Course Name- Nanophotonics, Plasmonics and Metamaterials

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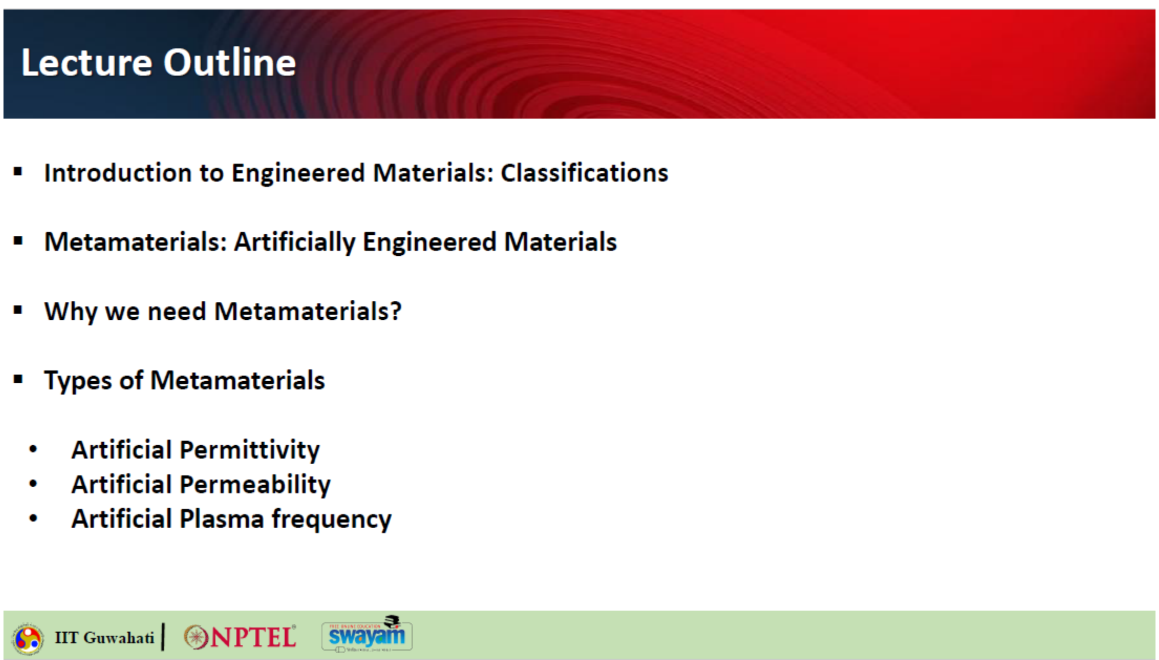
Institute Name- Indian Institute of Technology Guwahati

Week-08

Lecture -22

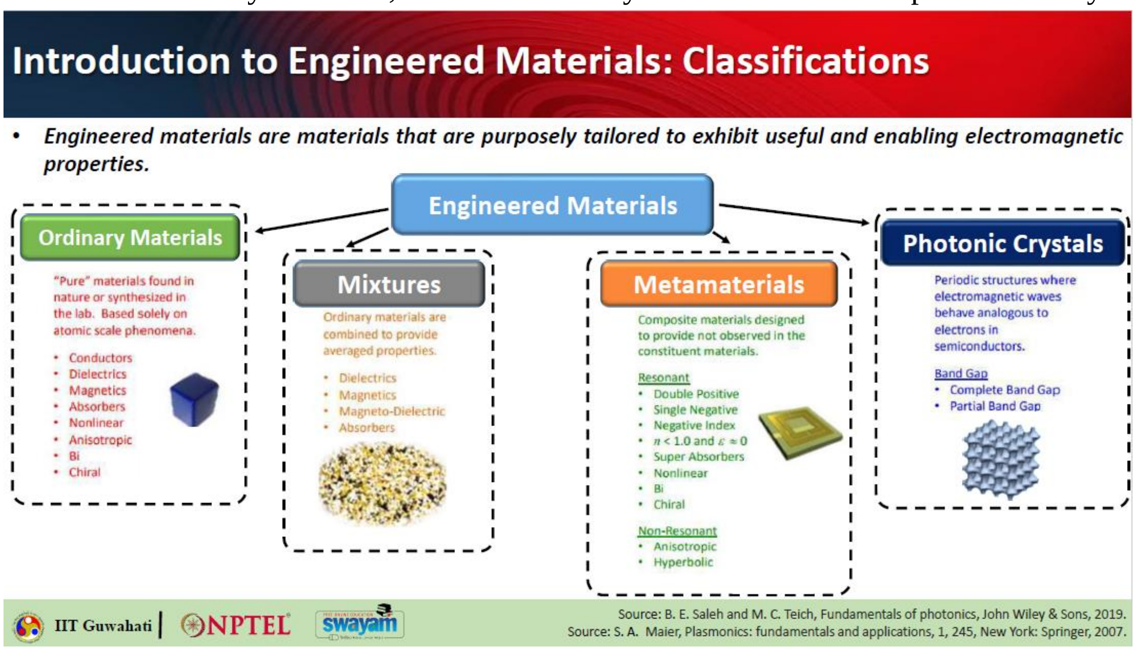
Hello everyone, welcome to lecture 22 of the online course on Nanophotronics, Plasmonics and Metamaterials. Today's lecture will be on Fundamentals of Metamaterials. We have already covered some basics of metamaterials and given you an overview of why metamaterials are needed. So, today we will actually go into bit more details of it and see how do we actually create materials with customizable properties which are dependent on the physical structure rather than the chemical composition of the materials that are being used for creating metamaterials. So, here is the lecture outline as you can see. First we will show you the engineered materials classification, then we will talk about metamaterials.



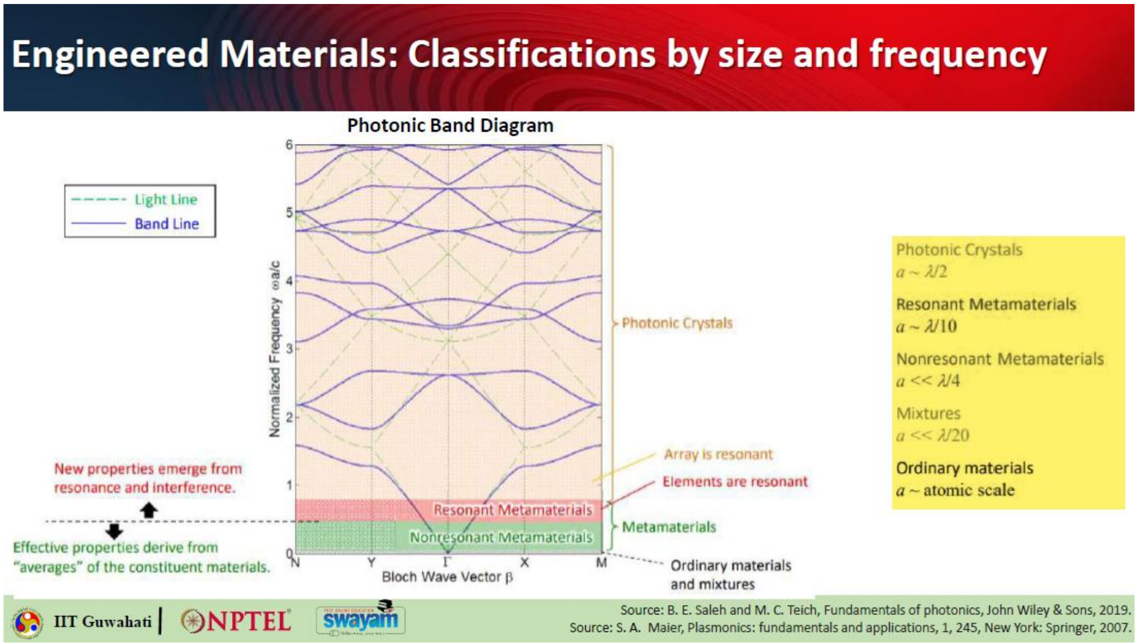
We will also discuss why we need metamaterials and then we will show you couple of designs of different types of metamaterials which has got artificial permittivity, artificial permeability and artificial plasma frequency. So, when we look into engineered materials, so engineered materials are basically those materials which are purposefully tailored to exhibit some useful and enabling electromagnetic properties. So, you can see that engineered materials can be put into four buckets. The first one is ordinary materials.

So, these are basically the pure materials which are found in nature or they are synthesized in lab. So, based on the atomic scale phenomena, the properties are defined. So, you can classify them as conductors, dielectrics, magnetics, absorbers, non-linear, anisotropic, biisotropic or chiral. Then you have another type of engineered materials which is basically a mixture. So, the name tells you that this is basically the ordinary materials are now combined to get some average property.

So, they can give you different type of dielectrics, magnetics, magneto-dielectrics and absorbers. And then we have also seen that photonic crystals. So, these are basically engineered structures which are periodic in nature. So, you are talking about periodic structures where the electromagnetic waves behave in a manner, the electrons behave in a electronic crystal. So, that is why it is called photonic crystal.



So, we have to understand that here the change in the permittivity in the photonic crystal is of the order of the wavelength of light. So, when you go to sub-wavelength, you can also get far more control on the properties of the materials and how you can actually design the properties that you want. So, that is the field of metamaterials. So, meta is beyond or it is like artificial. So, beyond natural materials, you can actually get metamaterials.



