

Course Name- Nanophotonics, Plasmonics and Metamaterials

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Week-10

Lecture -28

Hello students, welcome to lecture 28 of the online course on Nanophotonics, Plasmonics and Metamaterials. Today's lecture will be on Metasurfaces and Frequency Selective Surfaces. So here is the lecture outline, we will look into the basics of metasurfaces, their application towards phase modulation and some other applications and then we will move on to frequency selective surfaces. We will see their definition, we will look into the fundamentals and also discuss their applications. So metasurfaces when it comes to mind, it is basically a two dimensional metamaterial with sub wavelength periodicity and this metasurfaces are able to demonstrate unusual electromagnetic properties and it can vary over a frequency range from microwave to terahertz to optical. There are resonating metamaterials which can be tailored by tuning the geometry of its unit cells or meta atoms.

Lecture Outline

- **Metasurfaces:**
 - Introduction
 - Phase Modulation
 - Other Applications
- **Frequency Selective Surfaces:**
 - Definition
 - Fundamentals
 - Applications

So conventionally they are used for phase change and focusing of electromagnetic waves at optical frequency in the far field region. So here is an illustration of a typical metasurface. So you see it is a very thin, it is a basically 2D material with all these unit cells which are basically working on the phase of this incident terahertz wave. It can be

terahertz, microwave or optical depending on the frequency range you are looking at.

But what it can do, it can modulate the phase or the amplitude or the polarization. So that way you can actually get polarization, modulation, spatial beams, active control, focusing, hologram. All these different things can be generated by using metasurfaces. So a formal definition for metasurface is basically an ultra thin array of sub wavelength scale metallic elements which are deposited in periodic, aperiodic or random patterns on the surface of a dielectric substrate. And the shapes of the individual elements and their geometry of the layout on the surface, they endow the metasurfaces with distinctive optical properties.

Metasurfaces: Introduction

- **Metasurfaces:** *two-dimensional metamaterials with a sub-wavelength periodicity*
- They demonstrate unusual electromagnetic properties in various frequency ranges such as microwave, terahertz, and optical.
- Resonating metasurfaces can be *tailored by tuning the geometry of their constituent unit cells*
- Conventionally used for phase change and focusing electromagnetic waves at optical frequency in the far-field region.

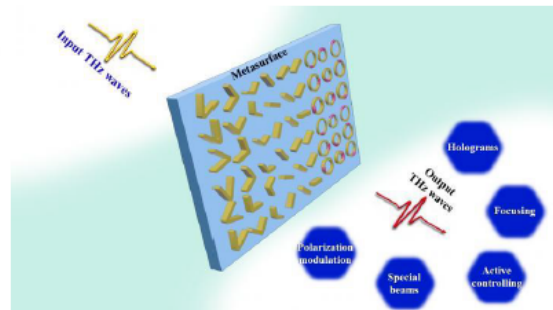


Figure: An illustration of metasurfaces.

And if you think of the origin of these spatial effects that comes from the metasurfaces, they are basically a consequence of coupling of light and the surface plus bond propagation waves that generate at the metal dielectric boundary. So today we will look at a comparison of metasurface with metamaterials. So if you remember metamaterials, they are basically 3D materials which are able to provide artificial permeability and permittivity. So if you remember the split tree resonator array positioned on a metallic wire, that was able to give you negative permittivity as well as negative permeability. So, what we do, we can actually model the system as an effective permeability, μ effective or ϵ effective.

Metasurfaces: Introduction

- **Metasurfaces** could be ultrathin arrays of subwavelength-scale metallic elements, deposited in periodic, aperiodic, or random patterns, on the surface of a dielectric substrate.
- *The shapes of the individual elements, and the geometry of their layout on the surface, endow metasurfaces with distinctive optical properties*
- These unusual properties are a consequence of the coupling of light and SPP waves generated at the metal-dielectric boundary.

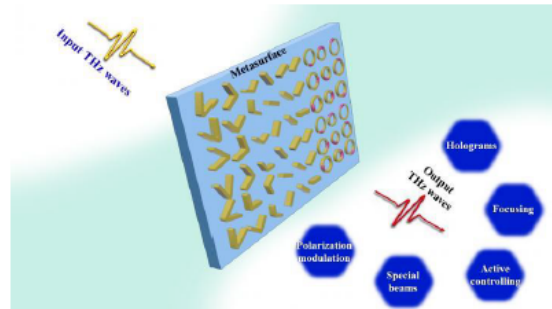


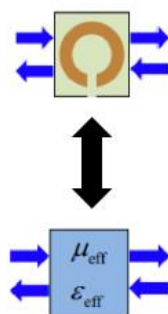
Figure: An illustration of metasurfaces.

So this is typically a 3D version, that is the periodic cells, sorry the unit cells are repeated in periodic fashion in all three dimensions. On the other hand, you can think of metasurface which is basically a 2D version. So here the periodic arrangement of unit cell happens in two lateral dimensions, the thickness is very little. So how it can help? So it can actually change the amplitude, frequency, polarization or phase of the incident wave. So here is an example, say if right circularly polarized light falls on the metasurface, it may give you a left circularly polarized light.

Metasurfaces: Comparison with Metamaterials

Metamaterials

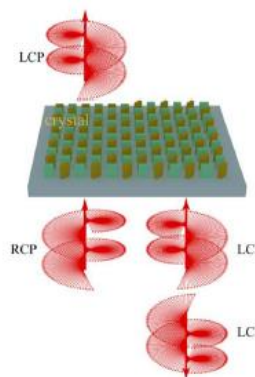
Provide artificial μ and ϵ



Metasurfaces

Modifies electromagnetic waves arbitrarily as a function of position:

- Amplitude
- Frequency
- Polarization
- Phase



You can also think of left circularly polarized light falling and some part of it getting reflected. So it is blocking the left circularly polarized light to go through. So this kind of applications may be possible. So let us look into the basics of how phase modulation

can be obtained through metasurfaces. So, let us assume a wave that is traveling along the z direction as shown in the figure.

Introduction to Phase Modulation using Metasurfaces

- A wave traveling in the z direction, on transmission through a dielectric plate of fixed thickness d and graded refractive index $n(x, y)$ in the $x-y$ plane, undergoes a spatially varying phase shift (modified its wavefront):

$$\varphi(x, y) = n(x, y)k_0d \quad (\text{L28.1})$$

- Achieving a phase shift of 2π requires a local thickness equal to the wavelength of light in the medium.
- A planar metasurface has the merit that it can introduce a phase shift of similar magnitude with far less thickness.

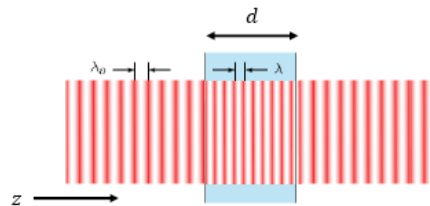


Figure: Transmission of a plane wave through a dielectric plate.

And on the transmission through a dielectric plate of fixed thickness that is d and graded refractive index, so the refractive index is given as $n(x, y)$, so it is in the xy plane. So this is z direction, so the xy plane is basically the vertical plane. And in that case the wave will undergo a spatially varying phase shift which modified its wavefront. So, you can write the phase shift $\varphi(x, y) = n(x, y)k_0d$. So here you can see this is the free space wavelength.

As soon as it enters the medium with a refractive index $n(x, y)$, so the wavelengths get shorter. So you can see the wave looks compressed. When it comes out it again retains the same kind of wavelength in vacuum. So here the variation is in xy plane. You are seeing this, this is the z direction.

Phase Modulation using Metasurfaces

- The metallic elements of the metasurface function much like optical antennas that modify the optical wavefront.
- A resonant antenna acts as a scatterer that introduces a frequency-dependent phase shift that ranges from $-\pi/2$ to $\pi/2$ for frequencies below to above resonance, respectively.
- A spatially varying phase shift $\varphi(x, y)$ may be implemented by making use of a metasurface comprising elements of spatially graded sizes and geometries that correspond to spatially varying resonance frequencies.

So this is how it changes. Now achieving a phase shift of 2π requires a local thickness which is equivalent to the wavelength of the light in that medium. So that is how you can actually get a phase shift of 2π . So d has to be equal to λ in that particular medium. Now a planar metasurface has the merit that it can introduce a phase shift of similar magnitude with far less thickness and this is where things become interesting.

