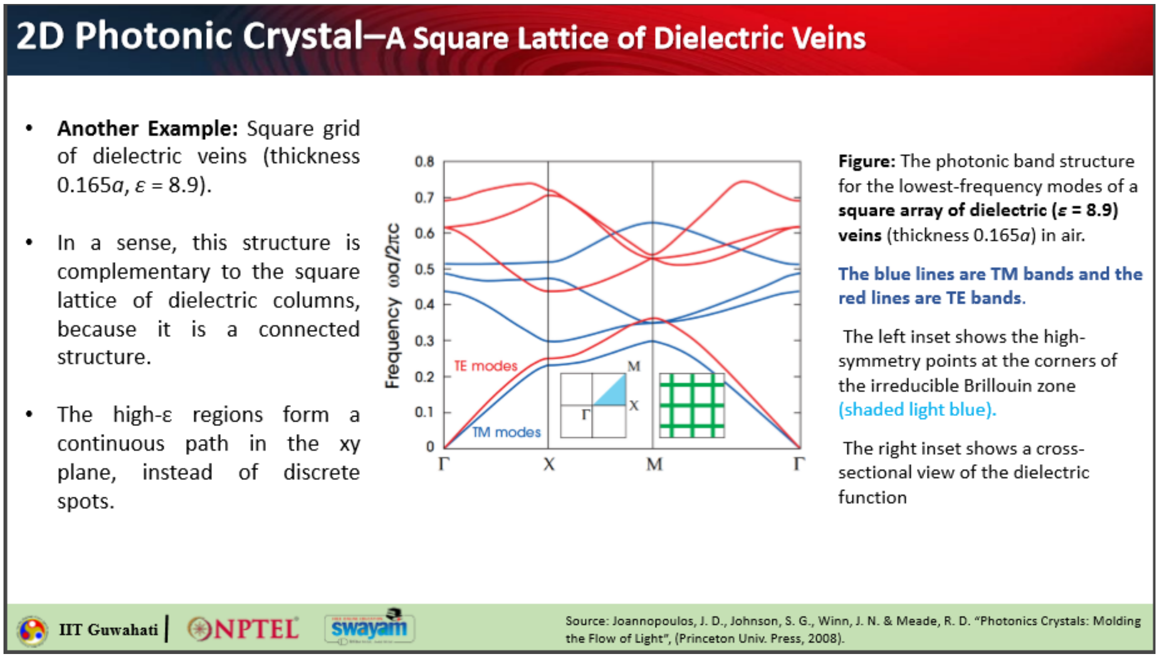
This is the X point and you see we are actually showing it for the dielectric one, this and this. So, this is dielectric band, the first one and this is the air band, okay. And here you can see that the TE modes are basically lying in the X-Y plane, okay. So, this is the top view. You can actually see these circles are basically your cylinders, right.



So, you are actually looking from the top. So TE modes are in the X-Y plane and the columns positions are indicated by this dashed green outlines and the color that you see red and blue they are basically the amplitude of the magnetic field strength. So here for TE mode we are plotting HZ whereas for TM modes we plotted DZ, okay. Now since D is largest along nodal planes of H the white regions are where the displacement energy is basically concentrated. So, this is where the displacement energies will be concentrated, fine. So, we understood that a square lattice of dielectric column can give us TM band gap but not TE band gap.

But we understood how the modes look like and what are the important points like gamma, X, M in those cases. Now let us look into another example which is a square lattice of dielectric vanes. So, let us show the figure. This is like a grid, okay.

You can see mesh grid or something like this. So, this is basically dielectric vane, okay. So here what is happening we have considered the thickness of the vanes to be 0.165a and the permittivity is taken to be 8.9. And once you do that, okay, you can actually plot the same dispersion relation with normalized frequency and K, okay.



And you see the blue lines are corresponding to TM modes, the red lines are corresponding to TE modes. So, what you see in this case that, yeah, so first of all this is again the irreducible Brillouin zone. So, if this is again a square lattice. So, for square lattice you will find a square Brillouin zone. From that you can find a triangular irreducible Brillouin zone which is marked in light blue, okay.

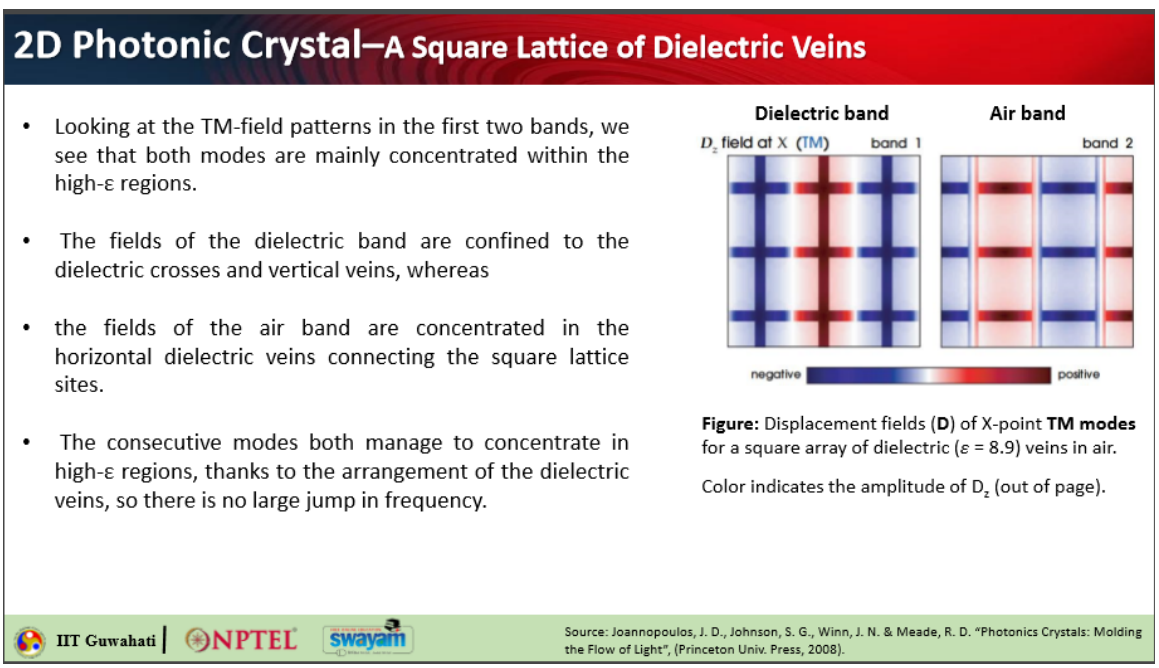
And then if you think of this structure, this structure is basically complementary to the square lattice of dielectric constant because this is a connected structure. In the previous case it was a disconnected structure. All the points dielectric rods were disconnected. Here all the, you know, dielectric materials are connected.