So, they can be classified in two manners. One is resonant, another is non-resonant. So, the non-resonant ones are like anisotropic and hyperbolic metamaterials. They are typically broad-panned in nature. But whereas you go for resonant metamaterials, resonance means there will be a sharp Q resonance.

So, there is a Q factor associated. So, you can make a double positive like you can have ordinary materials like double positive materials, but that some property which is not never found in nature, that kind of metamaterial is also possible. You can have single negative like you can have either permittivity or permeability negative. And then you can have double negative or you can get negative refractive index material. You can have n less than 1 or epsilon near 0.

You can make super absorbers, non-linear and other types of metamaterials. So, if you want to see the metamaterial, where it stands, you can actually bring back the photonic band diagram that we have seen in a couple of lectures before. So, you can actually see that this axis, the y axis is basically the normalized frequency. It has also got an element called a and this a is basically the size of or the lattice constant you can say or it is basically the size factor. So, if you see that when it is very small, almost atomic scale that is where the ordinary materials and mixtures come into picture.

So, you can actually see here. So, ordinary materials a is basically in atomic scale. So, inter atomic distance is typically, Armstrong, which is 0.1 nanometer in that scale. So, when you make mix those ordinary materials together, you get mixtures and those mixtures are also typically having the similar or slightly larger length scale.

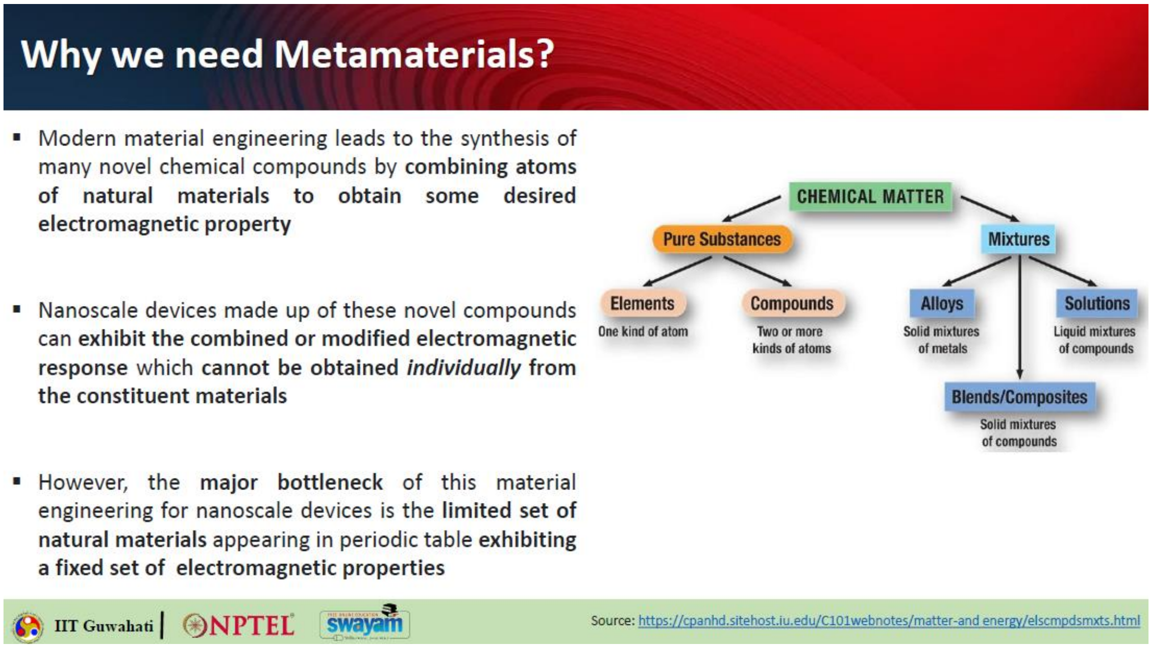
So, you can actually take that, mixtures are much much smaller than lambda by 20. What is lambda? Lambda is basically the wavelength of the interacting light. So, these are the cases where you can see along this y axis, you have ordinary materials and mixtures. Now, up to this case where it is like, the normalized frequency is typically half, that is the case. So, below this, you can say that the metamaterials are basically non resonant metamaterials.

That means the effective properties are basically derived as average of the constituent materials. And above this line, dashed line, you can say that the new properties that emerge from the resonance and interference effects. So, that is where, you know, the resonant metamaterials come into picture. Now, if you try to put into, you know, try to put a length scale, you will see that the non resonant metamaterials a is typically much much smaller than lambda by 4. Okay, and resonant materials, the metamaterials a is of the order of lambda by 10.

And what happens in photonic crystal, you already know, it is of the order of lambda or lambda by 2. So, this is the regime of photonic crystal. This is the regime where, you know, the light line and the blue line, they are overlapping to each other. You remember, if you remember from the from the photonic band diagram that we have discussed before, so these are basically the region where light line and the band line are matching. So, what is what is basically light line? Light line is basically when you take a crystal and approximate its properties based on some average value.

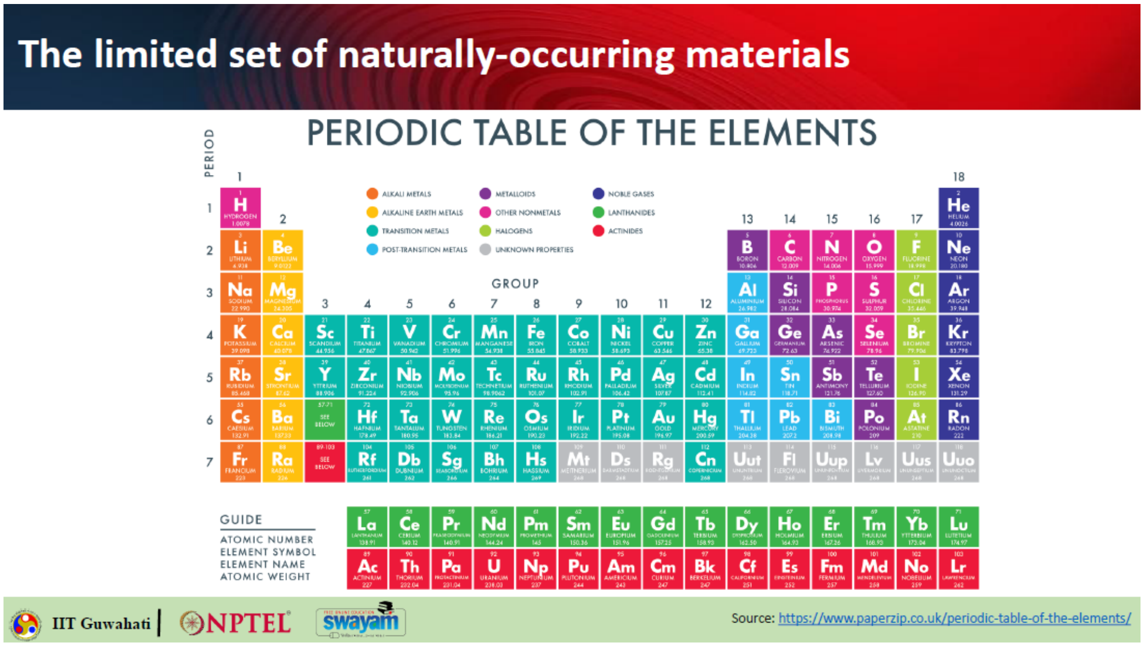
So, it means until this region, okay, this this particular case, okay, light is not able to see the lattice separately. Okay, so this is where the photonic crystal effects will come in. Okay, before that light actually sees everything like a averaged out property. Okay, so this is the case. And if you look into this line finally, you will actually say that, see that this is basically the boundary between the non-resonant and the resonant metamaterials.

So, when you go to resonant metamaterials, their new properties emerge from the resonance and the interference effects. So, keep this table in mind that will help you to understand the length scale when someone talks about photonic crystal, resonant metamaterials, non-resonant metamaterials, mixtures and all these things. Now, the question comes why do we need metamaterials? Now, if you look into the modern material science and engineering, they let you synthesis of many novel chemical compounds by combining the atoms of this natural materials to obtain some desired electromagnetic property. So, if you take any chemical matter, you can see there are pure substances like elements where only one kind of atom is there or you can have compounds where two or more types of atoms are there. And then you can also, so these are pure substances and on the other side you can have mixtures.



So, mixtures can be like, alloys where these are basically solid mixtures of metals, there can be solution which are basically liquid mixtures of compounds and there can be blend where these are basically composites, these are solid mixtures of compounds. So, these are different types of, mixture you can say where people have tried with different recipe to get different properties. But more or less the properties are basically coming from one of the constituent materials. There is nothing, that comes out completely random or unique which does not belong to any of the constituent materials. So, nanoscale devices made up of these novel compounds, they actually exhibit combined or modified electromagnetic response, okay, which cannot come from individual, which cannot be obtained individually from the constituent material.