There the same amount of phase shift can now be achieved using metasurface instead of using this dielectric plate. You can use a very very thin metasurface which is only couple of nanometers thin and still you can get a similar kind of effect. Normally λ in optics it is like in micrometers, orders of micrometers you can think of. So, if you think of telecommunication wavelength it is 1.55 micron. So in that case to achieve 2π phase shift you will require λ thickness of this material. So that is typically 1.55 divided by n that will be the λ in this particular case but that is also in micrometer range.

Phase Modulation using Metasurfaces

- An incoming wave of fixed frequency is then subjected to a spatially varying phase shift so that the metasurface acts as a phase modulator.
- A metasurface using an array of metallic elements whose shapes and resonance frequencies vary in the x direction.
- The shapes of the elements are engineered such that the phase shift they introduce is a linear function $\varphi = qx$ for one of the polarization components.
- Since the metasurface is ultra thin, it may be modeled mathematically as an optical component that introduces a spatially varying phase discontinuity (*i.e.*, a phase shift that takes place over a distance $d \rightarrow 0$).

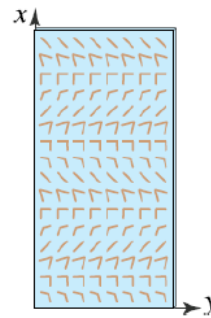
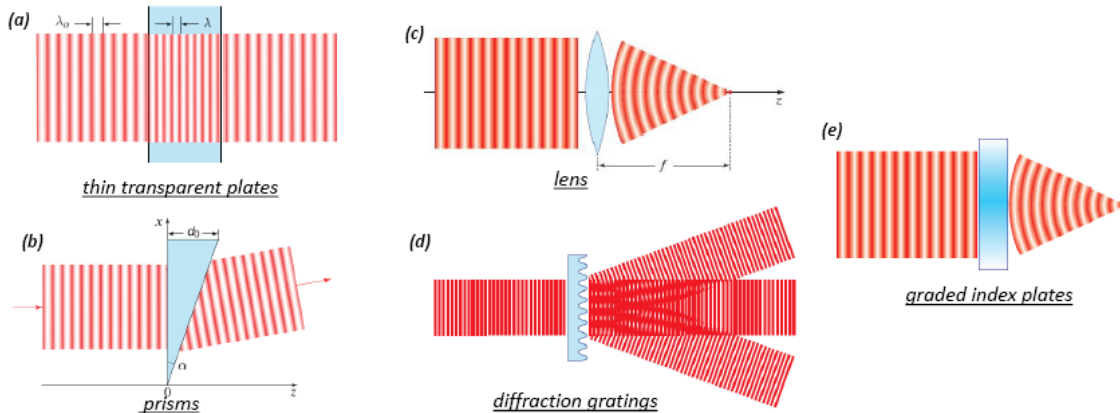


Figure: Metasurface for phase modulation.

But metasurface can help you achieve that with a nanometer scale thickness. Now what is the magic in this metasurface? So, the metallic elements that you see in the metasurface that function much like optical antennas which can modify the optical wavefronts. So, when you think of resonant antennas they act as scatterers and they can introduce a frequency dependent phase shift which can range from minus pi to pi for the frequencies below and above resonance. So a spatially varying phase shift like $\varphi = qx$ may be implemented by making use of a metasurface which comprises elements of spatially graded size because along x and y there is a variation in the phase shift. So the elements need not be same along x and y you have to change their size and geometry so that you are able to get spatially varying resonance frequencies. I will explain this with example very soon. So, an incoming wave of fixed frequency is then subjected to this spatially varying phase shift so that the metasurface can now act as a phase modulator.

Phase Modulation using Metasurfaces

- The phase modulation introduced by such a metasurface may modify an incoming optical wave in one of the many ways such as

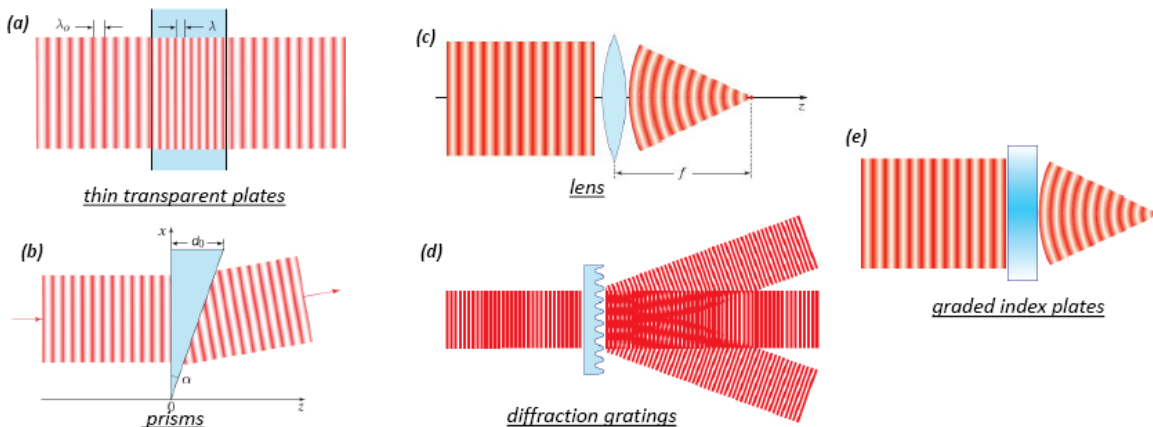


Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: H. Huang et al., Optics Express, 29(25), 40759-40769, 2021.

So let us take this particular example where the metasurface is basically an array of metallic elements. So these elements that you see these are metallic elements. So the metallic elements the element size and geometry are same along the y direction but along z direction they are different. So that way the phase will be changing along the x direction. So, the phases so this is just an example of how you can change phase along one particular direction.

Phase Modulation using Metasurfaces

- A salutory feature of this approach is that the wave undergoes *minimal spatial spread (diffraction)* as it crosses the *infinitesimally thin metasurface*.



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: H. Huang et al., Optics Express, 29(25), 40759-40769, 2021.

So here you can see the shapes of the elements are engineered such that the phase shift they introduce becomes a linear function like $\phi = qx$ for one of the polarization components. So here it becomes a linear function along x. Now since these metasurfaces are ultra thin they can be modeled as optical components that introduces a spatially

varying phase shift or you can say phase discontinuity because a phase shift that takes place over a distance d equals 0 can be thought of as a discontinuity and the thickness of this metasurface is like almost 0 . So you can say that they introduce spatially varying phase discontinuities. Now what is the benefit by doing phase modulation from these metasurfaces you can actually modify the incoming optical phase front.

So like this you have seen you can use in the same manner like plane transfer and plates they may allow the light to simply go through without modulating the phase front but when you take prism you can actually send it at a particular angle depending on the prism angle α . So all these functionalities can be achieved not only these two the work of a lens which does the focusing at a particular focal point at a distance f or diffraction grating or graded index plate which also does focusing kind of application all these components can be literally replaced by metasurfaces. So the design of the constituent elements need to be changed depending on the application. So this is what a solitary feature of this approach of using metasurface is that the wave will undergo minimal spatial spread or diffraction as it crosses the infinitesimal small metasurface. So, this is a very good benefit of using metasurface because the thickness is almost 0 so it will have very minimal diffraction or you can say minimal spatial spreading. So that is why people are trying to replace all these bulky optical components using this ultra-thin metasurfaces. Now let us look into how this phase modulation works. So let us consider a phase which is $\phi(x, y)$ that can vary linearly along this metasurface at a rate q . So q is nothing but the rate of phase change. So, you can write ϕ equals qx because in this case it is only changing along x direction.

Phase Modulation using Metasurfaces

- Consider, for example, a phase $\phi(x, y)$ that varies linearly along the metasurface at a rate q , so that $\phi = qx$.
- The complex amplitude of the incoming wave is then modulated by the factor $\exp(-jqx)$, which is a periodic function of the spatial frequency $\nu_x = q/2\pi$.
- An incoming plane wave of wavevector \mathbf{k}_1 will then generate refracted and reflected plane waves with wave vectors \mathbf{k}_2 and \mathbf{k}_3 , respectively.

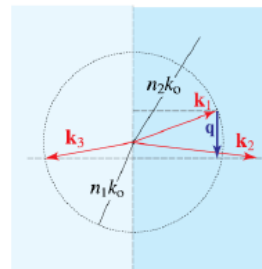


Figure: Phase-matching condition for the incident and refracted waves, and for the incident and reflected waves, at the metasurface boundary.