So this is like a complementary structure. So here the high permittivity regions they form a continuous path in the XY plane instead of the discrete spots. That was happening, the reverse happened in the previous structure. Now if you look into the band diagram here what you see TM modes, there is a band gap, yeah, and TE mode also here you are able to see a band gap. But the band gap is not at the same frequency for both TE and TM mode. So here you see one more time this is the blue line is the TM mode 1, band 1, this is band

2.

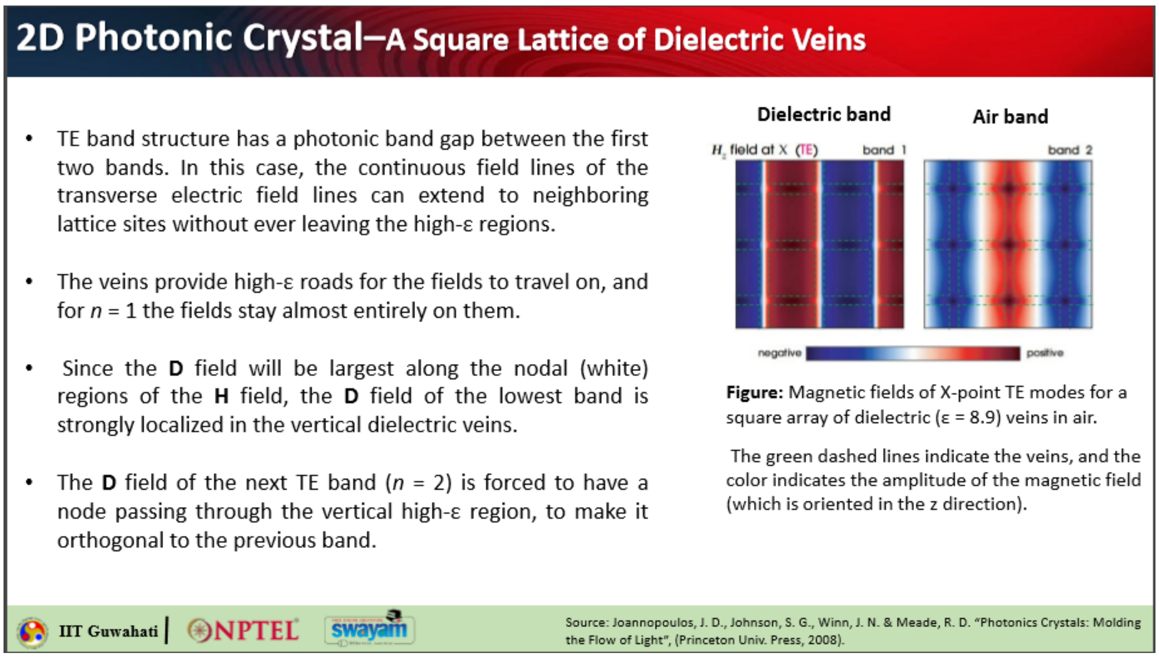
You see the band gap is around a very narrow band gap. More or less there is no, if you draw a line here you will be able to possibly see that there is no, yeah, there is no particular band gap, yeah. So, you can say that, you know, the TM band gap is not here but you are able to see, so I will actually remove this lines. But you are able to see if you look into the red ones you are able to find a good amount of band gap. You see here the red ones, yeah, this much and you can draw, okay, I am drawing it badly, yeah, this much is the band gap for TE modes. So, this particular structure can give you TE band gap and the previous structure could give you TM band gap, okay, keep this in mind.

So now let us do the, let us look into the electric field or displacement field pattern in this case. So, looking at the TM field patterns in the first two bands, so first band is called dielectric band, second band is called the air band. We see that both modes are mainly concentrated within the high permittivity region, that is within those vanes, okay, and that is what is clearly seen here, okay, and that makes sense also. And the field of the dielectric band are confined to the dielectric crosses and the vertical vanes, whereas the field in the air band here, they are concentrated mostly on the horizontal ones, not on the vertical ones, okay, only. So here you see along the vertical vanes you have less field concentration whereas the field concentration is very strong along the horizontal ones, okay.



