Course Name- Nanophotonics, Plasmonics and Metamaterials

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Week-05

Lecture -15

Hello students, welcome to lecture 15 of the online course on Nanophotonics, Plasmonics and Metamaterials. In today's lecture we will cover the emerging applications of photonic crystals. So, here is the lecture outline we will start with application of 1D photonic crystal, we will discuss Bragg grating briefly. Then we will move on to periodic dielectric waveguides, we will also look into some applications of 2D photonic crystals like photonic crystal slabs, photonic crystal fibers, we will also look into index guiding photonic crystal fibers and endlessly single mode fibers. So, let us look into the application of 1D photonic crystal as I mentioned, we will study Bragg grating. So, Bragg grating is nothing, but you know it is an set of uniformly spaced parallel partially refractive planar mirrors.

Lecture Outline

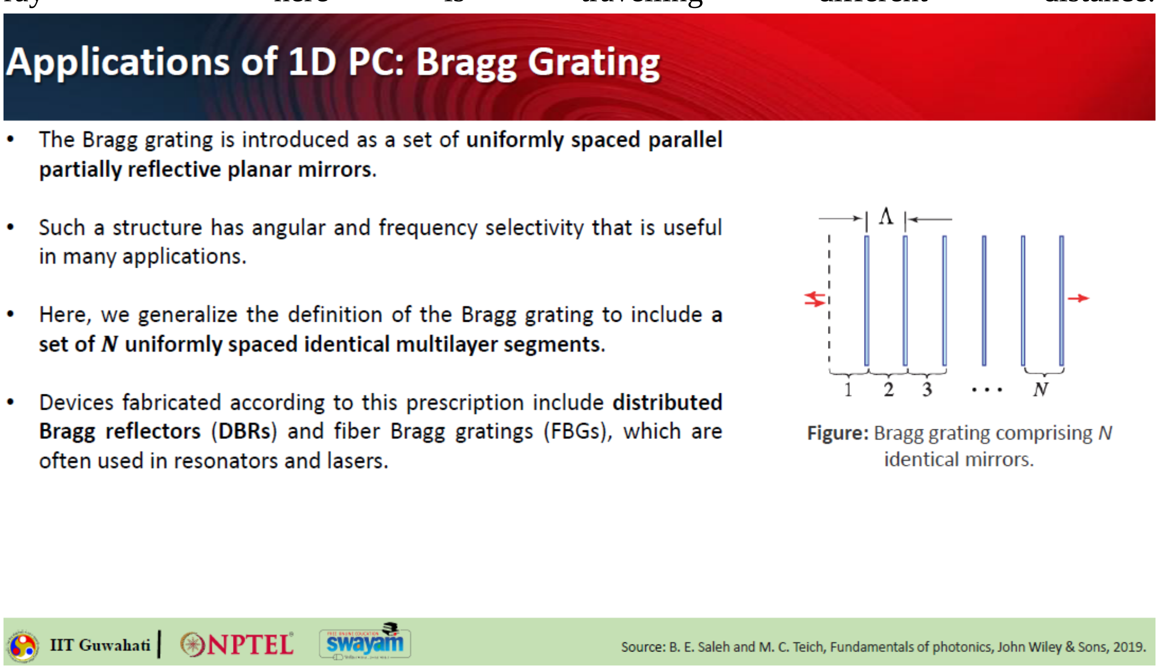
- Applications of 1D PC: Bragg Grating
- Periodic Dielectric Waveguides
- Applications of 2D PC: Photonic Crystal Slabs
- Photonic Crystal Fibers
- Index-Guiding Photonic-Crystal Fibers
- Endlessly single-mode fibers

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, you can see here. So, this is basically a Bragg grating. So, here we are showing that there are N identical mirrors. So, what happens in this grating when some incident light falls, some particular wavelength of frequency gets reflected remaining gets transmitted.

Now such a structure has angular and frequency selectivity and that is useful for many applications such as filter and then you know pulse compensation in optical fiber communication and so on. So, here we will generalize the definition of Bragg grating to include a set of N uniformly spaced identical multilayer segments. So, the devices which are fabricated using this particular grating are distributed Bragg reflectors (DBRs) and fiber Bragg grating which is FBGs in short and they are often used in resonators and lasers. FBGs are also very popular for sensing application because depending on the surrounding media, the refractive index of the surrounding media can change and that can change the wavelength of the reflected light and that can be used as a sensing mechanism. Now, let us look into Bragg reflection in details.

So, consider light reflected at a angle theta. So, here we are calculating the angle from the plane of the mirrors and here at the M parallel reflecting planes which are separated by a distance of capital lambda. So, what happens this light little bit of this will get reflected and remaining will get transmitted. So, we assume that only small fraction of the light is basically reflected from each plane and the amplitude of each of these reflected lights are assumed to be almost equal. Now, the reflected lights will have a phase difference because different light is actually travelling different distance, different ray here is travelling different distance.



So, you can calculate the phase difference phi small phi as k times 2 capital lambda and sine theta. So, k sine theta is basically the component of the wave vector into this particular multilayer structure. So, the phase difference will be k 2 capital lambda, why capital 2 capital lambda it has to travel this part twice ok. And the angle at which the intensity of the reflected light is maximum that is known as the Bragg angle. So,

$$\sin\theta_B = \frac{\lambda}{2\Lambda}$$

.

What is capital lambda? The spacing between the mirrors small lambda is that particular wavelength of light. Now, such reflections are encountered when light is reflected from a multilayer structure. And here I mention remember that theta is basically defined with respect to the parallel planes. Now, the reflection or reflectance of Bragg grating is basically determined under two assumptions. What are those? The first one is the mirrors are weakly reflective so that the incident light is not depleted when it propagates.

Bragg Reflection

- Consider light reflected at an angle θ from $\boldsymbol{M}$ **parallel reflecting planes separated by a distance $\boldsymbol{\Lambda}$.**
- Assume that only a small fraction of the light is reflected from each plane, so that the **amplitudes of the $\boldsymbol{M}$ reflected waves are approximately equal.**
- The reflected waves have a phase difference $\varphi = k(2\Lambda sin\theta)$ and that the angle θ at which the intensity of the **total reflected light is maximum satisfies:**

$$\sin\theta_{\mathcal{B}} = \frac{\lambda}{2\Lambda} \quad \textbf{Bragg Angle} \ (L15.1)$$

- Such reflections are encountered when light is reflected from a multilayer structure.
- **θ is defined with respect to the parallel planes.**

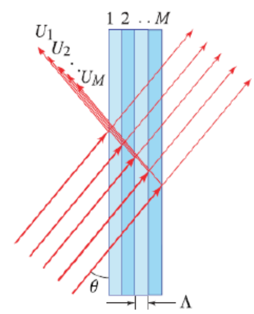


Figure: Reflections of a plane wave from M parallel planes.

IIT Guwahati | NPTEL swayam   Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, the reflection amplitude is basically very low. And second thing is that the secondary reflection that means, reflection of the reflected waves those are considered negligible because the first reflected wave itself is weak. If you take these two approximation in mind you can find out the reflectance of the N mirror grating which can be related to the reflectance R of the single mirror by this particular equation. So, RN, N corresponds to the N mirror grating here will be $\frac{\sin^2 N\varphi}{\sin^2 \varphi}\mathcal{R}$. And here phi denotes the phase between the successive phasors whereas, the phase is basically defined by 2φ and that is corresponding to a round phase or round trip phase ok.

Bragg Grating — A Simplified Theory

- The reflectance of the Bragg grating is determined under two assumptions:
    1. **The mirrors are weakly reflective so that the incident wave is not depleted as it propagates.**
    2. **Secondary reflections (*i.e.*, reflections of the reflected waves) are negligible.**
- In this approximation, the **reflectance $\mathcal{R}_N$** of an N-mirror grating is related to the **reflectance $\mathcal{R}$** of a single mirror by the equation:

$$\mathcal{R}_N = \frac{\sin^2 N\varphi}{\sin^2\varphi}\mathcal{R} \qquad \text{(L15.2)}$$

- The quantity φ denotes the phase between the successive phasors whereas here the phase is denoted by $\boldsymbol{2\varphi}$ since it represents **a round trip**.

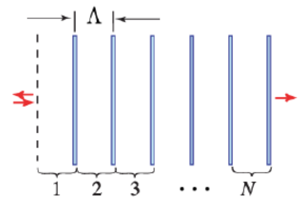


Figure: Bragg grating comprising N identical mirrors.

You can call it also round trip phase that is why it is 2φ. Now, this particular factor that you have seen here $\frac{\sin^2 N\varphi}{\sin^2 \varphi}$. represents the intensity of the sum of the N phasors of unit amplitude and phase difference 2φ ok. Now, the function can obtain a peak value which is N square when the break condition is basically satisfied. That means, you have to look for condition where 2 phi is equal to q times 2 phi. What is q? q is like integer 0, 1, 2, 3 and so on.