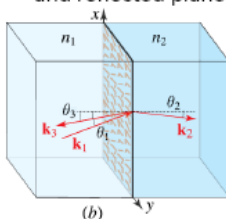


Figure: Negative reflection and negative refraction at a boundary between two media of refractive indices n_1 and n_2 by virtue of the presence of the metasurface between the two media

So this is the same metasurface that you have seen before. So it is changing only along x along y it is same. Now the complex amplitude of any incoming wave can then be

modulated by a factor exponential minus $j\mathbf{q}\cdot\mathbf{x}$. So this is the phase that will be modulate coming into the picture. So, which is basically a periodic function of the spatial frequency

$$v_x = q / 2\pi$$

Phase Modulation using Metasurfaces

- To ensure phase matching at both sides of the surface, as depicted in **Figure**, the component of the vector \mathbf{k}_2 parallel to the surface must match that of $\mathbf{k}_1 + \mathbf{q}$, where \mathbf{q} is a vector of magnitude q pointing in the x direction.
- Likewise, for the reflected wave, the component of the vector \mathbf{k}_3 along the surface must match that of $\mathbf{k}_1 + \mathbf{q}$.
- Hence, if the metasurface lies at the boundary between two ordinary media of refractive indices n_1 and n_2 , its presence causes the conventional Snell's law of refraction and reflection to assume the following modified form:

$$n_2 k_o \sin \theta_2 = n_1 k_o \sin \theta_1 + q \quad (\text{L28.1})$$

$$n_1 k_o \sin \theta_3 = n_1 k_o \sin \theta_1 + q \quad (\text{L28.2})$$

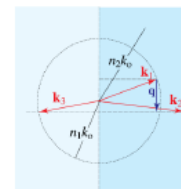
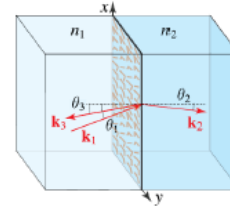


Figure: Phase-matching condition

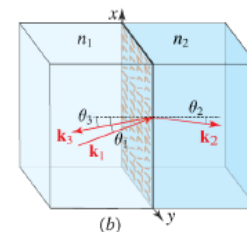
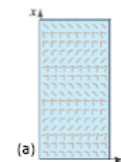
So this is the spatial frequency of those elements that introduces this phase. So now look into this figure. So, this figure basically shows a negative reflection and negative refraction at the boundary between the two media of refractive index n_1 and n_2 when the metasurface is basically present at the interface. So here an incoming plane wave of wave vector \mathbf{k}_1 which has got an incident angle of θ_1 and it will generate a refracted wave with wave vector \mathbf{k}_2 but it is basically a negative refraction. So instead of going that side it is basically coming towards on the other side of the normal.

Summary: Phase Modulation using Metasurfaces

Metasurface Refraction $n_2 k_o \sin \theta_2 = n_1 k_o \sin \theta_1 + q \quad (\text{L28.1})$

Metasurface Reflection $n_1 k_o \sin \theta_3 = n_1 k_o \sin \theta_1 + q \quad (\text{L28.2})$

where θ_1 , θ_2 , and θ_3 are the angles of incidence, refraction, and reflection, **respectively**.



- With appropriate choice of the magnitude and sign of q , the presence of the metasurface can result in *negative reflection* and *negative refraction* at the boundary, as illustrated in **Figure**.
- Equations (L28.1) and (L28.2) properly reduce to Snell's law when $\mathbf{q} = \mathbf{0}$.*

So the angle here is θ_2 . Similarly some part of the incoming wave will be reflected back but here also we are considering negative reflection. So k_3 is the reflected wave vector and the angle of reflection is θ_3 . So what has to be done for this wave to exist or this condition to exist you have to go through the phase matching condition for the incident and refracted waves as well as for the incident and the reflected waves at the metasurface boundary. So this is the metasurface boundary and you can see that all the wave vectors are drawn here. So, this is k_3 the reflected one this is k_2 the refracted one k_1 is the incident one and this is the vector of the phase change.

And this is $n_1 k_0$ that is $n_2 k_0$. So now to ensure the phase matching at both sides of the surface as we have seen in this figure what we have to do we have to look for the component of the vector k_2 parallel to the surface. So that will be basically the sine theta component and that should match the same component of k_1 the surface parallel component of k_1 that will be again the sine theta component plus that q vector. And q is basically the vector of magnitude small q that points in positive x direction or in the x direction because it is changing in x . So you can take it like that. So, if you do that you can actually put that for the reflected wave you can do the similar kind of exercise which is basically you have to look for the parallel component to the surface that will be the sine theta component here theta will be θ_3 and that should match your k_1 plus q .

Phase Modulation using Metasurfaces

Metasurface Refraction $n_2 k_o \sin \theta_2 = n_1 k_o \sin \theta_1 + q$ (L28.1)

Metasurface Reflection $n_1 k_o \sin \theta_3 = n_1 k_o \sin \theta_1 + q$ (L28.2)

- Equations (L28.1) and (L28.2) can also be written as:

Metasurface Refraction $n_t \sin(\theta_t) - n_i \sin(\theta_i) = \frac{\lambda_0}{2\pi} \frac{d\phi(x)}{dx}$ (L28.3)

Metasurface Reflection $n_i \sin(\theta_r) - n_i \sin(\theta_i) = \frac{\lambda_0}{2\pi} \frac{d\phi(x)}{dx}$ (L28.4)

where:

n_i and n_t : the refractive indices on the two sides of the interface

λ_0 : the free space wavelength, θ_i , θ_r , and θ_t : the incident, reflected, and transmitted angles.

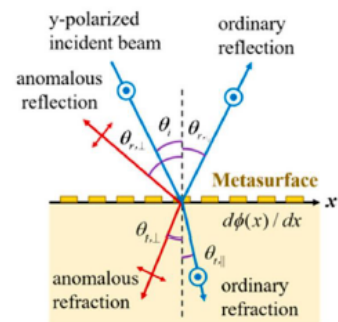


Figure: Generalized Snell's law of refraction and reflection: Schematic of anomalously refracted and reflected beams for cross-polarized scattered beams.

So this way you can actually obtain the conditions and find out the phase matching conditions. So hence if the metasurface lies at the boundary between the two ordinary medium of refractive indices n_1 and n_2 its presence can cause the conventional Snell's law of refraction and reflection to assume this particular modified form. So as I was talking about the phase matching so here you can see what is happening. So $n_2 k_0$ that is basically your k_2 the sine component that is sine θ_2 so that is the component of the

refracted wave parallel to the surface that is same as $n_1 k_0$ that is nothing but k_1 and its parallel component to the surface that is $\sin\theta_1 + q$. Similarly, for the refracted wave you can write $n_1 k_0$ that is basically $k_3 \sin\theta_3$ equals $n_1 k_0$ that is $k_1 \sin\theta_1 + q$.

Phase Modulation using Metasurfaces

Metasurface Refraction $n_t \sin(\theta_t) - n_i \sin(\theta_i) = \frac{\lambda_0}{2\pi} \frac{d\phi(x)}{dx}$ (L28.3)

Metasurface Reflection $n_r \sin(\theta_r) - n_i \sin(\theta_i) = \frac{\lambda_0}{2\pi} \frac{d\phi(x)}{dx}$ (L28.4)

- $\frac{d\phi}{dx} = q$ indicates the gradient of the phase discontinuity along with the interface, provided by the metaatoms of the metasurface.
- Equations implies that both the refracted and reflected beam can have an arbitrary direction, provided that a suitable constant gradient of phase discontinuity along the interface ($d\phi/dx$) is introduced.

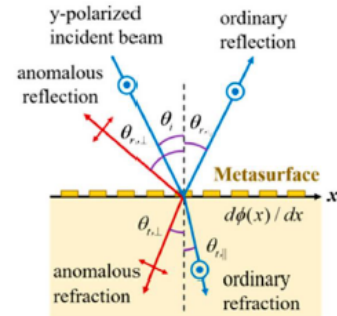


Figure: Generalized Snell's law of refraction and reflection: Schematic of anomalously refracted and reflected beams for cross-polarized scattered beams.

So this is how you are actually adding this particular factor q in your reflection and refraction equations. So these are the metasurface refraction and reflection equations. So these are basically modified Snell's law where you have introduced your particular design parameter into this law. So here we have already discussed θ_1 , θ_2 , θ_3 are basically the incidence, refraction and reflection angles and by appropriate choice of the magnitude and sine of q you can actually make this work like a negative refractive or negative reflection as well as negative refraction kind of surface as we have considered till now. So this is how a metasurface is able to get into the Snell's law and allow you that modification of the refraction and reflection characteristics.