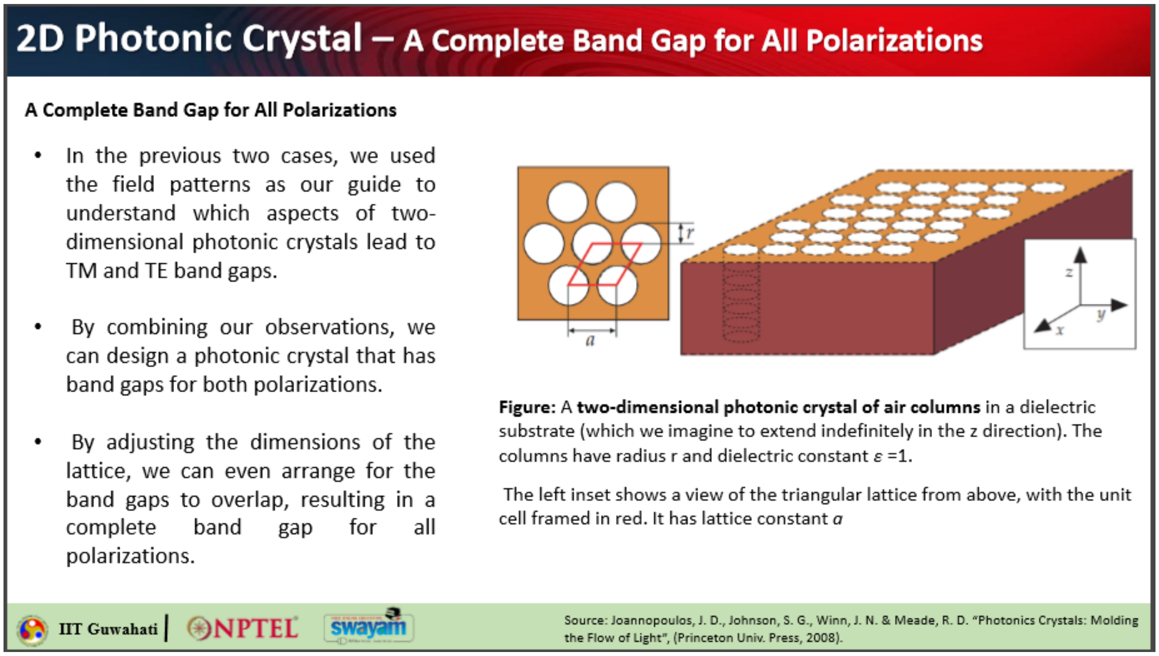
So, these are basically the vanes connecting the square lattice sites. The consecutive modes both manage to concentrate in high permittivity region thanks to the arrangement of the dielectric vanes. So, there is no large jump in frequency. So, they are all connected by these vanes. So, there is no discrete jump and that is why you do not see a band gap in this particular case of TM, the blue ones. But if you look into the T band structure, they have a photonic band gap between the first band and the second band, that is dielectric band and air band.

And in this case we will be plotting a jet like before. The continuous field lines of the transverse electric fields can extend to neighboring lattice sites without ever leaving the high permittivity region. So, you can see that from the high permittivity region they are able to extend to the next lattice site. The vanes provide high permittivity roads for the fields to travel and for n equals 1 the field almost stays entirely on them. So, this is what we are able to see here. And since D field will be the largest for the nodal cases that is the white region, the D field of the lowest band is strongly localized in the vertical dielectric vanes.



So that you can also see from here. And the D field in the next T band for this one is forced to have the node passing through the vertical high permittivity region. So, this will be the vertical high permittivity region to make orthogonal to the previous band. So, this is how you will get a band gap in these cases. So, I hope it is clear that this kind of structures allow you to have very good TM band gap sorry this one will have TE band gap but not TM band gap. But ideally what is required when you look for band gaps you basically want band gap a complete band gap for all polarizations.

Now a complete band gap means for both TE and TM you are able to overlap the band gaps. So irrespective of the polarization of the light, light should not be allowed to enter the crystal it should be completely reflected and that is what will be a complete band gap. Now in the previous two cases we have seen the field patterns as our guide to understand which aspect of the two dimensional photonic crystal leads to TM and TE band gaps. Now by combining our observation we are now in a position to design photonic crystals that can give us band gap for both polarizations.



So, this is one such structure okay. So, if you think of a two dimensional photonic crystal of air columns so you are basically having a hexagonal array or triangular array of air holes okay. So, in this particular system let us consider the radius of the holes to be r and they are having dielectric constant of 1 right. And this particularly shows you the triangular lattice or you can say hexagonal lattice okay. And this is the unit cell or you can take this one as a unit cell okay whichever way.

So you can call the lattice constant to be A. Now by adjusting the dimensions of this particular lattice we can arrange the two band gaps of TE and TM modes to overlap and that will give us complete overlap for all polarizations. So that is how it works. Now what are the ideas here? The idea is to put a triangular lattice of low permittivity columns inside a medium of high permittivity. So how does it help? So, you will see that this particular connectors of the material they work as the veins okay. So, if the radius of the column is large enough that the spot between the columns look like localized regions of high permittivity material which are connected through narrow squeeze between the columns to the adjacent spots.

2D Photonic Crystal–A Complete Band Gap for All Polarizations

- The idea is to put a triangular lattice of low-ε columns inside a medium with high ε.
- If the radius of the columns is large enough, the **spots** between columns look like localized regions of high-ε material, which are connected (through a narrow squeeze between columns) to adjacent spots.

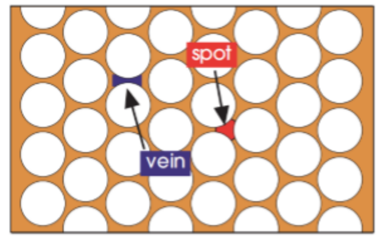


Figure: The spots and veins of a triangular lattice. Between the columns are narrow veins, connecting the spots surrounded by three columns.

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Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, you have to make sure these spots are very narrow okay. And how does it help? This will actually allow you to get the spots and veins of a triangular lattice. So once again between the columns are the narrow veins. So, between two columns you will see there is a narrow vein okay and connecting the spots surrounded by three columns okay. So, when you have three columns this one is called a spot. So, you have to kind of you know play with the size of the spot and vein to match your band gap of the two polarizations.

So here is an example. So, this is a particular high symmetry 2D photonic crystal you can say where you took a dielectric substrate of permittivity 13 and you have drilled a triangular array of air holes okay. And if you can see that this particular structure will have a hexagonal Brillouin zone and this particular triangular one marked with gamma K and M this is basically the irreducible Brillouin zone. So once again if you want to draw the dispersion relation you will have this normalized frequency on the Y axis and along the X axis you have the wave factor. So, what are the important points? You will start with gamma, gamma to M, M to K and back to gamma.