Bragg Grating — A Simplified Theory

- The factor $sin^2N\varphi/sin^2\varphi$ represents the intensity of the sum of N phasors of unit amplitude and phase difference 2φ.
- This function has **a peak value of $\boldsymbol{N^2}$** when the Bragg condition is satisfied, *i.e.*, **when $\boldsymbol{2\varphi}$ equals $\boldsymbol{q2\pi}$, where $\boldsymbol{q = 0, 1, 2, \dots}$.**
- It drops away from these values sharply, with a width that is inversely proportional to N. In this simplified model, the intensity of the total reflected wave is, at most, a factor of N^2 greater than the intensity of the wave reflected from a single segment.
- For a Bragg grating comprising partially reflective mirrors separated from each other by a distance Λ and a round-trip phase $2\varphi = 2k\Lambda\cos\theta$, where θ is the angle of incidence. Therefore, maximum reflection occurs when:

$$2k\Lambda\cos\theta = 2q\pi \quad \text{or} \quad \cos\theta = q\frac{\lambda}{2\Lambda} = q\frac{\omega_{\mathcal{B}}}{\omega} = q\frac{\nu_{\mathcal{B}}}{\nu} \qquad \text{(L15.3)}$$

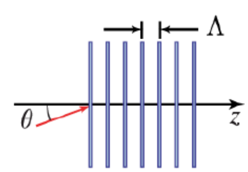


Figure: Bragg grating comprising N identical mirrors.

Now, the reflectance will also drop away drop away from these values sharply. So, there will be very sharp cut off with a width that is inversely proportional to N ok. So, larger the N sharper will be the cut off. So, that is why larger the Bragg reflectance you will get a higher order filter you can say ok. And in this simplified model you can actually see that the intensity of the total reflected wave is at most the factor of N square greater than the intensity of the reflected wave of from the single mirror or single segment.

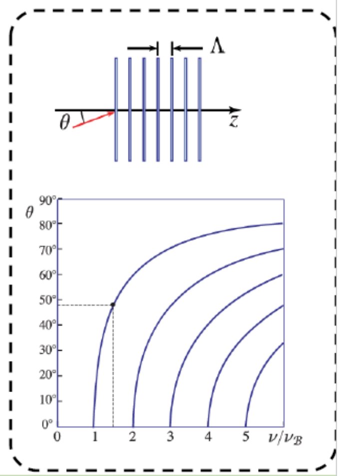
Now, for a break grating comprising partially reflective mirrors which are separated from each other by a distance of capital lambda. And we have seen that the round trip phase 2 small phi is basically $2k\Lambda\cos\theta$ and theta is considered to be the angle of incidence. So, here you see this theta is not considered from here it is basically this particular theta ok. So, in that case the equation will have cos theta ok. So, you can write $2k\Lambda\cos\theta = 2q\pi$.

This is nothing, but you know the phase round trip phase should be equal to integral multiple of 2φ ok. And from that if you write 2 pi by k equals small lambda you can actually get this in terms of small lambda and capital lambda. So, you can write $\cos\theta = q\frac{\lambda}{2\Lambda}$. This can also be written as omega B over omega that is the Bragg frequency over the normal frequency ok. Or you can this is the angular frequency you can also write them in terms of linear frequency ok.

Bragg Grating — A Simplified Theory

- **Bragg Condition:** $$\cos\theta = q\frac{\lambda}{2\Lambda} = q\frac{\omega_B}{\omega} = q\frac{\nu_B}{\nu} \quad \text{(L15.3)}$$

  where $$\nu_B = \frac{c}{2\Lambda}, \quad \omega_B = \frac{\pi c}{\Lambda}$$ Bragg Frequency (L15.4)



- At normal incidence ($\theta = 0°$), peak reflectance occurs at frequencies that are integer multiples of the Bragg frequency, i.e., $\nu_B = q\nu_B$.
- At frequencies such that $\nu < \nu_B$, the Bragg condition cannot be satisfied at any angle.
- At frequencies $\nu < \nu_B < 2\nu_B$, the Bragg condition is satisfied at one angle $\theta = cos^{-1}(\lambda/2\Lambda) = cos^{-1}(\nu_B/\nu)$.

Figure: Locus of frequencies ν and angles θ at which the Bragg condition is satisfied. For example, if $\nu = 1.5\nu_B$ (dot-dash line), we have $\theta = 48.2°$. This corresponds to a Bragg angle $\theta_B = (90 - \theta) = 41.8°$ (when measured from the plane of the grating.)

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Now, how we obtain this let us have a quick look. So, this is what you have seen this is called the Bragg condition ok. This particular condition of theta on the incident angle is basically the basically the Bragg condition. And here we have encountered 2 new terms which is omega b that is basically $\frac{\pi c}{\Lambda}$ and $\nu_B = \frac{c}{2\Lambda}$. So, what is the relationship between this and this? This is linear frequency and this is angular frequency ok.

So, these are both Bragg frequencies. So, what you see here that at normal incidence the peak reflectance occurs at frequencies that are basically integral multiple of the Bragg frequency that is you have to look for frequencies which are nu equals this is not nu B this will be nu equals q nu B ok. So, this is just nu ok. And at frequencies where nu is less than nu B means the Bragg condition is not getting satisfied at any angle. And for frequencies like which is less than this one nu B ok the Bragg condition is satisfied ok I think there is also one typo here it will be nu is here nu B should be here ok there is a typo again.

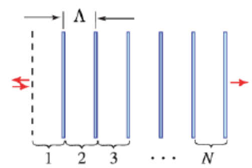
So, it is like you know the frequency between nu B and 2 nu B you can see that the Bragg condition is basically getting satisfied at one angle and that angle is basically theta equals cos inverse of lambda over 2 capital lambda ok or you can write it as cos inverse of nu B over nu. So, you can actually see those calculations here. So, here is a plot that shows the locus of frequencies. So, this is the frequency and the angle at which Bragg condition is satisfied. Now if you take nu and nu B the ratio to be 1.

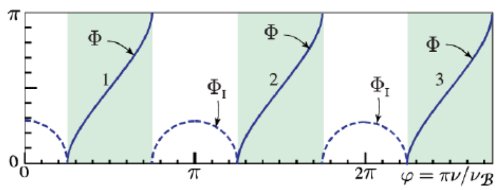
5. So, if you draw this dotted line you will see that you are getting theta equals 42.8 degree. So, in that case how do you get the Bragg angle? Bragg angle is basically this angle that will be 90 minus this which is 41.8 degree. So, with that we understand that how the Bragg grating works.

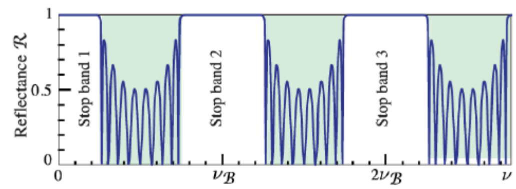
Now if you take an example specific example that N equals 10. So, you are considering only 10 identical mirrors and we have also restricted the performance of the mirrors like power reflectance modulus of small r square is taken as 0.5. And in that case ok, we will see how the performance of this mirror looks like. Now dependence of capital phi ok, on the inter mirror phase delay small phi is this one right.

Stack of Partially Reflected Mirrors

- Bragg grating comprising $N = 10$ identical mirrors, **each with a power reflectance** $|r|^2 = 0.5$.
- Dependence of Φ on the inter-mirror phase delay $\varphi = nk_o\Lambda$.
- **Within the shaded regions, Φ is complex and its imaginary part Φ_I is represented by the dashed curves.**
- Reflectance $\mathcal{R}$ as a function of frequency (in units of the Bragg frequency $\nu_B = c/2\Lambda$). **Within the stop bands, the reflectance is approximately unity.**







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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

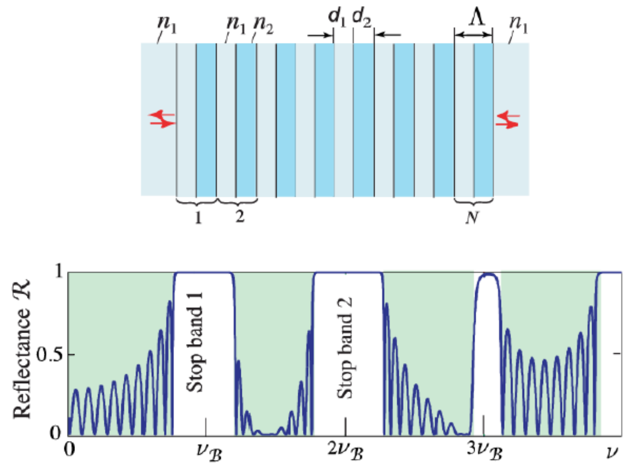
What is small phi? That is a phase delay $nk_o\Lambda$ that is like from one mirror to another what is the phase delay. This is not the round trip phase delay this is the inter mirror phase delay ok. Now if you plot this ok, you will see that within the shaded region this capital phi is basically complex and the imaginary part are actually shown here by dashed curves. So, what is understood here is that this is also periodically repeating.

Now if you try to plot reflectance R based on the previous formula that you have discussed as a function of frequency, you can actually convert them into the Bragg frequency which is nu b given by c over 2 capital lambda ok. We will see that you are getting clear stop band. Stop band means you are getting reflectance completely 1 ok and then suddenly drops and then again it goes back. So, at every nu B integral multiple of nu B ok and at 0 you will get this stop band ok. So, that that is how you can actually use Bragg grating as a particular band stop filter and you can actually make it reflect a particular wavelength.

