Course Name- Nanophotonics, Plasmonics and Metamaterials

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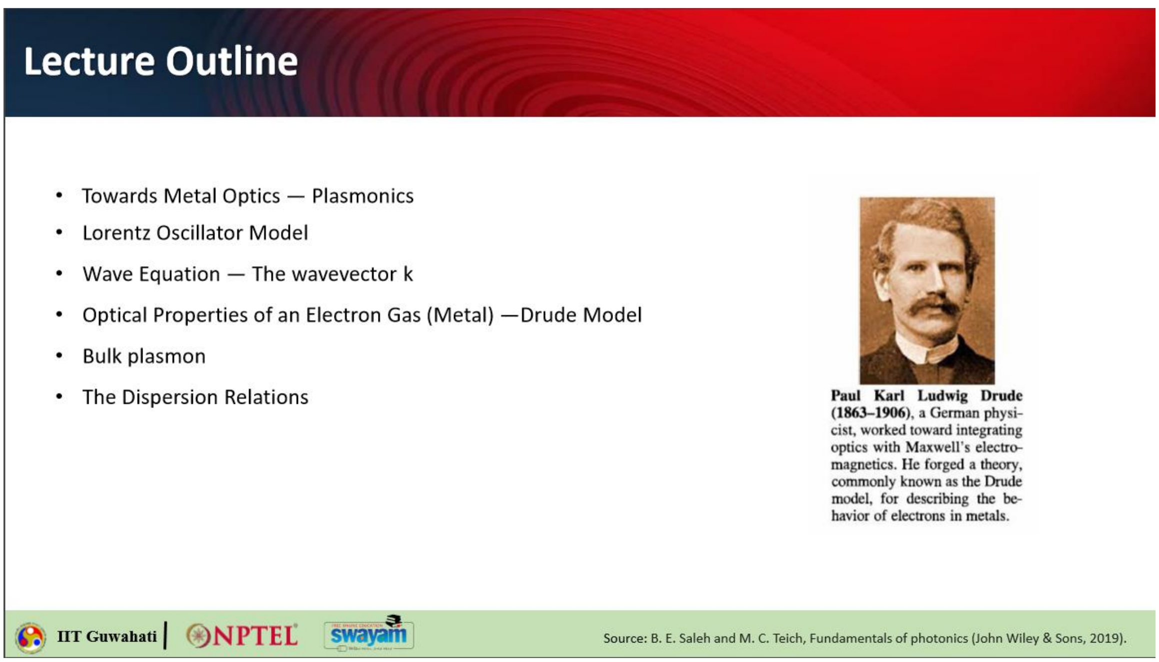
Week-06

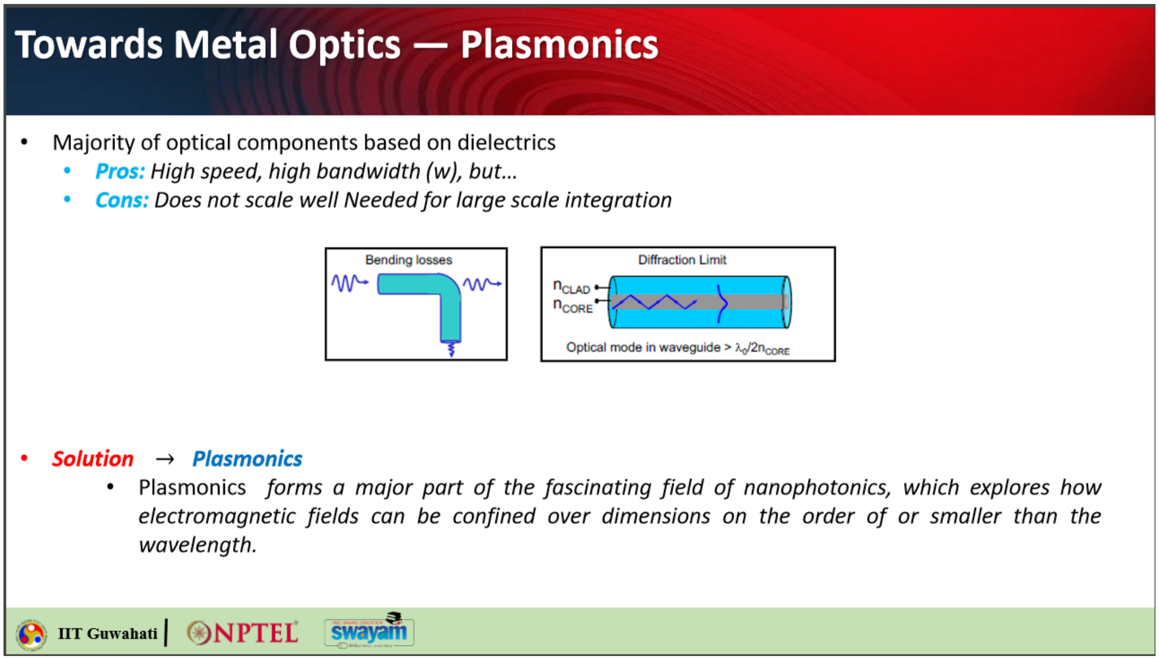
Lecture -16

Hello students, welcome to lecture 16 of the online course on Nanophotronics, Plasmonics and Metamaterials. We will be covering optical properties of metal in this particular course. Now here is the lecture outline as I mentioned we will be covering ah metal optics or plasmonics very briefly ok. The relevant models to explain the properties of metal that will also be covered. We will be discussing about the Lorentz oscillator model, we will discuss wave equation and the wave vector k. We look into the optical properties of an electron gas like that is present in a model metal using Drude model ok.



So, here is a picture of Paul Karl Ludwig Drude who was a German physicist and he has worked towards integrating optics with Maxwell's electromagnetics. And he has forced a theory which is commonly known as Drude model that wasable to describe the behaviour of electrons in metal. So, that was a very very important discovery and that is why the model is named after him. We will also look into bulk plasmon and different dispersion relations.

So, let us move towards metal optics or plasmonics. We have discussed dielectrics till now and we have understood that majority of the optical components are based on dielectrics. Now there are couple of pros and cons with dielectrics. First thing is that you know dielectrics allow high speed high bandwidth, but they have a problem that they do not scale well or you cannot make them very much miniaturized and that restriction comes from the diffraction limit of light that you have seen in the initial lectures. Other problems would be like bending loss.



So, when you bend it so the modes can actually leak out from this sharp bends. And also, if you look here the diffraction limit allows you to focus light only at a region which is lambda naught over 2 times the refractive index. So, if you look into an optical fibre that the core size is kind of limited ok and it is related to the optical wavelength that it is carrying and the optical mode in the waveguide will be larger than this. So, there will be some kind of mode extended into the cladding as well. Now what is the solution going ahead? So, these are some problems we know of dielectric photonics.

So, going ahead we can move towards plasmonics or metal optics. So, plasmonics forms a major part of the fascinating field of nanophotonics which explores how electromagnetic fields are confined over dimensions on the order or smaller than the wavelength. So, this is where you can go sub wavelength ok. Now in the past the devices were relatively slow and bulky. So, we have not seen most of you have not seen this particular era.

Towards Metal Optics — Plasmonics

- **In the past**, devices were relatively slow and bulky.
- The semiconductor industry has performed an incredible job in scaling electronic devices to nanoscale dimensions. Unfortunately, interconnect delay time issues provide significant challenges for electronic circuits operating above ~10 GHz.
- **Photonic devices** possess an enormous data-carrying capacity (bandwidth). Unfortunately, **dielectric photonic components** are limited in their size by the laws of diffraction, preventing the same scaling as in electronics.
- Finally, **plasmonics** offers precisely what we need
    - the size of electronics
    - the **speed** of **photonics**.

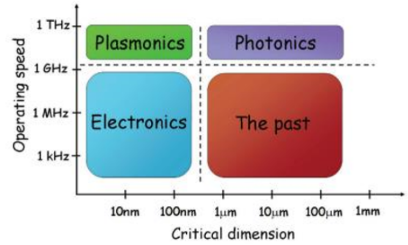


Fig.: Operating speeds and critical dimensions of various chip-scale device technologies, highlighting the strengths of the different technologies.

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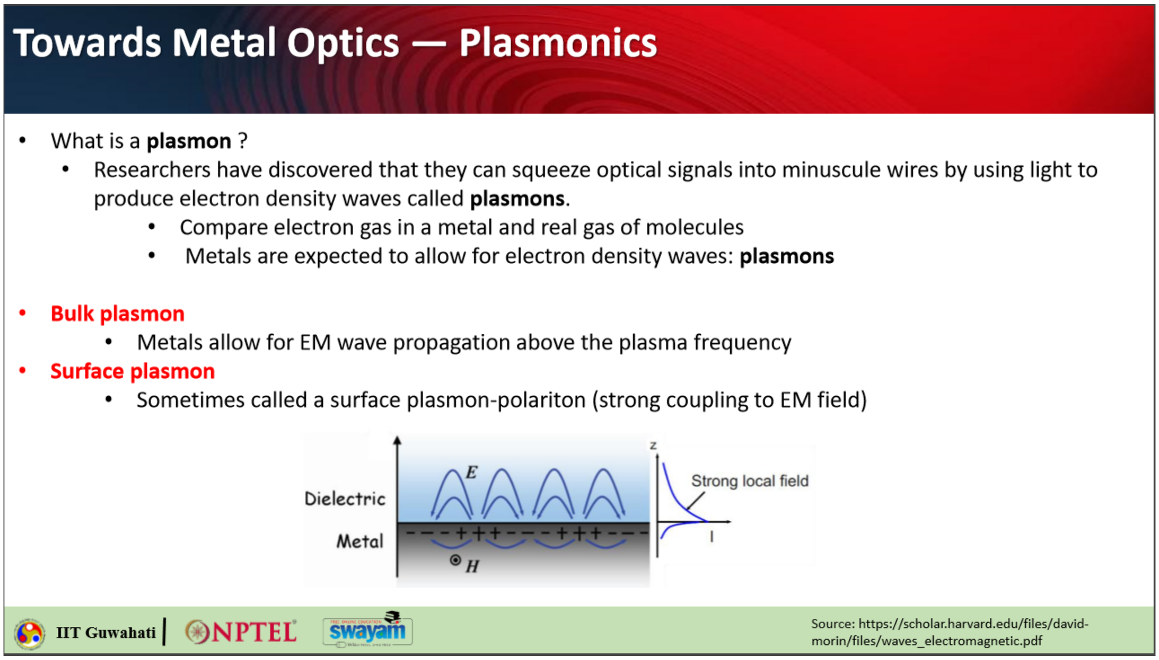
Source: R. Zia, J. A. Schuller, A. Chandran, and M. L. Brongersma, "Plasmonics: the next chip-scale technology," Materials Today, vol. 9, no. 7–8, pp. 20–27, Jul. 2006.

