Course Name- Nanophotonics, Plasmonics and Metamaterials

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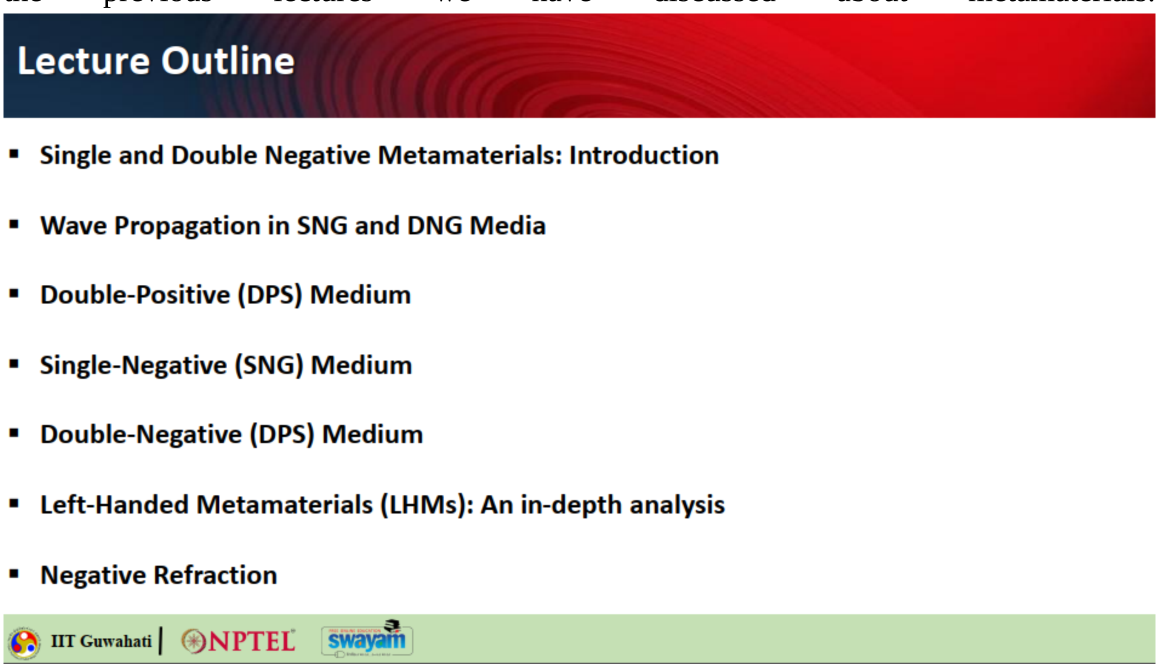
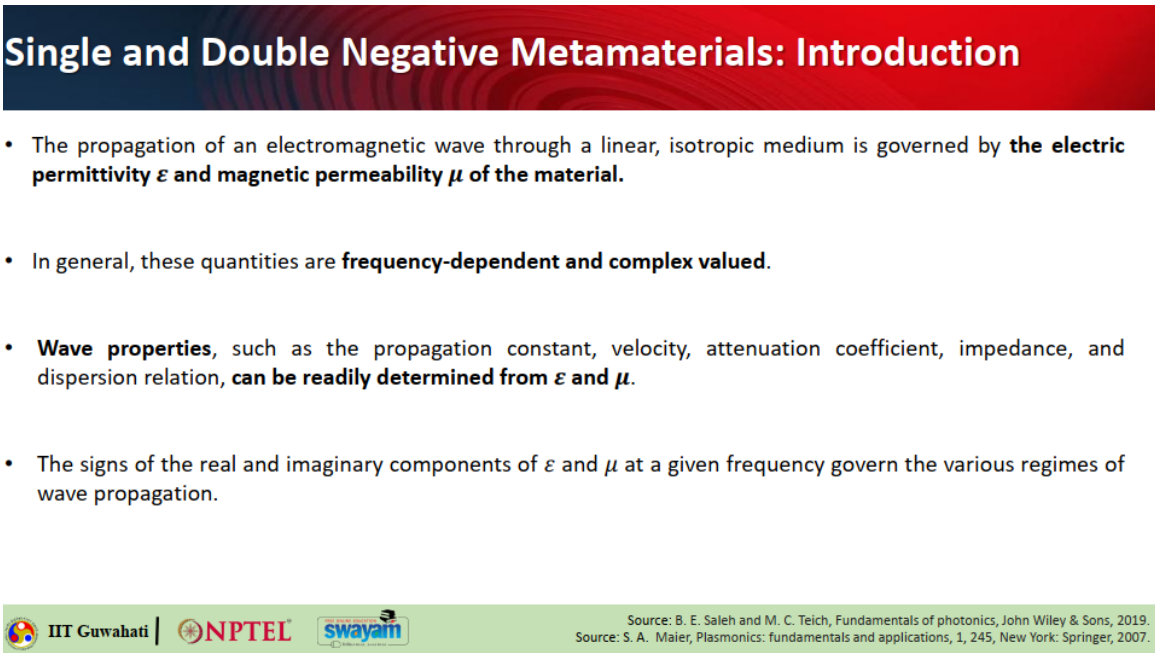
Week-08

Lecture -24

Hello students, welcome to lecture 24 of the online course on Nanophotronics, Plasmonics and Metamaterials. Today we will be going into more details of single and double negative metamaterials. So here is the lecture outline, we will give a brief introduction to this single and double negative metamaterials. We will also discuss about the wave propagation in single negative or double negative media that is SNG and DNG media. We will also go into bit of details of double positive medium that is DPS single negative SNG medium and double negative there is a typo this should be DNG okay DNG medium and we will also look for the in-depth analysis of left hand metamaterials and discuss about negative refraction and some of its applications. So in the previous lectures we have discussed about metamaterials.



We understood that metamaterials are basically those materials which have engineered unit cells and these unit cells basically give you some control on the properties which are otherwise only found in natural materials or you can completely come up with exotic materials which are not found in nature. Something like you can design materials with own customizable plasma frequency, you can also look for negative permittivity, negative permeability materials and so on. And why we are bothered about permittivity and permeability? The reason is the propagation of any electromagnetic wave through a linear isotropic medium is governed by these two parameters which is basically which are the electric permittivity epsilon and the magnetic permeability mu of that material. So in general these quantities are frequency dependent and they are complex okay and the wave properties such as propagation constant, velocity, attenuation constant, impedance, dispersion relation all these different properties can be easily obtained from epsilon and mu.



And the science of the real and imaginary part of this epsilon and mu at a particular given frequency govern the various regions of propagation. So as we mentioned that permittivity can be frequency dependent, mostly they are frequency dependent in almost every material and they are also complex values. So there is a real part, there is a imaginary part. So depending on those real and imaginary components you can find out that how the wave propagation takes place. Now if you consider a media which has got a permeability mu which is real and positive that means that particular medium does not support magnetic absorption or amplification okay.

Single and Double Negative Metamaterials: Introduction

- For media in which μ **is real and positive** (indicating that there is neither magnetic absorption nor amplification), the wave propagation characteristics depend on **the signs of the real and imaginary parts of** ε.
- Similarly, for media with **real and positive** ε, the magnetic properties, **described by** μ, dictate the nature of wave propagation.
- Magnetic media, including media with metal components that carry induced electric currents and generate magnetic fields, generally have complex values of μ, with real and imaginary parts that may be either positive or negative.
- In the most general case, the manner in which the signs of the real and imaginary components of ε and μ dictate the characteristics of wave propagation is more subtle.