Now if you take specific example of creating a Bragg grating like this by alternating two different dielectric material like d n 1 and n 2. So, here n 1 is 1.5 n 2 is 3.5 and d 1 and d 2 are taken to be equal and you have 10 such segments.

Dielectric Bragg Grating

- Power reflectance ($\mathcal{R}$) as a function of frequency for a dielectric Bragg grating comprising $N = 10$ segments, each of which has two layers of thickness $d_1 = d_2$ and refractive indices $n_1 = 1.5$ and $n_2 = 3.5$.
- The grating is placed in a medium with matching refractive index n_1.
- ***The reflectance is approximately unity within the stop bands centered about multiples of $\nu_B = c/2\Lambda$**, where $c = c_o/\bar{n}$ and $\bar{n}$ is the mean refractive index.*



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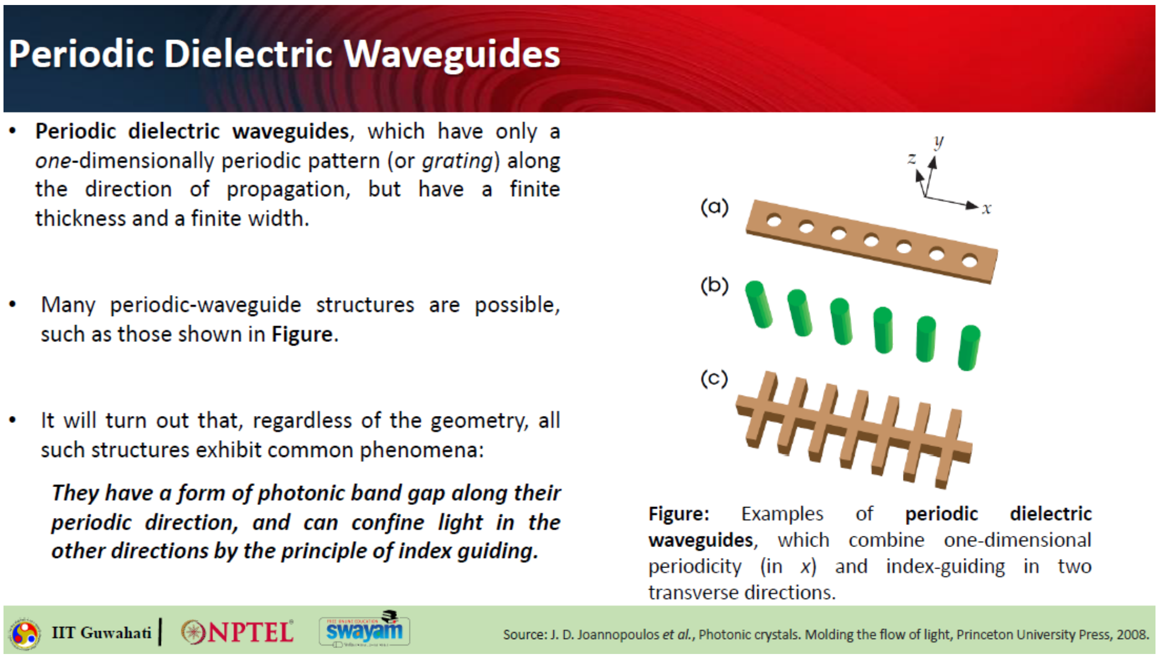
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, low high low high. So, this kind of low high segments you have repeated 10 times ok and you have placed this entire grating this is called dielectric Bragg grating you have put this entire grating in a medium of refractive index n 1 ok. And after you calculate ok, the reflectance versus frequency you can see that the stop bands are basically centered at this particular wavelengths ok lambda b sorry nu B 2 nu B 3 nu B and so on ok. So, here you can find out that nu B is nothing, but c over 2 capital lambda what is capital lambda that is basically the period here the period is marked clear. So, this way you are able to get good frequency selective performance from Bragg reflectors. Now let us move on to the next topic which is basically a periodic dielectric waveguide.

So, periodic dielectric waveguide which have only one dimensional periodic pattern or grating kind of thing ok along the propagation direction, but in this case they have finite thickness and width ok that is the only difference. So, in the previous case we have considered all to have infinite in this direction only in one direction we are bothered. So, let us look into some of these examples. So, many periodic waveguide structures are possible. So, few are shown here as you can see this is basically one dimensional array of holes ok and these are all like that.

So, these are basically array of dielectric columns. So, here index wave guiding is possible in the two transverse direction. So, while the periodicity is along this x direction in this case the index guiding is possible in z direction. So, based on because it will like high index material surrounded by low ah index air. So, you will be able to guide modes along the length of this wire ok.

So, that is how index guiding can take place along y direction or sorry z direction and in this case it is periodic along x and the index guiding can take place along the other transverse direction that is y direction. So, it will turn out that regardless of the geometry whatever is the geometry that you are seeing here this structures they exhibit a common phenomena and what is that they have a form of photonic band gap along their periodic direction. So, along this direction where they are periodic they have a photonic band gap and they can confine light in other directions like in this direction or in this direction they are able to confine light by the principle of index guiding. We can also look for two dimensional periodic patterns ok that will combine index guiding in one direction with photonic band gap in other direction.



So, examples are shown here. So, let us assume a two dimensional dielectric waveguide the permittivity material is permittivity is this one epsilon equals 12. So, it is a 2D material into the plane as you can see. So, we are only bothered about x and y dimension. So, the height or the width of this material is 0.

Periodic Dielectric Waveguides

- **Two-dimensional periodic pattern** that will combine index guiding in one direction with a photonic band gap in the other direction.

- **Two-dimensional dielectric waveguide** $(\varepsilon = 12)$ of width $0.4a$.

- Periodic waveguide: a period$-a$ sequence of $0.4a \times 0.4a$ dielectric squares. In both (a) and (b) there is a conserved wave vector k along the direction x of translational symmetry, resulting in guided modes.

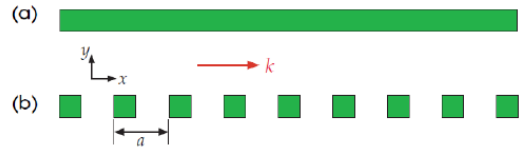


Figure: Dielectric waveguide.

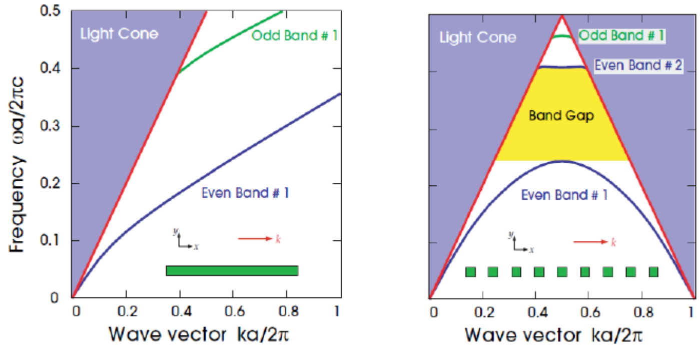
Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

4 a what is a? a is basically the period ok. So, this is a slab and this is or you can say this is a continuous waveguide and this is a periodic dielectric waveguide right. So, in the periodic dielectric waveguide we have used dielectric squares the dimension of the squares are given here 0.4 a times 0.4 a and the period is lattice period is a ok. So, in both case there is a conserved wave vector k that you can see along x direction ok because of the translational symmetry and that results in guided modes.

So, if you look into the band diagram of the waveguides ok for TM polarized modes ok that will be like kz equals 0. So, they are basically in plane. So, you are able to see that for the continuous one you do not actually have a band gap ok. You have a even band 1 here and then odd band 1 ok and this is the light cone ok. And in the case of the periodic structure you are able to see that even band 1 and even band 2 ok.

Periodic Dielectric Waveguides

- Band diagrams of waveguides, for TM-polarized in-plane ($k_z = 0$) light only.
- ***Left***: uniform waveguide. ***Right***: periodic waveguide including twice the irreducible Brillouin zone.



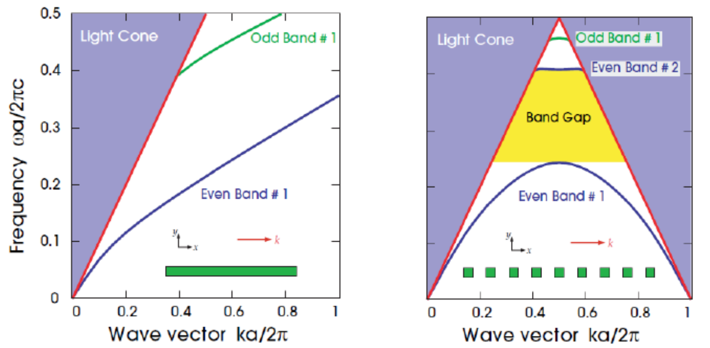
Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

We will come to this discussion later what is even band and odd band and this light cone. I am just trying to show you that with this kind of periodic structure you are able to get a band gap ok. And so, the left one as I mentioned it is a uniform waveguide and the right one is a periodic waveguide including twice the irreducible Brillouin zone. So, you could have actually done up to here, but just to show it you have taken twice the Brillouin irreducible Brillouin zone to show the entire curve and you are able to see a band gap in this case. So, as I mentioned the blue shaded region in this graphs are basically the light cone.