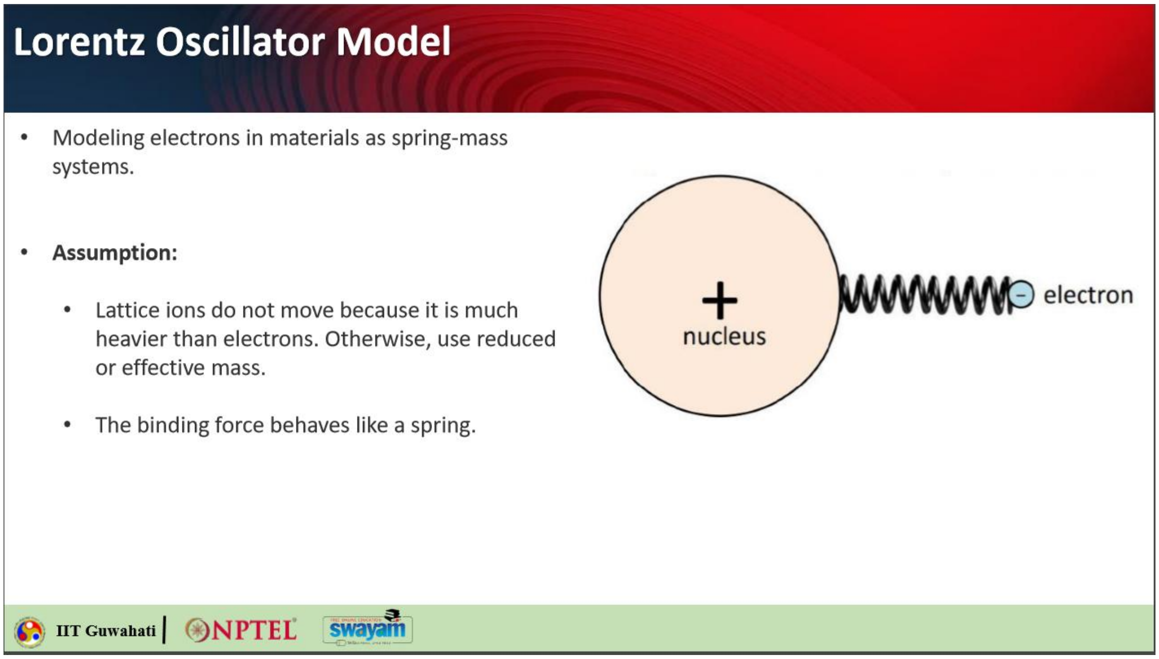
We are people of the era of electronics as well as photonics. So, we have seen semiconductor industry which has performed incredibly well to scale down the size of electronics component to nanometer scale ok. So, and that is how all these electronic gadgets and devices are becoming sleek, lightweight, compact and there are more and more electronic devices that is coming to the market. So, miniaturization is very well done by semiconductor industry, but there is a problem with the speed and we have seen that the interconnect delays in the initial lectures we have discussed it in more details that the interconnect delays typically restrict the speed of electronic devices to few gigahertz. So, how do you actually go to larger speed? The way is to go for photonics.

Now photonic devices they have enormous data getting capacity, but unfortunately this photonic components they are restricted in miniaturization because of the fundamental diffraction limit of light ok. So, you cannot scale them as small as the electronic devices. So, photonic devices cannot be scaled down to nanometer scale because optical wavelengths are in the order of micrometers right. So, we have to come to plasmonics area where we can have miniaturization as well as the high speed of the photonics. So, this is where plasmonics offers us the best that is the size of electronics and speed of photonics.

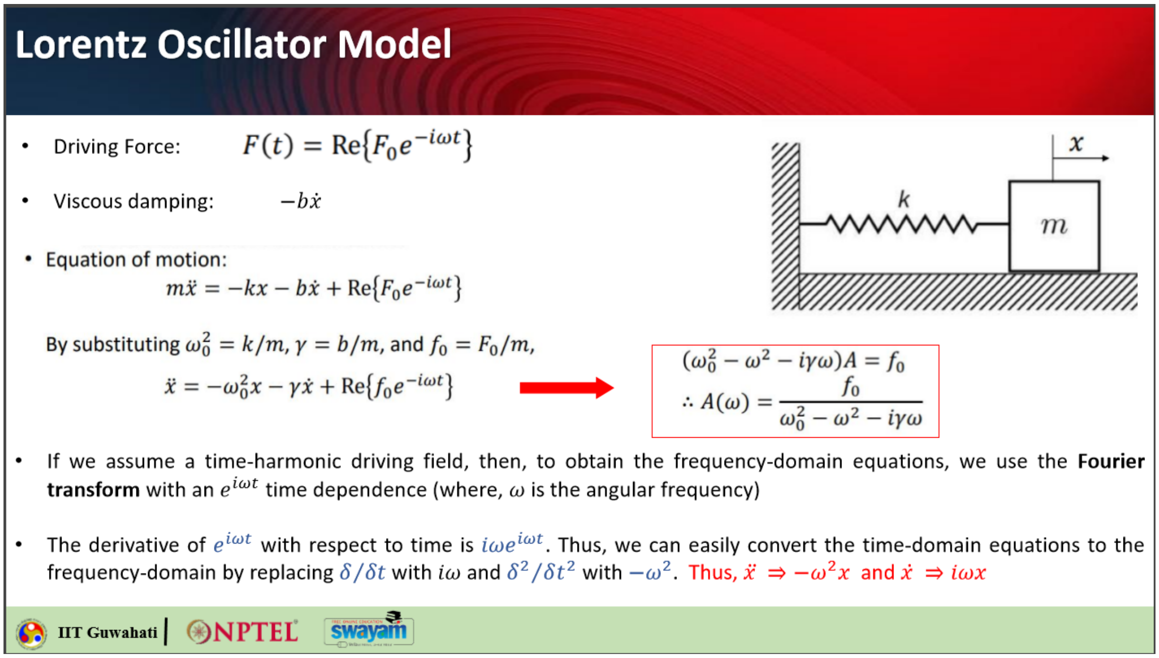
Now when we talk about plasmonics we have to understand what is plasmon. So, researchers have developed that this plasmon can squeeze optical signal into miniscule wires and how do you do it? So, you have to use light to produce electron density waves which are called plasmons. So, they can be compared like you can compare electron gas in a metal to a real gas of molecules ok and the metals are expected to allow for this electron density waves which are called plasmons. A simple analogy is like these are like sound waves. So, alteration of air molecules ok the way it how the way sound wave propagates in air you can think of surface density waves or electron density waves in metal that is nothing but plasmon.



Now bulk plasmons they are in the bulk metal in this case metals allow electromagnetic wave propagation above the plasma frequency ok. And there are surface plasmon where you they are also known as surface plasmon polariton which shows very strong coupling to the electromagnetic field. We have seen this briefly in the introduction lectures that you can have metal dielectric interface and you can have surface waves propagating along this particular interface. And the field extends more into the dielectric as compared to the metallic region. Now how do you model this kind of behaviour? So, we can actually start modelling the behaviour of electron in any material using a spring mass system.