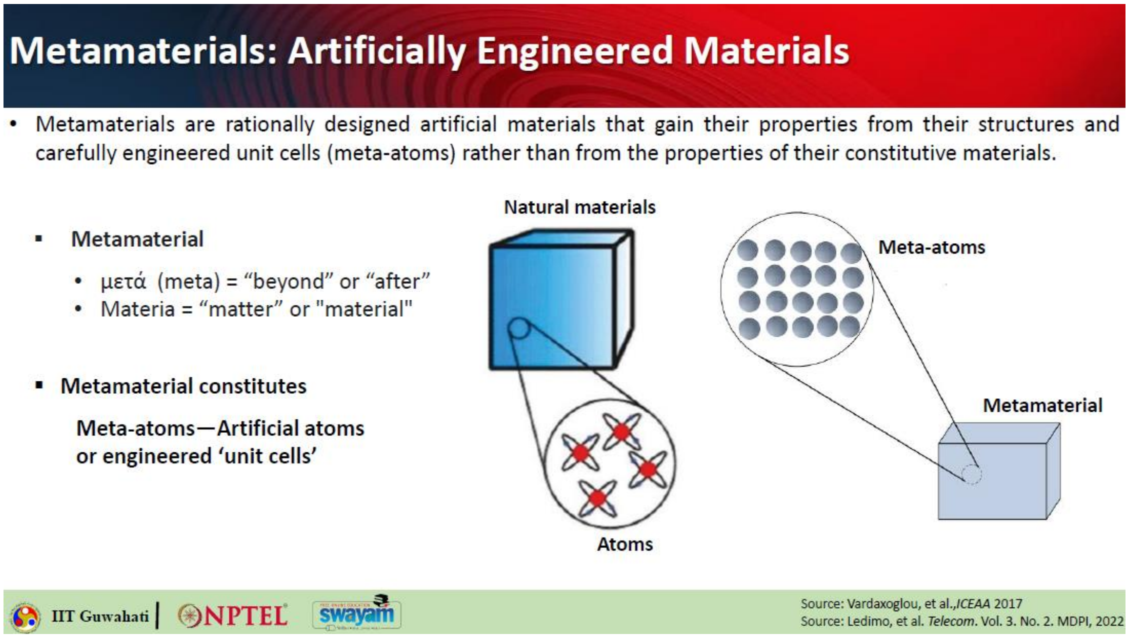
So, these mixtures actually give rise to some averaged or combined or modified properties, okay. Now, the major bottleneck in this kind of material engineering is that you have a very limited set of natural materials that you can use for some application. So, the number of trials and the combinations that you can try that also becomes very limited. So, if you remember this periodic table from your school days, okay, so only few materials you can use for one particular application. So, the combination, permutation combination of different mixtures are also very limited.



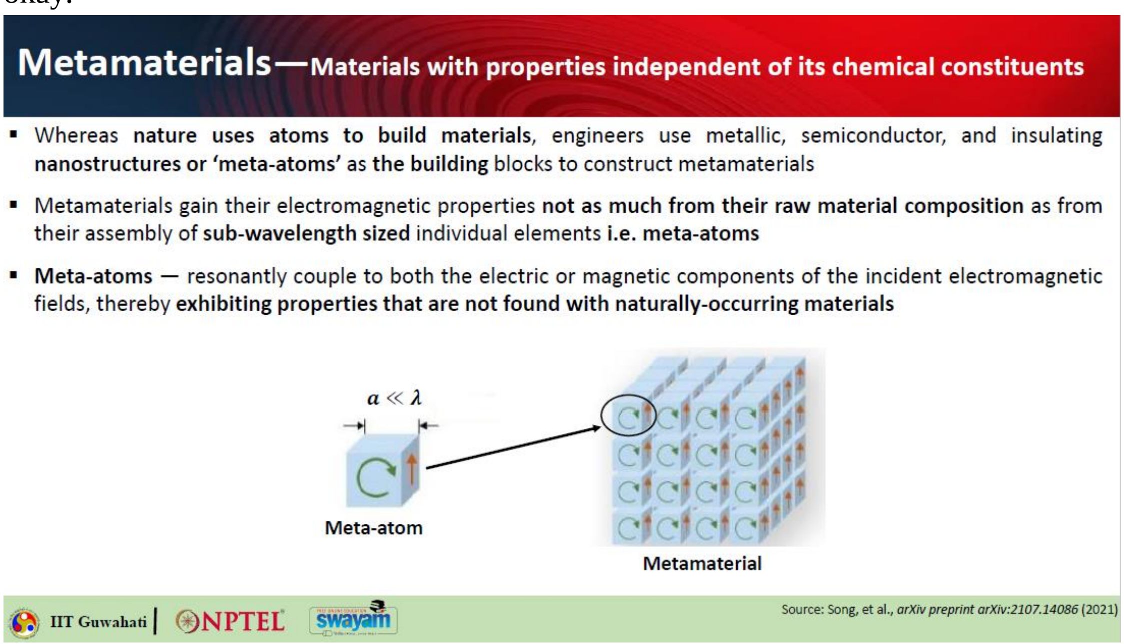
And this is where, metamaterials will come to your help because these are artificially engineered materials, okay. So, metamaterials, what are the definition? You can say metamaterials are rationally designed artificial materials that gain their properties from their structures and carefully engineered unit cells. So, here the unit cells behave like the artificial atom. So, you can call it as meta atom, okay. And these properties do not come from their constituent materials.

So, that is very important. And as I mentioned before, meta is a Greek word that means beyond or after and materia is nothing but materials. So, it is actually beyond natural materials. So, this, this diagram actually shows you the analogy between atoms and meta atoms, okay. Atoms constitute natural materials. Similarly, meta atoms, okay, these constitute metamaterials and you have the complete control on designing each of these meta atoms depending on the required property, okay.

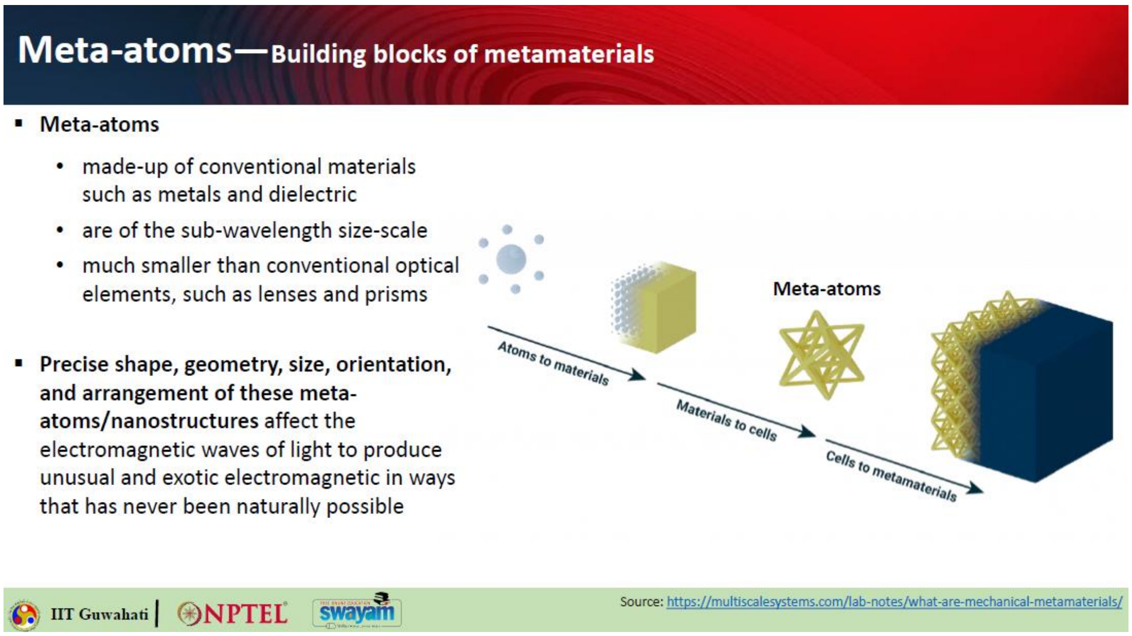
So, as nature uses atoms to build any materials, engineers use metallic semiconductor or insulating nanostructures in the form of meta atoms or unit cells to construct metamaterials. Now, this is the most important point. Meta materials gain electromagnetic properties not as much from their raw material composition as from their assembly of sub wavelength size individual elements which is meta atoms. So, the property of the metamaterial mainly depends on the structural design of the meta atom, not very much on the property of the material used for that structural design. The structural design material will play some role but not a major role.



The structure itself plays the most important role, okay. So, meta atoms can resonantly couple to both electric and magnetic components of the incident electromagnetic radiation and you can actually tune them to exhibit some properties that are not found in naturally occurring materials, okay. So this is how metamaterials will look like and this will be each meta atom looking like. Look at the scale here, a which is the size of this meta atom which is much much smaller than the wavelength of the incident electromagnetic radiation or light. So, this curve shows you nothing but the magnetic moment, okay and this shows you the dipole moment, okay electric dipole moment, okay.



So, we understood that meta atoms they are basically made of conventional materials such as metal and dielectric. So, you started with atoms from that you got some materials and that materials can be used to construct some meta atoms where the structural design plays the most important role, okay. And then when you put this meta atom and create a metamaterial which is either a 2D or a 3D array of this meta atoms then a completely new property come out which is not belonging to this atom or this kind of materials, okay. So, important factor is that these are sub wavelength in size, okay and they are much smaller than the conventional optical elements such as lenses, prism and all, okay. So, the precise shape, geometry, size, orientation and arrangement of these meta atoms or nanostructures can affect the electromagnetic waves, okay of light and they can produce some unusual and exotic electromagnetic response which was never seen before.