So, they are basically nothing, but the extended states propagating in air ok. These are not the guided modes ok. However, in this region you will have guided modes they are leveled as even or odd depending on the y equals 0 mirror symmetry plane. So, if you take this is y equals 0 is this particular plane. So, along this plane if it is symmetric you call it even, if it is asymmetric you call it odd and the waveguide is symmetric under reflection because if you cut it at the middle or if you put a plane at the middle you will see top and bottom are equal.

Periodic Dielectric Waveguides

- **Blue shaded region is light cone (extended states propagating in air).**
- Discrete guided bands are labelled *even* or *odd* according to the *y*=0 mirror symmetry plane.

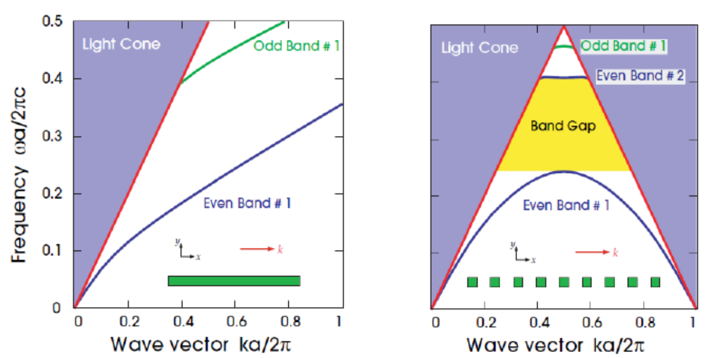


IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, you can say that the waveguide is symmetric upon reflection. So, consequently all the guided modes can be classified as either even or odd depending on the mirror reflection in this particular plane. And one can see that in this case we have only one even band and one odd band and even band is having the lower frequency. So, it is the fundamental mode. So, it has got the fewest mode nodes that is why it is fundamental and it is also having the lowest frequency.

Periodic Dielectric Waveguides

- The waveguide is symmetric under reflections through the plane *y* = 0 that bisects it. Consequently, all of the guided modes can be classified as ***even* or *odd* with respect to mirror reflections in this plane**.
- We see one even band and one odd band. The even band is the **fundamental mode**, for which the mode profile has the fewest nodes and the lowest frequency.

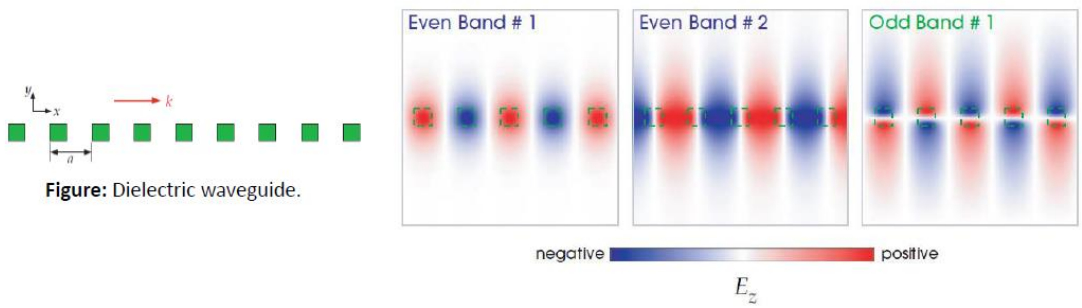


IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

Now, if you look into the field patterns let us look into the field pattern and understand what is even or not. So, at k equals pi by a that is the brilliant zone edge if you take the Ez electric field distribution and if you try to see along y equals 0 plane you will see top and bottom part are actually equal. So, this you can call as even band 1. In this case also if you take a you know y equals 0 plane and compare the top and bottom you will see they are symmetric.

Periodic Dielectric Waveguides

- E_z field patterns of the periodic waveguide at $k = \pi/a$, the Brillouin-zone edge.
- Left and middle panels correspond to the edges of the gap in the even guided modes, while the right panel has odd symmetry with respect to the $y = 0$ mirror plane. The dielectric squares are shown as dashed green lines.

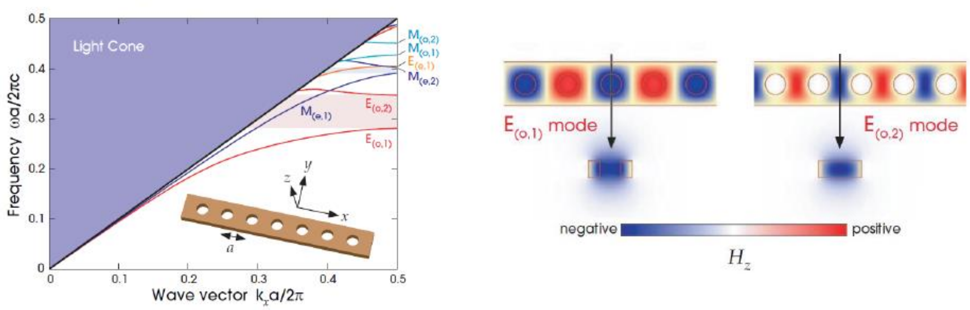


IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, that is also even. So, you can call it as even band 2. Why this is band 1? This is band 2 this has got 1, 2, 3, 4, 4 nodes this has got 1, 2, 3, 4, 5 nodes and so on. So, it has got more number of nodes and lowest energy. So, this is the lower fundamental node ok. However, odd mode you can see if you take a y equals 0 line here you will see that this part and this part are basically inverse.

Periodic Dielectric Waveguides

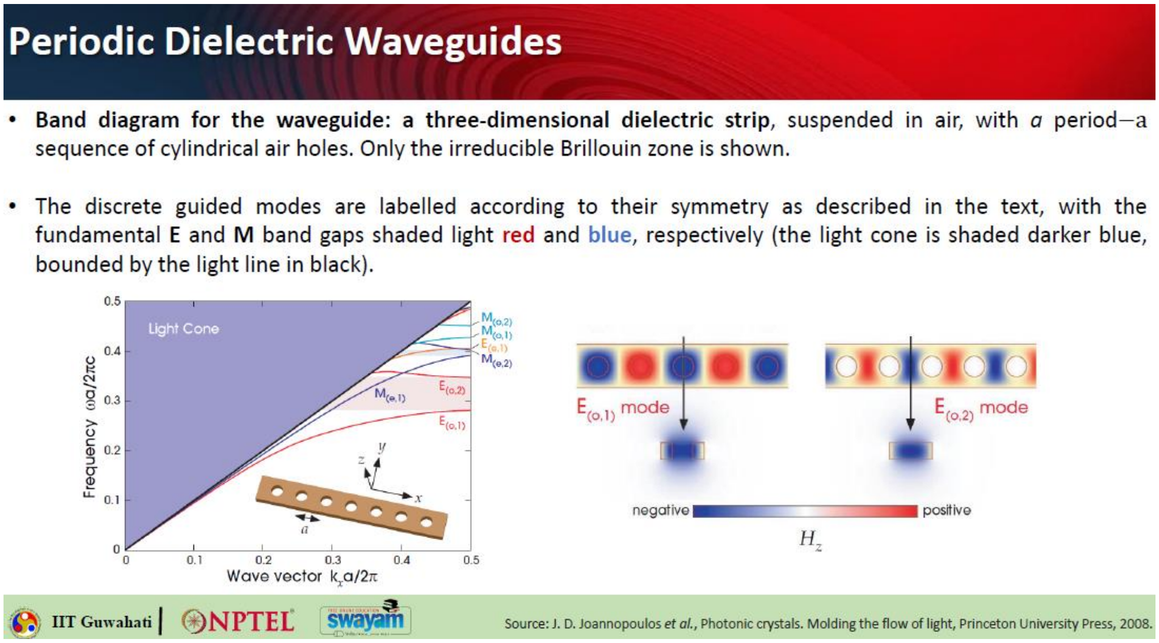
- **Band diagram for the waveguide: a three-dimensional dielectric strip**, suspended in air, with *a* period—a sequence of cylindrical air holes. Only the irreducible Brillouin zone is shown.
- The discrete guided modes are labelled according to their symmetry as described in the text, with the fundamental **E** and **M** band gaps shaded light **red** and **blue**, respectively (the light cone is shaded darker blue, bounded by the light line in black).



IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, 1 is positive, 1 is negative and so on the red one shows positive, blue one shows negative. So, they are basically anti symmetric. So, you can call this as odd mode ok.

Now, ok this we have already discussed.



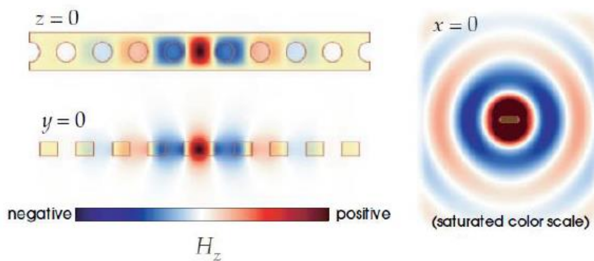
Now let us look into the band diagram for the waveguide ok. So, in case you take a 3 dimensional strip it means it has got a finite width and thickness. So, width is also finite, thickness also finite. So, this actually becomes a ah 3 dimensional dielectric strip. So, it has got it is basically suspended in air and then the holes are having a periodicity of ah a and this holes are cylindrical holes.