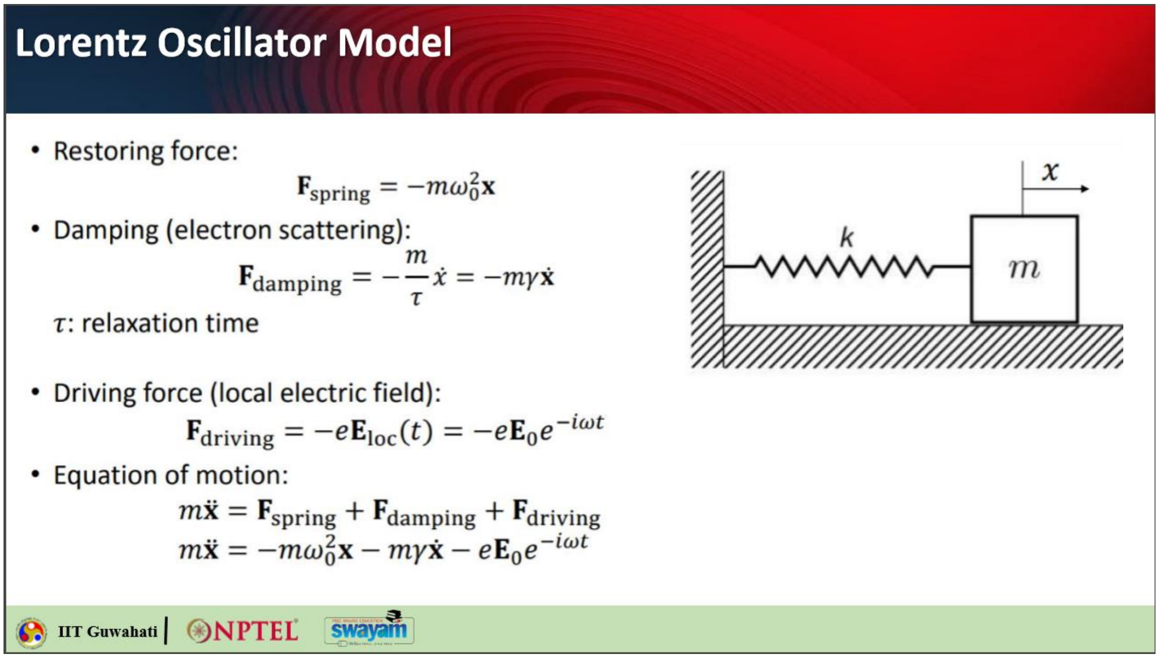
So, we can assume that the lattice ions do not move so are like they are like big bulky nucleus ok. And the electrons are connected using a binding force which behaves more or less like a spring and we assume that the nucleus or lattice ions do not move if they move you have to use reduced or effective mass ok. So, this is a kind of simplified assumption that we use. Now if you take this spring mass damper kind of arrangement for this system you will see that the damping force can be written as $F(t)$ which is $\mathrm{Re}\{F_0 e^{-i\omega t}\}$ ok. You can go for the viscous damping given as $-b\dot{x}$ ok. $\dot{x}$ is the velocity, x is the displacement as you can see $\ddot{x}$ will be the acceleration.



So, this is the force so overall force is nothing but $-kx$ the spring force minus the viscous damping this one $-b\dot{x}$ plus whatever is the driving force ok. So, that is the overall force acting on this system. Now if you substitute the following that you define k/m as ω_0^2, b/m as γ that is collision frequency and f_0 that is the amplitude of the force driving force F_0 over mass (m) or you can say per unit mass you can write in this equation takes this particular form ok. So, if we assume that the time harmonic driving field then to obtain the frequency domain equation what you can do you can use Fourier transform ok with $e^{-i\omega t}$ time dependence where ω is basically the angular frequency and if you take derivative of $e^{-i\omega t}$ with respect to time you get $i\omega e^{-i\omega t}$. In few books they call it jω. $i\omega$, jω they are the same thing ok.

Thus, we can easily convert the time domain equation this is this is a time domain equation. So, they are the first order partial derivative dot t can be replaced as $i\omega$ and second order derivatives can be written as this is basically $\delta^2/\delta t^2$ that is nothing but $i\omega$ whole square. So, you get $-\omega^2$. So, with that if you do this kind this apply this into this one and real part of this is nothing, but the cos term. So, if you take x divided by cos term and if you take that as capital A you can write this equation in this particular form ok.

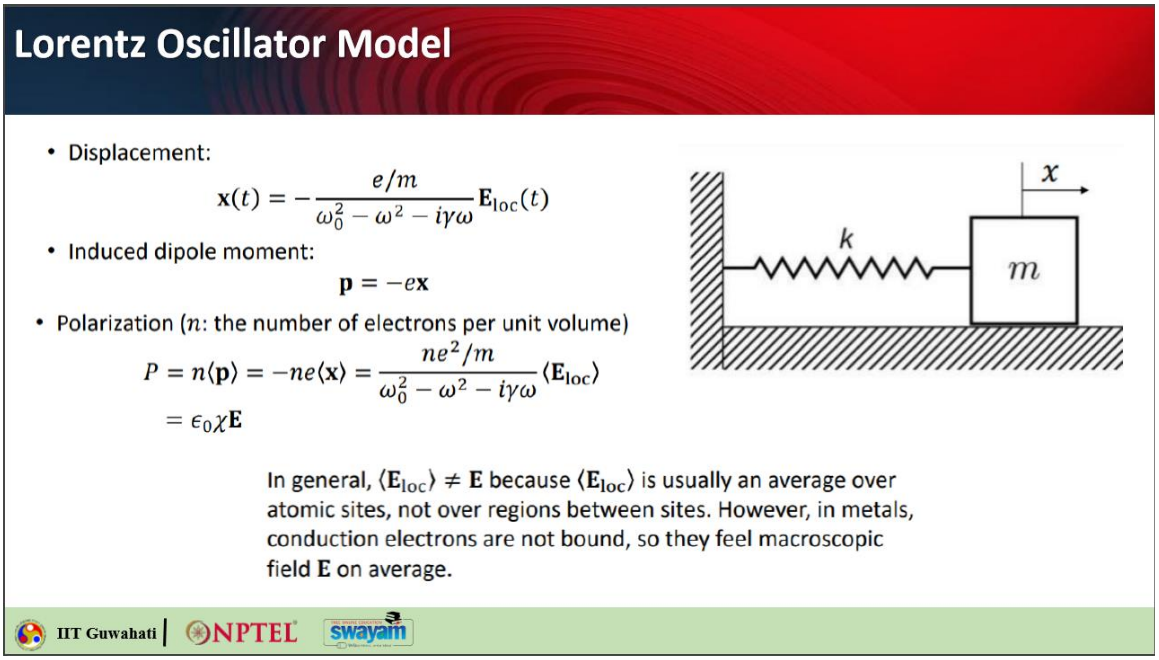
So, I am not I am not going to show each and every term here you can sit down and find out how it is done it is very simple. So, finally, you can write $A(\omega)$ that is how the displacement or the amplitude ok of the displacement is moving ok and this is in frequency domain. So, you have got $\frac{f_0}{\omega_0^2-\omega^2-i\gamma\omega}$. So, this is the equation you have obtained. Now, we can also write in terms of the restoring force what is the restoring force here.



So, restoring force is $-m\omega_0^2\mathbf{x}$ that is the restoring force ok. You can write in terms of damping, the damping is nothing but $-\frac{m}{\tau}$ or you can say $\frac{1}{\tau}$ is basically the collision frequency. So, τ is the relaxation time. So, $-\frac{m}{\tau}\dot{x}$ or you can write $-m\gamma\dot{\mathbf{x}}$ ok. And when you equate this with the ok one more term is left that is the driving force which is the local electric field.

Say if you have got an electric field in say the electron will be polarized by this electric field ok. So, usually what happens when the electric field is in the positive direction the cloud will be moved downwards, when the electric field goes in the downward direction the electron cloud will be pushed upwards because electric field will get tripled now the electron cloud will be get tripled by the electric field ok. So, you can think it in that direction. So, you can write F driving equals minus E is the electron charge and the local electric field this you can convert into the time dependence you can write here. And finally, you can put the equation of motion that is in this form you can write that *ma* this is the force.

So, *ma* acceleration is written as $\ddot{x}$ ok. What are the forces spring force then damping force that was the viscous force in the previous case ok. Here you have seen viscous damping. So, there is a damping force. So, here also you have the damping force component and the driving force ok.



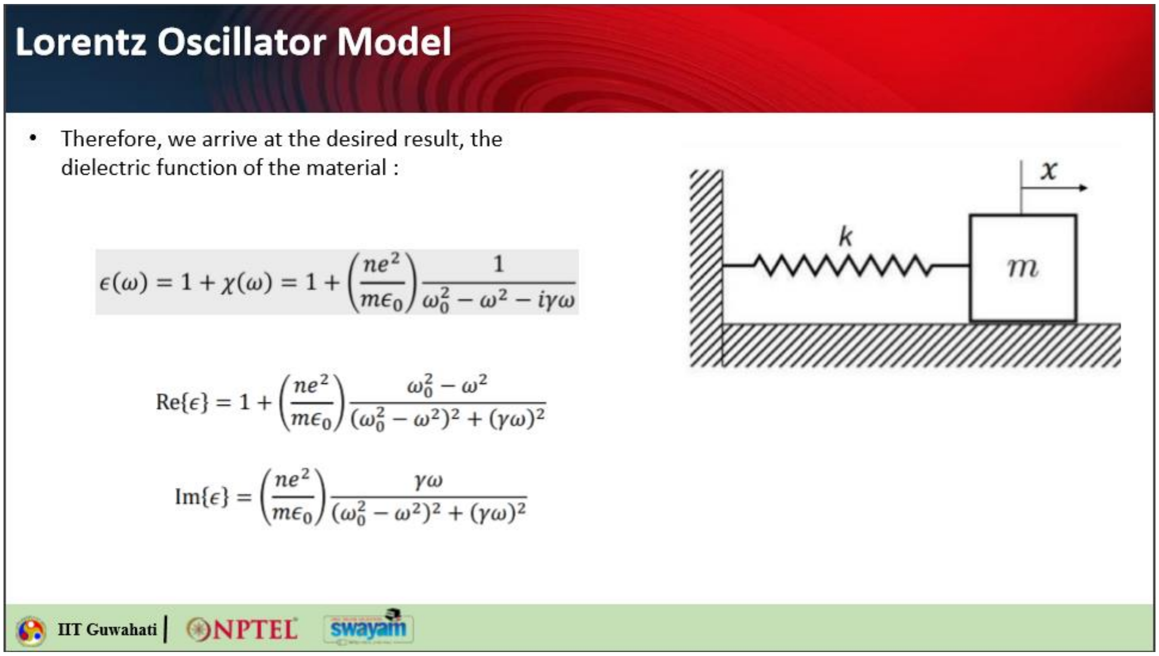
So, these are the three forces when you put them together ok in the presence of an electric field because here the driving force is from the electric field ok. So, when you do this you can find out $\mathbf{x}(t)$ that is the displacement in time domain can be written as this one $-\frac{\frac{e}{m}}{\omega_0^2-\omega^2-i\gamma\omega}$ ok.