So gamma, M, K, gamma and so on. So, once you do the calculation you will see that you know you get band gap for TE and TM modes to overlap. So here you will see this is basically the TM band gap yeah. So, it is not from the first band from the second band to third band you have a band gap okay. Whereas for TE the red colors okay you see there is a band from here to here. So, the TE mode band gap is much wider whereas the TM mode band gap is not that wider because this is the valley and this is the peak.

2D Photonic Crystal—A Complete Band Gap for All Polarizations

- The band structure for this lattice, shown here, has photonic band gaps for both the TE and TM polarizations.
- In fact, for the particular radius r/a = 0.48 and dielectric constant ε = 13, these gaps overlap, and we obtain an 18.6% complete photonic band gap.

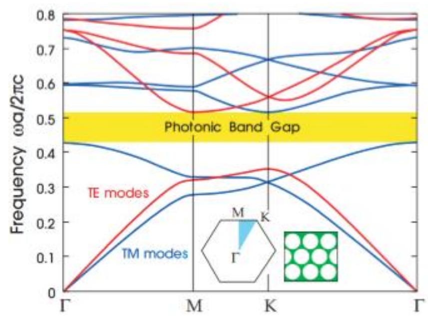

Figure: The photonic band structure for the modes of a triangular array of air columns drilled in a dielectric substrate (ε = 13).

The blue lines represent TM bands and the red lines represent TE bands.

The inset shows the high-symmetry points at the corners of the irreducible Brillouin zone (shaded light blue).

Note the complete photonic band gap.

Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, this is the minimum one. So, if you take the minimum of both TE and TM band gap so this is the photonic band gap for both polarization that you can get from this particular crystal. So, these are the values which are considered here r/a was taken as 0.48, the material substrate material was taken to be 13 okay and it gave you 18.6% overall you know band gap or you can say complete photonic band gap.

2D Photonic Crystal—Localization of Light by Point Defects

- we can remove a single column from the crystal, or replace it with another whose size, shape, or dielectric constant is different than the original.
- Perturbing just one site ruins the translational symmetry of the lattice
- Perturbing one column in the bulk of the crystal (yellow) might allow a defect state to be localized in both x and y.
- Perturbing one row in the bulk of the crystal (red) or truncating the crystal at a surface (green) might allow a state to be localized in one direction (x).

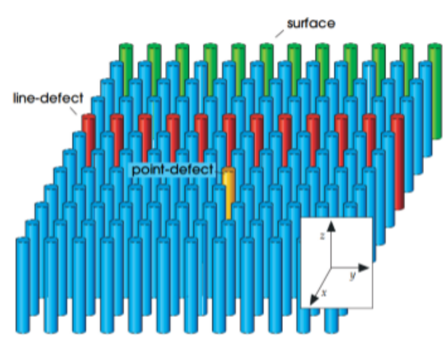

Figure: Schematic illustration of possible sites of point, line, and surface defects.

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Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

Now let us look into localization of light by point defects in a photonic crystal. So, I hope it is understood that this is the concept of complete band gap where light from any polarization any direction will not be allowed to enter the crystal. If you remove a single column from the crystal or replace it by another size shape or dielectric constant then the original that is how you will be able to ah perturb that particular site and that will ruin the translational symmetry of the lattice. And perturbing one column in the bulk of the crystal which is like this one this is called point defect whereas if you perturb the entire line that is called line defect okay. So as mentioned here the perturbing one row in the bulk of the crystal or truncating the crystal at a surface so here that is also possible you truncate it so these are basically line defects whereas only one point if you change the material or change the size of the hole or the array or the in this case it is a dielectric rod. So, if you change the rod material or the dimension that will introduce a point defect in your crystal.

2D Photonic Crystal–Localization of Light by Point Defects

- Perturbing a single lattice site causes a defect along a line in the z direction.
- But because we are considering propagation only in the plane of periodicity, and the perturbation is localized to a particular point in that plane, we refer to this perturbation as a **point defect.**
- Removing one column may introduce a peak into the crystal's density of states within the photonic band gap. If this happens, then the defect-induced state must be evanescent.

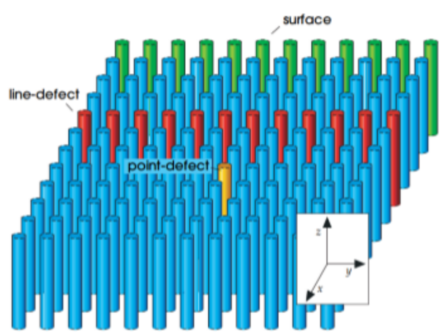

Figure: Schematic illustration of possible sites of point, line, and surface defects.