So, if you try to plot the irreducible Brillouin zone ok. So, you are actually plotting half of it if you want to get this entire thing like this in the previous case you can plot up to double of the irreducible Brillouin zone that is up to you. But here also you can see that in the case of odd modes you are able to get a band gap ok. So, here the odd modes are shown ok. So, this is basically EO 1 mode and EO 2 mode ok.

So, the plot is of Hz ok. So, let us look into the ah band diagram and here we can identify M and E modes. Now what are M and E modes? This is basically E mode or the TM mode and so these are basically E bands or you can say fundamental E bands or you can say TM bands and M bands are basically the T bands they are shown in red and blue ok. And ah this is the light cone ok and you can see these are the different modes which are possible. This is very much similar to the band diagram that you have seen in the previous ah lecture. Just that only difference here being ah that we are seeing it for a particular finite structure like a 3 dimensional dielectric step and we are strip and we are only calculating the half the Brillouin zone.

Point Defects in Periodic Dielectric Waveguides

- Now, let us consider a point defect. H_z field patterns of a localized resonant mode in a cavity **formed by a defect in the periodic waveguide** (suspended in air). The spacing between one pair of holes is increased from a to $1.4a$.
- **The strong localization, exponentially decaying in the waveguide**, is seen in the cross sections; the dielectric structure is shaded translucent yellow. The field decays only inversely with distance in the lateral directions, though, due to slow radiative leakage shown by the figure at right, which uses a saturated color scale to exaggerate small field values.



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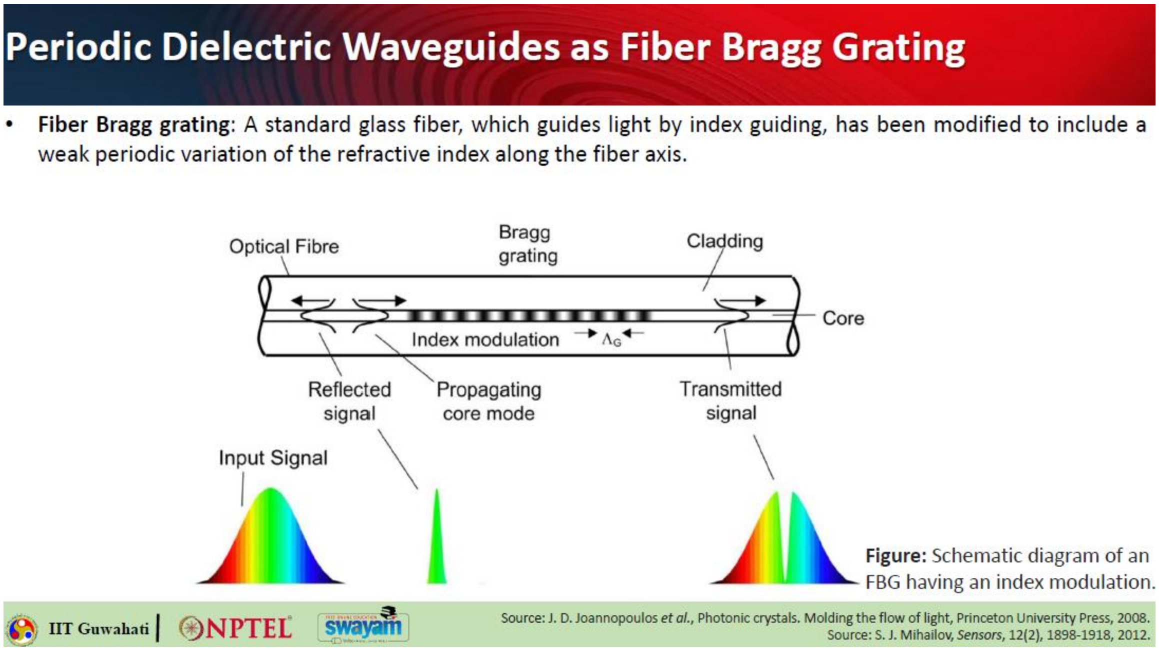
Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, these are the two odd modes and even modes ok sorry odd mode 1, odd mode 2 as we can see and this shows you the distribution. So, here you can see that in the first case the modes are concentrated in the dielectric sorry the air holes whereas, in the second case ah the modes are basically concentrated in the dielectric strip around the cylindrical holes ok. Now if you want to introduce a point defect in this dielectric periodic dielectric waveguide what you can do? You can actually make one of this hole a defective one means you can actually change the dimension of ah one particular hole. So, that the spacing between one pair of holes can be increased ok from say a to 1.

4 a. So, that way also you are actually introducing defect. So, what happens you know this is from the top you can see that this is how the electric field sorry the magnetic field pattern will look like and this is from the side ok. So, here you will see that the holes are actually ah the the magnetic field is actually getting decayed from the centre. So, there is a kind of confinement in the cavity ok. So, there are resonant modes possible in that particular point defect this is also shown from the top ok. So, the strong localization as you can see here as well ok or you can see in these two cases ok.

The dielectric structure is shown in this yellow pattern that you have already seen the field decays only inversely with the distance in the lateral dimension. So, the field decay is shown here you can also see this way that the field strength is decaying and this is because of the slow radiative leakage ok. And that is true because there is a defect after that the field is getting confined, but outside that it will be decaying. So, with that we understood that we are able to make periodic dielectric waveguides of different shapes to confine or ah to ah engineer the band gap and get some properties out of it.

So, one example from the bragg grating if you go back. So, it is basically this particular break grating you are making in the fibre. So, we are calling it as fibre bragg grating. So, you take a standard glass fibre ok which is basically guiding light by index guiding you just that you have to modify the you know index of the core in a you know particular pattern that high low high low high low and so on. So, you are able to get create a kind of break grating on the fibre itself. So, in that case what will happen one particular wavelength will get reflected or band can get reflected remaining will get transferred.

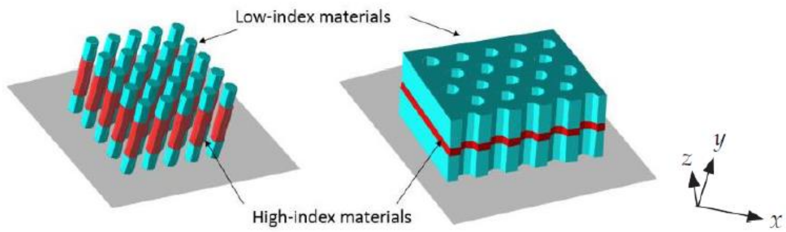


So, if you take this kind of a input signal red to blue all the colours are there, but say you have designed this break grating that is resonating at ah the reflection is maximum at say green. So, the green will be stopped and all other things will be passed other than green. So, this way you can use it for filtering purpose. Now, let us look into the another type of application of photonic crystals which are photonic crystal slabs. Now, these are simple structures with only one dimensional periodicity that that can be used to confine light in three dimension by combining combination of band gap and index guiding.

So, remember that photonic crystal slabs or planar photonic crystals with two dimensional periodicity, but a finite thickness ok. Now, because they have finite thickness they are not exactly two dimensional photonic crystals. In two dimensional photonic crystal we assume that to have you know they they can be very infinitely tall ok. But here despite there is resemblance with resemblance with the 2D photonic crystal the finite thickness in the vertical direction they introduce some qualitatively new behaviour. And because of which they are slightly different from that two dimensional photonic crystals.

Photonic-Crystal Slabs

- Simple structures with only **one-dimensional periodicity** can be used to confine light in three dimensions by a combination of **band gaps and index guiding**.
- **Photonic-crystal slabs** or **planar photonic crystals** with *two*-dimensional periodicity but a finite thickness. They are *not* "two-dimensional" photonic crystals, despite the resemblance: the finite thickness in the vertical (z) direction introduces qualitatively new behavior, just as the periodic dielectric waveguides differed from photonic crystals in one dimension.



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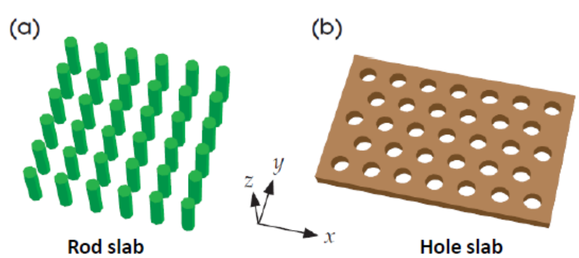
Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

Just as the periodic dielectric waveguide differ from photonic crystals in one dimension. So, if you look into the band structure or band diagram of a 1D photonic crystal and a periodic dielectric waveguide though they are more or less similar in nature, but there is certain difference. Here also you will find that that because of this finite height or thickness this photonic crystal slabs. So, this is basically a slab of the dielectric materials ok. So, you have low index material then high index material and then low index material.

Photonic-Crystal Slabs: Rod Slab and Hole Slab

- Examples of **photonic-crystal slabs**, which combine two-dimensional periodicity (in the xy directions) and index-guiding in the vertical (z) direction —
  1. **Rod slab**: a square lattice of dielectric rods in air.
  2. **Hole slab**: a triangular lattice of air holes in a dielectric slab.
- The rods have a radius $r = 0.2a$ and the slab has a thickness $2a$, whereas in the hole-slab example, the holes have a radius $r = 0.3a$ and the slab has a thickness $0.6a$.