And the localized electric field $\mathbf{E}_{\text{loc}}(t)$ is also time varying ok. So, with time it can change accordingly the displacement will also change. So, from that you can actually find out what is the induced dipole moment P.

P is nothing, but $-ex$, e is the electron charge ok. And then if you have n number of electrons per unit volume you can write down polarization which is P as n of this induced dipole moment: $n\langle\mathbf{p}\rangle = -ne\langle\mathbf{x}\rangle = \frac{\frac{ne^2}{m}}{\omega_0^2-\omega^2-i\gamma\omega}\langle\mathbf{E}_{\text{loc}}\rangle$ and you can put this equation here x you can write in terms of this ok and you can obtain this particular equation. So, here what we can see that this polarization can be written as $\epsilon_0\chi\mathbf{E}$, what is χ?

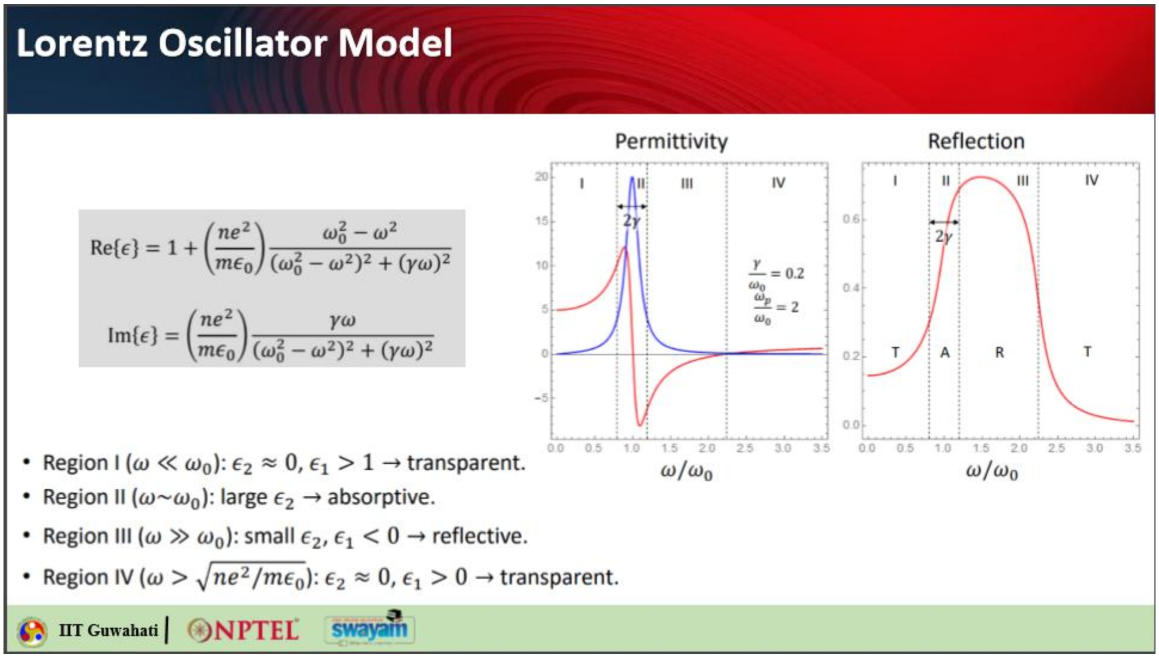
χ is a susceptibility of that particular material ok? Now, remember that in general this local electric field is not same as the applied electric field because this local electric field usually is an average over the different atomic sites not over the region between the sites. So, there may be slight difference, but in metals the conduction electrons are not bound ok.

So, they are they are allowed to freely move around. So, you can feel a macroscopic field of E on average. So, in metal you can safely take that these two things are equal ok. Now from that we arrive at a desired result we know that the value of permittivity is nothing, but 1 plus susceptibility and this susceptibility is we is what we got from here ok. So, this term we have correlated with $\epsilon_0\chi$.



So, you can find out what is χ here and you put it in this particular equation. So, χ is nothing but ne^2, n is the number of electrons per unit volume, e is the electron charge, m is the electron mass and ϵ_0 is the vacuum permittivity and this is what you have got. So, from this you can obtain what is the real part of the permittivity. So, there is an imaginary here. So, if you take the conjugate and multiply it on top and bottom on numerator denominator you can get the denominator completely real and then separate out the two parts.

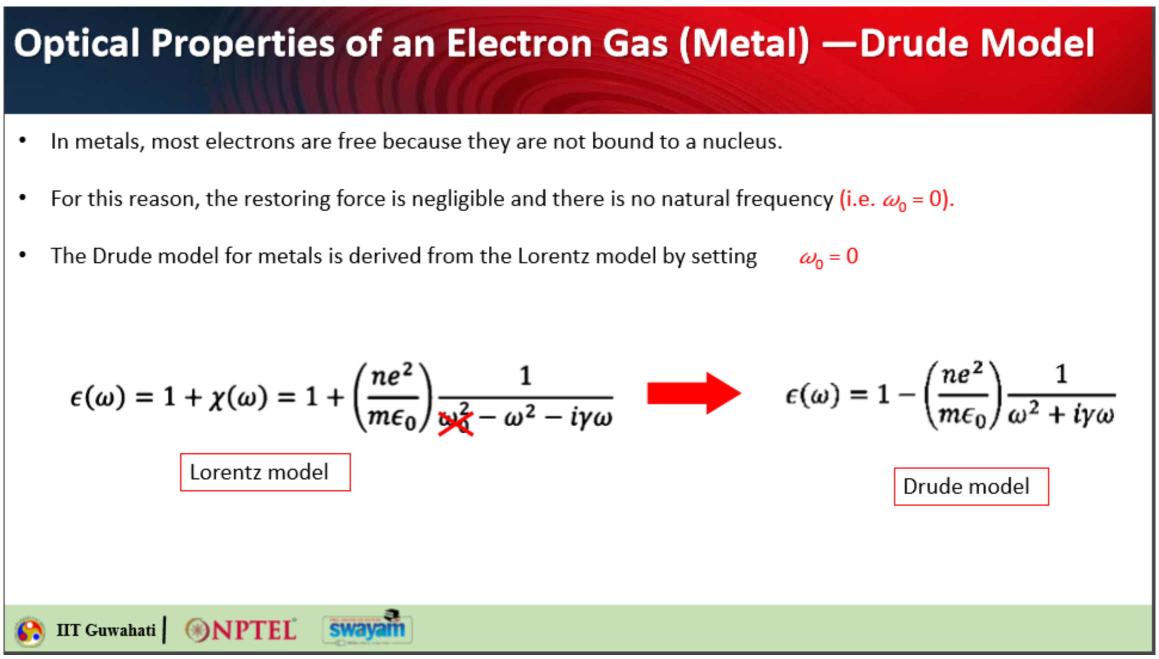
So, bit of sorry a bit of algebraic maths here and you will be able to find out what is the real and imaginary part of the permittivity. With that if you try to plot this as a function of frequency normalized to ω_0 you will see this particular graph. So, the red line here or the red curve corresponds to the real part of permittivity and the blue one is the imaginary part and this is the permittivity and this is the corresponding reflection curve. So, let us see region by region what happens. Let us look into the different regions.



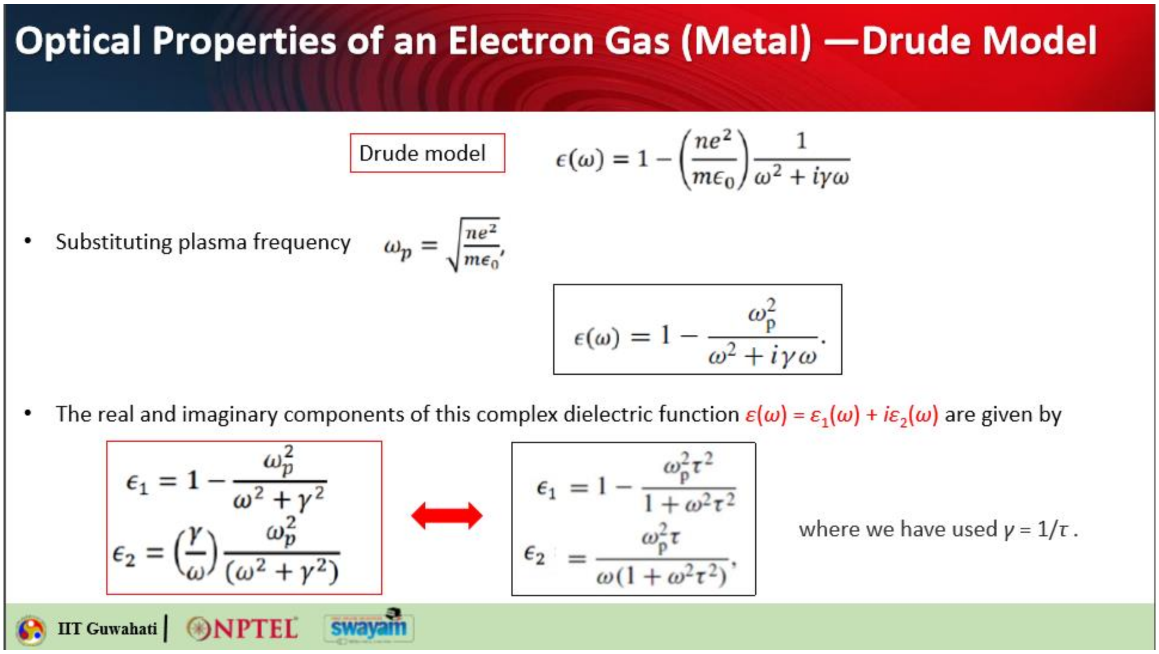
So, first one is region 1 as you can see here where ω is considered much much lesser than ω_0 that is this particular region. So, in that region look at the blue curve is almost 0 that means the imaginary part is almost 0. And what you see that that ε_1 that is the real part. So, epsilon this is called ε_1 and this is ε_2 in this particular plot. So, you can see ε_1 is greater than 1.