And as you can see in this equations when you put q equals 0 that means along the x axis there is no change in the phase that means it's become a normal dielectric material q is 0 this goes back to normal Snell's law. Now you can also write so if you take this k naught on the other side and n_2 you can write as n_t that is the transmitted one and one you write as n_i that is the incident one and θ_3 is basically the reflected angle so you can write a different notation something like θ_r and θ_i for the incident angle if you use this kind of notation. So here you are dividing so you are taking this term on the left side and taking k naught common and send it on the right side. So what you have k naught can be written as λ naught over 2π and q that is the rate of phase change can be written as $d\phi(x)/dx$ because only along x your phase is changing. So, this is also another form of this modified equations.

Phase Modulation using Metasurfaces

- For a phase discontinuity $\varphi(x)$ that varies slowly with the position x , the derivative $q = d\varphi/dx$ may be regarded as the local spatial frequency at x .
- This quantity determines the local tilt imparted to an incoming wavefront, and thus the angles of reflection and refraction as a function of x .
- This approach can clearly be generalized to metasurfaces that introduce a two-dimensional phase discontinuity $\varphi(x, y)$.
- In that case, the vector $\mathbf{q} = \nabla\varphi$ represents the magnitude and direction of the local spatial frequency of the phase modulation.

So here you can see the generalized Snell's law of refraction so this is one medium this is another medium and there is a metal surface that you can see here at the boundary. So, depending on how you are designing q you can actually make it work like ordinary surface where you will have ordinary reflection and ordinary refraction or you can choose the amplitude and phase of q in such a way that it can give you anomalous reflection the red line or anomalous refraction this is the red line again. So it depends on the design of the metal surface whether you can get a ordinary reflection refraction characteristic or some extraordinary thing anomalous means which is not the normal one something opposite to the normal one. So here q as I told you this is basically the gradient of the phase discontinuity along the interface and this is where the meta-atom design comes into picture. So this is given by the meta-atoms of the meta-surface.

So, this equations also tell you that if there is no change this part becomes 0 it is a typical Snell's law. So you can actually make them go in any particular direction depending on whatever is the value here. So if you choose a suitable constant gradient for the phase discontinuity along the interface that is whatever you will choose your $d\varphi/dx$ to be you can make the refracted or the transmitted and the reflected wave to go in any direction. So you get a complete control on the direction of reflection as well as refraction by introducing this $d\varphi/dx$ that is the q and that is how meta-surface is gaining so much of attention. So, for a phase discontinuity $\varphi(x)$ that varies slowly with the position x you can say that the derivative may be regarded as a local spatial frequency.

Metasurfaces: Summary

- The metasurface can therefore be designed to introduce desired local tilts of the wavefront in both the $x-z$ and $y-z$ planes, much like an antenna array or an optical phase plate.
- The metasurface can also be engineered to introduce position-dependent amplitude modulation, imparted by the shape of the local elements.
- The combination of phase and amplitude modulation can serve as a hologram with complex transmittance that is designed to simulate the wavefront of light generated by an object.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
Source: H. Huang et al., Optics Express, 29(25), 40759-40769, 2021.

So this quantity also determines the local tilt imparted on an incoming wave and thus the angle of reflection and refraction also becomes a function of x . So this approach can be clearly seen to be generalized to meta-surfaces that introduces a two-dimensional phase discontinuity. So right now we just saw one example of one-dimensional phase discontinuity you can actually make it 2d and that that gives you that ϕ_{xy} . So in that case the q vector will be basically the gradient grad of ϕ and this vector represents the magnitude and the direction of the local spatial frequency of the phase modulation. So that will determine which way the reflected and transmitted wave can travel.

So here is the summary of the meta-surfaces. So we understood that the meta-surface can be designed to introduce desired local tilts in the wave front of the incoming wave in both xz and yz planes much like the antenna array or an optical phase plate ok. So the meta-surface can be engineered to introduce position dependent amplitude modulation which can be imparted by the shape of the local elements or the meta-atoms. The combination of phase and amplitude modulation can serve as a hologram with complex transmittance that is designed to simulate the wave front of light generated by an object. So, here are the main understandings of this meta-surface. So, we understood light propagation with phase discontinuities which are basically introduced by meta-surfaces.

Light Propagation with Phase Discontinuities

- By engineering a phase discontinuity along an interface, one can fully steer light and accomplish unparalleled control of anomalous reflection and refraction described by the generalized Snell's law.
- As shown in Figure, V-shaped resonators support "symmetric" and "antisymmetric" modes, which are excited by electric-field components along \hat{s} and \hat{a} axes, respectively.

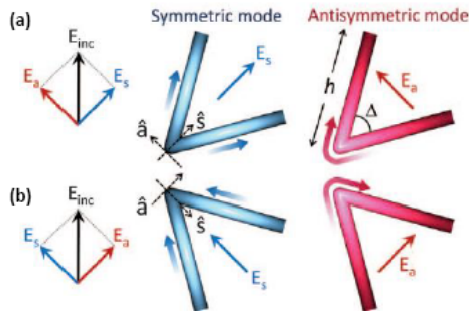


Figure: (a) An illustration of V-shaped resonators. The angle between the incident polarization and the antenna symmetry axis is 45° .

The schematic current distribution is represented by colors on the antenna (blue for symmetric and red for antisymmetric mode), with brighter color representing larger currents.

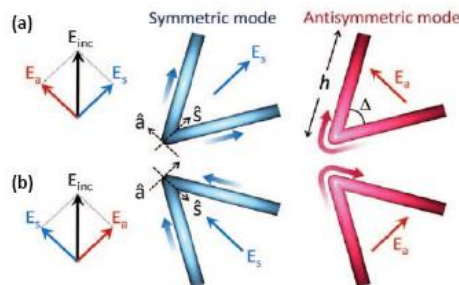
The direction of current flow is indicated by arrows with color gradient.

(b) V-antennas corresponding to mirror images of those in (a). The components of the scattered electric field perpendicular to the incident field in (a) and (b) have a π phase difference.

Now by engineering a phase discontinuity along an interface you are able to fully steer the light wave front and accomplish some unparalleled control of anomalous reflection and refraction which is described by generalized Snell's law ok. So we will take some example here. So as shown in the figure so you have got a V-shape resonator. So there are two ways light can fall one is this S that is this particular direction we can call it as a symmetric direction or A that is asymmetric direction. So, when light falls or the electric field is along this S vector you can excite symmetric mode on the two branches of this V-shape resonator.

Metasurfaces: Light Propagation with Phase Discontinuities

- In the symmetric mode, the current distribution in each arm approximates that of an individual straight antenna of length h (Fig. middle), and therefore the first-order antenna resonance occurs at $h \approx \lambda_{\text{eff}}/2$, where λ_{eff} is the effective wavelength.
- In the antisymmetric mode, the current distribution in each arm approximates that of one half of a straight antenna of length $2h$ (Fig. right), and the condition for the first-order resonance of this mode is $2h \approx \lambda_{\text{eff}}/2$.

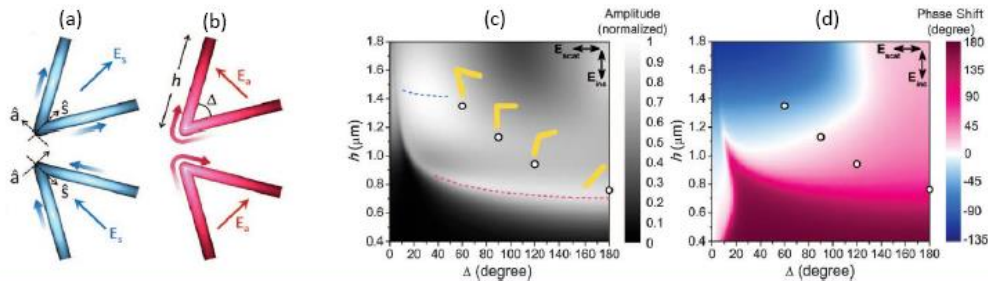


The angle is 45 degree ok and what you see here is basically the current distribution which is represented by the colors ok. So the blue line shows current distribution for the

symmetric case the red one shows for the asymmetric case and the brighter the color larger is the current. And the current flow direction is also shown here ok through this arrows. Now in this case what happens you can also take mirror image of this particular antennas and they also do similar kind of properties and you can get the components of the scattered electric fields just that they will be pi phase difference from this one. So just by rotating the antenna you can create a pi phase difference ok.

