Course Name- Nanophotonics, Plasmonics and Metamaterials

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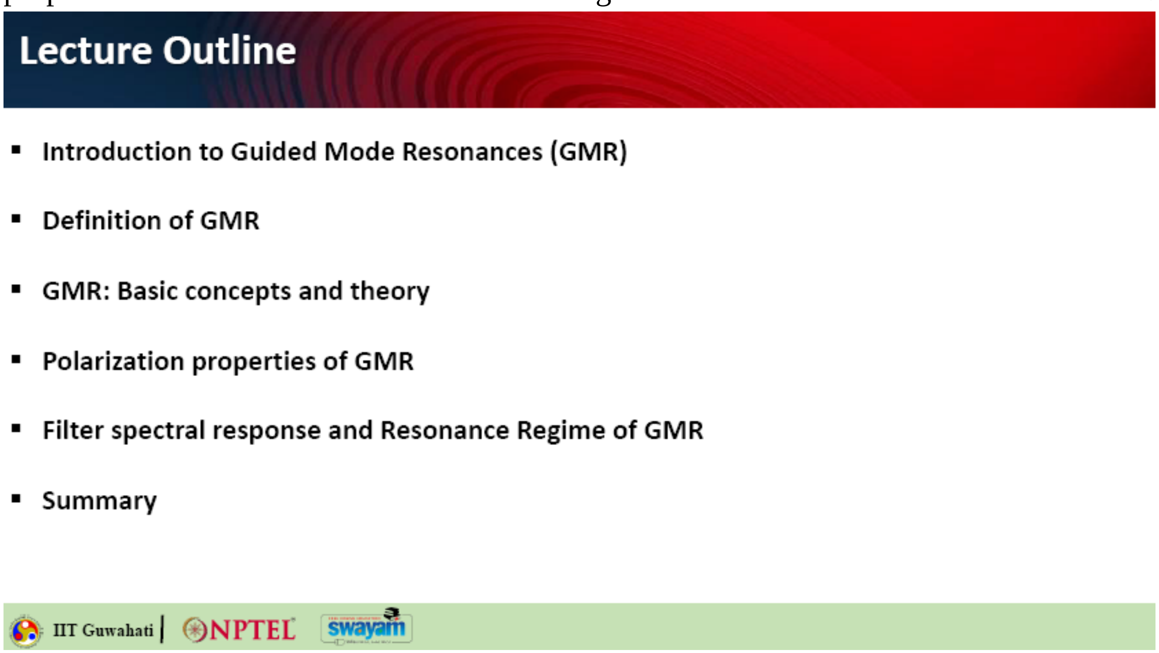
Week-10

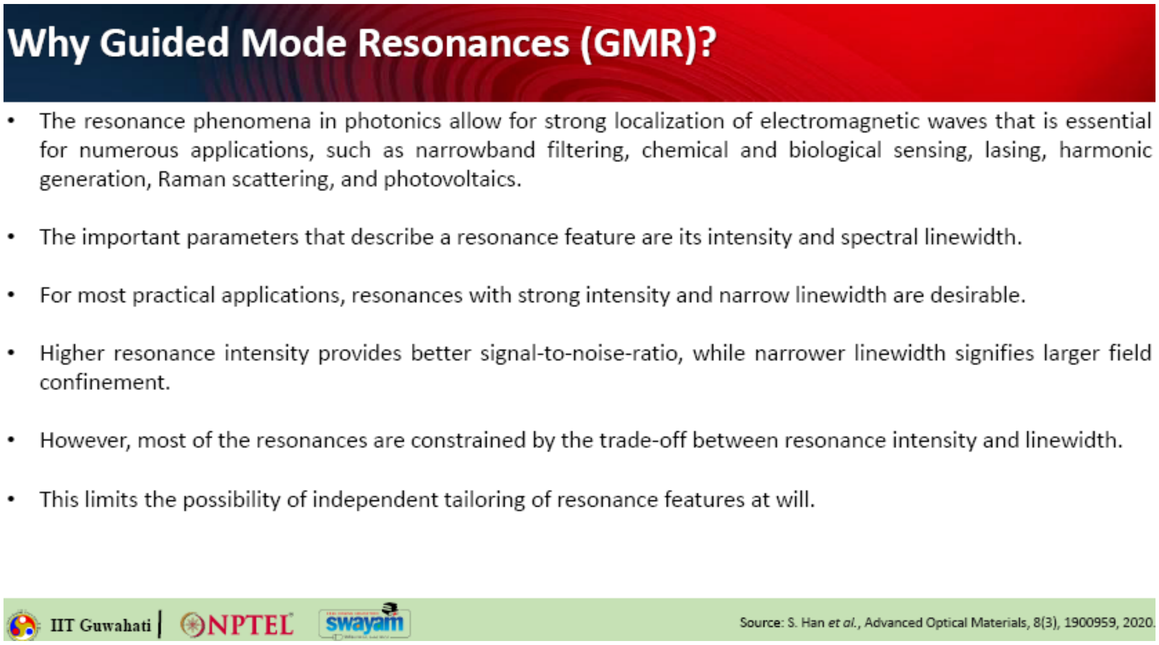
Lecture -29

Hello students. Welcome to lecture 29 of the online course on Nanophotonics, Plasmonics and Metamaterials. Today's lecture will be on guided mode resonance. So, here is the lecture outline. We will give a quick introduction to guided mode resonance. We will go through the definition, the basic concepts and theory, some of the polarization properties of the guided mode resonance.



And then we will see the filter design based on guided mode resonance. The filter spectral response and resonance regime of GMR and then we will provide a summary of this particular topic. So, the first question comes to mind is what is guided mode resonance? So, we have seen resonance phenomena in photonics which actually allow for strong localization of electromagnetic waves. And that has got numerous applications something like narrowband filtering, chemical and biological sensing, lasing, harmonic generation, Raman scattering, photovoltaics etcetera.

Now, the important parameters that describe a resonance feature are its intensity and the spectral line width. That will actually decide the Q factor, the quality factor of the resonance. In most practical applications, resonances with strong intensity and narrow line width are desirable. As you can understand high Q resonances are always desirable in most practical applications. There are applications where you actually look for broadband absorption or broadband resonance.

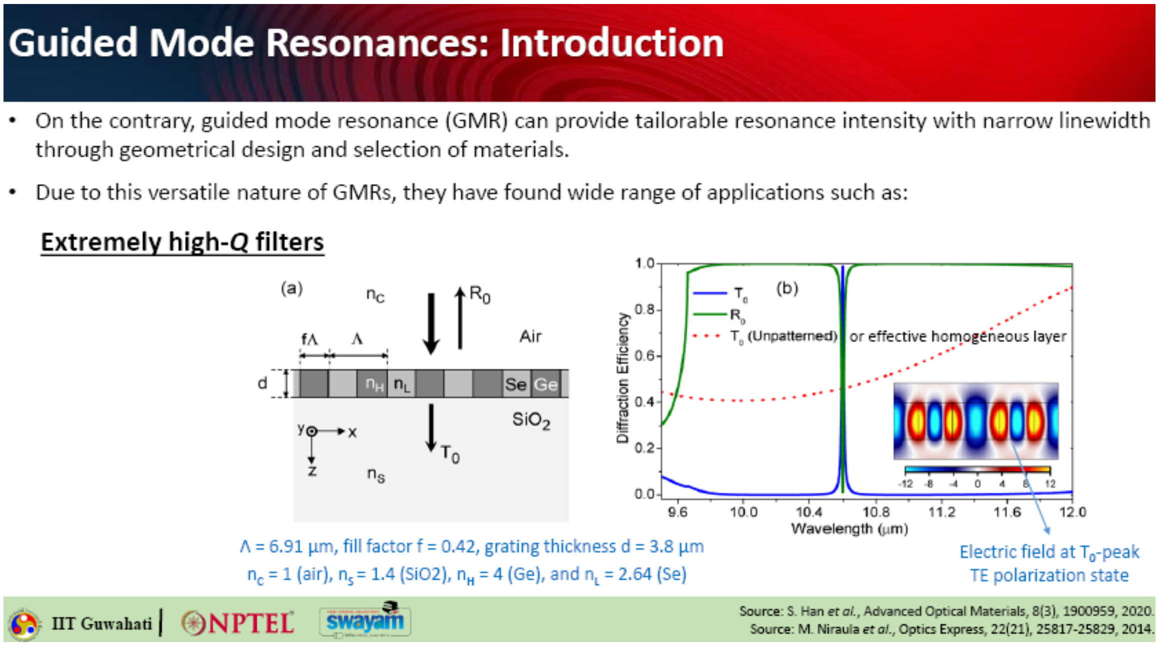


Those cases will come in the subsequent lectures. Today we are looking for very very narrow line width filters ok. So, high resonance intensity what are the benefits that can give you better signal to noise ratio. And when you have narrow line width that actually gives you very very strong field confinement. So, you are actually focusing your beam at a very very narrow spot.