So, it is behaving like a transparent material. So, you here also you can see in the reflection transmission regime you can see that this region is basically transmissive region. So, light can easily pass through. Now, let us look into the second region which is basically of the order. So, it is close to 1 it means ω/ω_0 is close to 1 it means frequency is of the order of ω_0.

So, in this case what you see that in this region your material is highly absorptive because the imaginary part of the permittivity is very high. So, you can denote this region as a and what is the good thing here you can see that gamma is the collision frequency or you can say damping constant and this width of this region is basically 2 gamma. In region 3 we are considering ω is much much greater than ω_0 that is this particular region. Here you can see that again the imaginary part is much smaller, but in this case you look at the real part the real part is basically negative. It means in this region the material will be reflective. So, you can see here it is showing reflection ok. And in region 4 you can actually see this is region 4 where ω is much much larger than ω_0 and we have taken ω to be greater than $\sqrt{ne^2/m\epsilon_0}$ that will define this term what is this particular term. And we have seen that in this particular region the imaginary part is almost 0 and the real part is greater than 0 that means the material again becomes transparent. So, these plots are actually done for these values of gamma. So, gamma is taken as 0.2 times ω_0 and ω_p where ω_p is this particular frequency where I know this real part crosses 0 ok. So, this is taken to be roughly 2 ok. So, here it is so ω_p over ω_0 is roughly 2. So, that is the case considered here. Now, with that let us try to see how we describe the optical properties of an electron gas in metal. So, that is the generic model for electron in any material, but in case of metal most electrons are free because they are not bound to any nucleus. In that case one important term that spring term or the restoring force term that becomes negligible. It means there is no natural frequency of oscillation that was actually given by ω_0 ok. So, you can take ω_0 to be 0 in the case of metal. So, Drude model is very simplified version of the Lorentz model where you can nullify this particular term omega naught.



So, if you simply remove this term what you are left with is called Drude model. I hope that is clear. So, Lorentz model actually tells you the electron optical property of electron in any material and then if you put this approximation that bound charges are not there in metal omega naught the natural frequency becomes 0. So, you can actually write $\epsilon(\omega) = 1 - \left(\frac{ne^2}{m\epsilon_0}\right)\frac{1}{\omega^2+i\gamma\omega}$ ok. So, in this case this particular term is taken as ω_p^2 or the plasma frequency. ω_p can be written as $\sqrt{ne^2/m\epsilon_0}$ ok.

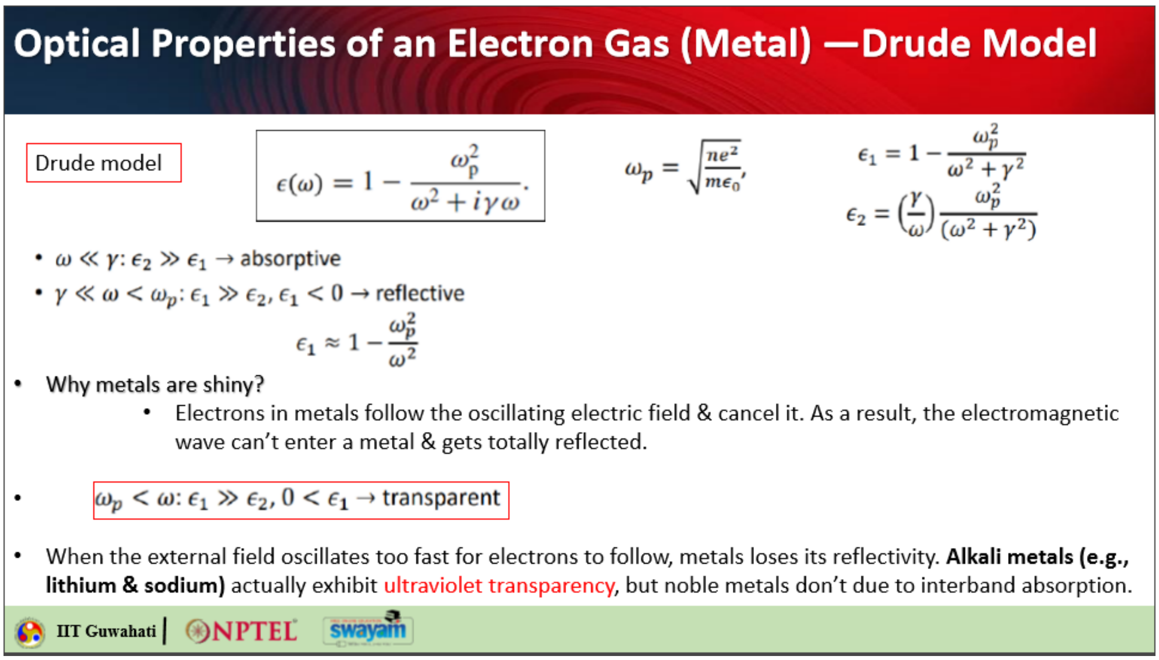


And in that case this equation looks like this. So, $\epsilon(\omega) = 1 - \frac{\omega p2}{\omega^2 + i\gamma\omega}$. So, again this $\epsilon(\omega)$ has got real and imaginary part. So, you can separate out the two components. So, real part or real components are named as ε_1 and imaginary part are named as ε_2. So, this actually tells you about the broadening at the loss or the absorption capability of a particular metal ok.

So, you can also replace gamma by the time constant or the collision time ok and you can replace gamma by 1/ τ and get this particular equation as well. So, they basically cannot be the same meeting. Now, this we have already seen that this is Drude model ω_p is given by this real and imaginary are written like this. Now, let us try to see that in what region how the metal is behaving. So, if you consider omega which is which is much much lesser than the collision frequency ok.

So, τ is what? τ is relaxation time. Now, relaxation time how do you define it? It is the time for the electron between the two collisions when it is roaming around in the lattice. So, inverse of the time is nothing, but the collision frequency ok. So, if the frequency is much much lesser than the collision frequency in that case you will see that your ε_2 is much much larger than ε_1 means the imaginary part will dominate it means the metal behaves like absorptive metal ok. It is it is a absorptive property of the metal in that case. Now, if you take frequency which is much much larger than the collision frequency, but lower than the plasma frequency in that case you will see that your ε_1 is much much larger than ε_2 .

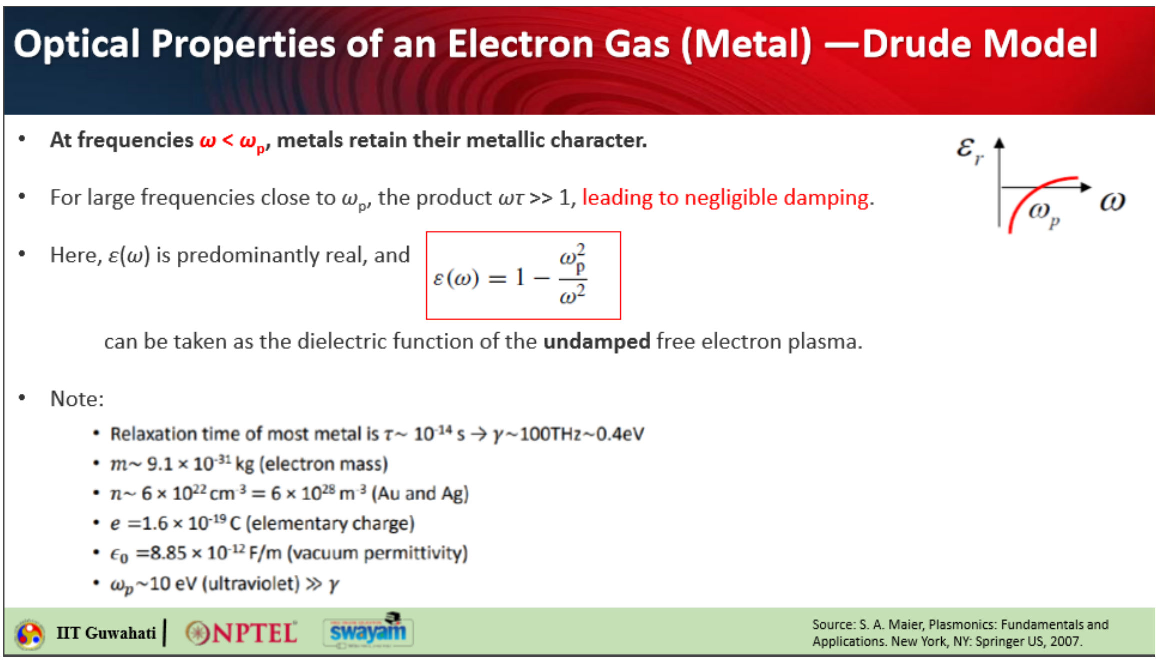
It means it is the material or the metal is not absorbing that much ok. The absorption is minimal and if you find that ε_1 is basically negative it means the metal is basically reflective. And this also tells you why metals are shiny. When you see something shiny it means it is reflecting very strongly. So, electrons in metal follow the oscillating electron field and basically it cancels it.