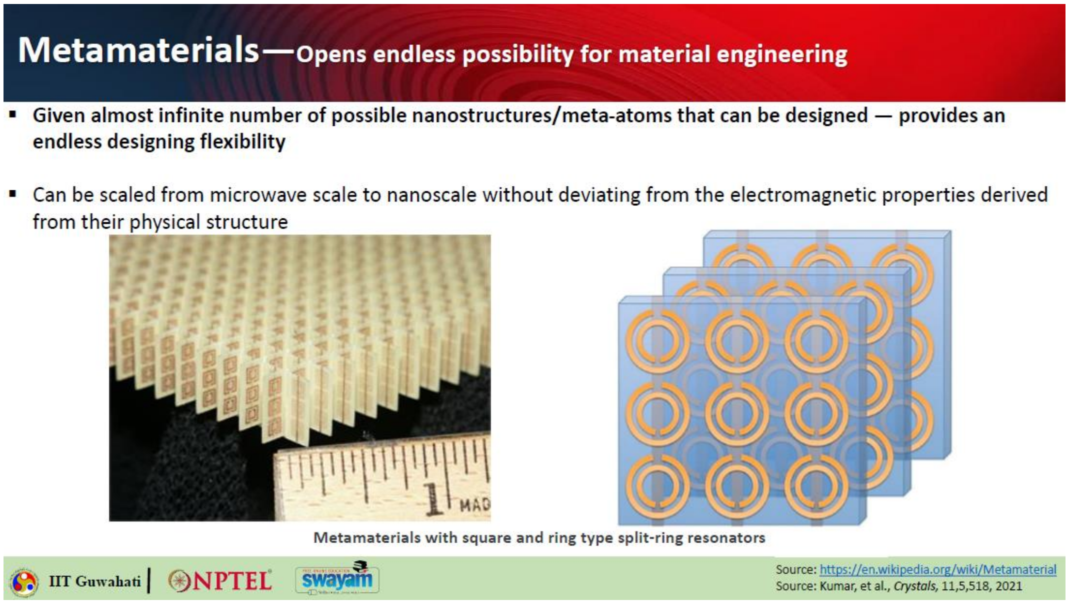


So this is one example of meta materials. Now, I believe now it answers the question that why we need meta materials. Now you can understand that you can change the design of this meta atom and can create any property. So that almost gives you infinite number of possibilities of designing meta atoms. So, you can actually create any number of metamaterials.

So this is one particular example of a meta material that has been made and fabricated, okay. So you can see the length scale here. So this is for a microwave frequency. So this is basically as you can see there is a rod and there is a square type of split ring resonator. This is again the same thing with a circular split ring resonator.

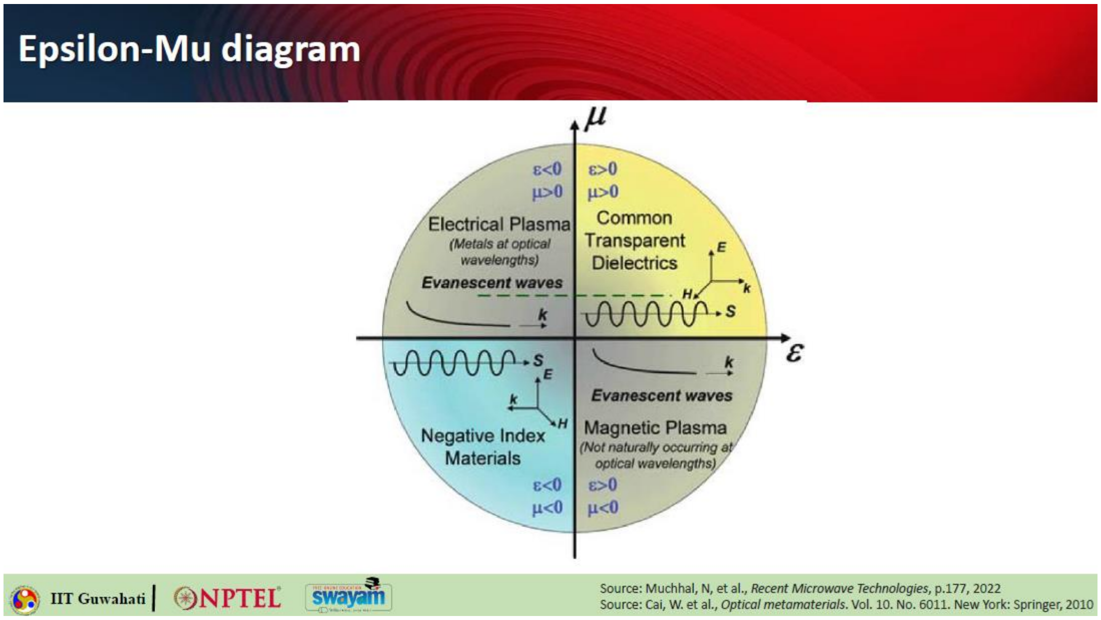
We will come to this why these things are needed. Now one good thing about meta material is that if you understand the concept of meta material for microwave domain, you can simply scale it to nanoscale, okay and they will still work, okay because if the electromagnetic properties are basically derived from the structure, okay. Just that you have to choose the correct dispersion function for the materials that you are using, okay.

So every material that you are using will have its refractive index of permittivity as a function of lambda. So, if you pick that correctly, the same design can give you response in microwave as well as optical frequency regime.



Now when you study meta materials, this is a very important diagram. It is called epsilon mu diagram. So you have got on the horizontal axis you have got epsilon, on the vertical axis you have got mu, permeability, okay. So this is the first quadrant where you have both positive, okay.

So this is also called DPS. We will see that in the next lecture, sorry next slide. So here these are basically the common dielectric, transparent dielectrics that we deal with, okay. So if this is E, this is H, this is k, okay. E cross H, the thumb gives you the direction of wave propagation and S shows you the Poynting vector. So, it means energy also flows in the same direction of your wave propagation.

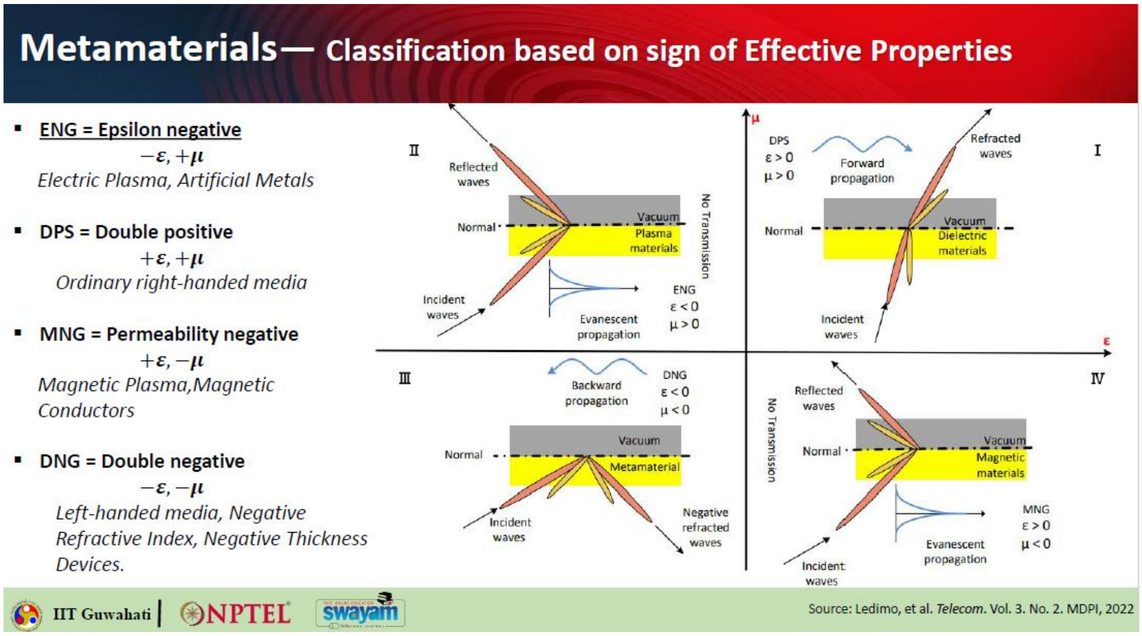


Moving on to the second quadrant, here epsilon is negative but mu is positive and that is the case of electrical plasma, okay or you can say that is what happens with metals at optical frequencies, okay. So here you will see that these are basically propagation does not happen or is not supported. There is basically evanescent wave, okay. So you can only have evanescent wave in metals. So metal does not support propagation of light, right.

So next is the third quadrant. So in this quadrant both epsilon and negative, epsilon and mu are negative. So this is also called negative index material because in this case the refractive index of the material becomes negative. So in such case, okay it becomes a left-handed system. So E cross H, you will see it follows a left-hand rule. So the k, the wave propagation happens in backward direction, okay whereas the Poynting vector goes in the forward direction, okay.

So you will be facing front but you will be moving back, something like that you can think of, okay. So that is bit unusual that that is not happening in the nature. So ideally when people thought of metal materials, they always thought of negative index materials. But now the community has loosened the definition of metal materials. So now any kind of materials that can give you some exotic properties which are not found in nature are also broadly classified as metal materials.

The fourth quadrant is basically where epsilon is positive but mu is negative. So that is the case of magnetic plasma. So that does not occur naturally at optical wavelength. However, if that is to occur, in that case also you will have evanescent waves.



So to keep things simple you can see here. So when we talk about ENG that is epsilon negative that is the case of this one, okay. So your epsilon is negative, mu is positive. So if you have light falling this way, okay you will get reflected light this way but there is only evanescent propagation, okay. This is the plasma material.