So, this is what is the thing you can relate to narrow line width. So, it can give you larger field confinement. However, most of the resonances are constrained by a kind of tradeoff between these two important parameters resonance intensity and line width. And this limits the possibility of independent tailoring of the resonant features at will. So, you are not able to kind of tune intensity and line width independently based on your requirement.

So, that that lecture is not there. Now, when you think of guided mode resonance this is where the guided mode resonance becomes very important. So, it can provide a tailorable resonance intensity with narrow line width through geometrical design and selection of material. So, this kind of filters the filters design based on guided mode resonance can give you very high quality factor. So, due to this versatile nature of GMRs they are found in wide range of applications.

One application we can see here is that high Q filter. So, this is how typically a GMR will look like. So, it is a guided mode resonance there is a grating and then there is a waveguide below this. So, as you can see there is light incident some part is getting reflected and the remaining is getting transmitted. So, the refractive index here if this is air.



So, you can take refractive index to be 1 this is silica. So, it is 1.4 we are considering the periodicity the lattice period to be 6.91 that is a grating period. You can think of high low high low and these are made of germanium and selenium.

So, that is n equal 4 and 2.64. And when you actually look into the grating thickness this is particular thickness of the grating that is 3.8 micrometer. And the fill factor of the high material is 0.

Guided Mode Resonances: Introduction

- The **GMR devices** primarily comprise of a *diffractive grating* and *an in-plane waveguide*.
- *The grating diffracts the incident light and couples it into the waveguide, which propagates as a guided mode.*
- However, this guided mode is designed to be leaky, which then interferes with the free-space propagating electromagnetic wave to give rise to the GMRs.
- Through the selection of material, grating design, dielectric layer thickness, and angle of incidence, **GMRs can provide a wide range of spectral features.**

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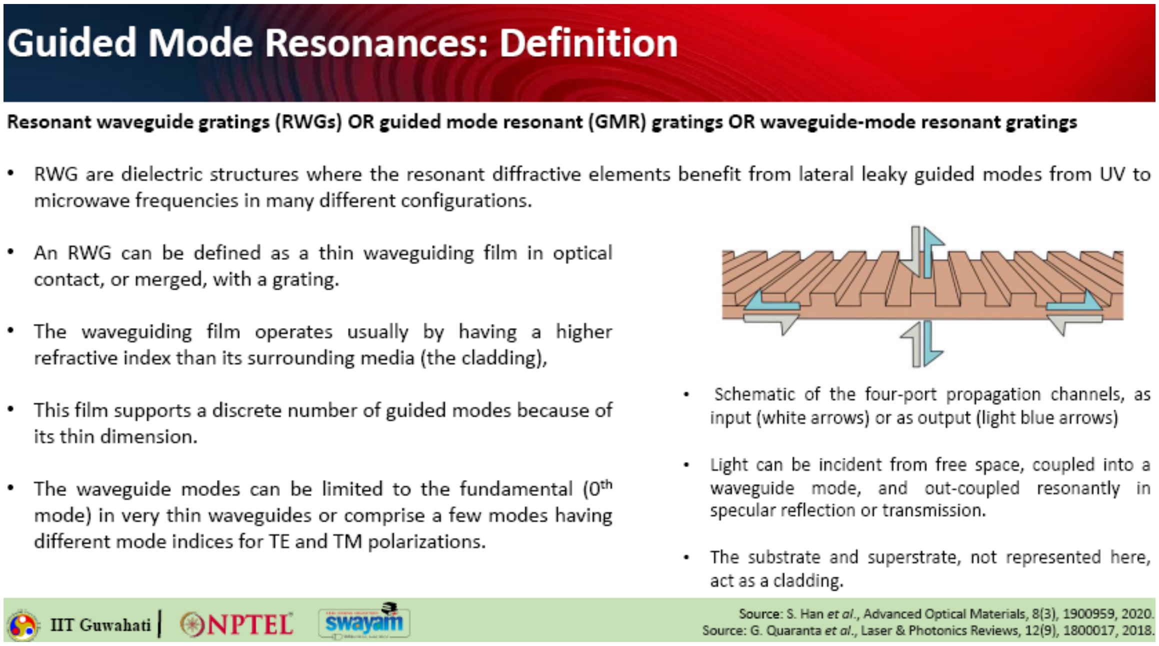
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.

42. So, 42 percent of the grating period is basically germanium the remaining is Se ok. So, with that when you see the diffraction efficiency we will see that over the wavelength if this is the wavelength thing. So, this actually gives the spectrum. So, the blue one tells you the transmission peak it is a very narrow peak as you can see ok. And it is giving also a reflection dip.

So, that particular wavelength only one particular wavelength is able to escape and remaining all are reflected. You can also look into the electric field profile at the peak wavelength of T0 and that corresponds to the T polarization state. And what is this dashed line that is basically the effective homogeneous layer. So, here you see you do not actually see any kind of resonance. So, the resonance comes from the grating and the grating does something which because of which you are able to only allow one particular wavelength to pass through it.

Remaining all are getting reflected. So, there is something very very interesting about this particular phenomena. So, as you see the GMR devices primarily consist of a diffractive grating and then you have a in plane waveguide. The grating will diffract the in fact incident light and some some diffracted wave will basically coupled to the waveguide and it propagates as the guided mode. And this guided mode is designed to be leaky and it leaks out when it interferes with the free space propagating electromagnetic wave and that gives that resonance GMR ok.



So, through the selection of material grating design dielectric layer thickness angle of incidence the GMR can provide a wide range of spectral feature. So, you can actually tune the transmission or resonance peak or deep as you can see here depending on all these parameters which are very easily tunable. So, that makes this GMR filters very very interesting. So, there are other names of this particular device it is also called resonant waveguide gratings RWG or we have already seen this name GMR grating or you can also call them waveguide mode resonant gratings ok. So, this RWGs as you can see they are basically dielectric structures where the resonant diffractive element they actually benefit from the leaky guided modes ok.

And they can be tuned from UV to microwave and other frequencies also ok for different different configuration. Mainly they are used in this particular range and they are they can also be tuned to optical and infrared ok. So, a resonant waveguide grating can be defined can be defined as a thin wave guiding film in optical contact ok or it is merged with a grating as you can see here ok. So, this particular one actually shows a grating and then there is this waveguide below it ok. So, this particular figure shows a 4 port configuration.

So, 1, 2, 3, 4 there are 4 ports and the white arrows are basically the inputs ok and the blue ones are basically the outputs ok. Light is considered to be incident from the free space which is coupled into a waveguide mode. And then that can out couple resonantly in a specular reflection or transmission and that is how it works. And here the substrate and super straight they are not shown, but they actually act as a cladding. So, when you think of this wave guiding film ok it operates usually by having a higher refractive index than the surrounding media.

Guided Mode Resonances: Definition

- Depending on the wavelengths, this leads to a very high reflection or transmission, giving rise to a zeroth-order reflection **Figure** (a,b).
- Those efficient resonances can be as narrow as 0.1 nm linewidth and are very sensitive to angle and wavelength, with a typical angular to spectral linewidth ratio of $0.1^{\circ}\ nm^{-1}$.
- Depending on the wavelength and phase delay accumulated during propagation in the waveguide, the destructive interference can occur either in reflection or in transmission.