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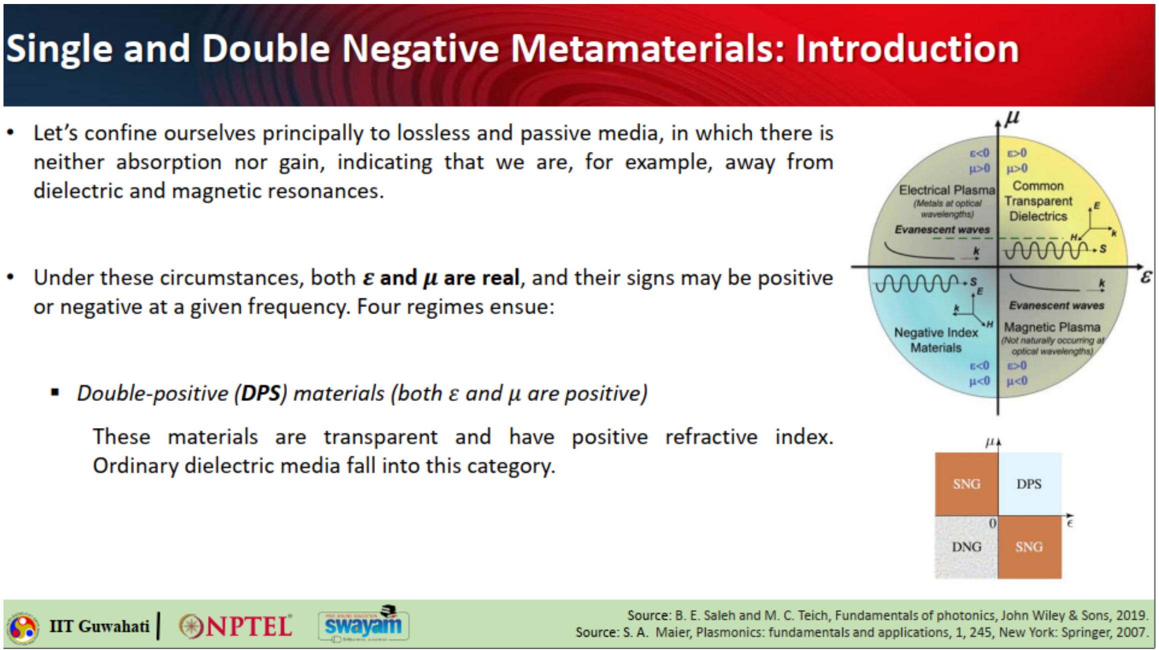
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

So in that case the wave propagation characteristics mainly depend on the science of the real and imaginary parts of the permittivity epsilon okay. So in those materials epsilon plays the most important role. Now if you remember initially we have discussed that at optical frequencies most natural materials known to be non-magnetic. So their mu you can simply take as mu r okay or you can say mu r is basically 1. So mu is same as mu naught okay they do not have any magnetic properties.

So in that case the real and imaginary part of the permittivity epsilon that depend or that dictates the wave propagation characteristics. Now similarly if you think of the other extreme that where your permittivity is real and positive and you have a fixed value there okay. In that case the magnetic properties which is described by mu okay that will dictate the nature of wave propagation. And magnetic media such as the media which includes some metal components that could carry induced electric currents and generate magnetic fields they generally have complex value of mu with real and imaginary part which may be either positive or negative right. So in most general case the manner in which the science of the real and imaginary components of permittivity epsilon and permeability mu they come up okay they will dictate the characteristics of wave propagation.

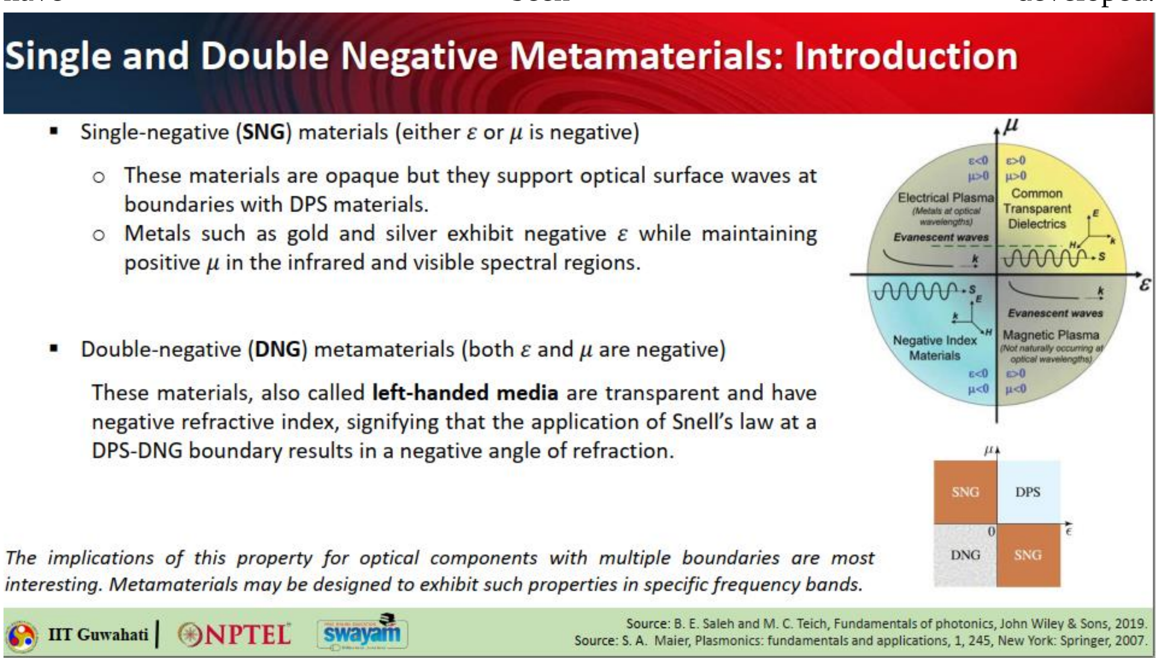
But usually the contribution is not that prominent it is subtle okay. We will look into some of these examples in this particular slide. So if you remember this epsilon mu diagram that actually classifies all different natural materials and metamaterials or everything can be classified from this epsilon mu diagram okay. So if we confine ourselves mainly to lossless and passive media in which there is neither absorption nor gain we can say that we are basically operating away from dielectric and magnetic resonances okay. And under these conditions you can say that epsilon and mu are real and their science may be positive or negative and when you think of those cases so when you have permittivity to be the permittivity and permeability if you consider them as real quantities you can have this four possible variations.



The first one or the first quadrant as you can see here basically denote epsilon greater than 0 and mu greater than 0 that means you are talking about double positive materials where both mu and epsilon are positive. So in this case this kind of materials are basically transparent and they have positive refractive index and ordinary dielectric medium whatever you see all the common dielectrics they all fall under this particular category okay of double positive materials. Then the next category belongs to single negative materials where either epsilon or mu is negative. So in this particular case the second quadrant okay here epsilon is negative mu is positive okay. So this is basically it is a negative permittivity materials this one but we can call this also as a single negative metamaterials.

This one also single negative metamaterials but here mu is basically negative and single negative metamaterials are basically opaque but they could support optical surface waves at the boundaries with double positive materials. So these are the simple example you can take as the metals at optical frequency they are basically this single negative metamaterials okay or you can say single negative materials and they support surface waves which are also known as surface plasmon polaritons to propagate along the metal dielectric interface right. So here are the examples so metals such as gold, silver they exhibit negative permittivity while maintaining positive permeability in the infrared and visible spectral region. So when we talk about electrical plasma those are basically we are talking about metals at optical frequency and one thing is that wave propagation is not permitted they basically support evanescent waves inside them. However remember that on the surface you can actually on the surface with a double positive material you can actually have some sort of surface waves which are basically the surface plasmon polaritons.

Now if you look into this particular third quadrant the third quadrant you see both permittivity and permeability are negative. So these are called as double negative metamaterials and they are also called as left-handed media we will see that in subsequent slides. So all other quadrants the media is basically right-handed media but this particular quadrant the third quadrant okay here you can actually see that the E cross H they follow left-hand thumb rule okay. So they also give rise to something which is completely unnatural that is like negative refractive index and you can if you apply Snell's law of refraction on a boundary between this kind of double negative and double positive kind of material you will see a negative angle of refraction. And that is something completely shook the world with surprise that this is something completely out of the world you can think of and you can also realize this using the techniques that have been developed.