Metasurfaces: Light Propagation with Phase Discontinuities

- Analytically calculated amplitude and phase shift of the cross-polarized scattered light for V-antennas consisting of gold rods with a circular cross section and with various length h and angle between the rods Δ at $\lambda_0 = 8 \text{ mm}$.
- The four circles in (c) and (d) indicate the values of h and Δ used in experiments. The optical properties of a rod and "flat" antenna of the same length are quantitatively very similar, when the rod antenna diameter and the "flat" antenna width and thickness are much smaller than the length.

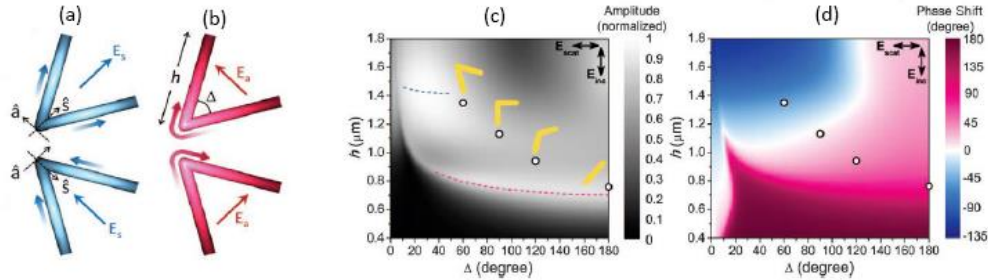


So in the symmetric mode if you look into the current distribution in each arm ok it approximates that of an individual straight antenna of length h . So this is length h ok and therefore the first order antenna resonance can occur at h equals $\lambda_{\text{effective}}$ by 2. So what is $\lambda_{\text{effective}}$ that is the effective wavelength. So in symmetric mode h equals $\lambda_{\text{effective}}$ by 2 whereas when you go for anti symmetric mode ok. So anti symmetric mode this is the direction of the electric field ok it is along this a axis.

So you can see the current distribution is actually in the entire arm ok. So in that case the total length is $2h$ and that is equal to $\lambda_{\text{effective}}$ by 2 ok. So you can also say that the current distribution in each arm approximates that one half of the straight antenna of length $2h$ that you can see here. So here the overall length antenna length is $2h$ and the condition is basically $2h$ equals $\lambda_{\text{effective}}$ by 2 ok. So, this is how the symmetric and anti symmetric modes will operate differently ok.

Metasurfaces: Light Propagation with Phase Discontinuities

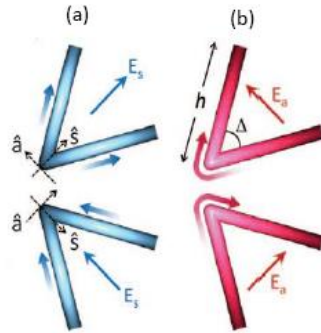
- In Fig. (c), the blue and red dashed curves correspond to the resonance peaks of the symmetric and antisymmetric modes, respectively.
- Four antennas detuned from the resonance peaks, as indicated by circles in (c), which provide an incremental phase of $\pi/4$ from left to right (d) for the cross-polarized scattered light.



So you can also see the analytically calculated amplitude so this is basically the amplitude shift and this is the phase shift. So this is different length or you can say height h of the antenna and these are the different Δ angle that is basically the opening between the two arms or you can say the angle between the two arms ok. So what we are seeing here so analytically it has been calculated what happens to the amplitude and phase shift of this cross polarized scattered light by this V antenna. Now if they are made of gold rods so gold rods they are basically having cylindrical or circular cross section. So, if you see take the cross section they are basically circle so these are rods ok and their height or length h is varied and the angle is also varied and the wavelength is kept fixed $\lambda_{naught} = 8 \text{ mm}$.

Metasurfaces: Light Propagation with Phase Discontinuities

- By simply taking the mirror structures (bottom row of (a) and (b)), one creates a new antenna whose cross-polarized radiation has an additional π phase shift.
- This is evident by observing that the currents leading to cross-polarized radiation are π out of phase in Fig. (a) and (b).



So, what happens in this case so you see the four circles ok so these four circles basically show that you are basically changing the angle ok between them. So the optical property of the rod of the same length so here you are keeping the length same, but you are changing the angle between them and you are comparing it with a flat antenna of the same length. So you can see how the amplitude as well as the phase changes ok in this two case. So here the blue and dashed curves they correspond to the resonance peaks of the symmetric and anti symmetric mode. So symmetric mode is this one sorry this one anti symmetric mode is this one.

So we are only talking about like this particular case so you can only think of the top case not the bottom one. But then if you look at these four antennas what is happening here you also correspondingly see the phase pattern. So when they are detuned from the resonance peaks ok as indicated by the circles in (c) here you can see the circles ok. You can see that there is a incremental phase shift this guy is having you know 90 degree then you have 45 degree then you have 0 degree and then you have minus 45 degree. So, this is how the phase of the cross polarized scattered light is changing you by changing only the design of the elements.

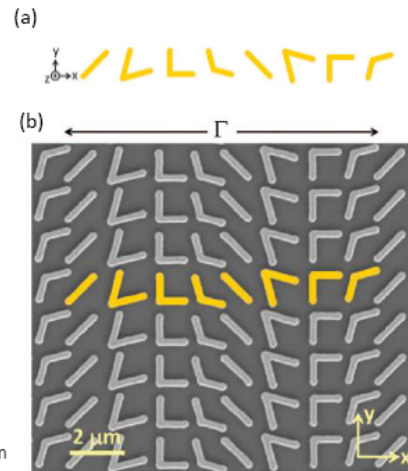
Experiments with Metasurfaces

- Schematic of unit cell of the plasmonic interface for demonstrating the generalized laws of reflection and refraction (Fig. a).
- The sample (Fig. b) is created by periodically translating in the x-y plane the unit cell.
- The antennas are designed to have equal scattering amplitudes and constant phase difference $\Delta\Phi = \pi/4$ between neighbors.

Figure: (a) A set of eight antennas were thus created from the initial four antennas.

(b) Scanning electron microscope (SEM) image of a representative antenna array fabricated on a silicon wafer.

The unit cell of the plasmonic interface (yellow) comprises eight gold V-antennas of width ~ 220 nm and thickness ~ 50 nm, and it repeats with a periodicity of $\Gamma = 11$ nm in the x dir. and $1.5 \mu\text{m}$ in the y dir.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019
Source: N. Yu et al., Science, 334(6054), 333-337, 2011

So if you start opening up this antennas so this is a V type of antenna and then you start opening them up you will actually get incremental phase of $\pi/4$ getting changed from this design to this design. Now if you do the mirror symmetry mirror structure that is this one and this one ok you can actually get additional π phase shift introduced. So you actually got many many elements which can give you your desired phase shifts. So, this is evident by observing the currents so because they do have completely different current pattern obviously they are 180 degree out of phase or you can say π is the phase different between this one and this one ok. So, they if this is giving you 45 degree this will give you 45 plus π ok like that.

So with that people have done some experiments with the matter surface. So this is a set of 8 antennas that has been taken but you see there are basically 4 antennas and then they are repeated. So this is how the antennas are taken and then you take this as a unit cell and then you repeat it. So this is an SEM image of the antenna array which is fabricated on a silicon wafer. The unit cell is basically this ones which are highlighted in yellow these are all gold V shaped antenna the width is 220 nanometer and the thickness is 50 nanometer and the repeat period with a periodicity of 11 nm ok.

So that is the periodicity in x direction. So this is x direction horizontal one so the periodicity is 11 nm ok the whole thing repeats like that and along this direction along the y direction you have 1.5 micrometer as the periodicity. So these antennas are designed in such a way that they have equal scattering amplitude so each of them will scatter the same amplitude but they will have a constant phase difference of $\pi/4$ between the neighbors from this one to this one $\pi/4$ difference from this one to this one $\pi/4$ difference and so on. So let us see what is the purpose of doing this so if you

look into the simulation study so this is the 8 antennas that you have seen ok so they are basically created from this 4 antennas repeated again. So you can also see that their amplitude is pretty much same just that they have a phase that increments as π by 4.

Metasurfaces: Light Propagation with Phase Discontinuities

- Simulations of the scattered electric field for the individual antennas composing the array in **Figure**.
- Plots show the scattered electric field polarized in the x direction for a y-polarized plane wave excitation at normal incidence from the silicon substrate.
- The silicon substrate is located at $z \leq 0$. The antennas are equally spaced at a subwavelength separation $\Gamma/8$, where Γ is the unit cell length.
- The tilted red straight line is the envelope of the projections of the spherical waves scattered by the antennas onto the x-z plane.