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Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

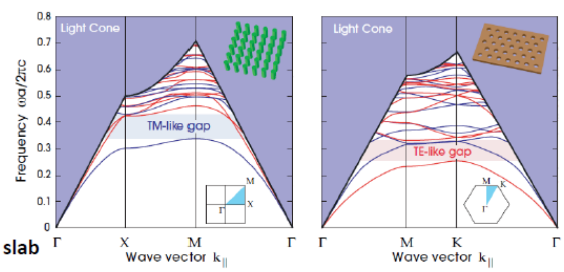
Here also you have something like that, but it is a inverse one then this structure. So, these two structures are basically complementary this is a array of rods this is an array of holes. So, you right now you might understand that why we need rod slab and hole slab one will give you band gap in one particular mode the other will give you in the other mode. So, these are the examples of photonic crystal slabs they basically combine the two dimensional periodicity that is in xy plane and they have index wave guiding along the z direction ok. So, if you take rod slab that is basically a square lattice of dielectric rods in air and you can consider hole slab these are basically triangular or hexagonal lattice of air holes.

So, ideally you should say cylindrical air holes in the electric slab. So, these are the dimensions we have taken. So, the rods are having radius of 0.2 a, a is standard a is the lattice constant right. Here everything is in terms of lattice constant.

So, they can be normalized to any other they are normalized. So, if you whatever value you pick as a lattice constant you can find the corresponding radius and the slab is been taken to have thickness of 2 a ok. So, this is the thickness for this one rod slab if you take the hole slab the radius of the holes are 0.3 a and the thickness of the slab is 0.6 a ok. So, these are the dimensions you have taken and then when you calculate the photonic band diagram for this rod slab and hole slab you will see the following.

Photonic-Crystal Slabs: Rod Slab and Hole Slab

- The rod slab favors a TM-like gap, and the hole slab favors a TE-like gap, as one might expect from our knowledge of the corresponding two-dimensional photonic crystals.
- Band diagrams for photonic crystal slabs suspended in air. The blue shaded area is the *light cone*, all of the extended modes propagating in air. Below it are the guided bands localized to the slab: **blue**/**red** bands indicate **TM**/**TE**-like modes, respectively (odd/even with respect to the $z = 0$ mirror plane). The rod/hole slabs have gaps in the **TM**/**TE**-like modes, which are shaded **light blue**/**red** respectively



Band diagram of Rod slab **Band diagram of Hole slab**

Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

The rod slab favors a TE like gap and the hole slab favors a TE like gap something very similar to what you have seen in the previous lecture for 2D photonic crystal of dielectric rods and hole array right. So, the band diagram for photonic crystal slabs these are suspended in air. So, again the blue lines are basically the air cones that means, these are basically all of the extended modes which are basically propagating in air ok. And below this whatever you see here is the guided mode in by this slab. Now, if you see the blue and red bands they indicate the blue indicates TM red indicates TE like modes ok.

And this is the Brillouin zone. So, we have actually gone through the Brillouin zone this is the square one. So, you have the important points of gamma X M and gamma this is a triangular array or hexagonal array. So, the Brillouin zone is a hexagonal shape, but the irreducible Brillouin zone is again a triangular shape as we have seen in the previous lecture. So, you can take gamma M K and gamma these are the points you go through.

So, you can see that there is a clear TM like gap and this is a TE like gap ok. So, we are saying like like here because they are similar to the ones we have seen for the 2D photonic crystals, but these are slabs the finite thickness ones ok. Now, in this particular slab array let us try to introduce some defects that we have discussed in the previous lecture. First thing let us introduce a linear defect in the slab. So, you can actually in 2 or 3 dimensional photonic crystals you can form a waveguide by removing one row of the rods that is possible. And how it is done in this case the row is removed gradually and to show how exactly how the defect is forming ok you can actually see that the radius on the top is getting shrink ok.

Photonic-Crystal Slabs: Linear Defects in Slabs

Reduced-radius rods

- **In two- and three-dimensional crystals, a waveguide can be formed by removing a row of rods.**
- In this case, **the row is removed *gradually***, to show exactly how the defect mode forms. Specifically, shrinking the radius of all of the rods in a particular row—a fabricated example of a similar waveguide is shown in **Figure.**

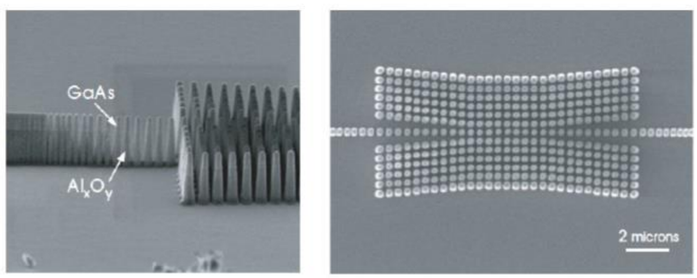


Figure: Two views of a reduced-radius waveguide fabricated in a rod slab by Assefa *et al.* (2004), designed to operate at near-infrared wavelengths. (GaAs rods on low-index aluminum-oxide pedestals.)

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Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

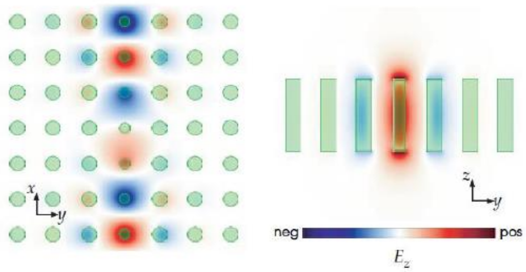
So, here you have all the rods are having identical radius, but towards the top they are getting all you know tapered fine. So, these are the 2 views of reduced radius waveguide fabricated in a rod slab ok. And this is designed to operate in near infrared wavelengths. So, what are the material? So, the material is gallium arsenide for the rods and those are having ah low index aluminium oxide as the pedestals ok stands.

So, this is the top view of that particular array. It is hard to see from the top view because you are actually from the top you will not be able to see the conical shape right. So, how does it help? Here you can see that if you have reduced radius rods say the rods radius has been changed to this one. So, for a defect ah rod radius of r equals 0.14 a and at a particular wave vector k x a by 2 pi is given as this. So, if you take this particular value and if you simulate the results you will see that you are able to waveguide light along this particular defect.

Photonic-Crystal Slabs: Linear Defects in Slabs

Reduced-radius rods

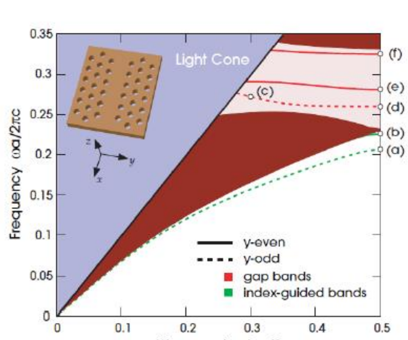
- E_z **field cross sections in reduced-radius line-defect waveguide**, for a defect rod radius of $r = 0.14a$ at a wave vector $k_x a/2\pi = 0.42$.
- The dielectric material is **shaded translucent green**. ***Left:*** horizontal ($z = 0$) cross section (in which E_z is the only nonzero electric-field component). ***Right:*** vertical ($x = 0$) cross section bisecting the rods perpendicular to the waveguide. The field decays exponentially away from the waveguide

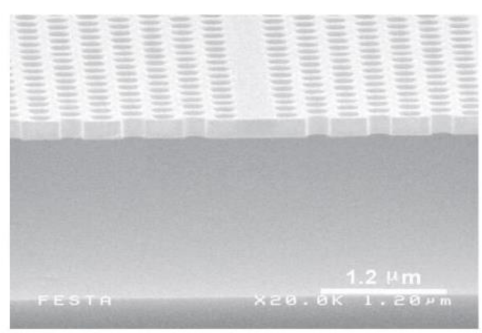


And here also you can see the confinement along this particular line defect or reduced radius rod. So, the electric field is mostly confined in this region ok. You can also have photonic crystal slabs without you know by removing or you can actually take a whole slab and remove a particular row of holes that can be can be also another defect.

Photonic-Crystal Slabs: Removed Holes

- The projected band diagram for a hole slab in which we have filled in a row of holes.
- A fabricated example of such a waveguide is in a suspended membrane.





SEM image of a waveguide formed by a missing row of holes

So, here you can see such an example. So, this is a beautiful slab of holes. So, and this particular ah line of holes have been removed. So, this is basically this one and this is the SEM scanning electron microscope image of this particular missing row of holes. So, how it helps again? This will help you to find out some particular modes can be guided through this and this modes need to be in the why we do the bend diagram because this modes should be in the gap bands. Gap band mean the frequency of those modes should lie in the band gap of the crystal.

So, that they are not able to leak out into the crystal. If they are within the like allowed bands what will happen? You excite or launch a particular mode they will be able to leak into this particular slab. So, that is not good you want to actually guide it. So, they should not leak into any of these cases. So, the frequency should be chosen from this particular gap band and that is why calculation of this photonic band structure for any of this particular slab is also very important.