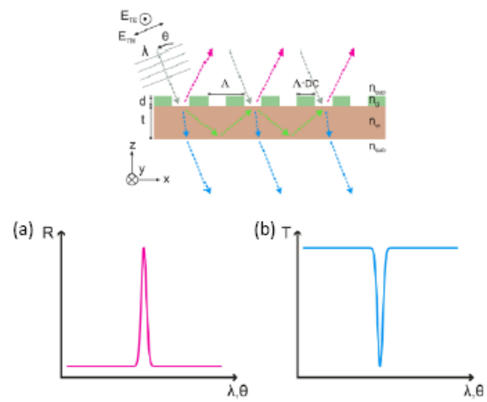


Figure: An example of a) reflection and b) transmission spectra for polarized light incident at normal at resonance of a monomode RWG.

Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020
Source: G. Quaranta *et al.*, Laser & Photonics Reviews, 12(9), 1800017, 2018

Because that is how wave guiding will take place. Wave guiding typically happens based on your total internal reflection for which the core, this will be the core in that case, waveguide core has to have a higher refractive index than the cladding. The thin film supports a discrete number of guided modes because of its thin dimension. So, the finite number of modes are allowed and the waveguide modes can be limited to the fundamental mode that is a zeroth order mode in case this is very very thin. Or it can actually go up to few modes if they are bit slightly thicker and for TE and TM polarization you can have different mode indices.

Now depending on the wavelength ah this leads to a very high reflection or transmission giving rise to zeroth order reflection as you can see here ok. So, this is how the grating is made on top of thin waveguide and you see this the incident plane wave. Some part is getting reflected some is some is getting deflected and when this deflected mode couples with the leaky mode ok it actually. So, this is how it travels ok and it leaks out as well. So, this is how you actually get the transmission ok.

So, in this particular example it has been designed to have a reflection peak and a transmission dip. So, you can actually design the filters or the guided mode resonance filters like that. So, those efficient resonances that you have seen here can be as narrow as 0.1 nanometer line width and they are very very sensitive to the incident angle and the wavelength. And you can see a typical angular to spectral line width ratio is like 0.

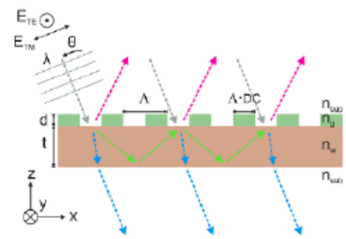
1 degree nanometer inverse. So, you can actually see they are very very sharp. So, they can give rise to very high Q resonance and they are also tunable. Now, depending on the length and the phase delay that is accumulated during the propagation in the waveguide

the destructive interference can occur either in reflection or in transmission. Now, this is why in the previous case we saw a transmission peak in this case we saw a transmission dip. So, you can actually so the modes that are coming the waves that are coming out if the destructive interfere they will cancel that particular transmission at that particular wavelength.

Guided Mode Resonances: Definition

- For a given polarization and wavelength, an RWG can support various guided modes having a different mode index and therefore transverse propagation speed and momentum.
- Light can be coupled into the waveguide modes by different grating diffraction orders, depending on the incidence angle and the wavelength (**Figure**).
- Some of this guided light is diffracted out of the guide while propagating, coupled back to radiation, and interferes with the non-coupled reflected (blue) or transmitted (magenta) waves.

Illustration of a standard RWG



- Figure shows propagation of light rays in the RWG
- A complete destructive interference happens in transmission at a specific angle and wavelength of incidence, resulting in a narrowband reflection.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020
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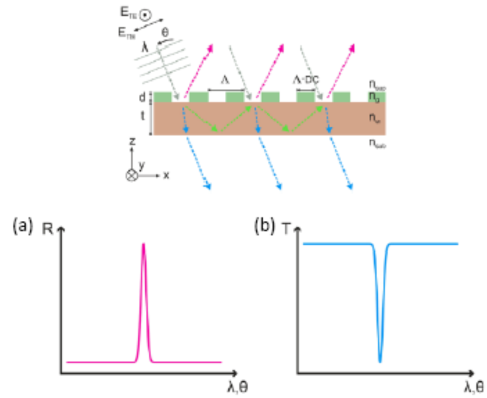
If they constructively interfere they will get a peak. So, you can actually design this particular phase delay and that is accumulated along the propagation of the waveguide. So, the length of this device also plays a important role. Now, for a given polarization and wavelength an RWG can support various guided modes as you can understand having different mode index and therefore, transverse polarization different transverse propagation speed and momentum. So, light can be coupled to the waveguide modes by different grating orders.

Guided Mode Resonances: Definition

- RWGs are therefore effective filtering structures, especially for collimated light.
- Further, RWGs can be designed to be extremely efficient diffraction elements.
- Additionally, because the structure consists usually of dielectric materials only, it can be highly transparent and therefore used either in transmission or in reflection.
- Moreover, RWGs do not suffer from thermal heating such as metallic structures, which enable their use in a variety of high optical power applications such as mirrors and diffractive elements.



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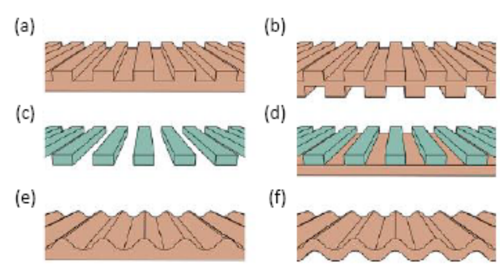
So, diffraction grating if you remember not only the zeroth order there will be plus 1 plus plus 1 minus 1 plus 2 minus 2 plus 3 minus 3 and so on. So, all the diffraction orders are possible. So, particular which angle that actually meet that that that is more than the critical angle at this interface that will be totally internally reflected and that will continue to propagate in this waveguide right. And some part of it will leak out and that that is how the leaky waves are also coming. So, in this particular case you can see that some of this guided wave is diffracted out of the waveguide while propagating coupling back to the radiation and interferes with the non coupled reflected lights.

Guided Mode Resonances: Definition

- Each ridge and groove corrugating the waveguiding layer (**Figure**), or each of the discrete ribbons (**Figure**) of RWG can be considered a scattering element connected to a thin-film waveguide, making a periodic array of scattering elements in which quasi-guided modes, or leaky modes, can propagate.
- RWGs can therefore be considered as temporal or spatial optical integrators as well as be used to enhance local electromagnetic field, as examples for sensing and nonlinear optics.

Figure: Different types of RWGs. (a) Single-sided rectangular corrugation of a waveguiding layer. (b) Double-sided rectangular corrugation with a thin-film waveguide. (c) Waveguiding layer corrugated over its full thickness, providing an array of discrete ribbons. (d) Array of ribbons on a waveguiding layer. (e) Single and (f) double-sided sinusoidal corrugation of a waveguiding layer.



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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
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So, here the reflected ones are basically ok the reflected ones this should be blue the reflected ones should be ok there is there is a mistake here. So, the reflected ones are basically magenta and transmitted ones are blue ok. So, yeah this this should be corrected. So, this is how it it works ok. So, here you can actually see this light yellowish kind of wave propagation inside.

So, when a complete destructive interference happens in the transmission that happens at the specific angle and the specific wavelength you can actually get a narrow band reflection. So, that is how you can actually get this particular feature as you have seen here. So, it is clear that this ah RWGs or GMRs are very good at filtering ok and for especially for collimated light they are very good filters and they have extremely efficient diffraction elements they can be designed to be ok. And the structures usually consist of all dielectric materials. So, they are highly transparent and they can be used either in transmission or reflection mode.

There is nothing absorbing here there is no metallic component. So, these structures do not suffer from thermal heating and they can be used for very high optical power applications such as mirrors or other diffractive elements. So, these are safe to use they will not heat up. Now, depending on the geometry there are different types of RWGs ok. So, each ridge and groove corrugating the waveguide layer that you see this one this is how you are corrugating the waveguide layer ok.

You can actually design different kind of RWGs. So, this one as you see this is a single sided rectangular corrugation of the wave guiding layer, but this one is a double sided corrugation ok. This one is a wave guiding layer corrugated to its full thickness. So, it is basically you have just etched the entire thing ok and this is actually giving you discrete ribbon kind of structure. And this is the same thing, but you are having this ribbons on top of a wave guiding layer.