So there is only evanescent propagation. So artificial metal or electric plasma they support this one. Then you can go to DPS that is the double positive. So double positive is basically all transparent dielectrics or you can say all ordinary right-handed media. So this is what we have seen from our childhood.

So there is nothing new here. So light comes, gets refracted and so on, okay. Now negative permeability is this one. So these two cases are typically same. Only thing is that here it has to be some magnetic material, okay or ferrites, okay. So magnetic plasma or magnetic conductor can behave like this although there is no optical material that can give you this kind of feature at the optical frequencies, okay.

And then you come to this one that gives you backward propagation as I mentioned. So here you can see when you have incident wave they are kind of reflected back, okay. And this is the case of negative refracted waves. So instead of going that side it remains here, okay, fine. So this is the case of metamaterials classification in four different axes, okay.

So now let us look into how do we create negative permittivity metamaterials. So it means we are now trying to create some artificial metal, right. So first system that we will be talking about is the case of metallic nanospheres in dielectric medium. Something like this. You take a metallic nanospheres of any known metal, okay, say gold or silver and then you put it in a dielectric medium.

Negative-Permittivity Metamaterial

Metallic Nanospheres in a Dielectric Medium

- A dielectric medium of ε uniformly filled with small nanospheres of complex permittivity ε_s.
- An isotropic medium is formed, obeying Maxwell's Garnett mixing rule, of effective permittivity ε_e:

$$\epsilon_e \approx \epsilon \frac{2(1-f)\epsilon + (1+2f)\epsilon_s}{(2+f)\epsilon + (1-f)\epsilon_s} \quad \text{(L22.1)}$$

where f is the volume fraction of the inclusions (filling ratio).

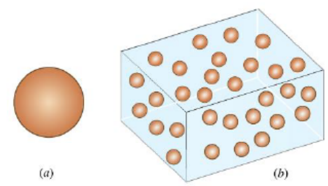


Figure: (a) Metallic nanosphere. (b) Metamaterial comprising uniformly distributed metallic nanospheres embedded in a dielectric medium.

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So you have got a dielectric medium of permittivity epsilon which is uniformly filled by small nanospheres of complex permittivity epsilon s, okay. And if you consider an isotropic medium is formed it means in this medium any direction you look it is more or less uniformly mixed. So it is a isotropic medium. Then you will get Maxwell-Garnett mixing rule defining the effective permittivity.

We will look into this rule in more details in the next lecture. But here I can simply tell you the formula that you will get effective permittivity which depends on the permittivity of the host medium. This will be the host medium and the nanosphere whatever is the material, the permittivity of the nanosphere material and the filling fraction of the nanosphere. So filling fraction is nothing but the fraction or the ratio of the volume occupied by these nanospheres over the total volume. So this is also called as filling ratio. Now what happens with that? Now let us assume that this is of a simple Drude model, right, material, Drude material.

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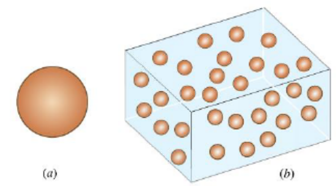

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So you can give the permittivity epsilon s as $\epsilon_s = \epsilon_o\left(1 - \omega_p^2/\omega^2\right)$. So what is omega p? That is the plasma frequency of that metal, right. Now with that when you put it into the equation of epsilon e you will get it is like $\epsilon_e = \epsilon_L \frac{1-\omega^2/\omega_1^2}{1-\omega^2/\omega_0^2}$. So what is omega naught? That is given by $\omega_0 = \frac{\omega_p}{\sqrt{1+\epsilon_{r0}}}$.

Omega 1 is basically $\omega_1 = \frac{\omega_p}{\sqrt{1+\epsilon_{r1}}}$. What is epsilon L ? That is this one, $\epsilon_L = \frac{1+2f}{1-f}\epsilon$. Epsilon r naught is $\epsilon_{r0} = \frac{2+f}{1-f}\epsilon_r$. And epsilon r 1 is $\epsilon_{r1} = \frac{2(1-f)}{1+2f}\epsilon_r$. What is this? This is basically a form which looks compact and these are the new variables which has come up, okay.

There are some significance associated with that. I will come to that, okay. So here you remember that epsilon r is basically the relative permittivity of the host medium. So we assume that epsilon was the permittivity. So when you do epsilon over epsilon naught that is epsilon r that is the host medium's relative permittivity. And we assumed it to be a frequency dependent one, frequency independent one, okay.

Metallic Nanospheres in a Dielectric Medium

$$\epsilon_e = \epsilon_L \frac{1-\omega^2/\omega_1^2}{1-\omega^2/\omega_0^2},$$

where

$$\omega_0 = \frac{\omega_p}{\sqrt{1+\epsilon_{r0}}}, \qquad \omega_1 = \frac{\omega_p}{\sqrt{1+\epsilon_{r1}}},$$

$$\epsilon_L = \frac{1+2f}{1-f}\epsilon, \qquad \epsilon_{r0} = \frac{2+f}{1-f}\epsilon_r, \qquad \epsilon_{r1} = \frac{2(1-f)}{1+2f}\epsilon_r,$$

- The effective permittivity ε_e has a pole at ω_0 and a zero at ω_1.
- Since $\varepsilon_{r0} > \varepsilon_r$, the resonance frequency ω_0 falls below that of the isolated nanosphere.
- Also, since $\varepsilon_{r1} < \varepsilon_{r0}$, we see that $\omega_1 > \omega_0$, so that ε_e is negative within the spectral band between ω_0 and ω_1, which lies below the plasma frequency ω_p of the metal.
- With μ positive, this metamaterial is therefore single-negative (SNG), much like a homogeneous metal below its plasma frequency.

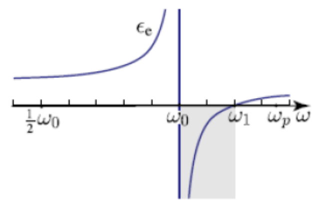


Figure: ε_e has a pole at the resonance frequency ω_0 and a zero at ω_1. Since ε_e is negative in the band lying between $\omega_0 < \omega < \omega_1$, this metamaterial is single-negative (SNG) if μ is positive.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

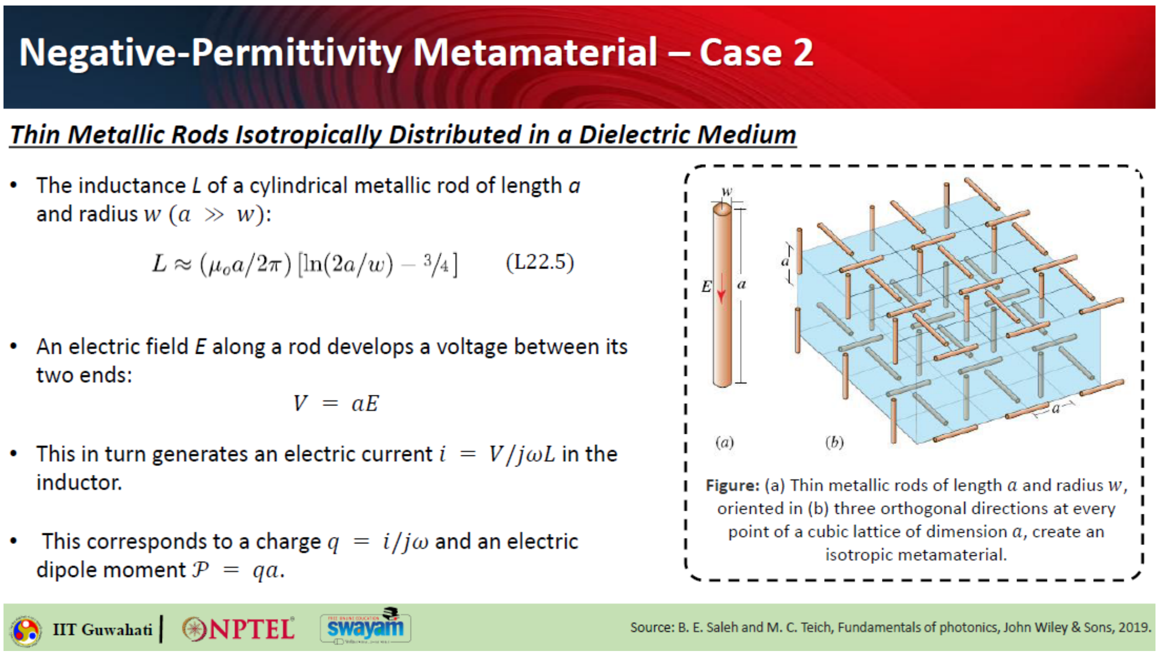
So it is same for all the wavelengths or all the frequencies. Now if you take this particular equation again, you will see that the effective permittivity has a pole at ω_0 and it has got a 0 at ω_1, okay. So, with that if you try to plot it, okay, so you will see that it has got a pole at ω_0, okay and it has got a 0 at ω_1, fine. And since from this equation you can see that ε_{r0} is basically greater than ε_r, okay. It means the resonance frequency omega naught, okay, falls below that of the isolated nanosphere. What is the resonant frequency of the isolated nanosphere? You can take it as ω_0, okay.

And you will see that the effective medium that you have created has a resonance frequency, okay, which is below that of the isolated nanosphere, okay. So, another application, another thing is ε_{r1}. If you look into this one, ε_{r1} is basically smaller than epsilon r, okay. And if you compare it is also smaller than ε_{r0}. From that you can find out and we can also see that o ω_1, ω_1 is greater than omega naught.