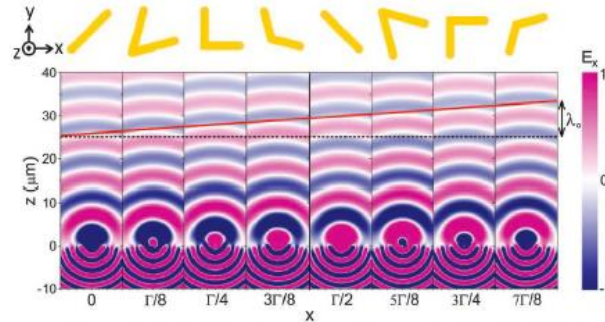
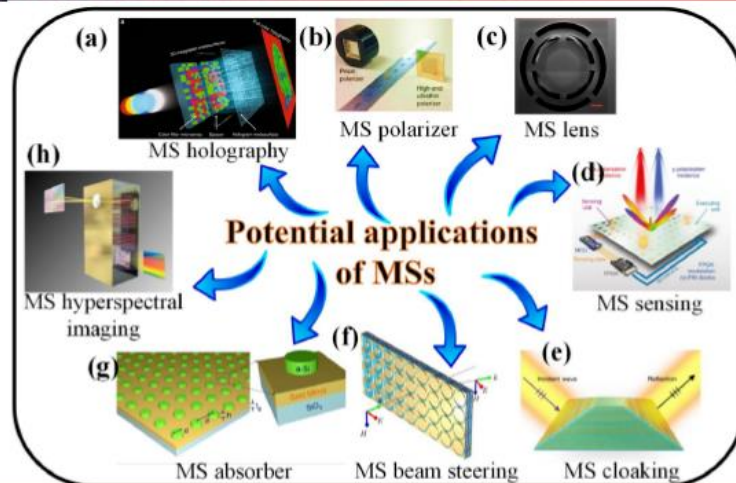


Figure: A set of eight antennas were thus created from the initial four antennas. Full-wave simulations confirm that the amplitudes of the cross polarized radiation scattered by the eight antennas are nearly equal, with phases in $\pi/4$ increments.

So this is the silicon substrate on which the antennas are kept so this is the point where the antennas are placed and you can actually see that when ok this particular plot shows the scattered electric field which is polarized in the x direction for a y-polarized plane wave excitation at the normal incidence from the silicon substrate. So this is the silicon substrate part and it is located below this z equals 0 line and another important thing to notice here is that the antennas are equally spaced at sub wavelength separation of γ by 8 where γ is basically the length of the unit cell. So, total length is γ so you have equally spaced them ok and this is how the spacing is 0 γ by 8 γ by 4, 3 γ by 8 and so on ok. Now what happens if you draw the phase front ok you will see that this particular tilted red straight line shows you the envelope of the projection of the spherical waves that are scattered by these antennas onto the x-z plane ok. So that way you are able to see that you are actually able to steer the beam.

Metasurfaces: Potential Applications



The beam normally it would have been this black dashed line but because they are incrementally adding π by 4 phase ok you are able to tilt the beam or steer the beam. So that way a very very thin surface can do beam steering ok. So that is only one particular application which has caught the attention initial attention of the metasurface or all the scientific community at large. There are other applications also like metasurface holography, metasurface polarizer, metasurface lens, sensing, cloaking, beam steering, absorber, hyperspectral imaging. I believe in the initial lectures when we are talking about the applications of metasurfaces I have discussed all these applications in details.

Frequency Selective Surfaces: Definition

- **Frequency Selective Surfaces (FSS)** is a periodic surface with two-dimensional arrays of identical elements arranged on a dielectric substrate.
- *An incoming plane wave will either be transmitted (passband) or reflected back (stopband), completely or partially, depending on the nature of array element.*

Advantages of FSSs

- **Broadband**
- **Robust to angle of incidence**
- **Resonance frequency depends on unit cells' shape and size.**

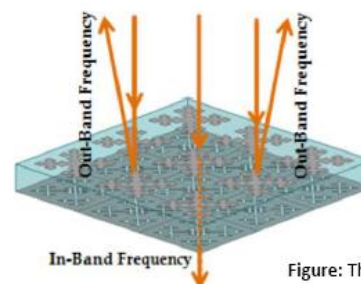


Figure: The functionality of an FSS.

Now you can go back and revisit that lecture and now you will be able to make more sense that how metasurface is allowing you to achieve all this. Now let us look into

another important topic which is frequency selective surfaces. So when we talk about frequency selective surfaces it is basically a periodic structure with two dimensional arrays of identical elements which are arranged on a dielectric substrate. This kind of surfaces when incoming plane wave falls on them they can be either transmitted so that is a pass band or reflected so that gives you a stop band which can completely block or selectively pass or something like that depending on the nature of the element that you have put in that periodic array ok.

So something like this. So what is the advantage of FSS? It can be broadband ok. You can have it can be robust to angle of incidence and most importantly the resonance frequency depends on unit cells shape and size. So here you can see this is a frequency selective surface so only the in band frequency is allowed to pass through all the other out of band frequencies are basically getting reflected. So at microwave and optical frequency ranges spatial filtering is most desirable in all signal processing systems and there frequency selective surfaces come into picture because they are called the spatial filters as they are used to modify the EM wave incident on such surfaces and then they can provide dispersive, transmissive or reflected characteristics. Now how FSS are designed? FSS are typically designed by periodic metallic arrays of elements placed on a dielectric substrate.

Frequency Selective Surfaces: Fundamentals

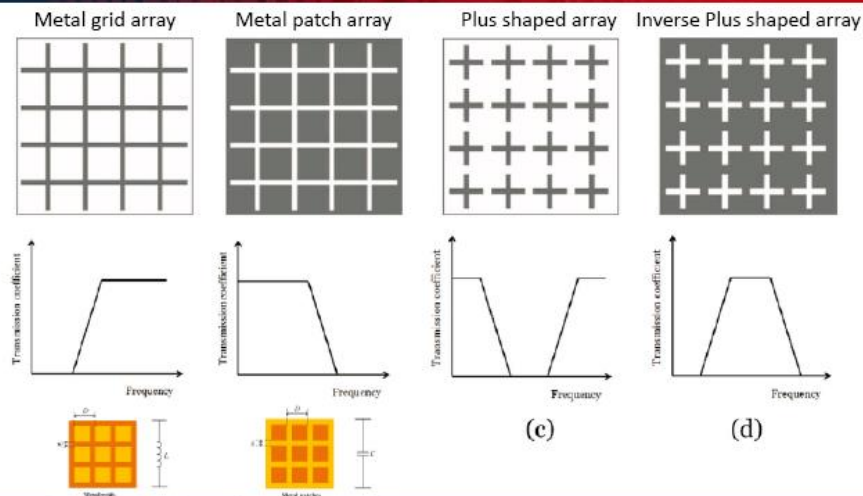
- At microwave and optical frequency ranges, spatial filtering is the most desirable operation in all signal processing systems.
- Frequency selective surfaces (FSS), also called spatial filters, are used to modify the EM wave incident on such surfaces and provide dispersive transmitted and/or reflected characteristics.
- FSSs are usually designed by periodic metallic arrays of elements on a dielectric substrate.
- The change brought to the transmitted wave can be both in amplitude or phase when compared to the incident wave.
- In any case, selectivity may be introduced against the incident polarization to improve the irregularities in the emission pattern, which is exhibited through a change of the phase or amplitude of the transmitted wave.
- A variety of applications according to different requirements can be facilitated, depending on the nature of modification added to the transmitted wave.

The change brought to the transmitted waves can be both in amplitude or phase when you compare it with the incident wave and in any case the selectivity may be introduced against the incident polarization to improve the irregularities in the emission pattern which is exhibited through a change of the phase or amplitude of the transmitted wave. Now there are different applications depending on the nature of modification that you have done to the transmitted wave. So some examples are you can think of metal grid

array so here the dark line shows the metal ok. So these are metal grids here also the dark one shows the metal grids. So this is basically a inductive element so you will get a high pass filter kind of characteristics when you see the transmission coefficient.

You can also make array of metallic patches so here the dark ones are the metallic patches. So metallic patches they behave like capacitive element so you can get low pass characteristics based on this. You can also make other shapes something like plus shape metallic plus shape so that actually gives you a band stop characteristics and if you try to make a inverse structure of that that is a complementary structure. So you take a metallic sheet and then you make this plus shape holes punch through them you can get the inverse characteristic of this band stop you can get a band pass transmission characteristics. So this is what we have seen typically the FSS patches they have resonance because you are using metallic elements and they can have inductance or capacitance depending on whatever the elements you are using.