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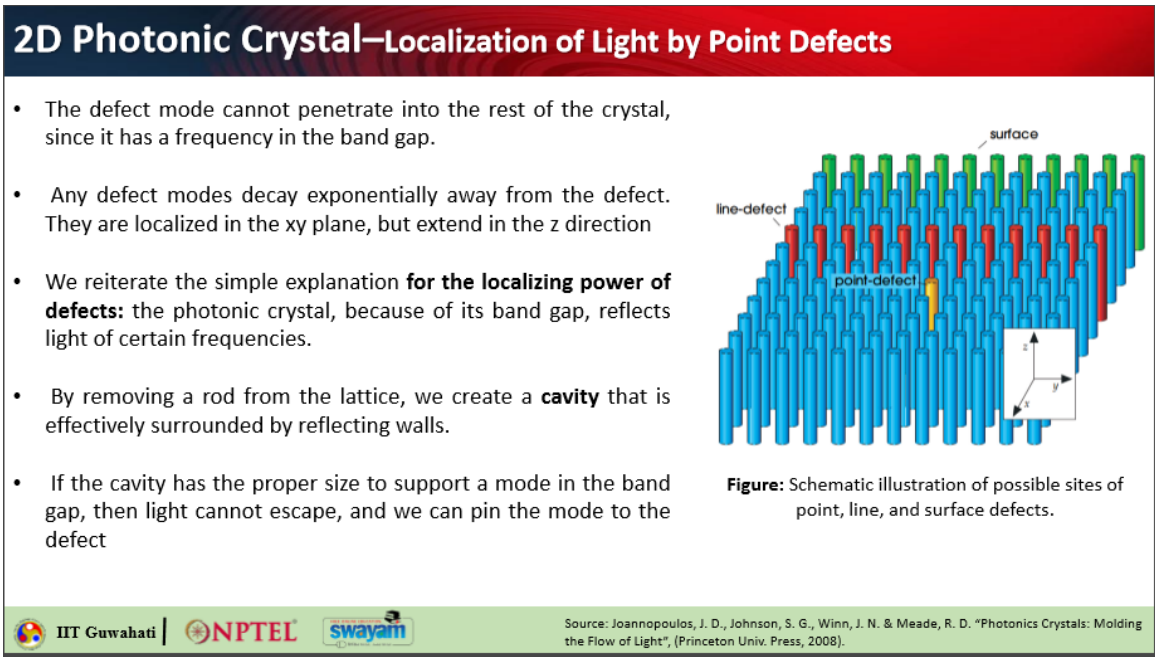
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

Now why these things are important because perturbing a single lattice site causes a defect along the line okay in the z direction. But because we are considering propagation only in the plane of periodicity the perturbation is localized at a particular point in the plane here okay and that is why this perturbation is called as point defect. Now removing one column may introduce a peak into the crystals density of states within the photonic band gap. If this happens then the defect induced state must be evanescent. So, the defect mode cannot penetrate into the rest of the crystal since it has got a frequency in the band gap.

So you can actually make it like a cavity so if you are able to have some kind of modes kept there that will not be able to leak into any of this surrounding crystal. So, any defect mode decay exponentially away from this defect they are localized in the XY plane but they are allowed to extend in the z direction only okay. So, we will reiterate the simple explanation for localizing power of the defects. The photonic crystal because of the band gap can reflect light from certain frequency and if you use point defect you are able to localize you know power at a particular point inside the photonic crystal okay. So, if you just think of this particular crystal if you remove this rod from the lattice you are able to create a cavity okay and this cavity is now surrounded by all reflecting walls because whatever is the frequency that you want to of the mode that you want to be stored here okay that is not allowed to enter this crystal.



So, it is like a reflecting wall all around okay. So, if the cavity has proper size to support a mode in the band gap the light cannot escape and we can pin the mode to that particular defect. So, it means we can use this point defects in photonic crystal to trap light this is light trapping okay and if you if you introduce line defect using line defect you are also allowed to guide light like this light will be allowed to propagate only along this particular defect but then this frequency of light is not supported in the band gap it means you have to choose those frequencies carefully so that they are in the band gap of that photonic crystal. So, light cannot leak into the photonic crystal so it will be guided. So whatever shape you make for your wave guide by introducing the line defect here it is like a line you can make it any curved shape okay light will actually follow that particular shape because it has got no option it cannot leak out into the photonic crystal because the frequency of that mode lies within the band gap of that crystal okay. So, the basic idea is to curve a waveguide out of an otherwise perfect photonic crystal by modifying a linear sequence of unit cell as I have just discussed.

2D Photonic Crystal–Linear Defects and Waveguides

- We can use point defects in photonic crystals to trap light.
- By using linear defects, we can also guide light from one location to another.
- The basic idea is to carve a waveguide out of an otherwise perfect photonic crystal by modifying a linear sequence of unit cells
- Light that propagates in the waveguide with a frequency within the band gap of the crystal is confined to the defect, and can be directed along the defect.
- An example is illustrated in figure, in which a column of rods has been removed along the *y* direction from the square-lattice crystal.

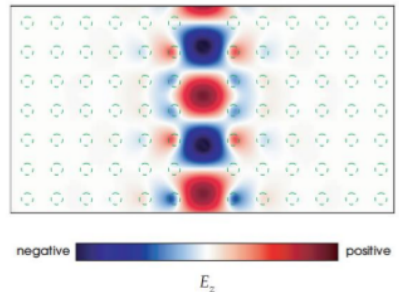


Figure: Electric-field (E_z) pattern associated with a linear defect formed by removing a column of rods from an otherwise-perfect square lattice of rods in air.

The resulting field, shown here for a wave vector $k_y = 0.3 (2\pi/a)$ along the defect, is a **waveguide mode** propagating along the defect. Rods shown as dashed green outlines.

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Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, light that propagates in the waveguide with the frequency within the band gap of the crystal is confined to the defect and it can be directed along the defect so you can actually make any kind of waveguide bands using this particular concept okay. Now with that we somewhat concluded the discussion on 2D photonic crystals now let us move on to the 3D photonic crystals. So here the periodic alteration of the dielectric happens in all three dimension okay. So, the optical analog to an ordinary crystal is basically a three-dimensional photonic crystal right.

Three-Dimensional (3D) Photonic Crystals – Introduction

- **The optical analogue of an ordinary crystal is a three-dimensional photonic crystal**: a dielectric structure that is periodic along three different axes.
- The band structure for a **lattice of air spheres** within a dielectric medium is shown here.
- To maximize the size of the band gap, the sphere radius *r* is chosen to be 0.325*a*, where *a* is the lattice constant of the cubic supercell.
- Between the second and third bands, there is a complete gap (shown in YELLOW)

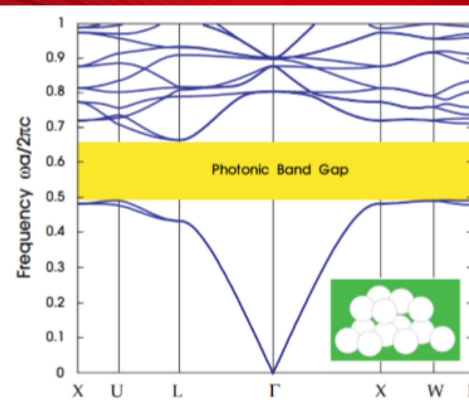

Figure: The photonic band structure for the lowest bands of a **diamond lattice of air spheres** in a high dielectric ($\varepsilon = 13$) material (inset).