So you can see from here, okay, if this is larger than this one, so this will be larger than this one, right. So, in that case ε_e is negative, okay. So, you can find from here that this is the condition where ε_e is negative and that will remain negative within this particular band where ε_0 and ε_1 is the frequency band. So if you look into this highlighted region, this is where this permittivity is negative.

And this is well below the plasma frequency of the metal. So, whatever was the metal that you started with, let us assume that gold or silver, so ω_p will denote the plasma frequency of that particular metal which is actually making the nanosphere. But when you put it in the dielectric medium uniformly, the effective permittivity will have a plasma frequency lower than this one. So, you are able to design an artificial metal. So, what will be the exact plasma frequency that will, that is here which is ω_1.

So, that is the case where it will cross from negative to positive. So, you can actually decide what is omega 1 depending on the materials that you are using, fine. So, that is how you are able to design artificial metal. Now if you consider that mu is positive, so the permeability is positive, then this particular material can be taken as a single negative metal material or you can say this is a kind of a artificial homogeneous metal which is having you know its plasma frequency below that constituent materials plasma frequency. Now there is another case where you can get a different way of calculating artificial plasma frequency that will be a system of thin metallic rods isotropically distributed in a dielectric medium.

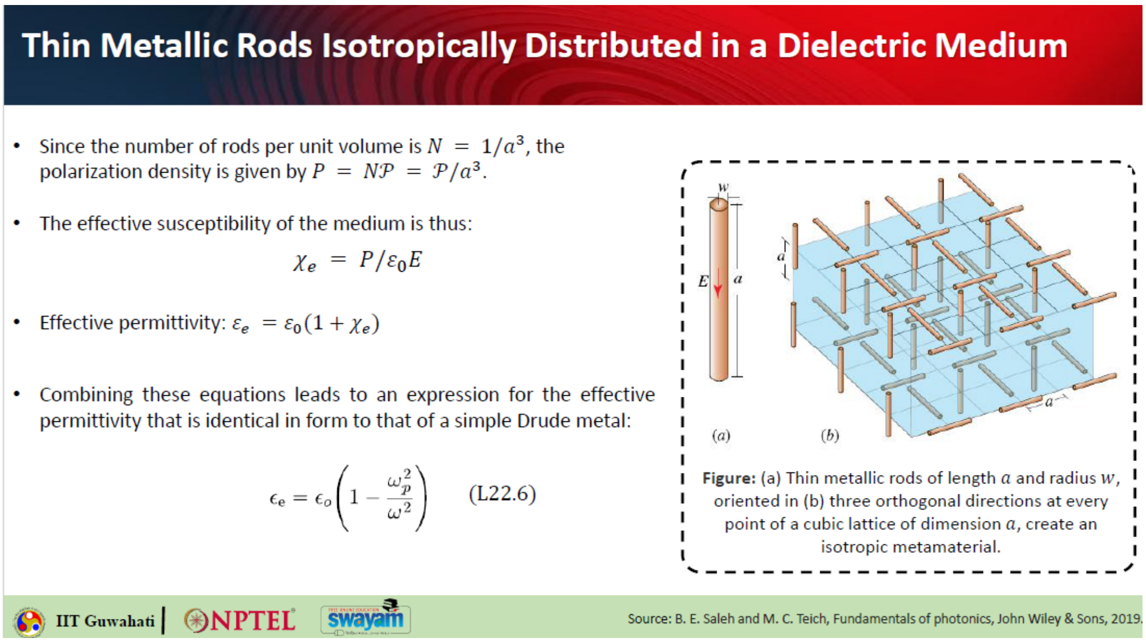


So this is this particular structure. So here you see we are considering thin metallic rods of length a and radius w, okay. And they are oriented in three-dimensional direction with a cubic lattice of dimension a. So from here to the mid it is a and that is how it is repeated in all three dimension and that creates an isotropic metamaterial. Now in this case what happens the inductance L of a cylindrical metallic rod which has got a length of a and radius of w and we consider that a is much much larger than w.

So you can approximate the inductance using this formula, okay. And when there is electric field along this rod so that will develop a charge difference or you can say the voltage difference between its two ends. So v can be taken as aE. What is aE? a is basically the length of this rod. And you will also then get a current.

So current i can be written as v over the impedance. v times yeah it is divided by the impedance of this rod which is $j\omega_L$, okay. And that is how you can find out, okay. So this also helps you to find out what is the charge associated. So charge can be taken as i over j omega, okay. So that gives you the charge and the electric dipole moment can be considered as P, curly P which is qa.

q is the amount of charge and a is the separation between the charges. Now if you consider the number of rods per unit volume to be capital N, okay. So per unit volume that is 1 over a cube, okay. So a cube is the volume. So, if you consider only one rod being there in a unit volume of unit cell of length a, okay, then the polarization density capital P can be written as N times curly P.



So it is basically curly p over a cube, okay. So from that you can find out what is the effective susceptibility of this medium. So, you can write $\chi_e = P/\varepsilon_0 E$. So, and once you know this electric susceptibility you can always find out what is the effective permittivity that is ε_e which is nothing but $\varepsilon_0(1+\chi_e)$, okay. So, by combining these equations you can actually find the effective permittivity to have a form which is very much identical to the simple Drude metal.

Thin Metallic Rods Isotropically Distributed in a Dielectric Medium

$$\epsilon_e = \epsilon_o\left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad \text{(L22.6)}$$

$$\omega_p = \frac{1}{\sqrt{\epsilon_o a L}} = 2\pi\frac{c_o}{a}\frac{1}{\sqrt{2\pi\ln(2a/w) - \frac{3}{2}\pi}} \quad \text{(L22.7)}$$

with a plasma frequency ω_p determined by the dimensions of the rod, a and w, via its inductance L.

- The dielectric medium has the permittivity of free space.
- With μ positive, this metamaterial is thus single-negative (SNG).
- Though the rod is assumed to be a perfect conductor in the calculations set forth above, loss is readily accommodated by adding a resistance R to the impedance $j\omega L$ of the rod.

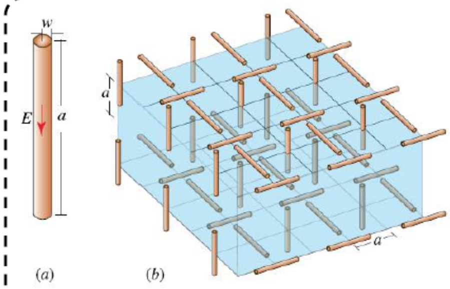


Figure: (a) Thin metallic rods of length a and radius w, oriented in (b) three orthogonal directions at every point of a cubic lattice of dimension a, create an isotropic metamaterial.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So you can find $\epsilon_e = \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$. So that is a very very well-known form of any Drude metal. So that makes this system easier to understand than the previous one. But it is definitely much more complicated to realize. But here what is omega p? That is a plasma frequency. In natural metal that is already decided by the electron concentration, the charge of electron, the mass of electron and all these things and you do not have any control.

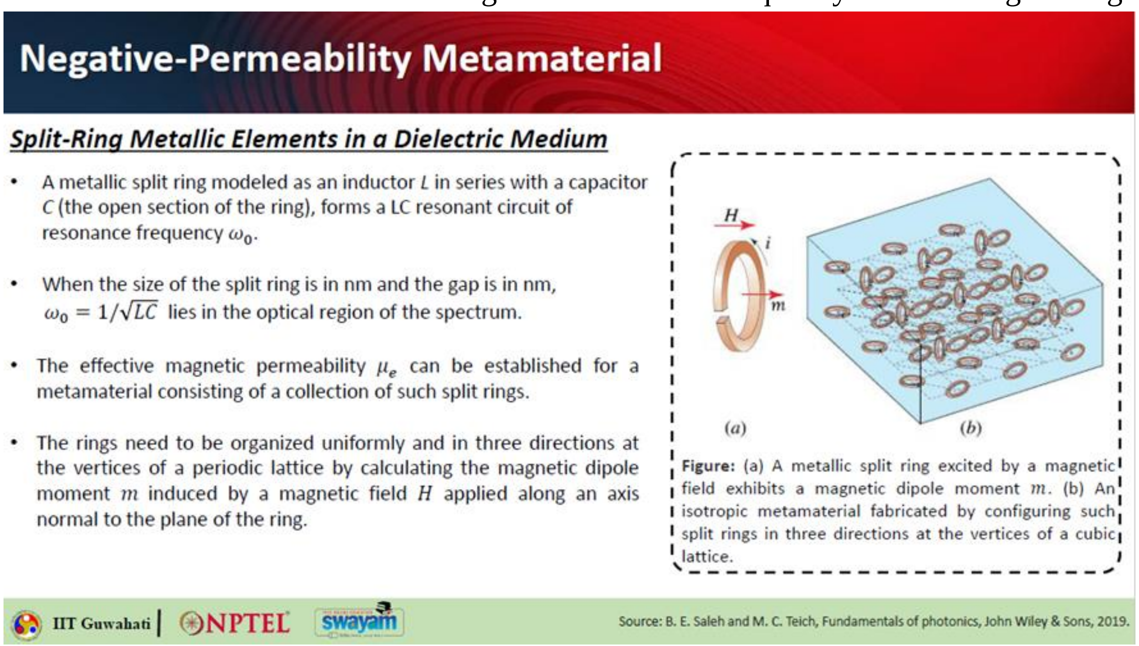
Whereas here epsilon p is basically $\omega_p = \frac{1}{\sqrt{\epsilon_o a L}}$. What is a? a is the length, L is the inductance, okay. So you have control of all the physical parameters and that is how you are creating a artificial metal using this kind of structure. So you are basically using a metallic rod but the property of the metal here does not play a significant role.