Frequency Selective Surfaces: Fundamentals



So, when you are using metallic patches you can think of a capacitive element so it can give you low pass characteristics and when you use inductive sorry when you use metal grids you can think of this as inductive elements and they will give you high pass characteristics that is basically the inductive response. So using this high pass and low pass you can always think of and you can combine them in series or parallel to make band pass or band stop characteristics as you learned already in circuit theory. Here also something like that can be used to make FSS based filters using this concepts. So physically when a unit cell of FSS is illuminated by electromagnetic wave you can convert that into a effective equivalent resonance circuit. So, in the case of metallic patch it is a capacitive element as I mentioned metallic grids is an inductive element but when you try to make a metal square loop array like this so the dark one is the metallic loop so

it can be modeled as a L loop and C loop.

Frequency Selective Surfaces: Fundamentals

- FSS patches create resistance (R) and inductance (L), while gaps among the FSS elements generate capacitance (C).
- Simple electrostatic principle applies to manipulate the physical significance of these passive values for different FSS elements, e.g., L of two parallel wires and C generated by a parallel plate capacitor.
- So, a required filter response is constructed by the combination of these capacitive and inductive elements.
- However, any change in the FSS dimensional parameters leads to an equivalent variation in the L and C values.

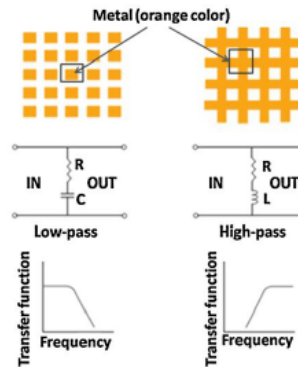


Figure: FSS periodic structure consisting of complimentary array elements, their equivalent circuits and the frequency responses (Left to right). The patch array element exhibits a capacitive response (low pass), whereas the grid array element shows an inductive response (high pass).

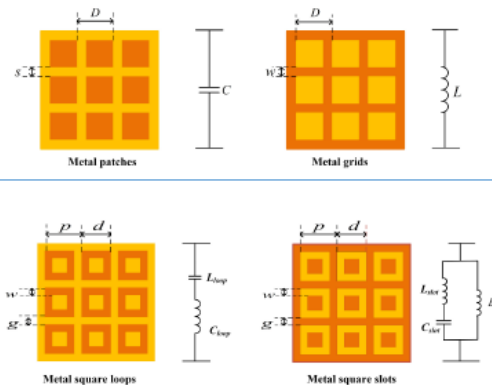
So there is gap between the loops that will give you that capacitance effect and this loop will give you that inductance. Similarly you can also make metal square slots so you are actually making slots like this so that can give you L slot and C slot in series parallel with another inductance that is coming from this particular grids. So that way you can actually make resonating elements and the resonance frequency of this kind of loops can be obtained as $f_r = \frac{1}{2\pi\sqrt{LC}}$. So that will tell you where the resonance of your band pass or band stop filter will be placed. So, by choosing appropriate array element you can choose the characteristics of your FSS.

Frequency Selective Surfaces: Fundamentals

- Physically, when a unit cell of an FSS is illuminated by the EM wave, it can be converted to an equivalent resonance circuit.
- The resonance frequency can be found as:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (L28.5)$$

where L and C represent the equivalent inductance and the capacitance of an FSS unit cell, respectively.



So different types of unit cell geometries have been implemented which are very

common to the FSS community. So let us classify this you can classify them based on their resonant properties. So, you can also see that there are some elements which are non resonant like patch and wire grid where you only have either capacitance or inductance or you can also have single resonator element something like a loop or a cross or a dipole kind of thing which can be modeled as a series combination of inductor and capacitor.

Frequency Selective Surfaces: Types

- Choosing an appropriate array element is very crucial in designing FSS. While various unit cell geometries have been implemented, out of which some are easily controllable and so more famous in FSS community.
- A classification of frequently used FSS elements types is summarized based on their resonant properties.
- For example, **non-resonant** element (patch, wire grid) can be modeled by a capacitance or inductance, whereas
- single resonant element (like loop, cross, dipole) can be represented by a series combination of capacitor and inductor.

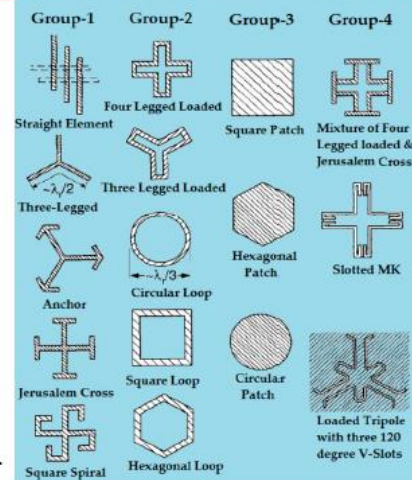


Figure. Typical shapes of FSS elements.

So, this will be resonating element and these are non resonating elements. So, the classification can be done like this the group 1 is basically center connected as you can see this is a center connected the length is basically kept as $\lambda/2$. So, this is the overall length as you can see here. So here larger elements relative to wavelength. Now in this one the loop type the circumference is basically of the order of $\lambda/3$ and in the solid type or the plate type as you can see there are different structures designs possible for patch you can have square patch hexagonal patch circular patch and so on.

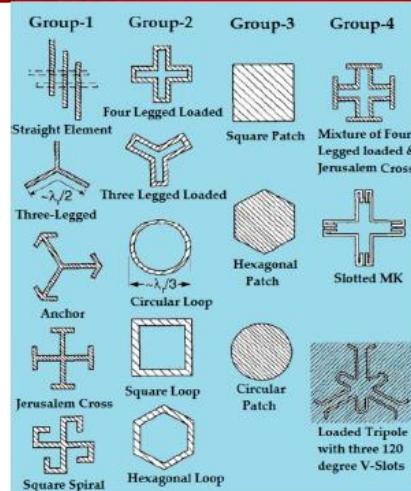
Frequency Selective Surfaces: Types

Group-1: Center Connected or N-Poles ($L \sim \frac{\lambda_0}{2}$)
Larger elements relative to wavelength

Group-2: Loop Types (Circumference $\sim \lambda_0$)

Group-3: Solid interior or Plate type ($L \sim \frac{\lambda_0}{2}$)
Larger elements relative to wavelength

Group-4: Combinations



So here the length of this is kept as $\lambda_0/2$. So again, the larger elements relative to the wavelength and group 4 can be combination of any of these elements from the group 1, 2 and 3. Now what is the main application of FSS one important application is towards electromagnetic shielding.

Frequency Selective Surfaces: Applications

Electromagnetic shielding using FSS

- Electromagnetic interference (EMI) may cause malfunction of electrical/electronic equipment in sensitive environments.
- Shielding the source of interference may not be the optimum solution.
- One of the most common approaches is to shield the sensitive device
- A metal foil may be employed as EMI shield, to protect RF circuitry from radiated fields.
 - Though, this technology has disadvantage in terms that it blocks all transmissions, regardless of their origin.
- However, an FSS does not suffer from such shortcomings.
- In addition, 2-D single layer FSSs have clear advantages in terms of their low profile and ease of fabrication.

Now electromagnetic interference is a very important factor because it may cause malfunction of any electrical and electronic component in a sensitive environment. Now it is important to shield the source of interference but that may not be the optimum solution. So one of the most common approach is basically to shield the sensitive device. And typically what people do they apply a metal foil that can be employed as an electromagnetic shield to protect the RF circuitry from the radiated fields. Although this

technology has some disadvantage because it blocks all kind of transmissions irrespective of the origin.

Frequency Selective Surfaces: Example

- The design layout of an FSS is illustrated in Figure.
- The proposed FSS is simulated, fabricated, and tested over flexible/conformable Rogers 5880 laminate of thickness $t = 0.127$ mm, relative permittivity of 2.2 and a dielectric loss tangent of 0.0009.
- The dimensions of the overall unit cell are 6.8×6.8 mm².
- The parameter D_x represents the inter element spacing.
- The parameter P , on rectangular lattice, represents the periodicity of the proposed unit cell in the fabricated FSS.

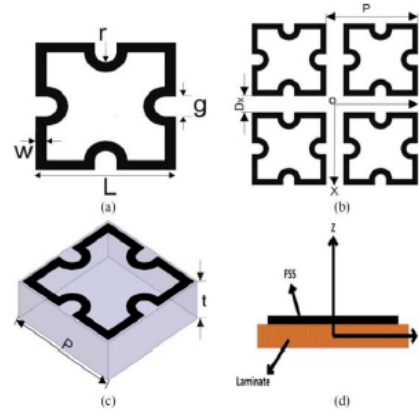


Figure: Geometry of proposed FSS. (a) FSS (b) Cu Elements array. (c) FSS perspective view of unit-cell. (d) Side view of unit-cell.