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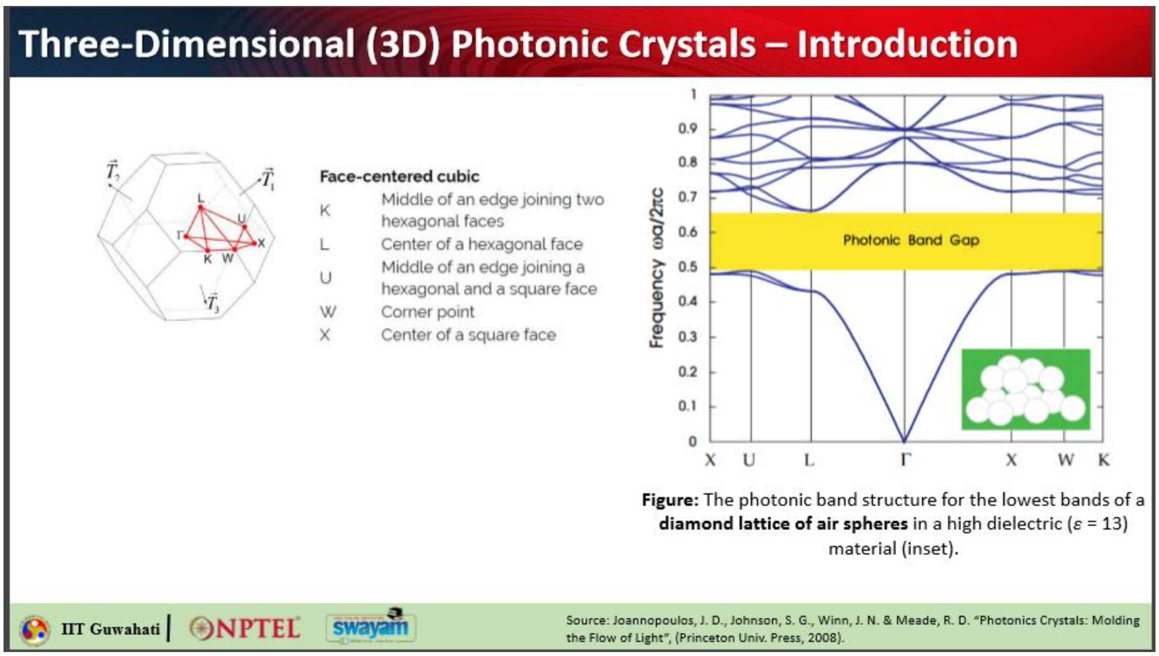
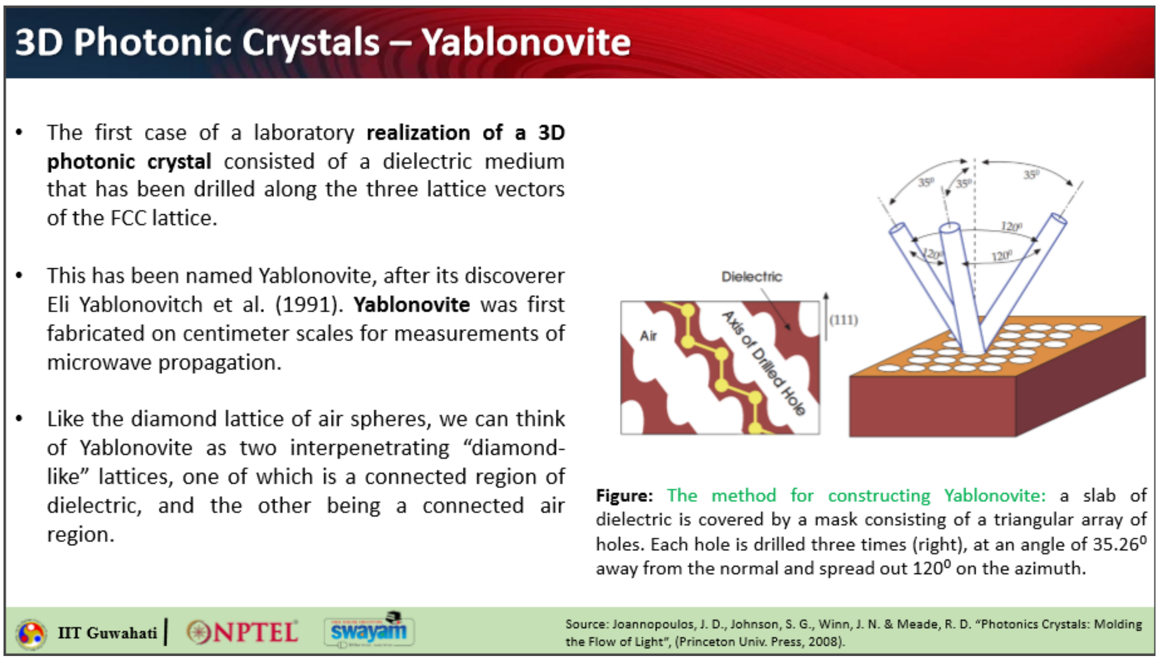
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, you can actually think of a lattice of air spheres within a dielectric medium okay. So, if you make a lattice of air sphere means holes so if you have a hole in a 3D object in a periodic pattern so this will be the band gap that looks like. So, what was the material? The material was having permittivity of 13 and then you have prepared a 3D photonic crystal by introducing a diamond lattice of air spheres in this material and you will see this is how beautiful big photonic band gap you are able to obtain. Now to maximize the size of the band gap the sphere radius r was chosen to be 0.325a where a is basically the lattice constant. So how this happened you have to optimize your design and see in which case you get the largest band gap okay.