So you see this kind of material also support you know wave propagation but the direction of wave propagation and the direction of the Poynting vector here are opposite they are not in the same direction okay as you can see here the in this case the k vector that is the wave propagation happens in the backward direction whereas the energy propagates in the forward direction. But if you look here in the case of double positive E cross H if you are using the left right hand thumb rule you are seeing that you know the wave propagation happens in the forward direction that is the right direction also energy propagates along the right direction S is basically the Poynting vector. So we will look into that very soon okay how the Poynting vector is calculated and why these things are different in case of double positive and double negative materials. Now we will let us look into some of the wave propagation characteristics in a single negative and a double negative media. So for simplicity we can consider a monochromatic plane wave with electric and magnetic field amplitudes which are given by these vectors okay.

Wave Propagation in SNG and DNG Media

- For simplicity, we consider a monochromatic plane wave with electric and magnetic complex-amplitude vectors given by $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0 \exp(-j\boldsymbol{k} \cdot \boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r}) = H_0 \exp(-j\boldsymbol{k} \cdot \boldsymbol{r})$, respectively, and with wave vector $\boldsymbol{k}$.
- Maxwell's equations then require:

$$\mathbf{k} \times \mathbf{H}_0 = -\omega\,\epsilon\,\mathbf{E}_0 \qquad \text{(L23.1)}$$
$$\mathbf{k} \times \mathbf{E}_0 = \omega\,\mu\,\mathbf{H}_0 \qquad \text{(L23.2)}$$

- The associated wavenumber (magnitude of the vector $\boldsymbol{k}$) is: $k = \omega\sqrt{\epsilon\,\mu}$ (L23.3)

  and the impedance (ratio of the magnitudes of E_0 and H_0) is given by:

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}} \qquad \text{(L23.3)}$$

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

So let us consider $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0\exp(-j\boldsymbol{k} \cdot \boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r}) = H_0\exp(-j\boldsymbol{k} \cdot \boldsymbol{r})$ okay. So what is k here? k is basically the wave vector. So Maxwell's equation then require these conditions to be satisfied which is $\begin{matrix}\mathbf{k} \times \mathbf{H}_0 = -\omega\epsilon\mathbf{E}_0 \\ \mathbf{k} \times \mathbf{E}_0 = \omega\mu\mathbf{H}_0\end{matrix}$.

So the knowledge of E can give you H the knowledge of H can give you E and that is how they are basically interconnected right. And from the information of the wave vector you can also know about the associated wave number k okay which is basically the amplitude of this vector that can be related to omega square root of epsilon mu.

Wave Propagation in SNG and DNG Media

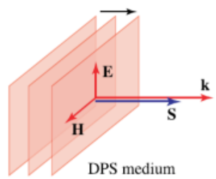
- Since k is in general complex, we write $k = \beta - j\gamma$, where β and γ are real, so that

$$\beta - j\gamma = \omega\sqrt{\epsilon\,\mu} \qquad \text{(L23.4)}$$

- The propagation constant $\beta = \omega/c$ determines both the wave velocity $c = c_o/n$ and the refractive index n, whereas γ represents the field attenuation coefficient.

- We now consider the implications of these equations for media in which ε and μ are real, where either or both may be negative.

- ***Double-positive (DPS) medium:***



Plane wave propagating in an ordinary double positive (DPS) medium. The vectors **E**, **H**, and **k** form a right-handed set and the wavefronts travel in the same direction as the power flow.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

And another parameter which is the impedance that is basically the ratio of the magnitudes of the electric field amplitude and the magnetic field amplitude and you can calculate it as eta which is given as omega mu over k or simply you can write square root of mu over epsilon. So eta can be directly written as square root of mu over epsilon. So this is the formula and these are the important characteristics of the wave propagation through any medium. Now the wave number that we have seen is generally complex right and we can always break it into two parts. You can write it as $k = \beta - j\gamma$.

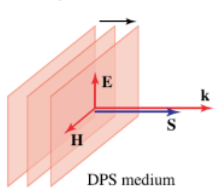
Double-Positive (DPS) Medium

- ***Double-positive (DPS) Medium:***

The double-positive (DPS) medium provides a simple and familiar benchmark. Both ε and μ are positive, so that k and η are real, whereupon:

$$\gamma = 0, \qquad \beta = nk_o\,, \qquad n = \sqrt{\frac{\epsilon}{\epsilon_o}\frac{\mu}{\mu_o}}\,, \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

Support transverse electromagnetic (TEM) waves for which the vectors $\mathbf{E}_0$, $\mathbf{H}_0$, and $\mathbf{k}$ are mutually orthogonal and form a right-handed system, as illustrated in **Figure**.



Plane wave propagating in an ordinary double positive (DPS) medium. The vectors **E**, **H**, and **k** form a right-handed set and the wavefronts travel in the same direction as the power flow.

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So here beta and gamma both are real and we can equate beta minus j gamma to the value of k we have seen in the previous slide that is omega square root of epsilon mu and from that you can determine that beta is basically this one and in this case it is a lossless or medium so you can take gamma equals 0. Because there is no imaginary part on the right hand side. So what is beta again? The beta is basically telling you that this is the propagation constant. So beta can be written as omega by c and determines both the wave velocity c. So c is basically c naught over n where n is the refractive index and gamma that we have discussed gamma is basically the field attenuation constant.

In this case the gamma is 0 ok. Now we can consider the implications of this equations for media in which epsilon and mu are real and they can be either or both negative. So let us investigate these cases. So the first simple case that comes to our mind is double positive medium. So here both epsilon and mu are positive.

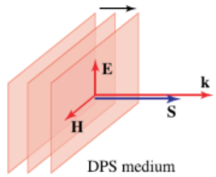
So you can simply write the E and the H vectors and wave propagation as I told you that the wave vector can be determined from the right hand rule ok. So they actually form a right handed set E H and k they form a right handed set and the wave front will travel in the same direction of the power flow ok that is the pointing vector. So S and k are in the same direction. So when you consider the double positive medium as we understood that epsilon and mu both are positive so that your k and eta are also real and that gives you very simple case where gamma the attenuation constant is basically 0. It means the amplitude of the wave will remain same as it propagates through this particular medium.

Beta is a propagation constant that can be given as $n \times k_0$, k_0 is the propagation constant in vacuum and is the refractive index of this medium that is basically the square root epsilon r mu r. So epsilon over epsilon naught is basically epsilon r, mu over mu naught is basically mu r ok. So from this you can find out these are very general and known formula and you can also write that eta is basically square root of epsilon by mu by epsilon ok. So this is basically the impedance ok. So support this kind of medium they support TEM waves.

Double-Positive (DPS) Medium

- ***Double-positive (DPS) Medium:***

The Poynting vector $\boldsymbol{S} = \frac{1}{2}\mathbf{E_0} \times \mathbf{H_0}^*$ points along the same direction as the wave vector $\boldsymbol{k}$, and the intensity of the wave (power flow per unit area) is given by $I = Re\{S\} = |E_0|^2/2\eta$.



Plane wave propagating in an ordinary double positive (DPS) medium. The vectors **E**, **H**, and **k** form a right-handed set and the wavefronts travel in the same direction as the power flow.

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Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

So TEM means the transverse electromagnetic waves. So in this case both electric and magnetic fields are perpendicular to the direction of propagation that means your E, H and k are mutually orthogonal and they will form a right handed system which you have discussed already ok. Now in this medium the last thing that remains is in the what is the direction of the energy flow. So that you can determine from the pointing vector S and S is calculated as $S = \frac{1}{2}E_0 \times H_0*$ ok. So this is how you can compute what is the direction of energy flow or power flow per unit area ok.

So you can see that because E and H they are forming a right hand system so you can actually see that S will also propagate in the same direction because both are positive in this case ok. And they point in the same direction as the wave vector k and the intensity of the wave that is basically the power flow per unit area can be given as I which is real power of S and that can be given as modulus E naught square over 2 eta. So that is how the intensity can be found out ok. So intensity once again what is intensity it will be the power flow per unit area ok and power is nothing but energy per unit area sorry energy per unit time ok. So here in power you get the time dependence and in intensity you are getting the area dependence as well ok.