Photonic Crystal Fibers

- The most important conduit for modern telecommunications is the **optical fiber**: a long filament of glass (or sometimes plastic) that guides light, often for a distance of many kilometers.
- Optical fibers are also used in a range of other applications, ranging from astrophysics to medicine.
- A traditional optical fiber consists of a central **core** that is surrounded by a **cladding** of slightly lower dielectric constant, which confines the light by index guiding.
- New regimes are opened for fiber operation by incorporating periodic structures in the cladding: **Photonic-crystal fibers**

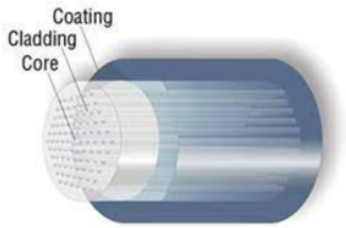


Figure: Structure of a Photonic Crystal Fiber.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

The next important topic is photonic crystal fiber. Now, why it is important because you understand the importance of optical fibers that is the backbone of this telecommunication and internet and everything right. So, you you actually require optical fiber from astrophysics to medicine everywhere. So, a traditional fiber has got a central core which has got a cladding of slightly lower refractive index right and the light is basically guided through index cladding. So, the light travels through the higher refractive index material surrounded by a region of lower refractive index material.

So, this is a typical photonic crystal fiber. So, what happens here in the cladding region you are basically incorporating a periodic structure or a photonic crystal and that photonic crystal should be having band gap of the wave length that is being carried in the core. So, how it will help? It is a simple explanation that light from the core should not be allowed to leak into the cladding. So, if that particular wavelength falls within the band gap of that photonic crystal that has been made in the cladding.

Photonic Crystal Fibers

- **Photonic-crystal fibers**, also called **microstructured optical fibers**, can be divided into a few broad classes, according to whether they use **index guiding or band gaps** for optical confinement, and whether the periodicity of the structure is one-dimensional or two-dimensional.
- **Photonic-bandgap fibers** confine light **using a band gap rather than index guiding**.
- *Band-gap confinement is attractive because it allows light to be guided within a hollow core.*
- This minimizes **the effects of losses, undesired nonlinearities, and any other unwanted properties of the bulk materials** that are available.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, there is no way out. So, light will be guided through the core endlessly right. So, photonic crystal fibers they are also called a microstructured optical fiber they can be divided into a few broad classes ok. According to whether they use ah index guiding methods or band gap methods for optical confinement. So, there are different ways of working. So, let us see what is that.

Photonic Crystal Fibers

- Three examples of **photonic-crystal fibers**:
  - **a) Bragg fiber**: a one-dimensionally periodic cladding of concentric layers.
  - **b) Two-dimensionally periodic structure** (a triangular lattice of air holes), confining light in a hollow core by a band gap.
  - **c) Holey fiber** that confines light in a solid core by index guiding.

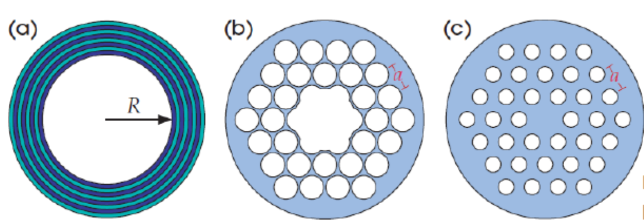


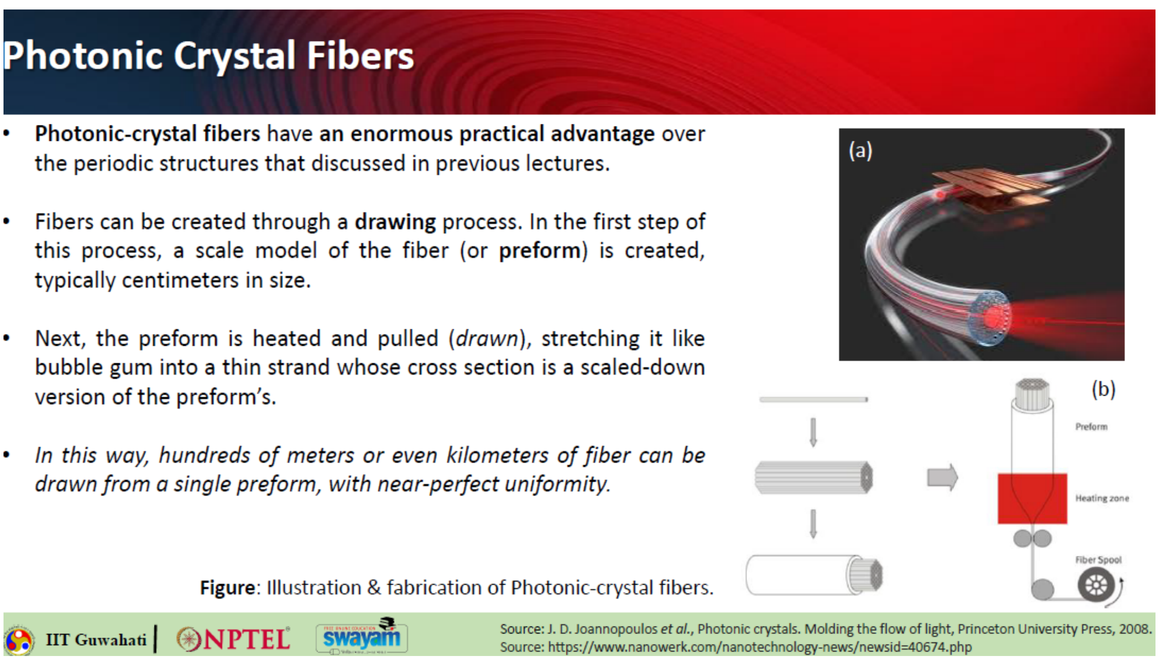
Figure: Three examples of photonic-crystal fibers.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, first one is photonic band gap fibers confined light using a band gap rather than index guiding ok. And band gap attractive is a band gap confinement is very attractive because it allows light to be guided within a hollow core. So, the attenuation will be not there ok. So, in a silica core fiber when light travels there is a attenuation, but if light is able to travel in a hollow core fiber that is the core is made of air ok, in that case there is very low attenuation. So, as I mentioned this minimizes the effect of loss, undesired non-linearity and other unwanted effect which comes from the bulk material.

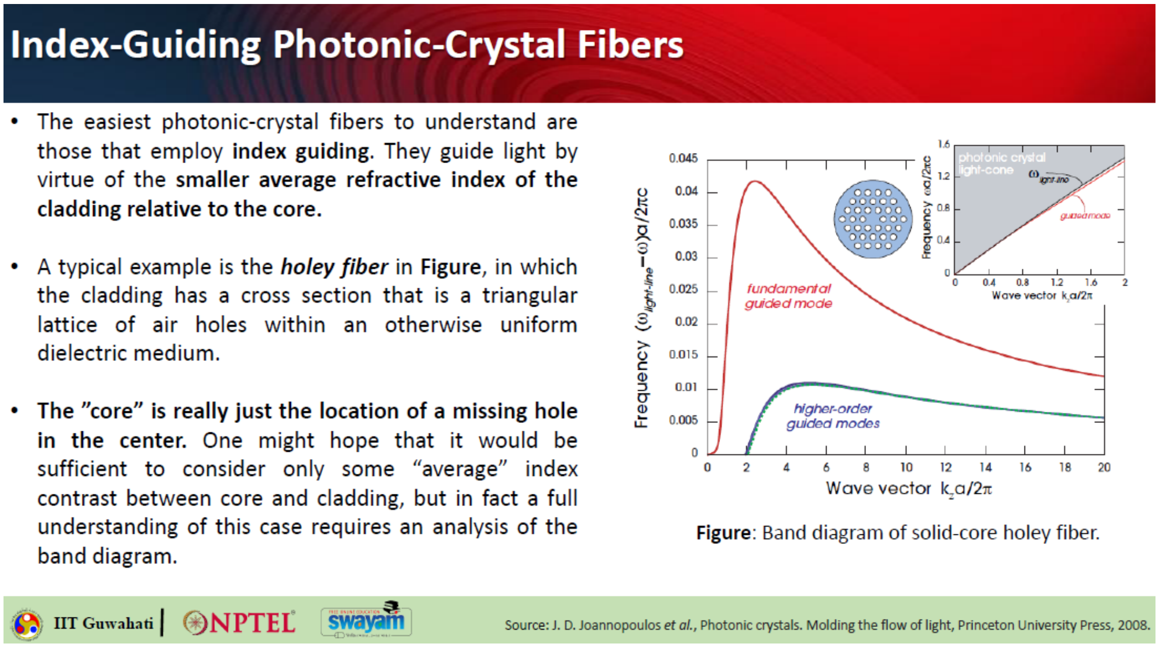
Like silicon induced effect in fiber that can be removed if you use a hollow core fiber ok. Now, there are 3 types of photonic crystal fiber. So, one can be a black fiber that is one dimensionally periodic cladding of concentric layer. So, you can actually design the cladding using a bragg grating ok these are called as bragg fibers. Then you have seen ah this one hollow core fiber by a band gap. So, here it is actually doing a band gap kind of guiding because you have a hollow core and you have a photonic crystal around it.