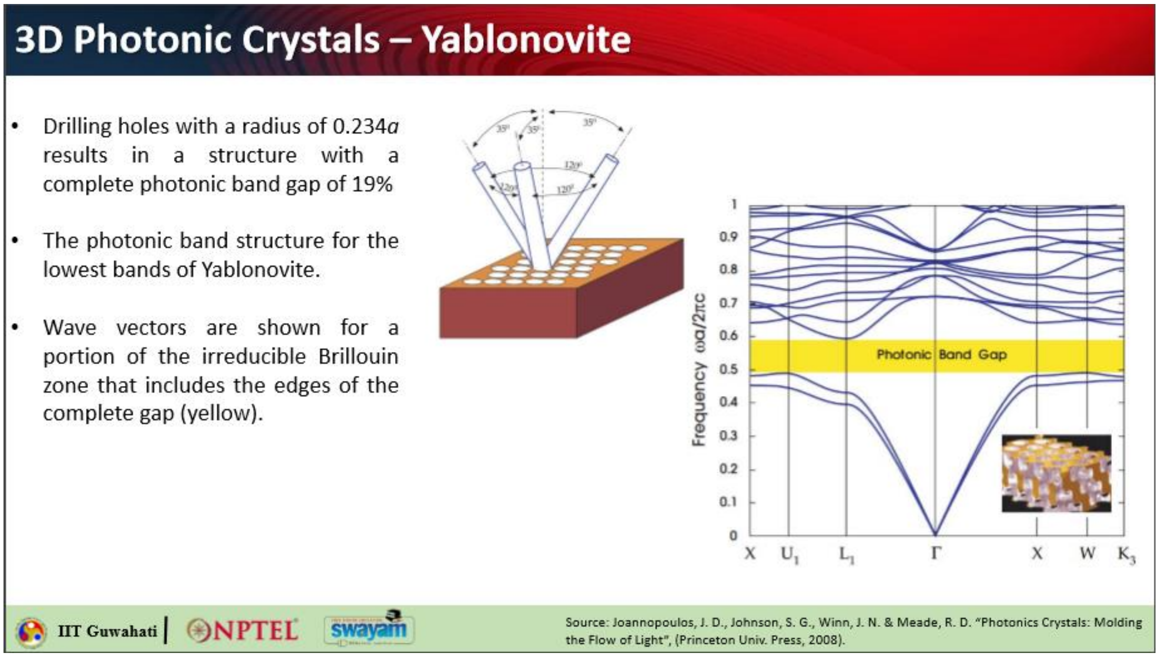


So, and one more thing you can see this band gap is between 1, 2 no it is between 2 and 3 okay. So, this is how you know sometimes you may not have a band gap between the first two bands but you will be able to get the bands between band gap between band 2 and 3 okay. Now as I mentioned this is the FCC lattice so you can actually see this is the Brillouin zone which we discussed in the previous lecture and this area is the irreducible Brillouin zone. So here the important points are basically K, L, U, W and X. So, if you start from X okay and you go to U then you go to L then you go to gamma then back to X, W, K you kind of traverse all the points. So that is the whole idea of traversing through all the points of the irreducible Brillouin zone so that will give you the band diagram okay.

Now let me show you the first ever laboratory realization of a 3D photonic crystal. So, it was basically a dielectric media that has been drilled along the three lattice vectors of the FCC lattice. So, this is how different way it was drilled. So, you have to understand that it has to be material air not only in one direction along X, Y and Z okay. So, this was done for FCC lattice and this was the first demonstration done by Yablonovite in 1991 after he discovered that photonic band gap was indeed possible and this particular crystal was named after him Yablonovite, and he did the experiments in microwave domain to show that there is band gap possible okay. So, like the diamond lattice of air spheres we can think of Yablonovite as two interpenetrating diamond like lattices one of which is connected region of dielectric and another one is a connected air region and those two regions are mixed and that is how you are able to get this particular crystal.