Single-Negative (SNG) Medium

- ***Single-Negative (SNG) Medium:***

  In a single-negative (SNG) medium, either ε or μ is negative so that k and η are both imaginary, whereupon (L23.4) provides:

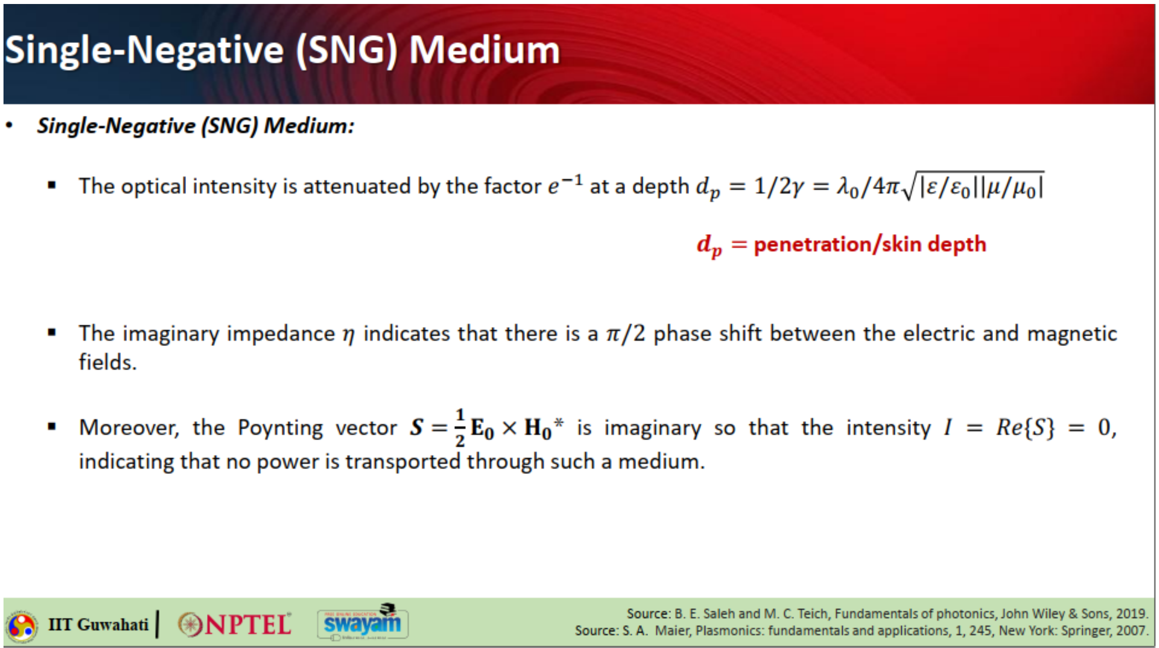
$$\gamma = \omega\sqrt{|\epsilon||\mu|} \qquad \beta = 0 \qquad \eta = j\sqrt{\frac{|\mu|}{|\epsilon|}}$$

  - These parameters correspond to an exponentially decaying field that behaves as $\exp(-\gamma z)$, where z is the propagation distance.
  - Since $\beta = 0$, a SNG medium does not support propagating waves.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

And this diagram we have already discussed so this is how you got your pointing vector and wave vector both going in the same direction. Now when you move to a single negative medium few things change. Now depending on whether your permittivity or permeability which one is negative ok. you can have k and eta both as imaginary ok. So you can calculate gamma which is now omega square root of modulus of epsilon times modulus of mu okay.

And when you compare it with the wave vector you will see that the real part is not there so beta becomes 0 ok. So immediately you can conclude that this kind of medium they do not support wave propagation because the propagation constant is 0 ok and there is a gamma, gamma is basically the attenuation factor. So this tells you the rate at which the wave will decay when the wave will enter this kind of a medium. And this is the impedance of this particular medium that is given as j square root of modulus mu over modulus epsilon. This parameters correspond to an exponentially decaying field that behaves as exponential minus gamma z.



So z is a propagation direction and as I told you gamma is basically the attenuation constant ok and this we have already discussed that because beta is 0 no propagating waves are supported in this medium. And you can also find out the depth at which the wave will the amplitude of the wave or you can say the intensity of the wave is attenuated by a factor of 1 by e and that happens at a depth of d_p so that can be correlated to 1 by 2 gamma. And when you put the other important factors in this equation that determines gamma you can write $d_p = 1/2\gamma = \lambda_0/4\pi\sqrt{|\varepsilon/\varepsilon_0||\mu/\mu_0|}$

So this actually gives you the penetration depth or the skin depth of that particular material and this is what we know about the skin depth of metal and other things ok. The same concept is there in the microwave domain as well ok.

Double-Negative (DPS) Medium

- ***Double-Negative (DNG) Medium:***
  - In a double-negative (DNG) medium, both ε and μ are negative so that (L23.4) leads to $k = \omega\sqrt{|\varepsilon||\mu|}$, which is real, whereupon

$$\gamma = 0 \qquad \beta = nk_o \qquad n = -\sqrt{\frac{|\epsilon|}{\epsilon_o}\frac{|\mu|}{\mu_o}} \qquad \eta = \sqrt{\frac{|\mu|}{|\epsilon|}}$$

  - Indicating that the refractive index is negative.
  - Since $\gamma = 0$, the medium sustains wave propagation without attenuation. The choice of signs for the square roots is established by examining the directions of the vectors $\mathbf{E_0}$, $\mathbf{H_0}$, and $\mathbf{k}$, which may be determined directly from Maxwell's equations.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

And the imaginary impedance that we see here that indicates that there is a pi by 2 phase shift that j factor is there so that gives you a pi by 2 phase shift between the electric and magnetic fields. And when you calculate the pointing vector the pointing vector is basically half of E cross H, E naught cross H naught conjugate and this also comes out to be imaginary and in that case if you try to calculate what is the intensity of this particular wave you will see that the intensity will be real part of S and that is 0 that means no power is basically transported through this medium and that makes sense because the propagation constant is also 0. So from that when you move on to the next category which is basically double negative medium. So that your permittivity and permeability epsilon and mu both are negative. So when you do that you can actually see that what happens to k the k is dependent on omega square root of modulus epsilon times modulus mu.

Double-Negative (DPS) Medium

- ***Double-Negative (DNG) Medium:***
  - The DNG medium, (L23.1) and (L23.2) yield:

$$\mathbf{k} \times \mathbf{H}_0 = \omega\,|\epsilon|\,\mathbf{E}_0 \qquad \text{(L23.5)}$$
$$\mathbf{k} \times \mathbf{E}_0 = -\omega\,|\mu|\,\mathbf{H}_0 \qquad \text{(L23.6)}$$

  - As with the DPS medium, the vectors $\mathbf{E_0}$, $\mathbf{H_0}$, and $\mathbf{k}$ are mutually orthogonal.
  - However, the reversal of signs in (L23.5) and (L23.6), relative to those in (L23.1) and (L23.2) for the DPS medium, is tantamount to exchanging the roles of the electric and magnetic fields.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: S. A. Maier, Plasmonics: fundamentals and applications, 1, 245, New York: Springer, 2007.

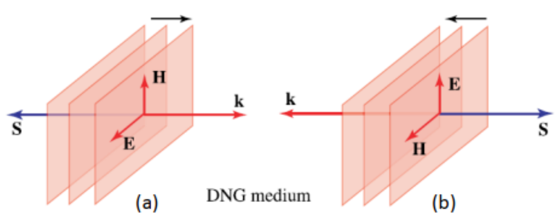
If both are negative, but there are modulus so actually k is real. So you are actually having a k that is real that means beta will be n k naught and you are actually able to have wave propagation. Here gamma is 0 the attenuation constant is 0 ok and then n is given as minus square root of modulus e over epsilon naught times modulus mu over epsilon naught and you will see that is how you are actually getting a negative refractive index ok. And the impedance of this media is given by square root of modulus mu over modulus epsilon fine. So these are the important things so always remember that in this kind of media wave propagation is possible you can get negative refractive index you can actually have no attenuation when the wave propagates ok.