Guided Mode Resonances: Basic Concepts

- The most elementary structure is a planar, unslanted grating in asymmetric waveguide geometry, shown below.
- The relative permittivity (dielectric constant) of region 2 is spatially modulated as:

$$\epsilon(x) = \epsilon_g + \Delta\epsilon cosKx$$

where ϵ_g is the average relative permittivity, $\Delta\epsilon$ is the modulation amplitude, and $K = 2\pi/\Lambda$, where Λ is the grating period.

- Region 2 is a waveguide grating, and thus $\epsilon_g > \epsilon_1, \epsilon_3$.
- We call this waveguide-grating resonance structure a **guided-mode resonance filter, or GMRF.**

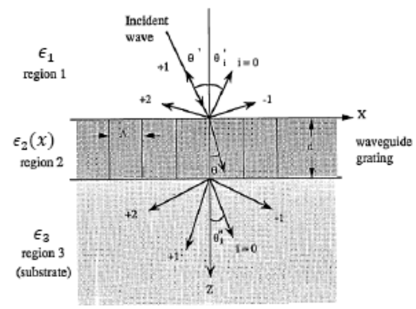


Figure: Basic planar waveguide-grating model used.

- The angle of incidence (θ') is arbitrary
- The angles θ_i' represent the angles of the wave vector of the i^{th} backward diffracted wave with respect to the z axis
- θ_i'' are the corresponding angles for the forward-diffracted waves

Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

This is a single sided sinusoidal corrugation of the wave guiding layer whereas, this one is a double sided sinusoidal corrugation. So, these are different kind of corrugations that you can make ok. And they actually decide the pattern of your quasi guided or leaky modes and also tell what kind of mode can propagate. So, RWGs can therefore, be considered as temporal or spatial optical integrators as well as they can be used to enhance the local electromagnetic field as example for sensing or non-linear optics applications. Now, let us have a look at the basic concepts of GMR ok.

So, this is a grating structure and this is the waveguide that you have seen. Now, in most elementary structure in this case is a planar unslanted grating ok in a asymmetric waveguide geometry. Why we are calling asymmetric? Because epsilon 1 and epsilon 3 are not same here that we take them as different. And the relative permittivity or the dielectric constant of this region 2 can be specially modulated like this.

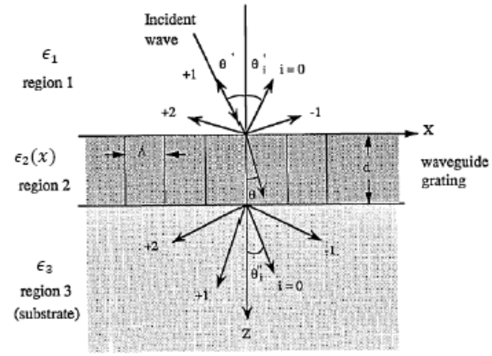
So, this is x direction this is z. So, epsilon x can be varied like this epsilon g is basically the average hm relative permittivity. And delta epsilon is the modulation amplitude and it is having cosine variation, where k is basically 2 pi by lambda where lambda capital lambda is the grating period. Now, we understand that the waveguide grating ok this is the waveguide grating. So, it should have a permittivity which is larger than both epsilon 1 and epsilon 3 ok.

Guided Mode Resonances: Basic Concepts

- For TE polarization (the electric field vector is normal to the plane of incidence in Figure), the coupled-wave equations governing wave propagation in the waveguide grating can be expressed as:

$$\frac{d^2\hat{S}_i(z)}{dz^2} + [k^2\epsilon_g - k_2(\sqrt{\epsilon_g}\sin\theta - i\lambda/\Lambda)^2\hat{S}_i(z) + \tfrac{1}{2}k^2\Delta\epsilon[\hat{S}_{i+1}(z) + \hat{S}_{i-1}(z)] = 0 \quad \text{(L29.1)}$$

where $\hat{S}_i$ is the amplitude of the inhomogeneous plane wave of the i^{th} space harmonic, $k = 2\pi/\lambda$, λ is the free-space wavelength, and θ is the internal angle of incidence.



Basic planar waveguide-grating model

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
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So, this is where the waveguide grating is ok. So, this is also called as guided mode resonance filter or GMRF. Now, for TE polarization you can consider the electric field vectors to be normal to the plane of incidence in this figure ok. The coupled mode equations which govern the wave propagation in this ah waveguide grating can be written in this form. I will not go into the details of this, but I will just highlight the few important factors that is this S cap is basically the amplitude of the inhomogeneous plane wave of the ith space harmonic. So, you have got this i index, k is basically the free space wavelength and theta is the internal angle of reflection and that is this one ok.

So, you start with the coupled mode equations and when you put delta phi equals 0 in this equation ok you actually make this to be a unmodulated dielectric waveguide it simply becomes a normal waveguide. So, in that case the equation also simplifies and it looks like a normal wave equation of this form where beta is the propagation constant. Now, a guided wave can be excited if the average or if the effective ah waveguide index of refraction N that is given by beta by k is in this range. So, it has to be the modulus of n should be lower than square root of epsilon g, but it has to be greater than or equal to the maximum of the refractive index of this or this whichever is maximum. So, this is how you can ensure that the mode will be propagating guided ok.

Guided Mode Resonances: Basic Concepts

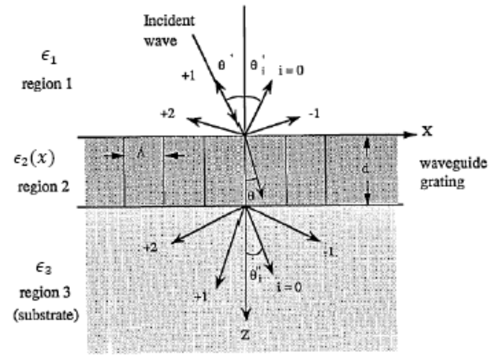
- As $\Delta\epsilon \rightarrow 0$, this equation has the appearance of the wave equation associated with an unmodulated dielectric waveguide, given by:

$$\frac{d^2E(z)}{dz^2} + (k^2\epsilon_g - \beta^2)E(z) = 0 \qquad \text{(L29.2)}$$

where β is the propagation constant.

- A guided wave can be excited if the effective waveguide index of refraction $N = \beta/k$ is in the range:

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \leq |N| < \sqrt{\epsilon_g} \qquad \text{(L29.3)}$$