As a result, the electromagnetic fields are not able to enter the metal ok and gets totally reflected and that is why metals are always shiny. You can look at your silver or gold ring or any other any other metal for that matter and you will see that it is more or less reflecting in the visible wavelengths ok. Now, if the frequency is larger than omega p we have seen what happens right. In that case your ε_1 becomes much much larger than ε_2. It means again the absorption is minimal, but in this case because ε_1 is positive it allows the light or wavelength to pass through that means it will become transparent ok.

When the external field oscillates too fast that is when the frequency becomes larger than the plasma frequency of that particular metal. So, what is this frequency? This is the frequency of the electromagnetic field that is falling on that metal ok. And ω_p is the plasma frequency of that metal. So, the metal loses its reflectivity it is not able to reflect it back ok rather it gives up and it allows light to pass through. So, alkali metals such as lithium, sodium they actually shows this kind of transparency and these are known as ultraviolet transparency.

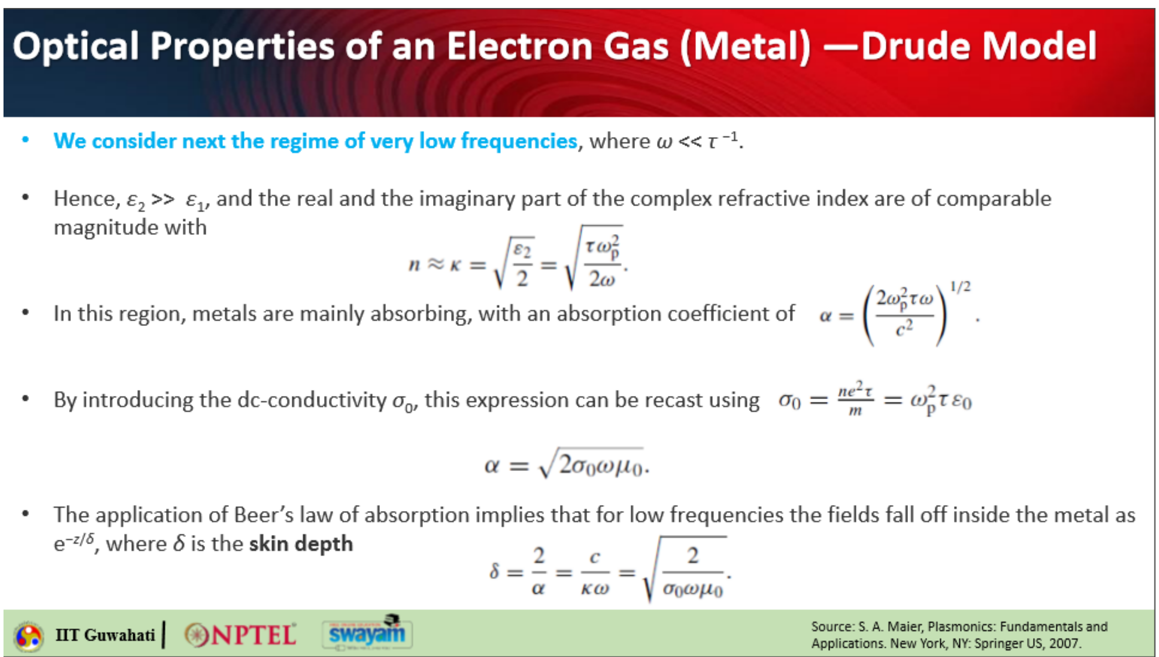
So, this high frequencies are typically in the ultraviolet range, but novel metal they do not show this transparency due to inter band absorption. So, novel metal like gold, silver, copper they will not become transparent at this wavelength because they actually absorb they do not let the light pass through ok. So, this is the diagram of permittivity versus frequency and you can see where it crosses 0 it is basically the plasma frequency. Now, we are actually discussing when the frequency is less than ω_p it means the metal will retain its negative permittivity that is it will be reflective.



So, that is the metallic character. Now, for large frequencies close to ω_p. we will see that the product of $\omega\tau$ is much much greater than 1 that leads to negligible damping ok. And in this case $\varepsilon(\omega)$ will be predominantly real because there is no damping is negligible means that imaginary part is also negligible. So, you can simply write $\varepsilon(\omega)$ as $1 - \frac{\omega_p^2}{\omega^2}$ and this is this particular graph. So, why it is crossing 0 at ω equals ω_p you can see from here.

So, if you take ω equals ω_p this term becomes 1. So, permittivity will become 0 ok. So, this is basically the case as the dielectric function of undamped free electron plasma because there is we are ignoring the damping term here gamma term here ok. Now, these are certain values that is important to kind of remember that the relaxation time of most metal that is τ is in the order of you know 10^{-14} second. So, from that if you find out what is gamma, gamma is basically 100 terahertz or in energy you will see it is around 0.4 eV ok. So, mass of electron charge of electron mass of electron charge of electron density of electron in case of gold and silver is this one 6 into 10 to the power 28 per meter cube you can see how many electrons are there in 1 meter cube of gold and silver. Permittivity and all these values are already known to you. Now, what is this omega p turns out to be 10 eV. So, it is typically in the ultraviolet range ok and you can see this is much larger than the collision frequency ok.

So, you can actually see this approximation is fine ok. Now, if we consider the regime of low frequencies it means where ω is much much lesser than tau inverse or gamma. If we consider that particular case here you will see that you know your imaginary part is becoming much much larger than the real part. That means, the real and imaginary part of the complex refractive index are now also getting comparable. So, from the imaginary part real and imaginary part of the permittivity you are also able to find out the real and imaginary part of the refractive indices that can be written as n and kappa ok. And you can see that they are more or less having equal values in this kind of case.



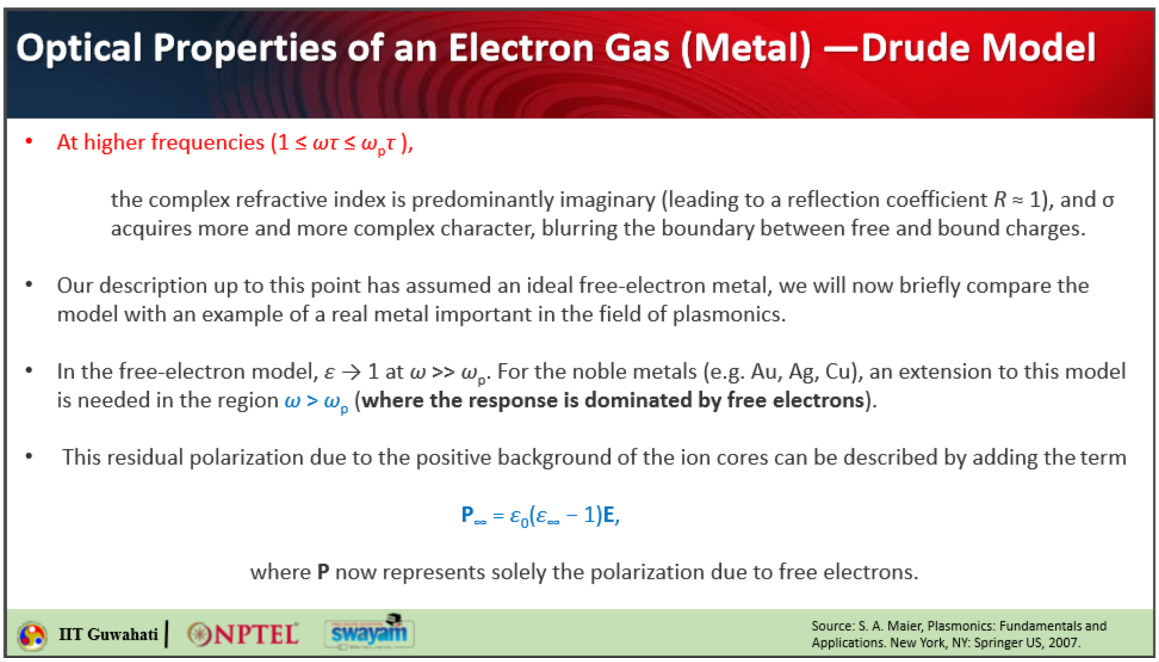
So, it is $\sqrt{\frac{\varepsilon_2}{2}}$ and you can write it as $\sqrt{\frac{\tau\omega_p^2}{2\omega}}$ ok. And in this region the metal is mainly absorbing because ε_2 the imaginary part is much larger and it can be defined using a absorption coefficient α that is given as $\left(\frac{2\omega_p^2\tau\omega}{c^2}\right)^{1/2}$. We are not going to the derivation of each of this because that will be time consuming. I just want you guys to understand the physics that in what region the metal is reflective in what region it is absorptive and that is how ok.

So, that will tell you the overall behavior of the metal in a particular electromagnetic field. Now, by introducing the DC conductivity σ_0 you can write σ_0 as $\frac{ne^2\tau}{m}$ ok.

And that can also be correlated to the plasma frequency by this formula $\omega_p^2 \tau \varepsilon_0$. So, once you know that you can put this into your absorption coefficient formula and you can find out that α is $\sqrt{2\sigma_0 \omega \mu_0}$ ok. Now, this α actually allows you to find out the skin depth ok. By application of Beer's law ok or Beer-Lambert law of absorption it implies that for low frequencies the field falls off inside the metal as e to the power minus z by delta.

So, that is happening in low frequencies. So, e to the power minus z by delta what is delta? Delta is the skin depth. So, skin depth delta is defined as 2/ α and you can write it as $\frac{c}{\kappa\omega}$. So, kappa is this imaginary part ok in the refractive index and you can finally write it as $\sqrt{\frac{2}{\sigma_0 \omega \mu_0}}$ ok.