So, whatever is being guided cannot escape into this periodic photonic crystal. So, this is band gap kind of ah or you can say this is band gap method or band gap confinement. And the third one is holy fiber where you have a solid core that supports light propagation through index guiding, but the cladding is now made of photonic crystal again. So, in this 2 case this is the difference is a holy core ok and this one the core is solid, but it is called a holy fiber ok. Because the cladding is basically made of holes because when you actually have holes around that will effectively lower the refractive index.



And this is how you get a high and low refractive index material in the fiber ok. So, this is how the fibers are drawn. So, photonic crystal fibers they have enormous practical advantage over the periodic structures that you have discussed. First the these fibers can be ah created through a drawing process fiber drawing method. So, what do you do in this case a scale model of the fiber or preform is created which is typically centimeters in size.

So, you actually make the actual shape of the fiber like this and then you heat the preform and pull it. So, it is like you know bubble gum being stretched and you can draw the micrometer thin fibers ok. So, in this way hundreds of meters or even kilometers of fiber can be drawn from a single preform and as you are drawing you can actually make a roll or coil out of it ok. So, here is some ah band gap calculation of the index guiding fiber of the or the holy fiber as you can see ok. So, the easiest photonic crystal fiber to understand are those that employ index guiding as you see here.



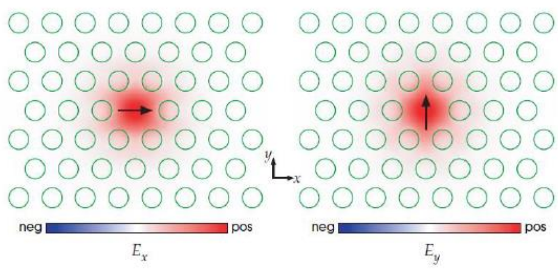
They guide light by the virtue of smaller average refractive index of the cladding relative to the core. So, by introducing this air holes you have brought down the refractive index of the surrounding medium. So, that works as the cladding and the light is basically guided in the core. So, here we have taken like triangular array of air holes. So, this is the core the core can be thought of as a missing hole in the center and how does it help? One might hope that it would be sufficient to consider only the average index contrast between the core and the cladding in this case, but in fact, you have to understand the band diagram to find out that which all modes are basically guided in such a particular fiber.

So, that way calculating the photonic band structure for this one is also very important. We will not go into that much details because this is not that advanced course. I am just showing you the possibilities. So, here are two possibilities of the fundamental mode that can be generated in such a fiber as you can see one can have this kind of electric field orientation the other one can have the orthogonal one. So, these are basically doubly degenerate fundamental mode.

So, their polarizations can be nearly orthogonal in one case it is Ex another case it is Ey ok. Now, in an ordinary index guided fiber one can go up to higher and higher frequencies ok or smaller wavelengths and more and more guided modes can be pulled below the light line right. So, higher order modes are also possible, but in that case if you see here in the solid core Holy fiber ok when you reduce the wavelength or increase the frequency ok. So, the effective index basically touches that of the silicon ok. So, they can actually remain endlessly single mode ok regardless of the wavelength.

Index-Guiding Photonic-Crystal Fibers

- Doubly degenerate band is localized in the core—**fundamental mode**, whose field patterns are shown in **Figure.**
- **Electric-field pattern for the doubly degenerate fundamental mode**. Their polarizations are nearly orthogonal everywhere: the mode pictured at left is mostly E_x, and the mode pictured at right is mostly E_y. The green circles show the locations of the air holes.



IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, that is something very interesting that in this particular case you can have single mode throughout. So, the question will come why are the higher order modes absent. There you can see from here the reason is that the effective index contrast between the core and the cladding in the Holey fiber keeps on decreasing at smaller wavelength ok rather than remaining fixed if it was a effect like homogeneous cladding. So, in this case the confinement for higher energy modes become very weak because the delta n between core and cladding is very less.

Endlessly single-mode fibers

- In an ordinary index-guided waveguide, **as one goes to higher and higher ω (smaller wavelength λ), more and more guided modes are pulled below the light line.**
- Eventually, one approaches the ray-optics limit, in which the guided modes are described by a continuum of angles greater than the critical angle for total internal reflection.
- This need not be true of photonic-crystal fibers: they can remain *endlessly single-mode*, regardless of wavelength (limited only by the material properties).
- So, why are the higher-order modes absent?

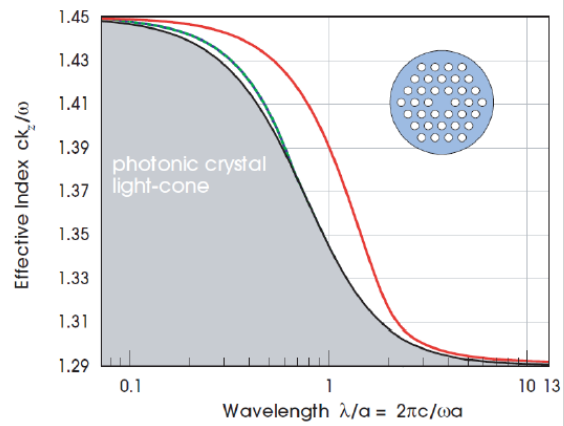


Figure: Bands of the solid-core holey fiber.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, those higher order modes leaks out. So, they will not be guided ok. So, the higher order guided modes they remain below the light line. So, they are basically air. So, they will be leaked. So, in the limit of small lambda in this particular limit you can see that effective indices of both the modes and the light line they approach the index of 1.

5 which is of the bulk silica right. So, that is how you can say that in this kind of solid core Holy fiber higher order modes cannot be sustained. So, only single mode can propagate. So, that is also very very good there is a multimode interference and all these things. So, there is a huge promise of photonic crystal fibers in the future.

Endlessly single-mode fibers

- The reason is that the **effective index contrast between the core and the cladding in the holey fiber *decreases* at smaller wavelengths, rather than remaining fixed as it would for a homogeneous cladding.**
- Thus, the strength of confinement is weaker at smaller wavelengths, and higher-order guided modes remain above the (lowered) light line.
- In the limit of small λ, the effective indices of both the modes and the light line approach the index 1.45 of the bulk silica.

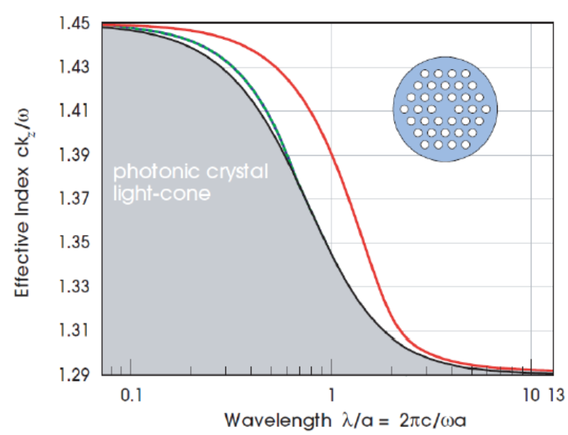


Figure: Bands of the solid-core holey fiber.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, here are some electron microscope image showing you how exactly the holo-core fibers look like. So, this is holo-core fiber ok and this is a omnidirectional mirror this is again holo-core ok omnidirectional because in all the direction you have a mirror kind of thing based on Bragg cross section. So, you have alternating layers of high low high low high low dielectric and so on. So, this is one particular fiber and this is another fiber you have silica glass and air ok. So, these are the air holes ok.

These are the zoomed one you can see this is electron microscope image of holo-core Holey fiber ok. So, this part remains as it is. So, there is no hole air hole is missing here ok. So, this is the Holey fiber section right. So, this way you can understand that there are so much of applications of photonic crystals 1D and 2D photonic crystals and even 3D are also being used as cavity. So, by introducing point defect they they are very good resonators will not go into all those applications here, but in short these are the more popular usage of 1D and 2D photonic crystals and with that we will stop our discussion here.

Electron Microscope Images of Photonic Crystal Fibers

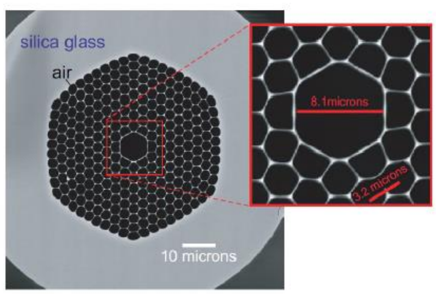


Figure: Electron-microscope image of hollow-core holey-fiber cross section.

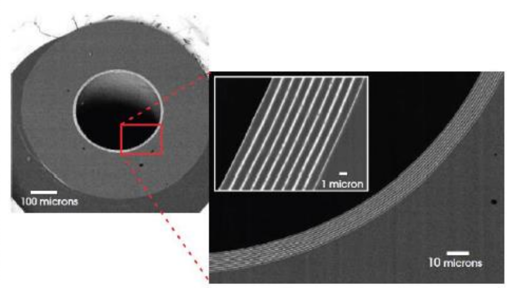


Figure: Electron-microscope image of a hollow-core omnidirectional-mirror Bragg-fiber cross section.

Source: J. D. Joannopoulos *et al.*, Photonic crystals. Molding the flow of light, Princeton University Press, 2008.

So, in the next lecture we will go into the basics of metal optics or plasmonics and this will be the final thing. This lecture concludes the discussion on photonic crystals. If you have any questions on photonic crystal feel free to email me at this particular email address mentioning MOOC on the subject line. Thank you.