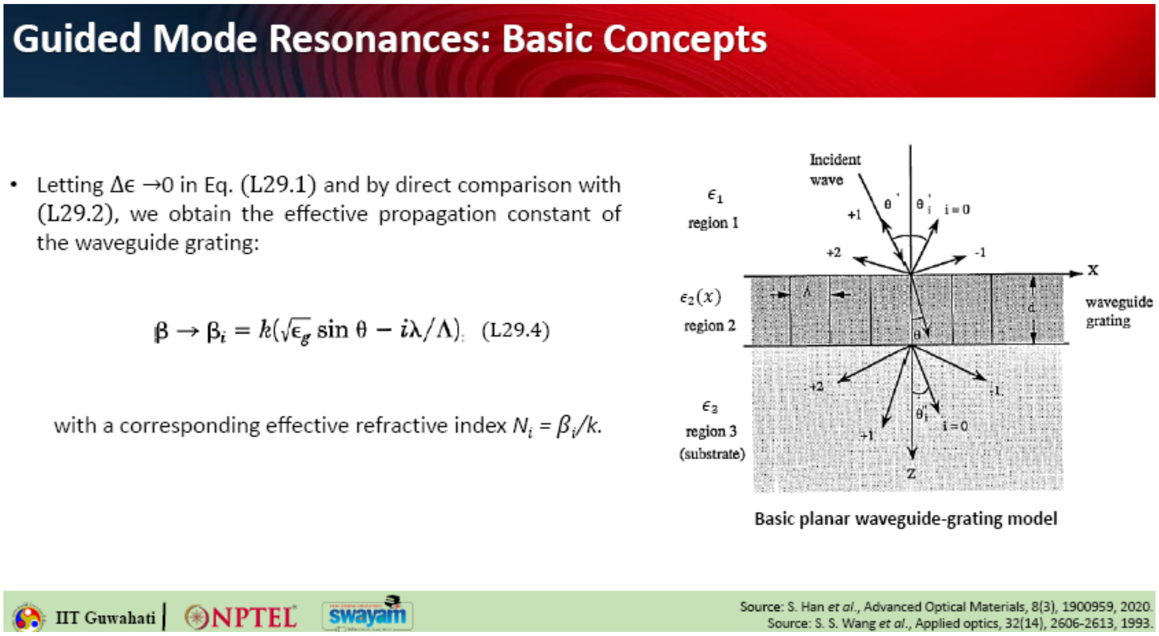


Basic planar waveguide-grating model

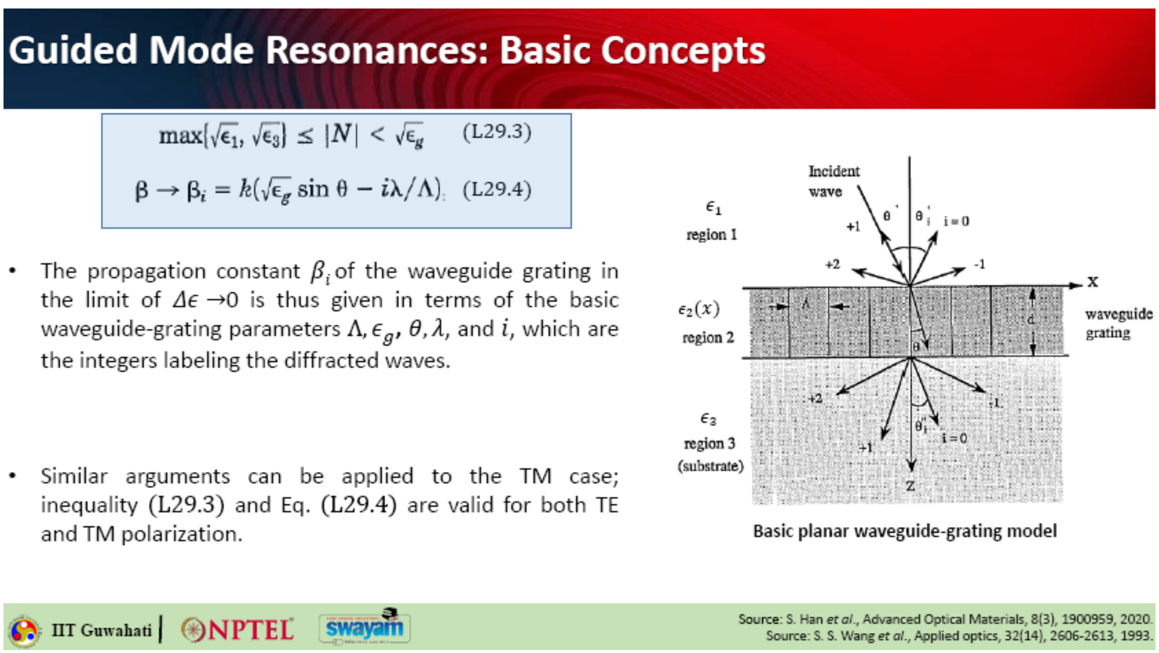
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, this is very this is a very common kind of requirement that comes from the waveguides that you can study in any other course ok. But one important thing is that when you put this delta epsilon to be 0 in that equation and when you compare that with the equation 2 that we have seen here ok you can find out that beta can be written as this ok. So, that is basically the effective propagation constant in the waveguide grating and this also corresponds to a effective refractive index, but there it corresponds to the ith mode ok. So, n i can be written as beta i over k. Now, the propagation constant beta i of the waveguiding in the limit that delta epsilon is 0 or tending to 0 ok is thus given in the terms of the basic parameters something like basic wave guiding grating parameter that is capital lambda epsilon g theta lambda i and all these things.

So, this is how you can actually see these 2 equations and you can put the same arguments here also for the TM case and you will see that this equations are valid this conditions are valid TE and TM polarization ok. So, here let us refer to the eigen mode equations of this unmodulated slab waveguide ok. So, this will become slab waveguide when delta epsilon tends to 0 as you can understand the corresponding eigen value equation of the modulated waveguide can be written as this. So, you can actually find out that tan delta tan kappa i d will be given by this one. So, kappa i is related to the propagation constant in the different regions ok.



So, this these are again all coming from the waveguide theory. So, we will just have a quick look we will not go into very much details of this just I am just showing you the formula here that will give you some idea that how this the GMR effect has come. So, for TM polarization the eigen value equations looks like this. So, the previous one was for TE this one is for TM.



So, in the limit of delta epsilon tending to 0 ok. So, the equation that you have seen for TE or TM and the range that is equation 3 this one they are all holding good. So, they all govern the mode coupling. So, it all tell you that which all modes are possible ok this that equation 3 and this also governs the resonant behavior of this waveguide grating filter with the modified propagation constant beta i ok. So, here in this case ah this will contain the grating parameters explicitly. What is d? You remember d is basically the thickness of this waveguide grating.

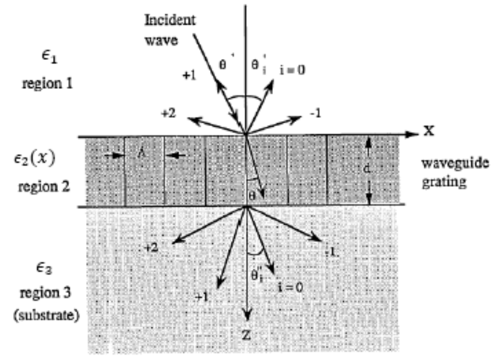
Guided Mode Resonances: Basic Concepts

- By referring to the eigenvalue equation of the unmodulated slab waveguide, the corresponding eigenvalue equation of the modulated guide for TE polarization can be calculated as:

$$\tan(\kappa_i d) = \frac{\kappa_i(\gamma_i + \delta_i)}{\kappa_i^2 - \gamma_i\delta_i} \qquad \text{(L29.5)}$$

where

$$\kappa_i = (\epsilon_g k^2 - \beta_i^2)^{1/2}$$
$$\gamma_i = (\beta_i^2 - \epsilon_1 k^2)^{1/2}$$
$$\delta_i = (\beta_i^2 - \epsilon_3 k^2)^{1/2}$$



Basic planar waveguide-grating model

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
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So, these are the conditions that you have seen ok and for any parameter varied. In fact, the resonance free ah parametric range in the limit of epsilon tending to 0 can be found from these two equations.

Guided Mode Resonances: Basic Concepts

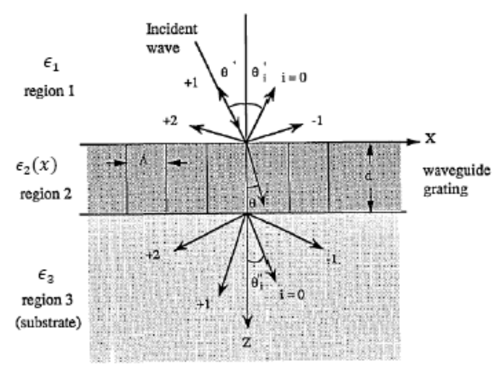
$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \le |N| < \sqrt{\epsilon_g} \qquad \text{(L29.3)}$$

- For TM polarization the eigenvalue equation is:

$$\tan(\kappa_i d) = \frac{\epsilon_g\kappa_i(\epsilon_3\gamma_i + \epsilon_1\delta_i)}{\epsilon_1\epsilon_3\kappa_i^2 - \epsilon_g^2\gamma_i\delta_i} \qquad \text{(L29.6)}$$

- In the limit of $\Delta\epsilon \to 0$, the eigenvalue equation [Eq. (L29.5) or Eq. (L29.6)] and the range condition [inequality (L29.3)] govern the mode coupling.

- This governs the resonance behavior of the waveguide-grating filter with the modified propagation constant β_i, which contains the grating parameters explicitly.



Basic planar waveguide-grating model

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So, these are the eigen mode equations that tell you the modes which are allowed to propagate in this particular grating and this you can find because all these coefficients gamma i delta i and kappa i they depend on epsilon g theta lambda this is the wavelength of light this is the grating period and this is the index ok. So, the resonance free region ah especially represents a separation between the two modes make sense. So, every resonant mode is there and there is also a region between those where different modes can propagate in a equivalent unmodulated slab waveguide that corresponds to them waveguide grating.