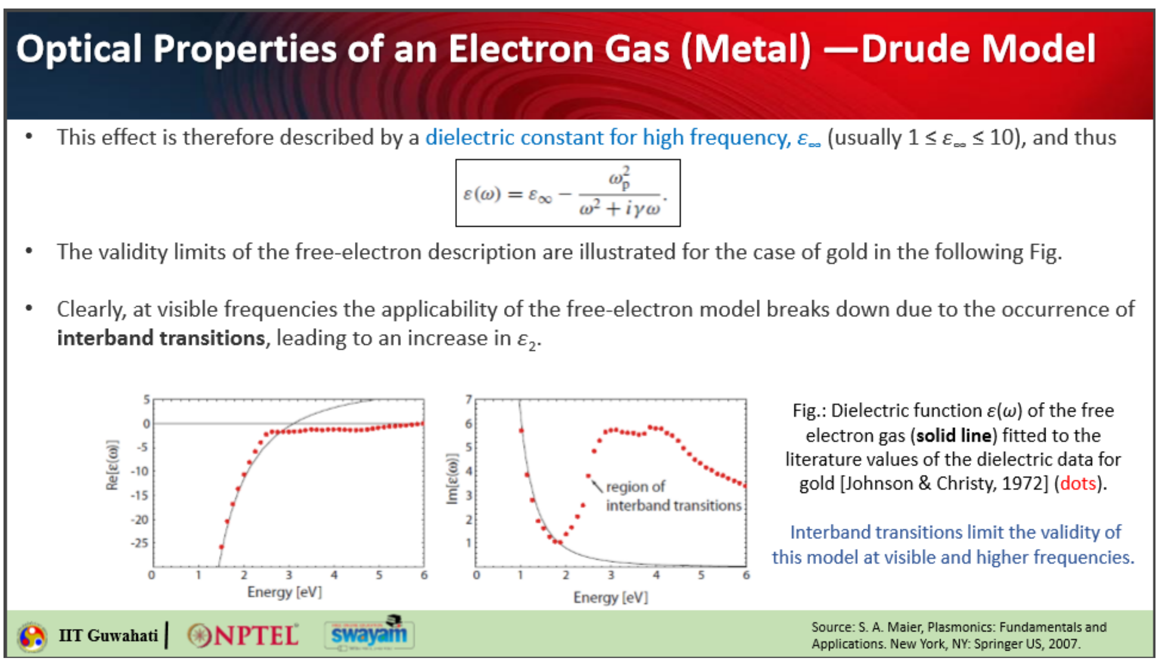
Now, if you look at higher frequencies that is in the case when you know $\omega\tau$ is larger than equal to 1, but smaller than it could be smaller than equal $\omega_p \tau$. If this is the range of the frequencies, you will see the complex refractive index is predominantly imaginary ok.



It means in that case reflection coefficient R will be almost 1 ok. And sigma that is the conductivity part acquires more and more complex character and that will blur the boundary between the free and bound charge ok. So, more or less you can actually use the same model for both the cases. Now, our discussion up to this point has assumed that in ideal free electron metal we will briefly compare the model with an example of real metal important in the field of plasmonics. So, let us take one example. So, in the free electron model we know that epsilon goes to 1 when your frequency is much much larger than ω_p .

So, you can actually take noble metals like gold, silver, copper ok. An extension to this model is needed in the region when omega is greater than ω_p ok. So, this is the region where the response is mainly dominated by the free electrons. So, this residual polarization due to the positive background of the ion cores they can be described by adding the term $\mathbf{P}_\infty$ ok. So, that can be written as $\varepsilon_0(\varepsilon_\infty - 1)\mathbf{E}$. So, what is P here this is this represents solely the polarization due to the free electrons ok.

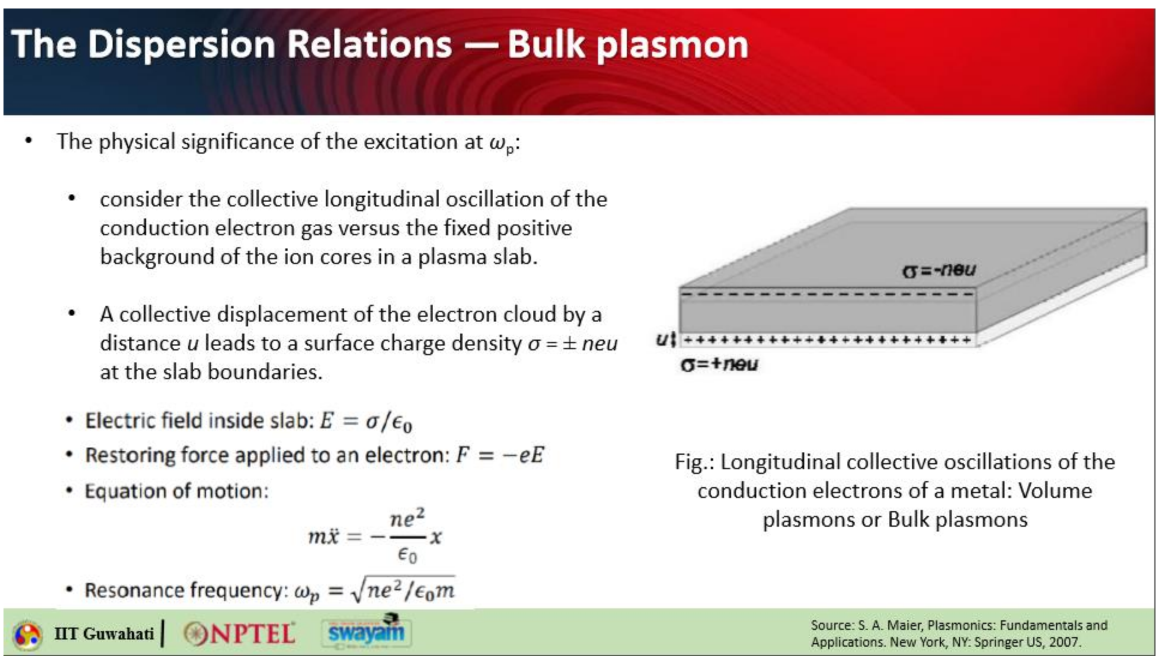
So, what you are finding out you are able to find out the contributions from both free electrons and the background ok. So, the effect therefore, is described by a dielectric constant of high frequency which is called epsilon infinity ok. And usually the value of ε_∞ is from 1 to 10. So, instead of 1 you can actually so 1 minus this was for the free electrons only, but the at high frequency there is a background because of that you will get this permittivity ok from the positive background.



So, you can write that as ε_∞ ok. So, this is the final equation that should describe the permittivity properties of electrons in metal. So, this is the Drude model ok. So, you can see that the validity limits of the free electron description in the case of gold. So, this particular curved line and these dots are the experimental ones. So, you see after a particular frequency range ok this free electron description which was with 1 minus this that fails very badly ok.

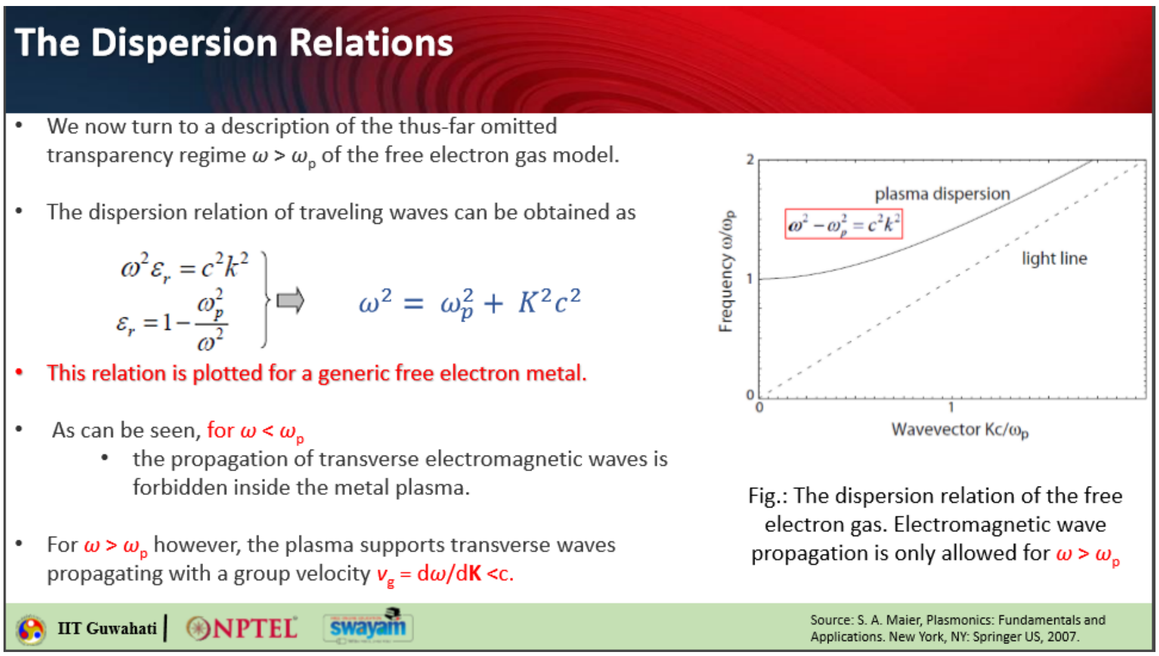
So, till here it is fine, but after that the background plays an important role. Similarly, in the case of imaginary part also up to this the free electron 1 means $1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ ok that is doing descently well, but as soon as the region of inter band transitions enter ok you are not able to see a good fit with the experimental data and the free electron description. So, you should actually use this particular model that covers the entire thing. So, these red dots are basically taken by experimental data measured by Johnson and Christy in 1972. This is a very old paper, but this is one of the most highly cited papers in this field because every they are the first one to measure the dielectric constant of gold and silver and copper ok and this was the starting point of Plasmonics research. Now, inter band transition limits the validity of this model at visible and higher frequencies that you have seen.