And the choice of sign for the square roots is established by so these are basically the sign right of the square roots whenever you take square root is plus minus right, but then when both are negative you can actually take the negative one ok. So these are actually chosen in a process by examining the direction of the vectors E naught, H naught and k and they may be directly determined from the Maxwell's equation as well to satisfy the Maxwell's equation ok. So if you see the equations for electric field and magnetic field how they are interrelated in a double negative medium this is how the equations are now changed ok. So you have k cross H naught which is basically omega modulus of epsilon E naught ok and k cross E naught is minus omega modulus of mu H naught ok. So just in the case we have seen for DPS medium like double positive medium that E naught, H naught and k are mutually orthogonal So here also you can try to establish this fact that they are also orthogonal ok.

Just that there is some kind of sign that is reversed in the previous case the negative sign was here now the sign has come here right and that is where you have to exchange the roles of the electric and magnetic fields ok and that is how you can think of how to draw the E, H and k vectors in the case of a left handed medium right. So it is apparent that E naught, H naught and k in a double positive media they form a right handed set of vectors right. So when you actually switch E and H field that is what we see here right that because the negative sign was previously here now it is here so you can actually exchange the roles of the electric and magnetic field and if you do that this is how you will get ok. So in the double positive medium E was here, H was here but now you have switched their roles ok and you actually got a left handed system. Why it is a left handed system because E cross H ok you have to you take your fingers in the direction of e and roll it towards H you will see that your thumb is directing towards the k vector and this you can only do using your left hand ok.

Left-Handed Metamaterials

- It is apparent that $\mathbf{E_0}$, $\mathbf{H_0}$, and $\mathbf{k}$ in a DPS medium form the usual right-handed set of vectors, whereas in a DNG medium they form a left-handed set (the medium is then said to be **left-handed**).
- Since the impedance η positive, the wavenumber k is taken to be negative, and therefore so too is the refractive index n. The DNG material is therefore a **negative-index metamaterial (NIM)**.



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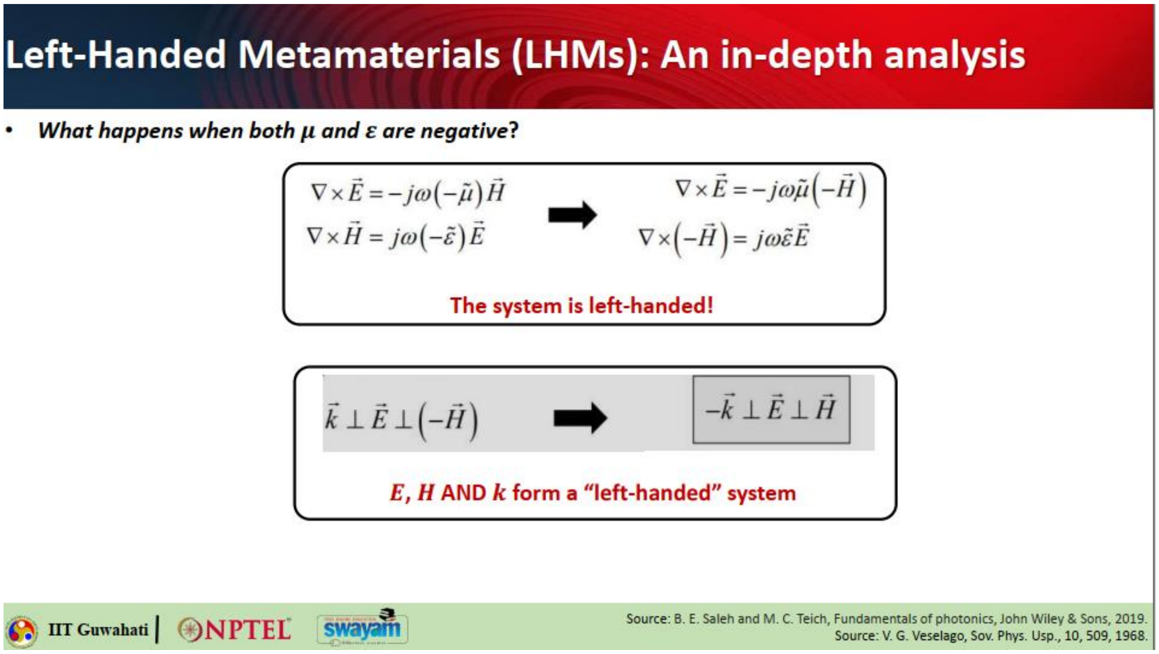
So this is a left handed system. Now when you compute eta you will see that the impedance eta is positive ok. However the wave number k is taken to be negative and therefore the refractive index also comes out to be negative. So this kind of double negative materials are called DNG materials and they are also called NIM materials negative index materials. So ok before we go there yeah here you can also see that the wave propagation happens in this direction but when you take E cross H ok that happens in this direction right. So the wave propagation and the pointing vector they are in opposite direction.

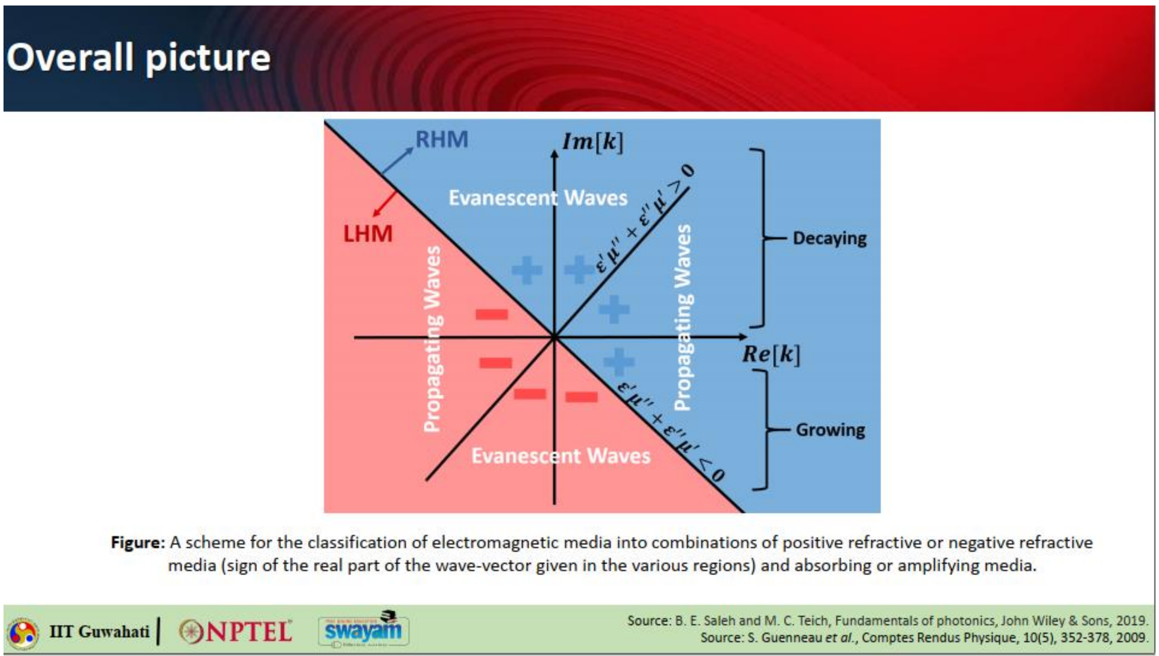
So the energy flow will happen opposite direction of the wave propagation. So if you put this two figures are actually showing the same thing just that if you put it in the same manner the way you kept it for the double positive system. So here you will see that everything else remains same other than the wave propagation direction which was on the towards right in the double positive system here it changes to left that is the only difference. Now why this has happened we have discussed that the equation has got a negative sign and because the permittivity is here and negative. So if you put if you absorb the negative sign into the magnetic field only ok.