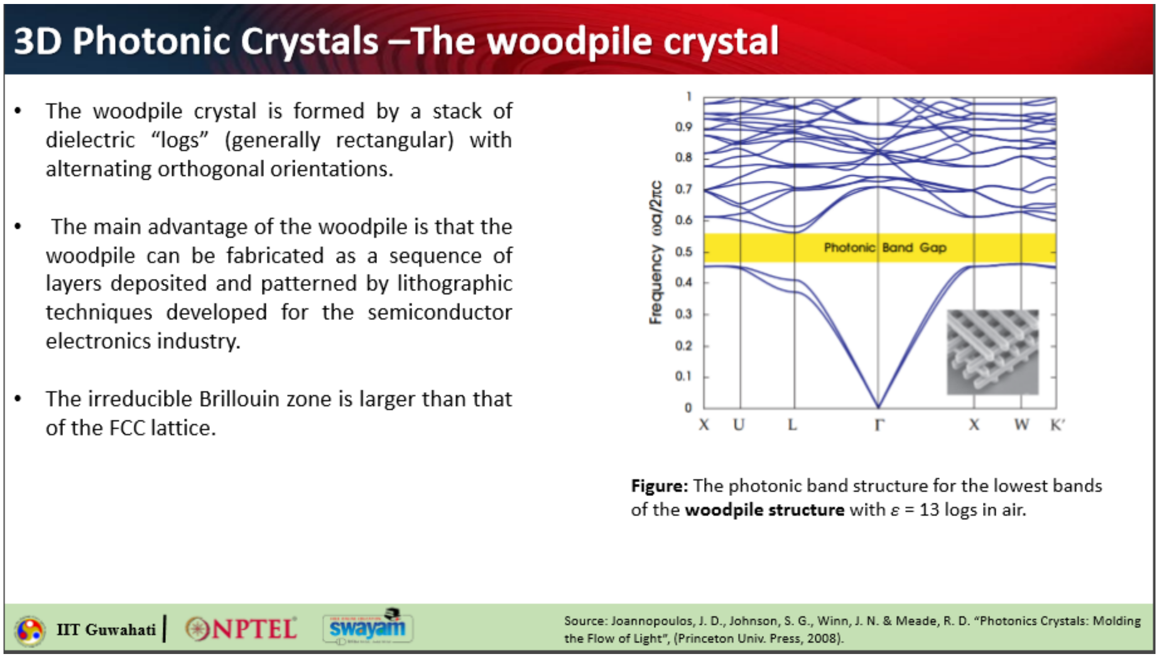


So, this is how it was the drilling was very complicated to make this kind of a crystal. So, this is the method for constructing Yablonovite. A slab of dielectric is covered by a mask okay consisting of triangular array of holes. Each hole is drilled three times okay right at the angle of 35.26 degree away from the normal and spread out at 120 degree on the azimuth and that is how the drilling was done and you will be able to get FCC face centered cubic kind of crystal arrangement and that is repeating everywhere. So, it is not very simple to imagine it is a very pretty complicated structure but this gave a very very good band gap as you can see over here right.

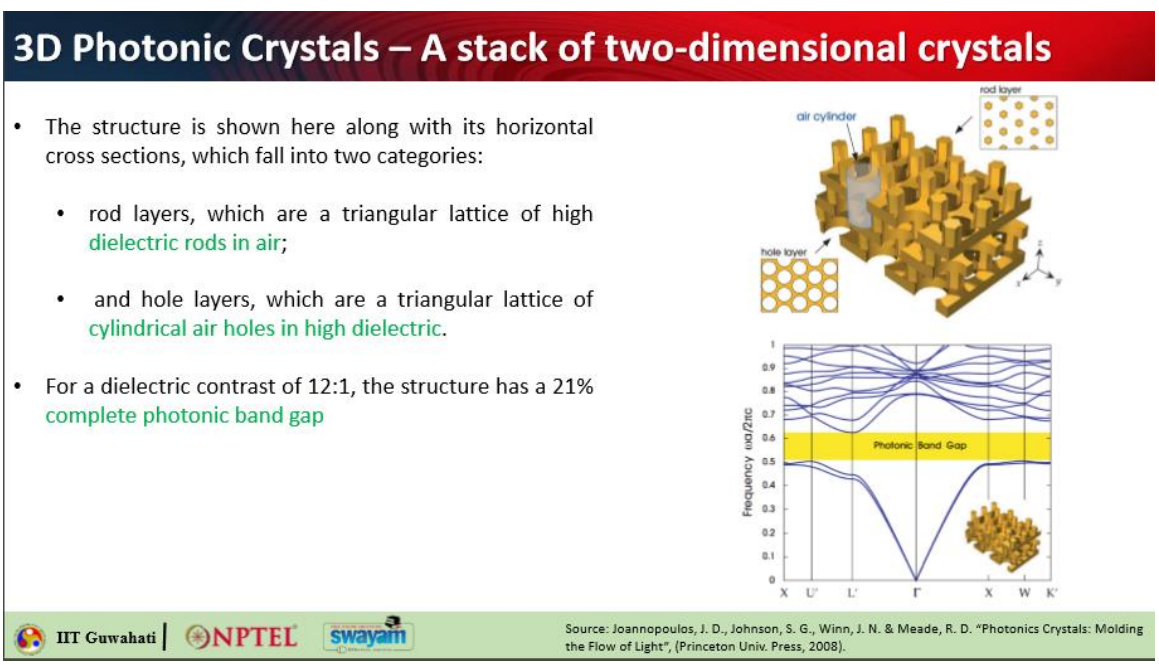


So, drilling holes so what was the size the radius was taken as 0.234a and that could give you a photonic band gap of around 19% okay. Now the photonic band structure for the lowest bands of Yablonovite okay and the wave vectors are shown here they are basically for the portion of the irreducible brilliant zone right. So, we have already discussed this okay. So other structure was woodpile structure this is structure which is slightly easier to make this is like you know you have logs in this and then this and then this.

So, this is how you know woodpile because this is how wooden logs are piled okay they are kept. So, this kind of structure as you can see so that is far less complicated than the Evlonovite structure. So here the logs are made of the dielectric material with permittivity 13 and they are kept in air. So, when you do that you actually get this kind of a photonic band gap. Now what is the main advantage of woodpile is that woodpile can be fabricated as a sequence of layers deposited and patterned by lithographic techniques which are developed for semiconductor electronics industry. So that way this kind of fabrication of this kind of photonic band gap crystals are much easier and they can be commercially done okay.



The only thing is the irreducible brilliant zone is larger than that of the FCC lattice. So, we have to now look for a wider band here not band like directions okay because the irreducible brilliant zone size is larger. So, in that case what will happen you will require more time to compute all these points and come up with this band diagram okay. And this is the other one this is basically a stack of two dimensional crystals. So, if you take two-dimension crystal one is this kind of rod layer so you just have layer of rods okay which are placed in a triangular lattice okay.



So, this rods are made of high dielectric and they are in air so this is one and another one is a whole layer like this. So, it is a triangular lattice of cylindrical holes in a high dielectric substrate. So, you are just drilling holes in a triangular lattice or hexagonal array whichever way you want to call it and then you mix them together you blend them together. So, once you do that okay you will for a dielectric contrast of 12:1 okay so if you take the material in this case the dielectric rod material permittivity to be 12 and surrounding is air in this case the holes are air permittivity is 1 and the other material is 12.

So if you take that case you are able to get 21% complete band gap. So, this is also another structure that is possible. So once again complete band gap means for both TE and TM polarization you should get the same band gap okay. So, with that we will stop the discussion about 2D and 3D photonic crystals in the next lecture we will look into some emerging applications of photonic crystals and if you have got any query regarding this lecture you can drop me an email to this particular email address mentioning MOOC on the subject line. Thank you.