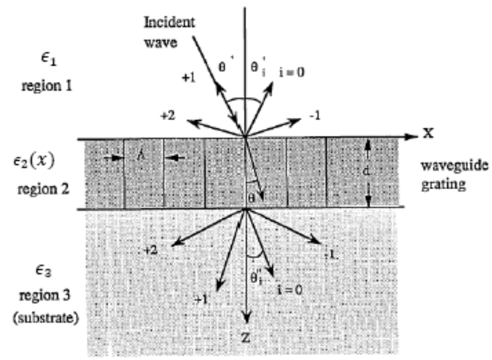
Guided Mode Resonances: Basic Concepts

$$\tan(\kappa_i d) = \frac{\kappa_i(\gamma_i + \delta_i)}{\kappa_i^2 - \gamma_i\delta_i} \qquad \text{(L29.5)}$$

$$\tan(\kappa_i d) = \frac{\epsilon_g\kappa_i(\epsilon_3\gamma_i + \epsilon_1\delta_i)}{\epsilon_1\epsilon_3\kappa_i^2 - \epsilon_g^2\gamma_i\delta_i} \qquad \text{(L29.6)}$$

- For any parameter varied, in fact, the resonance-free parametric range in the limit $\Delta\epsilon \to 0$ can be found from Eqs. (L29.5) and (L29.6) since,

  in general, the coefficients γ_i, δ_i, and κ_i depend on $\epsilon_g, \theta, \lambda, \Lambda$ and i.



Basic planar waveguide-grating model

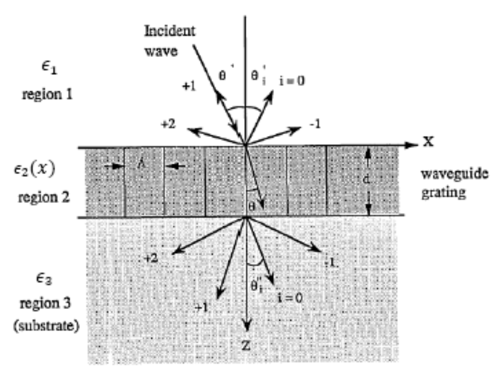
So, whenever there is grating there are some specific modes. So, in between there will be some resonance free region. So, we will also look into those how to identify those particular region. Now, if the i th diffracted wave corresponds to the guided mode then the resonance free range in the thickness in thickness for both TE and TM polarization can be given as delta d which is pi over kappa i and you can write this as expand this in this particular form. So, let us take one example.

Guided Mode Resonances: Basic Concepts

$$\tan(\kappa_i d) = \frac{\kappa_i(\gamma_i + \delta_i)}{\kappa_i^2 - \gamma_i\delta_i} \qquad \text{(L29.5)}$$

$$\tan(\kappa_i d) = \frac{\epsilon_g\kappa_i(\epsilon_3\gamma_i + \epsilon_1\delta_i)}{\epsilon_1\epsilon_3\kappa_i^2 - \epsilon_g^2\gamma_i\delta_i} \qquad \text{(L29.6)}$$

- The resonance-free range essentially represents the separation between the various modes propagating in the equivalent unmodulated slab waveguide that corresponds to the waveguide grating.



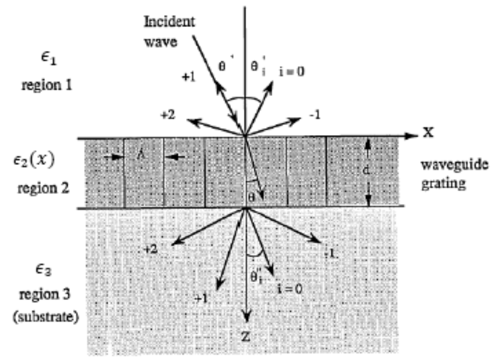
Basic planar waveguide-grating model

So, if you are looking at the TE eigenvalue equation. So, the TE eigenvalue equation was written as this ok and solve when you solve for lambda that gives a free spectral range that can be written as delta lambda FSR nu and that is basically the resonance at nu plus 1 minus the resonance wavelength at nu. So, nu is basically your 0 1 2 and so on these are the integers which are used for labeling the waveguide modes.

Guided Mode Resonances: Basic Concepts

- If the i^{th} diffracted wave corresponds to a guided mode, the resonance-free range in thickness for both TE and TM polarization is:

$$\Delta d = \pi/\kappa_i = \frac{\lambda}{2}\left[\epsilon_g - \left(\sqrt{\epsilon_g}\sin\theta - i\lambda/\Lambda\right)^2\right]^{-1/2} \qquad \text{(L29.7)}$$



Basic planar waveguide-grating model

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

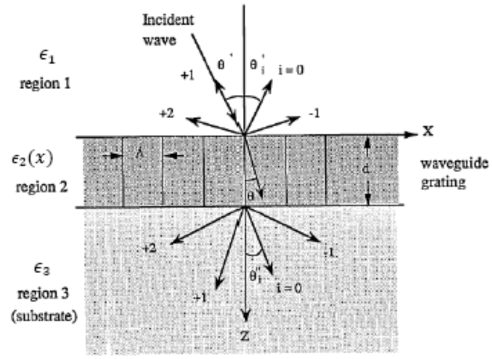
Now, since the eigenvalue expressions for TE and TM modes are different and the resonance occur at different parametric locations ok. So, that that is that is easily understood that the eigenvalue equations for TE and TM modes are different. So, the resonance for TE and TM mode will occur at different parametric location So, these are the two equations for your quick reference.

Guided Mode Resonances: Basic Concepts

- Expressing, for example, the TE eigenvalue equation as

$$\tan[\kappa_i(\lambda)d] = \frac{[\gamma_i(\lambda) + \delta_i(\lambda)]\kappa_i(\lambda)}{\kappa_i^2(\lambda) - \gamma_i(\lambda)\delta_i(\lambda)} \qquad \text{(L29.8)}$$

- Solving for λ gives the free spectral range as $\Delta\lambda_{FRS,v} = \lambda_{v+1} - \lambda_v$, where v = (0, 1, 2, .. .) are the integers labeling the waveguide modes.



Basic planar waveguide-grating model

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

Now, you can consider this as parameters ok and that that will allow you to calculate. These are like some parameters that you can take and you can calculate what will be the positions of the TE and TM resonance modes. Now, to quantify the TE, TM polarization separation of the filters you can apply this equations to produce this particular graph. So, this is how you are plotting the TE0 and this TM0 ok.

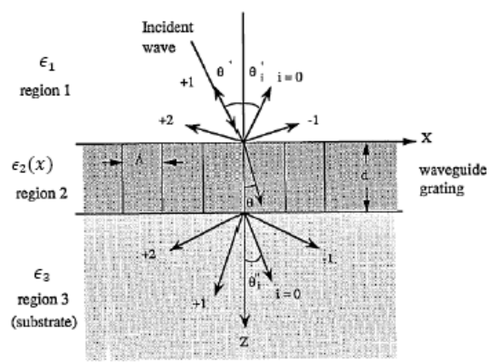
Guided Mode Resonances: Polarization Properties

- Since the eigenvalue expressions [Eqs. (L29.5) and (L29.6)], for the TE and TM modes are different, the resonances occur at different parametric locations for TE- and TM-polarized incident waves.

$$\tan(\kappa_i d) = \frac{\kappa_i(\gamma_i + \delta_i)}{\kappa_i^2 - \gamma_i\delta_i} \qquad \text{(L29.5)}$$

$$\tan(\kappa_i d) = \frac{\epsilon_g\kappa_i(\epsilon_3\gamma_i + \epsilon_1\delta_i)}{\epsilon_1\epsilon_3\kappa_i^2 - \epsilon_g^2\gamma_i\delta_i} \qquad \text{(L29.6)}$$

- The parameters are $\epsilon_1 = 1$, $\epsilon_g = 3$, $\epsilon_3 = 2.161$, and $\theta' = 0^o$ (normal incidence).



Basic planar waveguide-grating model

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

This is TE1, TM1 and this is TE2 and TM2. So, what are these? These are basically showing you the TE, TM polarization separation and this all these are normalized.