What plays a significant role is the a and w that is the physical structure of this meta-atom. And that is why this field of meta-materials gives you infinite possibility, right. So the dielectric medium has the permittivity of free space here. So you are just assuming that this rods are nicely oriented but in free space. So when you consider mu to be positive, this is again a single negative meta-material or it is a negative permittivity meta-material. Now there are some ifs and buts that though the rod here has been assumed to be a perfect conductor in the calculations as you can see, okay.

But there will be usually loss associated in this rods and you can accommodate that loss by adding a resistance R to the impedance where you have calculated j omega l you can add R that is basically the loss factor. Now if you change one material to another material for making this particular rods there will be a change in the R, okay. What about l? So let us see if l has got any physical parameter, not really. So mainly it will not depend on the kind of material that you are using, okay, fine. So with that let us move on to the next part like that is like can we realize negative permeability meta-materials now.

So that can be done using split ring metallic elements in a dielectric medium. So it is like this. So you have a ring, okay and then it is cut. So this is called a split ring. So when a metallic split ring is excited by a magnetic field, okay, it exhibits a magnetic dipole moment M, okay. So this is the magnetic field H that is exciting this. So this will induce the current in this loop and this circular current will give you a magnetic dipole moment small m.

And then what you have to do, you have to actually, just like the way you have done in the previous case, you have to make an isotropic meta-material, okay by configuring this split ring in three dimensions at the vertices of a cubic lattice. So you have to place them at the vertices of a cubic lattice and once you do that you can actually form a isotropic meta-material that gives you negative permeability. So let us see how it works. So a metallic split ring can be modeled as an inductor L and it has to have a, this gap will behave like a capacitor. So there has to be capacitor C, okay and that will give you a resonant LC circuit which has got a resonant frequency of omega naught.



So when the size of the split ring is in nanometer and the gap is also in nanometer, you can calculate and see that the resonant frequency omega naught which is calculated as 1 over square root of LC will fall in the visible region of the spectrum. So effective magnetic permeability mu E then can be established for a meta-material consisting of a collection of such split rings, okay. So you make a collection in by placing them in this fashion you will get that, okay. The rings need to be organized uniformly and in three dimensions at the vertices of a periodic lattice by calculating the magnetic dipole moment M induced by the magnetic field capital H along an axis normal to the plane of the ring.

Split-Ring Metallic Elements in a Dielectric Medium

- The voltage V induced in the loop is equal to the rate of change of the magnetic flux so that:

$$V = -j\omega\mu_0 AH$$

where A is the area of the ring.

- This voltage generates an electric current $i = V/Z$, where the circuit impedance is $Z = j\omega L + 1/\omega jC$.
- This electric current in turn results in a magnetic dipole moment $m = Ai$.

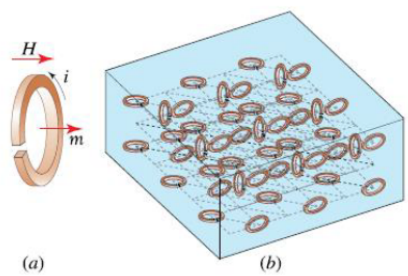


Figure: (a) A metallic split ring excited by a magnetic field exhibits a magnetic dipole moment m. (b) An isotropic metamaterial fabricated by configuring such split rings in three directions at the vertices of a cubic lattice.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So this is what I have discussed initially. So, once you do that you can find out what is the voltage induced in the loop that is equal to the rate of change of the magnetic flux and you can write them as $V = -j\omega\mu_0 AH$. What is A? A is basically the area of the ring, okay. H is the magnetic field, mu naught is the vacuum permeability, okay.

Split-Ring Metallic Elements in a Dielectric Medium

- A density of N split rings per unit volume gives rise to a magnetization density $M = Nm$ so that the effective magnetic permeability $\mu_e = \mu_0(H + M)/H$ is given by:

$$\mu_e = \mu_o \frac{1-\omega^2/\omega_1^2}{1-\omega^2/\omega_0^2}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad \omega_1 = \frac{\omega_0}{\sqrt{1-\mu_o N A^2/L}}$$

- The inductance of the ring is: $L \approx \mu_0 b\,[\ln(8b/a) - 7/4]$
- The capacitance of the gap is: $C \approx \varepsilon_0 A'/g$

where b and a are the ring and wire radii, respectively ($b \gg a$). A' is the cross-sectional area and g is the gap

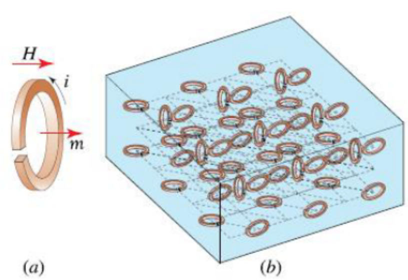

Figure: (a) A metallic split ring excited by a magnetic field exhibits a magnetic dipole moment m. (b) An isotropic metamaterial fabricated by configuring such split rings in three directions at the vertices of a cubic lattice.

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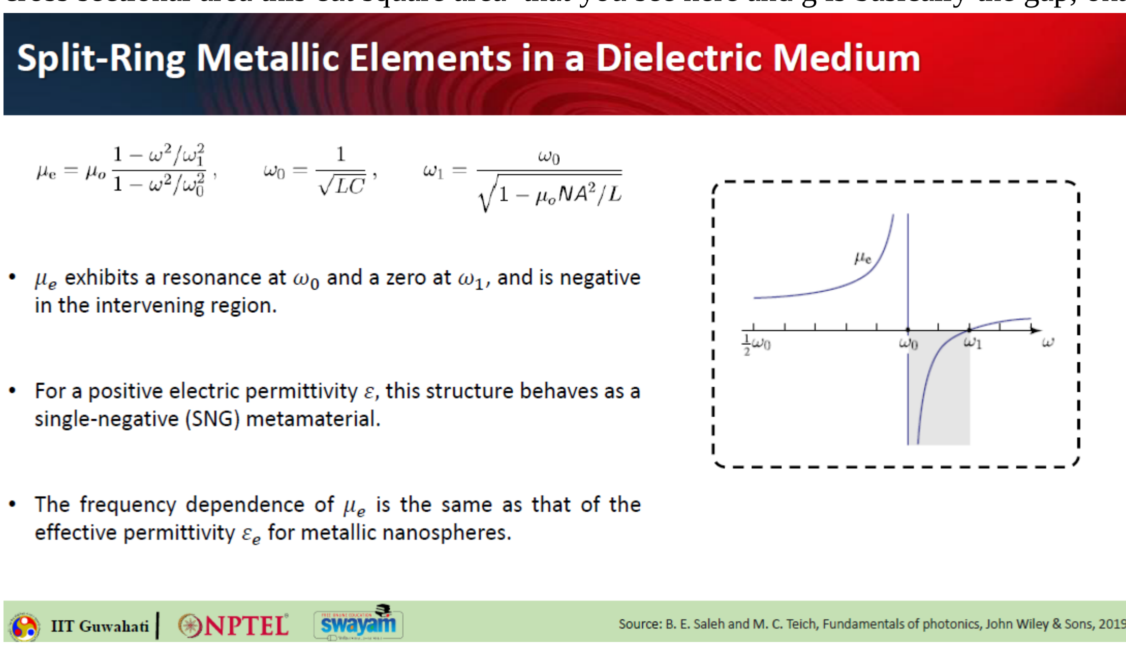
J is the j omega. So omega is the frequency of the incident wave, okay. And from that voltage you can also find out what is the current. So I equals V by Z. So Z is basically the circuit impedance. So here that there is the inductor and the capacitor in series so you can find out $Z = j\omega L + 1/\omega jC$, okay. And this particular current in turn results in a magnetic dipole moment which is given as $m = Ai$. And this small m is basically the magnetic moment dipole moment for one ring and when you have a density of capital N split rings per unit volume, okay, you can think of the magnetization density capital M to be capital N times small m. So, you can write down the effective permeability to be $\mu_e = \mu_0(H + M)/H$. So, this is the new term that you have brought in by doing this kind of assembly and you will see that there is a bit of maths involved but it can be seen that you are able to obtain $\mu_e = \mu_o \frac{1-\omega^2/\omega_1^2}{1-\omega^2/\omega_0^2}$. What is omega naught? That is 1 over square root of LC and what is omega 1? That is given $\omega_1 = \frac{\omega_0}{\sqrt{1-\mu_o NA^2/L}}$.