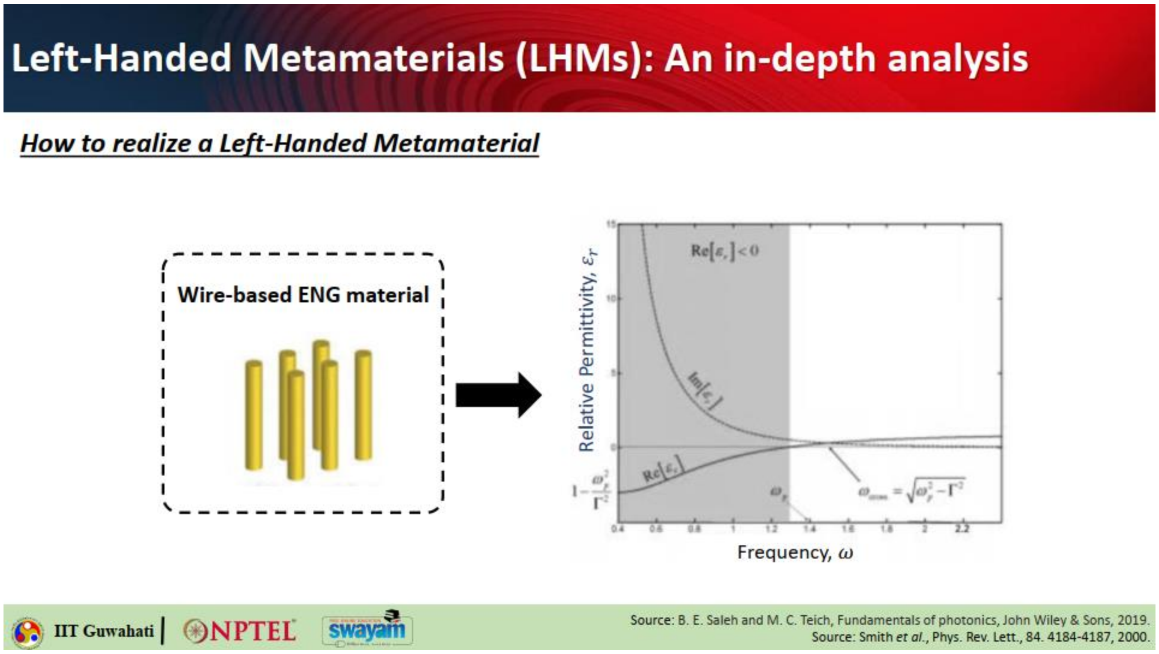
So first we will look into the Maxwell's equation here to start with that you have a material which has got mu to be negative. So mu is basically minus mu and you have got a negative epsilon as well. So epsilon is basically a minus epsilon. So you have negative signs on both cases ok and you are trying to push this negative sign to the H vector ok H field. So when you do that you actually can get this kind of a system.

So this system is basically a left handed system. So only thing that you have done here is that negative sign of permittivity and permeability you are pushing it to the H vector so that the H vector will flip its sign but then when you put it in the handedness so K is perpendicular to e and that should be now perpendicular to minus H. In a double positive system this was basically H here it is a minus H ok and that is all. So this gives you a left handed system. If you consider the k vector in more details and try to split it in the real part and the imaginary part ok you can actually get this kind of a diagram where you can also consider your epsilon to be complex mu to be complex.

So this is the overall situation. So epsilon complex will have a real part that will call as epsilon prime it will have an imaginary part that is epsilon double prime ok. Similarly mu will also mu is a complex number so mu can also have real and imaginary that is mu prime and mu double prime and you can see here that these are the region so this is the boundary that tells us ok where that this portion will be right hand material so you have epsilon prime mu double prime plus epsilon double prime mu prime less than 0 ok and when this is satisfied here it is a right hand material below this in the different color that you see this is basically left hand material right ok. And another important factor that you can see from this graph is that this is the portion where you can actually have so this is the region where you can have propagating waves. These two cases so you can actually find out how you land up here by the choice of materials ok and in these two cases you have propagating waves if you do not satisfy so here what will be the line here so this particular line is basically telling you that on this side it is epsilon prime mu double prime plus epsilon double prime mu prime has to be greater than 0 ok. So it is a positive one and this also tells you that you have this this quantity should be negative ok and if this two satisfies you actually get a evanescent wave here ok and that is where the waves are like evanescent waves cannot propagate they will be simply dying ok.



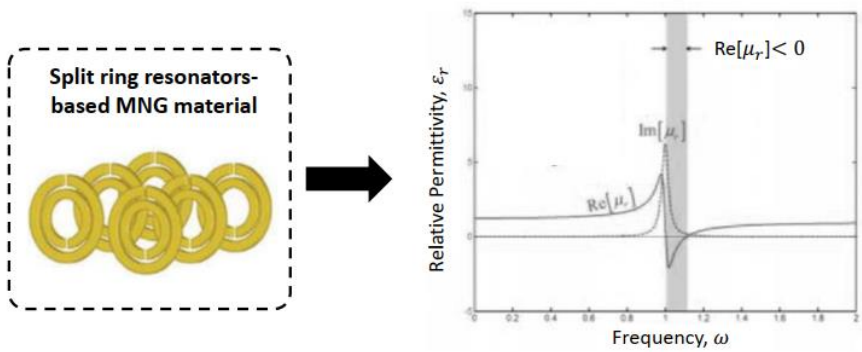
You can get propagating waves here you can also get propagating waves here ok and you can get evanescent wave in this particular region again ok. So that is how you can actually read this particular graph that depending on because in all other cases we have not considered epsilon and mu to be complex but if you have epsilon mu to be complex you can actually calculate this and see where your material is landing and you can from this chart you can find out whether it will be a evanescent wave it will be a propagating wave if it is a propagating wave sorry if it is an evanescent wave in a normal medium it will be a propagating wave in a normal medium it will decay right. Normal medium in the sense if it is a DPS medium or if it is a single negative medium it will decay, but if it is an evanescent wave in a negative index material or double negative material the evanescent waves will grow ok instead of decaying they will grow. So all this information you can actually obtain from this particular graph which is a plot between the real part and the imaginary parts of the wave number. So if you remember from the previous lecture that how do we realize this kind of negative handed or left handed metamaterials the first thing is that we can actually go with wire based arrays and this can give us negative permittivity over a certain frequency range and you can also decide by the by changing the dimension and the array periodicity that what will be the plasma frequency below which the material will behave like a metal.



We have also seen this case where you we have designed an array of split ring resonators and that could give us negative negative permeability, but always remember these are kind of complex values so we are basically there is a typo here this is basically a mu r ok and we are looking into the real part of the permeability ok. In the previous case also we have actually seen into this region the shaded region which is basically showing the real part of the permittivity ok. So here also we are bothered about the real part of the permeability which is negative for a very small for a very small frequency range. Now if you want to make a metamaterial which has got both negative permittivity and negative permeability in the same frequency window what you have to do you have to take this one and also this one both are having negative values at the same wavelength range ok and you can add them up together.

Left-Handed Metamaterials (LHMs): An in-depth analysis

How to realize a Left-Handed Metamaterial

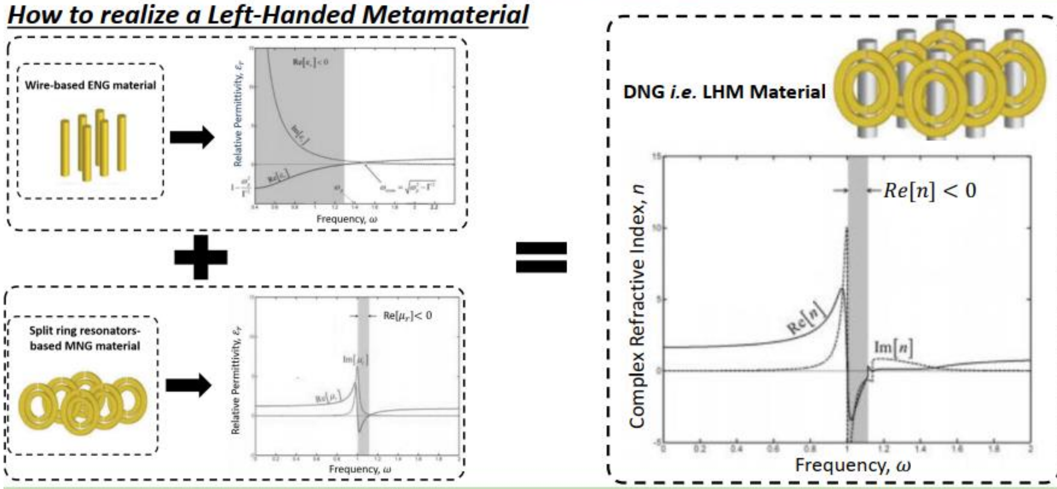


Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So here you can see so it is starting from 1 to say 1.5 here also 1 to 1.5 these are normalized frequencies so they can be matched and you can see that this and this if they are combined they will probably give you what you are looking at. You can make a combined array of this and you will be able to get negative permittivity as well as negative permeability over a certain frequency window and that will give you negative refractive index. Now this negative refractive index also gives rise to some exotic effects something like negative or inverse Doppler shift. Now what is Doppler shift? I believe all of you are aware of Doppler shift.