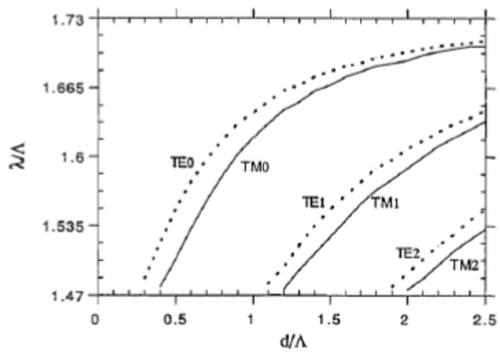
Guided Mode Resonances: Polarization Properties

$$\tan(\kappa_i d) = \frac{\kappa_i(\gamma_i + \delta_i)}{\kappa_i^2 - \gamma_i\delta_i} \quad \text{(L29.5)}$$

$$\tan(\kappa_i d) = \frac{\epsilon_g\kappa_i(\epsilon_3\gamma_i + \epsilon_1\delta_i)}{\epsilon_1\epsilon_3\kappa_i^2 - \epsilon_g^2\gamma_i\delta_i} \quad \text{(L29.6)}$$

- To quantify the TE/TM polarization separation of the filters, we can apply Eqs. (L29.5) and (L29.6) to produce the plot shown in **Figure**.

TE/TM polarization separation and the normalized resonance-free ranges in wavelength and in thickness for a guide-mode resonance filter



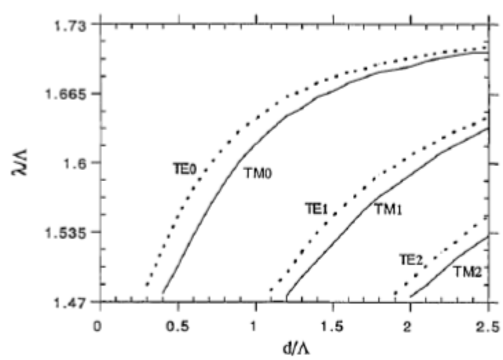
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, this is basically the grating thickness normalized by its period this is the lambda normalized to the grating period. So, these are normalized one this also tells you about the separation between the ah modes ok and it also tells you the resonance free regions. So, these are the regions which are free of resonances. So, for a given normalized filter thickness ok d lambda the normalized wavelength separation between TE and TM mode you can find from here ok.

Guided Mode Resonances: Polarization Properties

- For a given normalized filter thickness d/Λ, the normalized wavelength separation between the TE and TM modes can be read off the plot.
- For example, if we assume $d = \Lambda$ = 1000 nm, the wavelength separation between the fundamental (v = 0) TE and TM resonances is 24 nm.
- Separations of other modes such as TE_0 and TE_1 can also be found from **Figure**.

TE/TM polarization separation and the normalized resonance-free ranges in wavelength and in thickness for a guide-mode resonance filter



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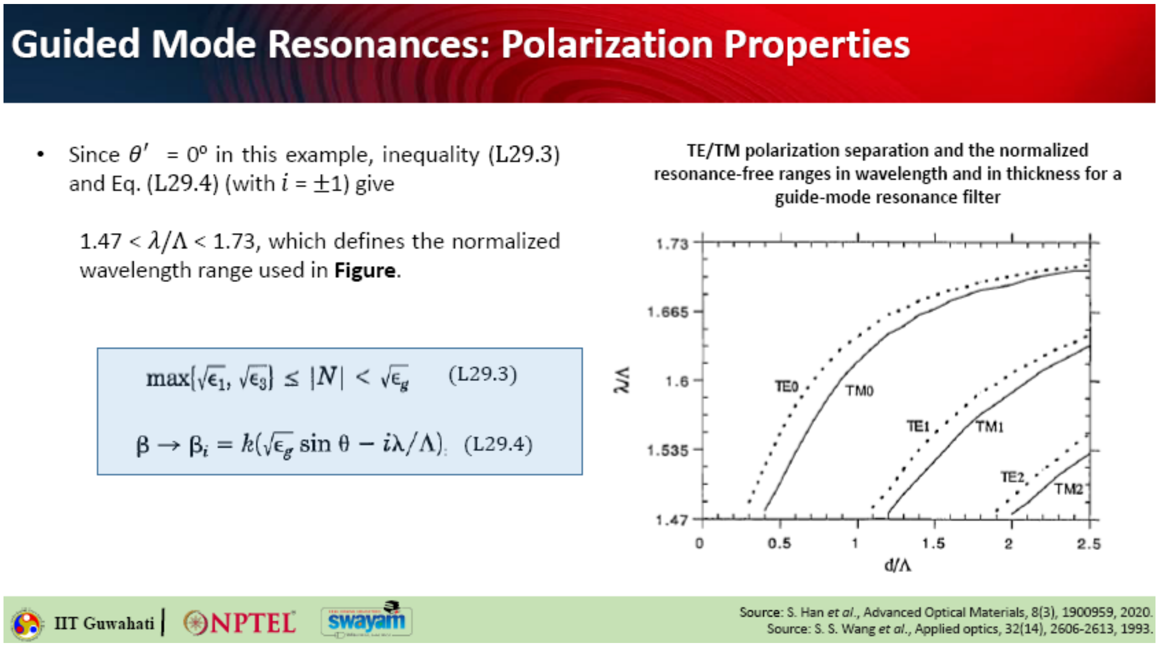
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, for a particular value you can find out what is the lambda separation for the zeroth order TM mode and zeroth order TE mode you can use this graph and find it out. So, if you assume that d equals say 1000 nanometer ok and you can see that the fundamental the mode that is nu equals 0 and if you consider the separation between the TE0 and TM0 you will find that the resonances are separated by 24 nanometer.



So, that you can find from this particular graph ok. Other other kind of separations like what is the separation between TE0 and like this particular when you take this value you can actually have TE1 mode as well as TE0 mode you can actually find out what is the lambda corresponding to this two resonances. So, this particular figure is very important because it gives you all this resonance positions and also it tells you about the resonance free regions ok.

Now as we have considered normal incidence in this example. So, here what are the parameters if you see the parameters this will be the parameters considered ok. So, we are considering normal incidence and other parameters are also given. So, the inequality actually boils down to this. So, this is the condition when you put epsilon 1 and epsilon 2 and other parameters you will see this is the condition that will actually give the normalized wavelength range that you see.

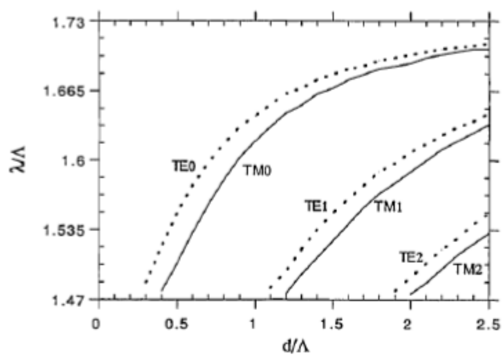
Guided Mode Resonances: Polarization Properties

- **Figure** can additionally be used to find the resonance-free ranges in filter wavelength (for d/Λ = const.) and in thickness (for λ/Λ = const.)
- The analytical expression for the resonance-free range in thickness given by

$$\Delta d = \pi/\kappa_i = \frac{\lambda}{2}\left[\epsilon_g - \left(\sqrt{\epsilon_g}\sin\theta - i\lambda/\Lambda\right)^2\right]^{-1/2} \quad (L29.7)$$

- This is in agreement with Figure.

TE/TM polarization separation and the normalized resonance-free ranges in wavelength and in thickness for a guide-mode resonance filter



Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, this particular range 1.47 to 1.73. So, this is the range that you have plotted here. So, this has basically come from this one ok. So, this actually tells you. So, you have actually put i equals plus minus 1 in this equation ok and you have obtained epsilon 1 epsilon 3 are the values that you have shown there.

So, once you put that you will get this particular range. So, this is why this particular range is considered because in this configuration or the material choice this will be the normalized wavelength that you have to consider. Now the figure also additionally can be used to find out the resonance free region as I told you. So, you can actually see this regions ok in filter wavelength and thickness in both parameter which region is resonance free you can find out. So, you can actually take d by lambda equals constant like this and obtain those region where resonance is not there or you can consider the thickness and see whichever portion has no resonance.