So, higher the frequency this model will get increasingly bad. Now, clearly at visible frequencies the application of free electron model breaks down due to the occurrence of inter band transition and this inter band transition actually gives rise to ε_2 ok. Now, with that let us move to the dispersion layer. So, now how to describe the permittivity or the material property of any dielectric using Lawrence model and we have seen how to do it for Drude metals ok. And now let us look at bulk Plasmon ok. So, what are the dispersion relation of bulk Plasmon? So, the physical significance of the excitation at ω_p.



So, let us consider the collective longitudinal oscillation of the conduction electron gas versus the fixed positive background of the iron cores in a plasma slab. So, this kind of slab you can think of where the electrons are kind of electron cloud is kind of dislocated by u in that case sigma(σ) can be the charge density ok you can you can write it as $\pm neu$ ok. So, a collective displacement of the electron cloud by distance u leads to a surface charge density sigma given as $\pm neu$ at the slab boundaries. Now, the electric field inside the slab can be defined as $E = \sigma/\epsilon_0$ ok. And the restoring force applied to an electron is F and that can be written as minus e that is the charge of electron and the electric field E that has been applied.

So, from that you can also write down what is the equation of motion here it is *ma* is this particular force ok. So, $m\ddot{x}$ can be written as $-\frac{ne^2}{\epsilon_0}x$ ok. So, resonance frequency is nothing, but ω_p that is the plasma frequency which is $\omega_p = \sqrt{ne^2/\epsilon_0 m}$ fine. Now, let us take time to describe the transparency regime which is ω greater than ω_p for the free electron gas model.



Now, the dispersion relation for the travelling wave can be obtained in this form. So, you have omega epsilon r omega square epsilon r equals c k ok. So, what is this particular formula if anyone is able to guess this formula is nothing, but what is the dispersion relationship omega equals ck right in vacuum. Now, omega equals c by or c by n into k that will be the relation in any medium with refractive index n. So, instead of n you can also write it in terms of permittivity you can write square root of epsilon right. So, in that case if you take square on both sides you can get this equation which is omega square epsilon r equals c square k square.

And you also already know that in this region there is this transparency region you can actually write epsilon r that is the relative permittivity can be written as $1 - \frac{\omega_p^2}{\omega^2}$. So, when you put this one into this equation you will get this is your dispersion relation of the travelling wave because this is the transparency region ok. So, the region is plotted for a generic free electron metal. So, you can see this is this particular dispersion relation.

So, this is the normalized frequency curve. So, omega over ω_p and this is the wave vector K c over ω_p and this is basically light line ok. Now, as can be seen when omega is less than ω_p that is in this region ok. So, propagation of transverse electromagnetic wave is not allowed ok into the metal plasma. And when it is above this particular region ok plasma supports transverse wave propagating into the inside the metal and what will be the group velocity in that case? It is vg which is d omega by dk and you can say that it is less than c.

The Dispersion Relations

- The significance of the plasma frequency ω_p can be further elucidated by recognizing that in the small damping limit, $\varepsilon(\omega_p) = 0$ (for $\mathbf{K} = 0$).
- This excitation must therefore correspond to a collective longitudinal mode.
- In this case, $\mathbf{D} = 0 = \varepsilon_0\mathbf{E}+\mathbf{P}$.
- We see that at the plasma frequency the electric field is a pure depolarization field, with $\mathbf{E} = -\mathbf{P}/\varepsilon_0$.

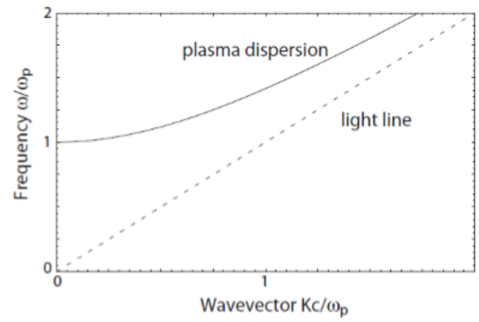


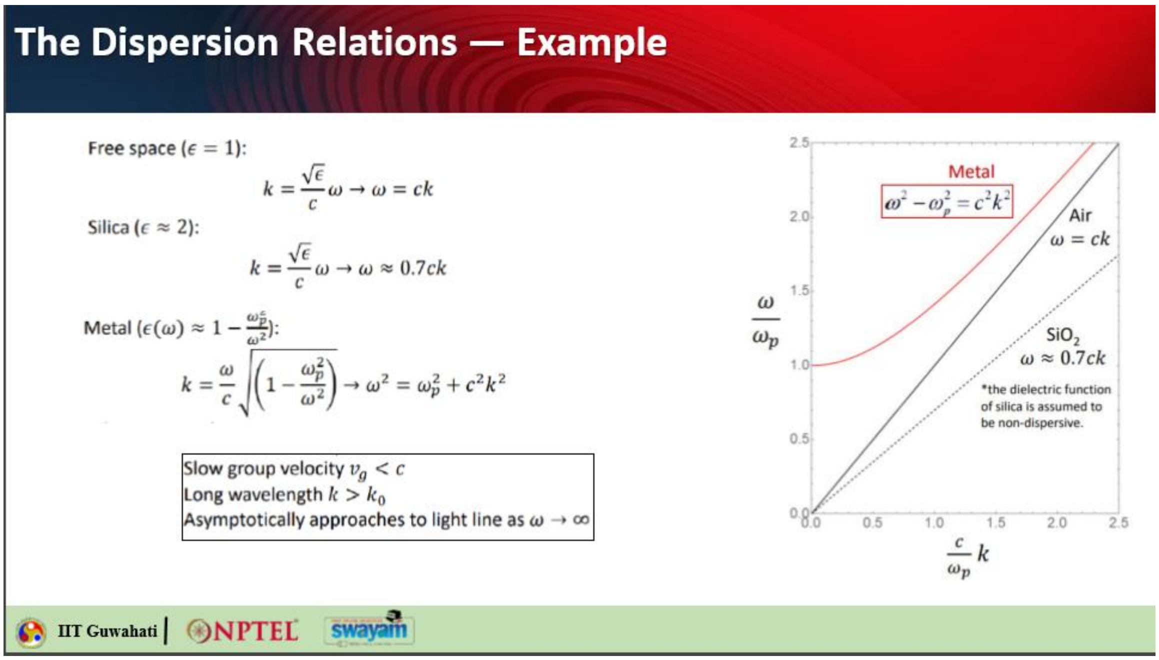
Fig.: The dispersion relation of the free electron gas. Electromagnetic wave propagation is only allowed for $\omega > \omega_p$

Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, it is all in this particular case. So, this is the plasma dispersion ok. So, it is clear that electromagnetic wave propagation is only allowed for omega greater than ω_p this is the boundary. So, this is where exactly omega equals omega p. So, anything above that the electromagnetic wave propagation is possible. Now, the significance of the plasma frequency omega p can be further elucidated by recognizing that in the small damping limit you can write that $\varepsilon(\omega_p)$ for K equals 0 ok.

So, this is the case right. So, both are 0 in this case. This excitation must therefore, correspond to a collective longitudinal wave. And in this case you can write that $\mathbf{D} = 0 = \varepsilon_0\mathbf{E} + \mathbf{P}$. So, from that we will see that at the plasma frequency the electric field is a pure depolarization field that is electric field will be -P/ε_0 ok. And that is the essence of plasma frequency.

Now, this was the dispersion relation I was talking about in the previous case. So, for different dielectric like free space where you can take epsilon equals 1 these are basically epsilon r ok do not mistake it these are epsilon r ok. I am just writing epsilon here as a short form. So, $k = \frac{\sqrt{\epsilon}}{c}\omega$. So, if you take epsilon equals 1 for air this is ω equals ck this is this particular solid straight line. Now, if you consider silica and you want to see what is the propagation inside that ok and find out the dispersion you put epsilon equal to 2 here you get this particular dispersion relation ok.



So, omega is around *0.7ck* that is this particular dotted line ok. So, in this case remember that the permittivity or the dielectric function of silica is assumed to be non dispersive means it is assumed to be a fixed value for all the wavelengths, but actually it is not like that. However, the importance here is to show that if you have a dielectric constant larger than air the slope of the dispersion curve is reduced. And in the case of metal you can consider this one where $\epsilon(\omega)$ is $1 - \frac{\omega_p^2}{\omega^2}$ ok. So, if you put this value here you actually get this one you take square on both sides.

So, you will find $\omega^2 = \omega_p^2 + c^2k^2$. So, this is the dispersion relation in metal. So, this is how the wave propagation has to satisfy this condition that you have seen before that ω has to be greater than ω_p for the plasma to be able to propagate ok. So, slow group velocity obviously because you will get v_g which is lesser than c you will also find wavelengths which are longer than the wavelength of light because you are able to get k values for the same one. So, you are getting k values larger than k naught and asymptotically this will approach the light line when ω goes to infinity. So, somewhere down towards infinity these 2 lines will kind of merge. So, what is important to remember here is that you will get slow group velocity. How do you get group

velocity? You take doh omega by doh k. So, if you calculate that you will see that in this case it is c in this case it will be smaller than c. So, metal will have or support a lower or slower group velocity and for the same frequency it will also support long wavelengths because the wave number is larger ok.
So, k is greater than k naught ok. k naught is the one in vacuum. So, k will be the one in the metal ok. So, k is larger to check this looks like there is a typo but yeah. So, it also can support a long wavelength ok. So, with that we will stop here. So, in the next lecture we will look into the surface plus bond polar item fundamentals and if you have got any query regarding this lecture you can drop an email to this particular email address. Thank you.