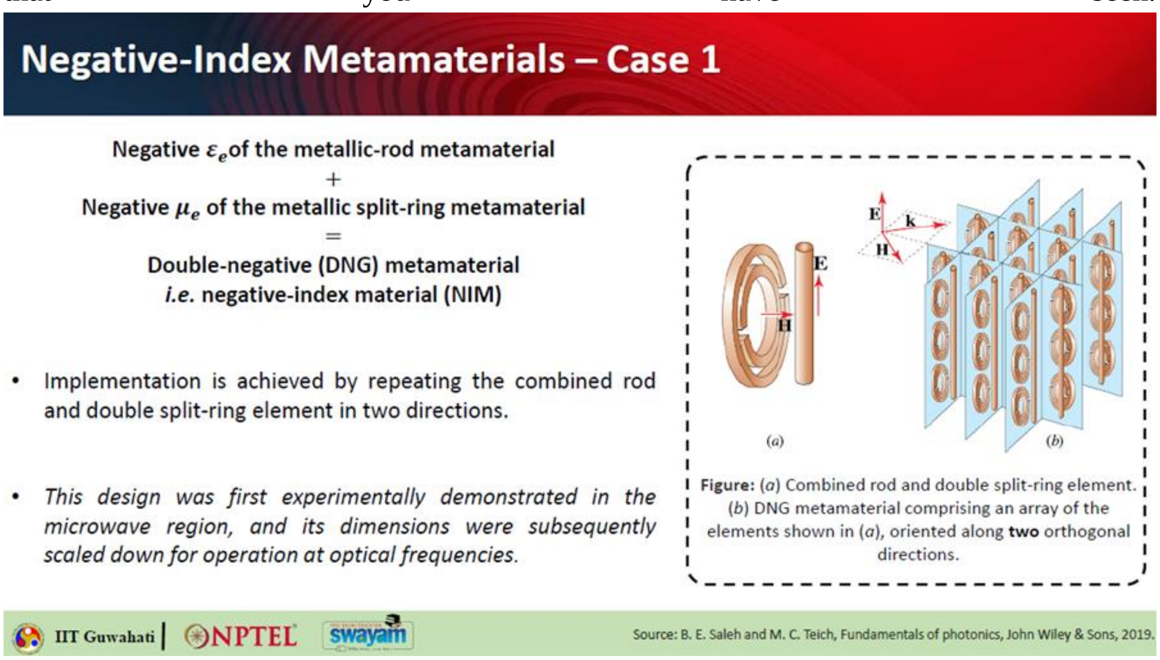
So here you see this is basically the term that defines this particular ring, okay. So it tells you that based on the design of the ring you are able to change what is omega 1, okay. So omega 1 has got L that is the inductance of the ring that is given as mu naught b ln of 8 b by a minus 7 by 4. What is b and a? Small b and small a they are basically the wire ring and the wire radius. So that is the ring, okay and b is the ring and wire radius so you can actually see these two parameters here and the capacitance of this gap will be given as epsilon naught A prime over g, okay. What is A prime? A prime is basically the cross sectional area this cut square area that you see here and g is basically the gap, okay.



So that tells you what is C, you knew what is L you can put it here and also because you have designed this so you can find out what is ω_1. So it will give you a very similar kind of response that you have seen in the first case, okay. So, mu e exhibits a resonance at omega naught because it has got a pole at omega naught and it has got a 0 at ω_1, okay and it will be negative in the intervening region. So this is the region where it behaves like a metal. So again, I told you that ω_1 in this case is programmable you can change not programmable you should say tunable because you are having the complete control on setting this value depending on the size and the gap and all these things all the parameters of the ring, okay.

And if you consider the material to have a positive electric permittivity this structure can then give you a negative single negative metamaterial. So in this case it is a negative permeability metamaterial. So as you see this particular graph the frequency dependence of mu e is same as that of the effective permittivity epsilon e which you have seen for the metallic nanospheres, okay. So they actually have very similar kind of dependence, okay.

And for reference these are the values which are shown here, okay. This is the value of omega 1 this is omega naught, okay. So omega naught is also ln c that is also tunable so you get complete control on this particular graph, right. So, you can have the control on the resonance frequency as well as the range till which it will behave like a negative permeability metamaterial, okay. Now let us move on to the next case where you have negative index metamaterial. Now as I told you negative index will have negative effective permittivity, okay of the metallic rod metamaterial you can take that and you can add a negative permeability of the metallic split ring resonator kind of metamaterial that you have seen.



And when you combine these two structures together you will be able to get a double negative DNG metamaterial, okay. And DNG metamaterials are also called negative index metamaterials or NIM, okay. So the implementation is achieved by repeating the combined rod and double split ring in two dimension, okay. And the design was experimentally demonstrated in microwave region first and then its dimensions were subsequently scaled down for operation at optical frequency and this is the structure. So, in this structure you can see that you are actually making this kind of array, okay.

Negative-Index Metamaterials – Case 2

- *Alternative designs for optical NIMs that are easier to fabricate have subsequently been developed.*
- One such design is a **"fishnet" metal–dielectric multilayer structure.**
- In this configuration, the optical wave is normally incident on the fishnet surface and the electric and magnetic fields are aligned with the metal strips, as shown in **Figure**.

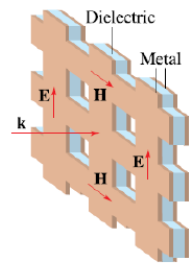


Figure: Simplified version of a "fishnet" metal–dielectric nanostructured composite metamaterial consisting of a stacked network of intersecting subwavelength plasmonic waveguides.

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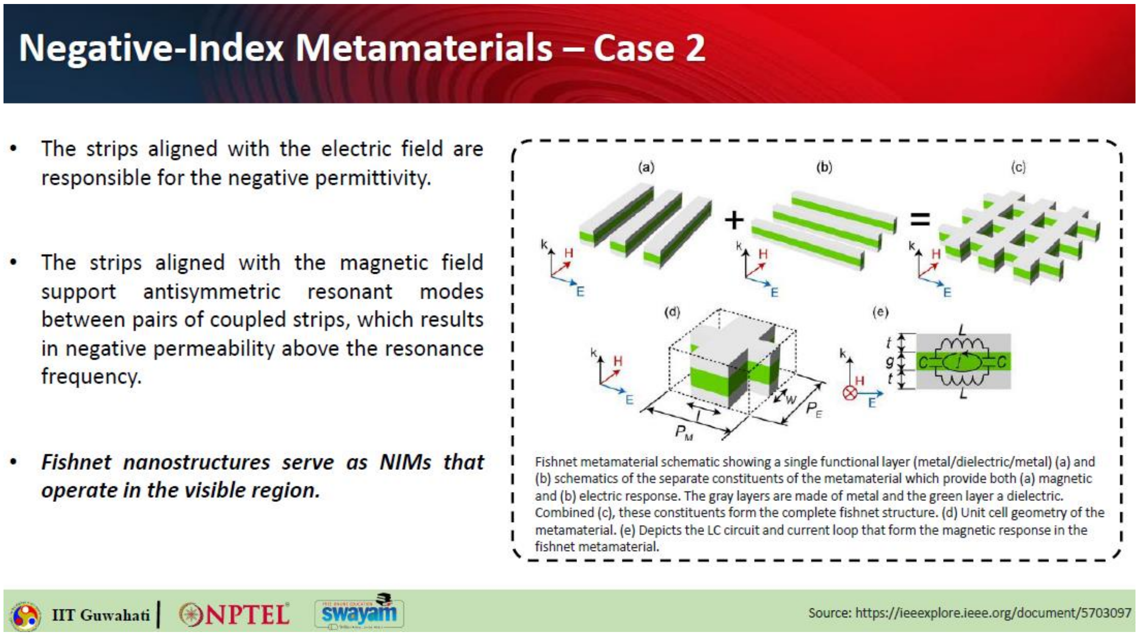
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So it is a 2D array and then you have some vertical column so that makes it 3D, okay. And this is the combined structure you have a rod and you have a double split ring resonator, double split ring resonator gives you more parameters to control over a single split ring because you have two gaps here and there will be coupling between the inductors, coupling between the cap the rings that will give you some capacitance effect. So you have much more design parameters to play with when you go for double split ring resonator. Because here the challenge is that you have to get negative permittivity and negative permittability both should be negative over the same frequency range and that might not be possible with just one split ring you may not be able to completely overlap them. So, you can actually take the help of this double split ring resonator that has got more tuning parameters which allow you to get a overlap of this negative permittivity window with negative permeability window, okay.

Now yeah so there are some alternative designs as well for achieving negative index metamaterials. So this is the second case that we will be discussing. So alternative designs are something like for optical negative index metamaterials they are easier to fabricate if you follow this kind of design. So as you can see this is called a fishnet metal dielectric multilayer structure, okay. So, it looks like a fishnet so there is a metal dielectric metal layer and in this configuration the optical wave falls like this that is basically normal incident on the fishnet structure and the electric and the magnetic fields the electric and the magnetic fields are basically aligned with the metal strips, okay.



So they actually give you this particular negative permittivity and negative permeability effects. So let us look into more details of this structure. So as you can see here so that this is the k vector so you have strips which are aligned along the H that is the magnetic field there are strips which are aligned along electric field and then you combine this together. Now you can understand that which strips are responsible for the electric fields and or the permittivity and which strips are responsible for permeability, right or you can say electric response or magnetic response, right. So the strips which are aligned with the magnetic field they support anti-symmetric kind of resonant modes between the pairs of the couple strips and this gives you two negative permeability above the resonance frequency. So we are not going to too much of details into the mathematical foundation of this because it will be mostly done based on simulation, numerical simulations on CST or FDTD or COMSOL, okay.

But I just want to tell you here is that in this kind of structure you have the inductance on one side of metallic layer you have another inductance here, okay and then you have capacitances between these two layers, okay and these are the factors that give you this negative refractive index, okay. So fishnet nanostructures they serve as NIMs that can operate in the visible region so these are good optical metamaterials, okay. So with that we will stop here and in the next lecture we will discuss the effective medium theories, okay. So if you have got any queries on this particular lecture you can drop an email to me at this particular email address. Thank you.