Left-Handed Metamaterials (LHMs): An in-depth analysis

How to realize a Left-Handed Metamaterial



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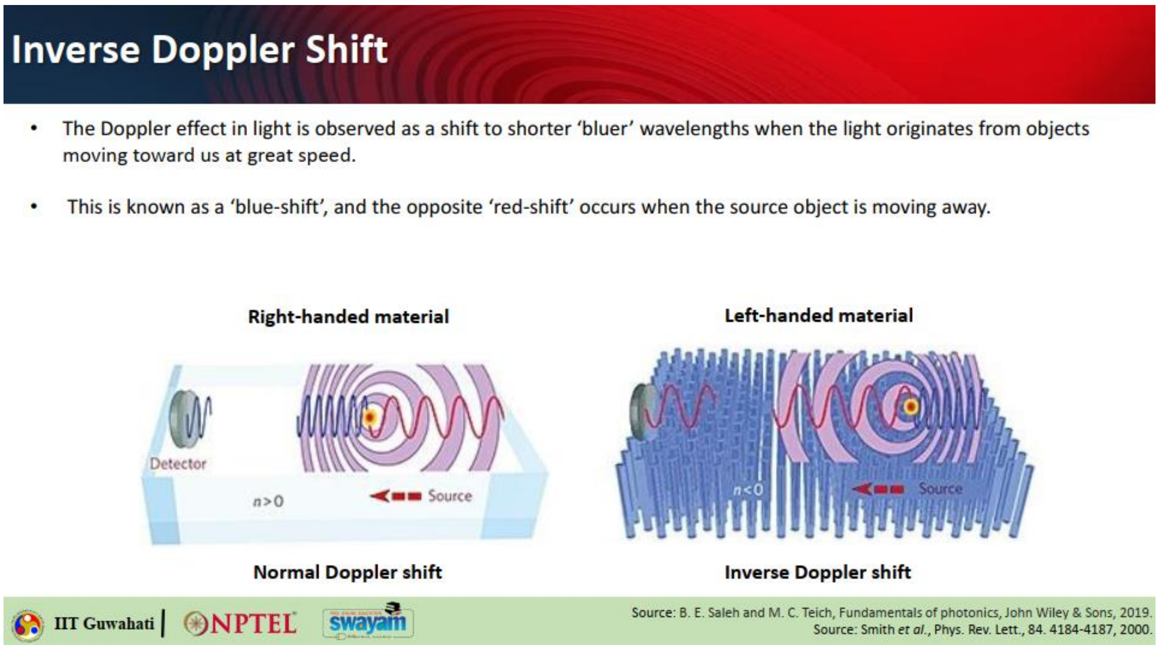
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

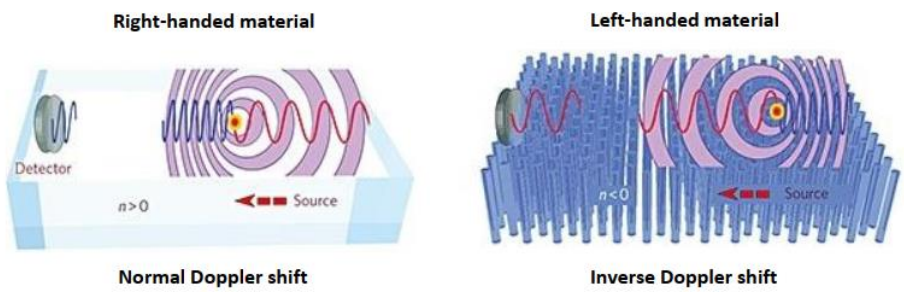
It is basically the effect in light or sound ok you can have Doppler shift and sound as well. So if you consider Doppler shift in light here it is observed as a shift to shorter or bluer wavelengths when the light originates from objects which is moving towards us at a very great speed ok. In terms of frequency you can say the frequency increases or the wavelength is decreasing and that is why it is shifting towards shorter bluer wavelengths ok. The opposite effect will happen when the source moves away from us the frequency decreases that means the shift will be towards longer or red wavelength. So that is what the blue shift is seen when the object moves towards us and red shift is seen when the object moves away from us and this is how a normal Doppler shift works in right handed material.



Now if you want to observe inverse Doppler shift it is possible it means there the particles that make up a wave need to move in one direction but the intensity variation of the light's peak and valleys in the wave that should shape the other ok. So that should move in the other direction. So that can only happen in some kind of material which is not naturally found ok. So usually there are natural materials in which intensity variation moves slower than the particles in it but materials in which they will move in the opposite direction they do not exist in nature ok. It means you can only realize them using some artificial structures and that is where some unique things are seen using this kind of left handed materials ok.

Inverse Doppler Shift

- To observe the inverse Doppler effect, the particles that make up a wave need to move in one direction, but the intensity variation of the light's peaks and valleys in the wave shape the other.
- There are natural materials in which the intensity variation moves slower than the particles in it, but materials in which they move in opposite directions do not exist in nature and can only be realized using artificial structures.



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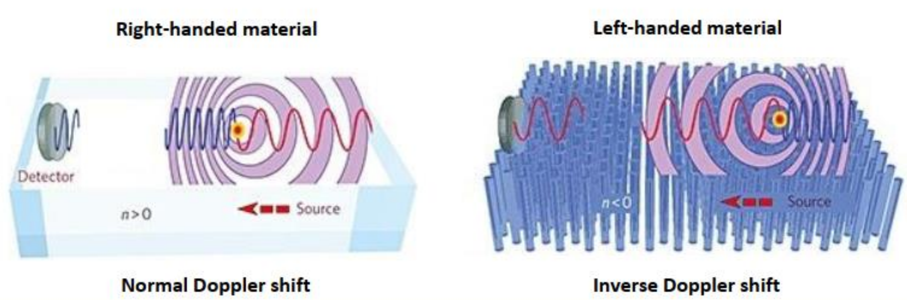
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So here is one example so this is one beam directed at the detector via a medium which has got a positive refractive index and in this case it is again identical beam which is passing through a photonic crystal with negative refractive index and it can actually show us the reverse or inverse Doppler effect. Now how it works let me explain here the photonic crystal was mounted on a platform that moved toward the detector at a constant speed and the Doppler shifts were measured from the interference patterns of the beams at the detectors and this could give us a clear verification of the inverse Doppler effect at optical wavelength.

Inverse Doppler Shift

- Consider, one beam is directed at a detector via a $n>0$ medium, and a second identical beam was passed through a photonic crystal with negative refractive index, in which the reverse Doppler effect was predicted to occur.
- The photonic crystal was mounted on a platform that moved toward the detector at a constant speed, and the Doppler shifts were measured from the interference pattern of the beams at the detector, which led to the first clear verification of the inverse Doppler effect at optical wavelengths.



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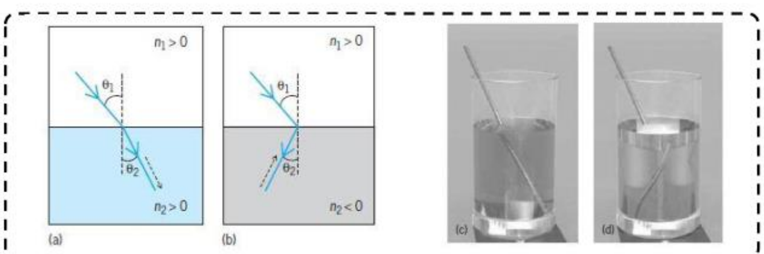
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So here when the source was moving towards the detector you were seeing red shift instead of a normal and red shift instead of a normal blue shift. So that is what you were able to see so here you will see red shift and when it is going away you are supposed to see blue shift but you are seeing sorry you are supposed to see red shift but you are seeing blue shift ok. So I believe this diagram is clear so when the source is moving towards the detector with some velocity ok the frequency should increase or the wavelength should get shorter so it should become blue you can see the squeezing of the pulse here that means the frequency has increased or the wavelength has decreased and when the source is moving away from the detector in this direction if it moves the frequency reduces it means the wavelength increases ok.