Now that you may not like. So in that case FSS may actually help you from getting rid of this kind of problems. So if you design a 2D single layer FSS they will definitely have clear advantages because they will be easy to fabricate and they can only block or emit a selective frequency. They will not shield all the frequencies. So those are the different scenarios. So here is a design layout of an FSS as you can see these are the different structural parameters.

Frequency Selective Surfaces: Example

Optimized parameters for resonating FSS		
Resonant Frequency: 10 GHz		
Parameter	Optimization Range (mm)	Optimized Dimensions (mm)
L	5.49-9	6.5
$g = 2r$	0.18-1.98	1
w	0.09-0.99	0.5
r	0.09-0.99	0.5
D_x	0.09-0.45	0.3
P	5.4-9.45	6.8

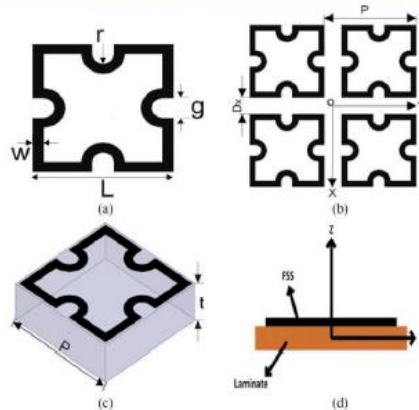


Figure: Geometry of proposed FSS. (a) FSS (b) Elements array. (c) FSS perspective view of unit-cell. (d) Side view of unit-cell.

This is the period that is periodic arrangement. This is the overall 3D view of the unit cell and this is the side view of the unit cell. So, this is basically FSS made of copper on

a dielectric substrate and it is Rogers 5880 unit which is called a thickness of 0.127 mm permittivity of 2.2 and dielectric loss tangent is given here.

Frequency Selective Surfaces: Example

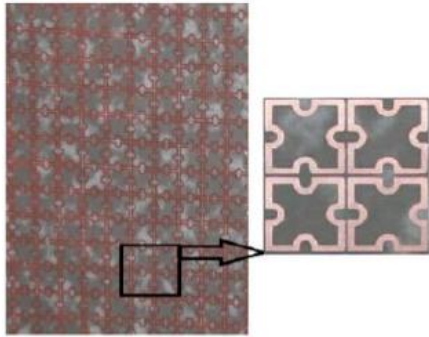


Figure: Fabricated FSS.

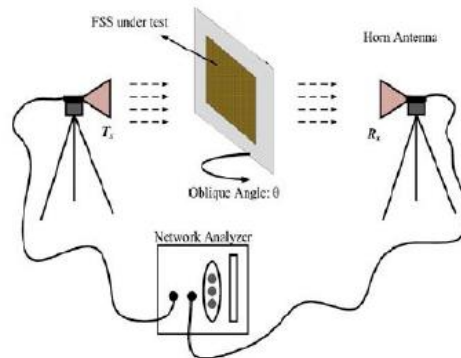


Figure: The measurement setup. Two horn antennas with operating bandwidth of 8–12 GHz are employed to measure the transmission response.

The two horn antennas are connected through a Network Analyzer. Then, the transmission measurements are carried with the FSS placed between two horn antennas.

And this is the overall dimension of the unit cell 6.8 by 6.8 millimeter square and d_x is the parameter that gives you inter element spacing. So what I am showing here is a design of a FSS and how what will be the response for that. So we are putting this parameter p on the rectangular lattice that tells you about the periodicity in of the proposed unit cell in this fabricated FSS. Then you want a resonating frequency at 10 gigahertz so you actually optimized all these physical parameters L , g , w , r , D_x and P all are these physical parameters. These are the range over which they are optimized and these are the basically the optimized dimensions of the FSS that has been obtained.

Frequency Selective Surfaces: Example

- Figure shows the TE scattering parameters, S_{11} and S_{21} at normal incidences.
- The transmission and reflection coefficients of a FSS printed on Rogers 5880 is shown.
- The FSS printed on Rogers provides effective shielding with attenuation of at least 56 dB as shown in Figure.
- Owing to design symmetry, the proposed FSS gives identical response in both TE and TM incidences.

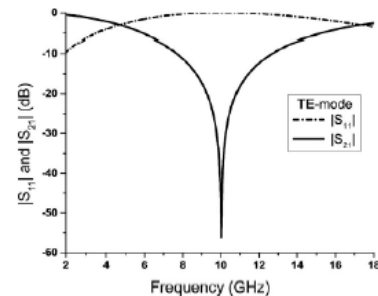


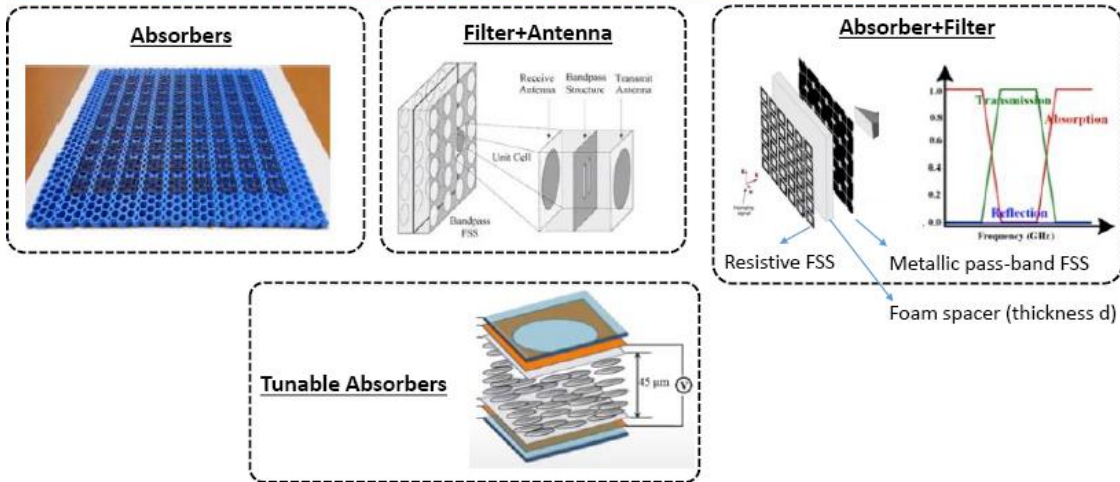
Figure: Scattering parameters at normal incidence. TE wave mode characteristics of FSS on Rogers 5880.

Once you optimized you have fabricated the FSS and this is how the fabricated FSS looks like. So this is how you can create a measurement setup. So you can put two horn antennas operating with operating bandwidth of say 8 to 12 gigahertz for measuring the transmission characteristics. Here the antennas are connected to a network analyzer and you can actually measure the transmission characteristics by placing a FSS between these two horn antennas. And what you will see you can measure the S_{11} and S_{21} parameter. So, this is the plot of the direct line shows the transmission characteristics and the dotted line shows you the S_{11} .

So, here you can see for the 10 gigahertz it is giving a very effective shielding of almost 56 dB. It is minus 56 dB here you see. So it can block 10 gigahertz frequency up to 56 dB. So that is very very good shielding and it will not actually block other frequencies. So this is very very good electromagnetic shield this FSS at 10 gigahertz. So because of the design symmetry along x and y you can say that this FSS will give identical response in both TE and TM incidences.

So that makes this a very very useful one. So here are the other potential applications. You can think of absorbers, you can think of filter plus antenna. So here you have receiver antenna you can have a band per structure made of FSS and then you have a transmitter antenna. You can also have absorber plus filter where you can have a resistive FSS here which can absorb. You can have a pass band metallic FSS here and in between there is a spacer.

Frequency Selective Surfaces: Further Potential Applications



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
 Source: M. Nauman et al., IEEE Transactions on Electromagnetic Compatibility, 58(2), 419-428, 2016.

So these two combined can give you something between transmission or absorption. So you are switching between transmission and absorption and you are keeping the reflection more or less flat. You can also have tunable absorbers. So based on this FSS where you can also include some of the this one liquid crystals. So, I will not go into details of this but these are different different applications apart from shielding you can have absorber, filter plus antenna, absorber plus filter and all these things made out of FSS.

So with that we will stop here today and in the next lecture we will consider guided mode resonance. So regarding this lecture if you have got any query you can always drop an email to me at this address mentioning MOOC on the subject line. Thank you.