Whereas it happens completely opposite in case of a negative and negative index material or left handed material. Now left handed materials also produce negative refraction. Now refraction is already known to all of us in school days it is a very very interesting phenomena if you look into this particular image this is what we have seen in our school days many many times also naturally if you stick a you know pipe in a juice glass you will be able to put you will see this kind of a effect right. So there is a bending starting from the interface. And this particular diagram actually indicates or compares the case where both the materials are positive materials double positive materials but in this case they are not positive materials case B here one material is basically a negative refractive index material.

Left-Handed Metamaterials (LHMs): Negative Refraction

LHMs produce Negative Refraction



- Diagrams of (a) positive refraction and (b) negative refraction and calculated images of a metal rod (c) in a glass filled with "negative index water" (n = -1.3).
- In parts a & b, solid lines with arrows indicate the direction of energy flow, broken line with arrows show the direction of the wave vectors.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: G. Dolling *et al.*, Optics express, 14(5), 1842-1849, 2006.

So when you are taking that what happens in the normal case you see the arrow ok is showing the direction of wave vector and now the broken line the broken line with arrows this one the broken line with arrows show the direction of the wave vector that is that will show you the k vector and the solid lines with arrows indicate the direction of energy flow ok. So here you can see the here the energy flow is happening this way but the wave vector will move in the opposite direction in this case the energy flow also happens this way but and the wave vector is also the wave is also moving in the same direction but here the wave will be seen moving in the backward direction. So when you actually produce a image of the same straw in the glass but this time it is not water instead of water water has got refractive index of what 1.3. So here we are considering negative index water that means n is minus 1.

3 and in that case this is how the bending is going to take place. So it will bend on the same side of the normal and it will look like this. So let us look into an in-depth analysis of the negative refraction. So first of all that we have studied in our school days that the refraction of light at the boundary between two ordinary DPS media they obey the Snell's law. So Snell's law as it can be written as n1 sin theta 1 equals n2 sin theta 2 and in this case this particular equation is coming from matching the components of the wave vectors k1 and k2 along the direction of the boundary. Now if one of this media say medium 2 is replaced by a double negative medium which has got a negative refractive index of n2 in that case the equation looks like this.

Negative Refraction: An in-depth analysis

- The refraction of light at the boundary between two ordinary dielectric (DPS) media obeys Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which results from matching the components of the wavevectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ along the direction of the boundary.

- If one of the media, say medium 2, is instead a DNG medium with negative refractive index n_2 then we have:

$$n_1 \sin\theta_1 = -|n_2| \sin\theta_2$$

This reveals that the angle of refraction θ_2 must be negative and the refracted and incident rays both lie on same side of the normal to the boundary.

- This outcome also can be understood as arising from matching the components of the wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ along the direction of the boundary.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So you will have $n_1\sin\theta_1= n_2\sin\theta_2$. Now this reveals that the angle of refraction theta 2 in the case of a double negative medium that angle should be negative and the reflected waves and the incident waves they will lie on the same side of the normal to the boundary and this outcome can also be understood as it is arising from matching the components of the wave vectors k_1 and k_2 along the direction of the boundary. So let us look into one figure that shows that. So it is understood and clear that the optics of planar boundaries and lenses they get significantly altered when a DPS medium is basically replaced by a DNG medium. So if you for example if you take a convex lens which is made of DNG material that will behave like a concave lens made of DPS material and vice versa. And if you look pictorially what happens the planar boundary ok between positive and negative materials they actually focus they they possess focusing power.

So let us look into this figure in little bit more details. So here we are considering two positive refractive index materials so n_1, n_2 both are positive. So n1 is shown in a lighter shade so this is basically a lower refractive index than n_2 and this is the direction with which k_1 the wave is incidenting ok this is theta 1 ok. So you can find out this is the cos theta component and this is the sin theta component ok. And this is the refracted ray this is angle theta 2 so you can also see that this is the sin theta component. So this is how you are basically matching the components of the wave vector along the boundary and that gives you that Snell's law equation ok.

Negative Refraction: An in-depth analysis

- It is thus clear that the optics of planar boundaries and lenses is altered significantly when a DPS medium is replaced by a DNG medium.
- Indeed, a convex lens of DNG material behaves like a concave lens of DPS material, and vice-versa.
- A planar boundary between positive- and negative index materials possesses focusing power (shown below) for the special case $n_2 = -n_1$, which provides $\theta_2 = -\theta_1$.

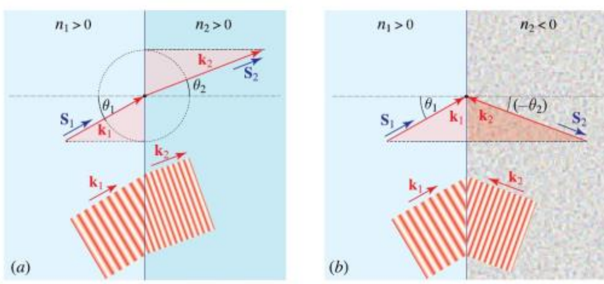

Figure: Negative Refraction.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So here one important thing to mark is that though the direction slightly bends but the propagation of wave k_1 happens in this direction and k_2 slightly bends but it still moves in the similar direction. Now if you replace this medium this n_2 medium with a negative index medium and say you are having n_2 same as minus n_1 in that case what can be seen that theta 2 will be same as minus theta 1. So the refracted ray will also lie on the same side of the normal it will not go to the other side it will lie on the same side energy flow will happen along this direction but the wave propagation in this media will happen in the backward direction. So here you can see k_1 is falling on the interface but you will see k_2 it is it looks like as if it is propagating backward but the energy is flowing in the opposite direction.

Negative Refraction: An in-depth analysis

- The planar boundary then acts on optical rays in the same way as does a convex spherical boundary between two DPS media.
- Moreover, for DPS and DNG media with permittivities and permeabilities that have the same magnitudes ($\varepsilon_2 = -\varepsilon_1$ and $\mu_2 = -\mu_1$), the impedances $\eta_1 = \sqrt{\mu_1 \varepsilon_1}$ and $\eta_2 = \sqrt{\mu_2 \varepsilon_2}$ are of equal magnitude and sign, so that no reflection occurs at the boundary, at any inclination, regardless of the polarization.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: Smith *et al.*, Phys. Rev. Lett., 84. 4184-4187, 2000.

So, this is how negative refraction takes place ok. So the planar boundary that acts on the optical ray in the same way as does a convex spherical boundary between two DPS medium. So, here using a planar boundary between a DPS and DNG medium you are basically creating a lens which is otherwise created by a convex spherical boundary if you consider only DPS medium. So, this kind of metamaterials will encourage you to have planar optical components and that saves a lot of space when you try to design compact optical or integrated photonic systems. Moreover, for DPS and DNG medium with permittivities and permeabilities having the same magnitudes so if you consider ε_2 to have same as ε_1 but negative so ε_2 is $-\varepsilon_1$ and μ_2 you can consider as $-\mu_1$. In this cases the impedance $\eta_1=\sqrt{\varepsilon_1\mu_1}$ and $\eta_2=\sqrt{\varepsilon_2\mu_2}$ and they both will be equal and the same sign.

So in that case what happens no reflection will occur at the boundary at any inclination regardless of the polarization that means you are able to then pass all the light ray that is falling on that kind of interface to the other side of the interface and you can get rid of all the reflection losses. So that is also a big big advantage of this kind of materials. So with that we will stop here and in the next lecture we will go into more details of the examples using negative index materials such as perfect lens. Thank you.