So, the analytical expression you can obtain for resonance free range. So, when you calculate. So, you will see the delta d that is the in thickness what is that thickness where there is no resonance you can obtain it like this. I am not going to the details of this equations there are complicated equations even you can obtain them from this particular paper, but this actually tells you how this separations are basically obtained. And when you combine this inequality of this equation and this one you will actually get this particular equation because the relationship between beta by k is basically your N.

Guided Mode Resonances: Resonance Regimes

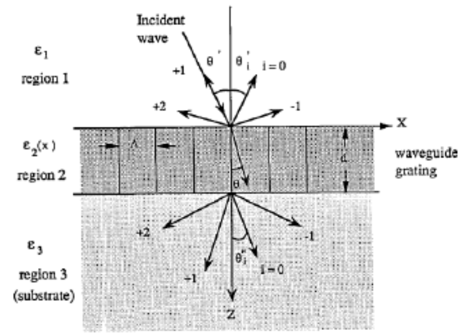
- Inequality (L29.3) and Eq. (L29.4) can be combined to give:

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \le |\sqrt{\epsilon_1}\sin\theta' - i\lambda/\Lambda| < \sqrt{\epsilon_g} \quad \text{(L29.9)}$$

where $\sqrt{\epsilon_1}\sin\theta' = \sqrt{\epsilon_g}\sin\theta$ with θ' being the external angle of incidence.

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \le |N| < \sqrt{\epsilon_g} \quad \text{(L29.3)}$$

$$\beta \rightarrow \beta_i = k(\sqrt{\epsilon_g}\sin\theta - i\lambda/\Lambda) \quad \text{(L29.4)}$$



So, you can put this guy here and you will get this particular expression. So, here you can also see that if you want to make epsilon square root of epsilon 1 that is n 1 sin theta 1 equals the refractive index of this one that is square root of epsilon g sin theta ok. So, that way you will be able to make a range which are the allowed angles for your particular device. So, with that you can identify the resonance regime ok. So, this expression allows you to define the parametric regime where the guided mode resonance actually occurs.

Guided Mode Resonances: Resonance Regimes

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \le |\sqrt{\epsilon_1}\sin\theta' - i\lambda/\Lambda| < \sqrt{\epsilon_g} \quad \text{(L29.9)}$$

- This expression permits the definition of parametric regions within which the guided-mode resonances occur.
- **Figure** gives the result for selected values of average permittivities with the diffraction-order index i as a parameter.
- The parameters are $\epsilon_1 = 1$, $\epsilon_g = 3$, and $\epsilon_3 = 2.161$.

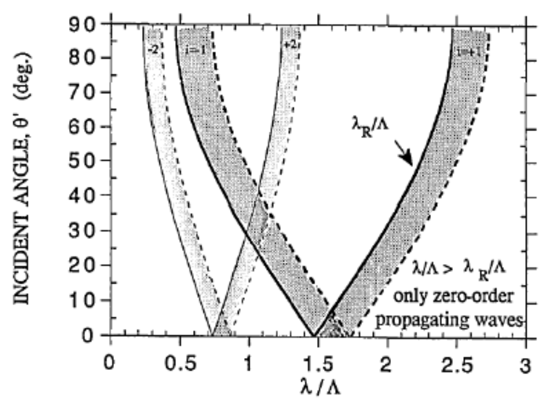


Figure: Resonance regimes of waveguide gratings.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

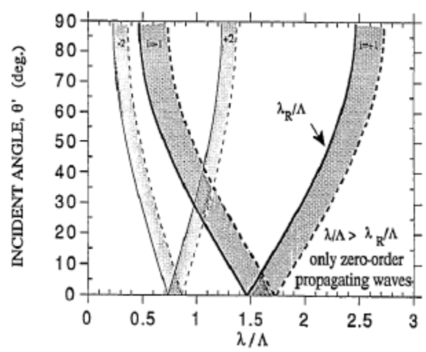
So, this is particularly a plot that shows those resonance regime ok. So, on the x axis you have the normalized wavelength and on the y axis you have angle theta okay. So, this actually gives you the selected values of the average permittivities with the deflection order i as a parameter. So, here you see this is for i equals plus 1, this is i equals minus 1, this is i equals plus 2, this is i equals minus 2 ok. And again, the parameters that you have considered here is ε_1 is 1, ε_g is 3 and ε_3 is 2.161. So, the solid curves that you see here on the left side of the inequality ok. So, here the left side of the inequality is actually coming from this solid line ok and the right side is shown by this dashed line that is coming from this one. So, the right side inequality gives you this one for a particular i ok and the left side in equality gives you this solid line ok for that particular i. And the shaded region between the two lines are basically those parameter values for which resonance can takes place. So, this can be called as resonance regime ok and these are the boundaries for i equals1 right. So, the solid boundaries correspond to the deflected order i at the grazing angle ok at which the classic Rayleigh anomaly is associated.

Guided Mode Resonances: Resonance Regimes

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \leq |\sqrt{\epsilon_1}\sin\theta' - i\lambda/\Lambda| < \sqrt{\epsilon_g} \quad \text{(L29.9)}$$

- The solid curves indicate the left-hand side of inequality (L29.9), and the dashed curves represent the right-hand side of inequality (L29.9).
- The shaded regions between these lines indicate the parameter values for which a resonance can occur; *i.e.*, the resonance regimes.

Resonance regimes of waveguide gratings



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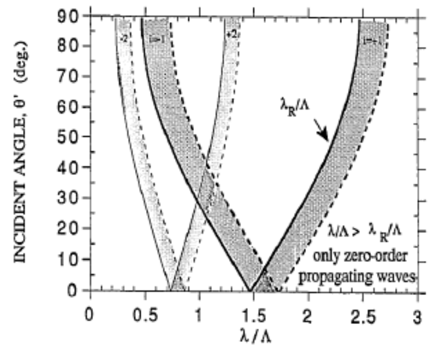
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, we will see what what is that Rayleigh anomaly in the next slides. So, here one important thing is to note that at the intersection of the two solid curves a double Rayleigh anomaly takes place ok something like this. Now when you say about Rayleigh anomaly this is nothing, but a anomalous or abnormal behaviour of light that is being reflected from a periodically corrugated surface or diffraction grating that was observed in the form of rapid variation of intensity of the diffraction orders as a function of the wavelength. So, it happens like this a sharp intensity variation and related exclusively to the emergence of diffracted beams when the incident beam is at glazing angle and you will also get beams which are parallel to the surface of the grating. So, Rayleigh was the first to represent this particular effect and he was able to explain this.

Guided Mode Resonances: Resonance Regimes

- The solid boundaries correspond to the diffracted order i at the grazing angle with which the classical Rayleigh anomaly is associated.
- At the intersection of two solid curves, a double Rayleigh anomaly occurs.

Resonance regimes of waveguide gratings



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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993

So, that is why the anomaly is named after him. It was pointed out that the anomaly in reflection occurs at wavelengths for which one of the diffracted order becomes parallel to the main plane of the grating like this ok and then eventually ah it will vanish at greater wavelengths because after that you cannot actually like the reflected one cannot go inside. So, that has to vanish ok. So, this changes the power distribution among the remaining diffraction orders including the specular reflection or the zeroth order reflection right. So, if one mode is not allowed or one particular order is vanishing.

Guided Mode Resonances: Resonance Regimes

Rayleigh Anomaly

- Anomalous behavior of light reflected from the periodically corrugated surfaces or diffraction gratings was observed in a form of rapid variation of intensity of the diffraction orders as a function of the wavelength.
- Rayleigh presented the first explanation of this effect.
- It was pointed out that the anomaly in reflection occurs at the wavelengths for which one of the diffracted orders becomes parallel to the main plane of the grating and vanishes eventually at greater wavelengths.
- This changes the power distribution among the remaining diffraction orders (including the specular reflection or the zeroth-order).

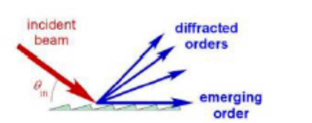

Figure: Rayleigh anomaly: A sharp intensity variation and related exclusively to the emergence of the diffracted beam.

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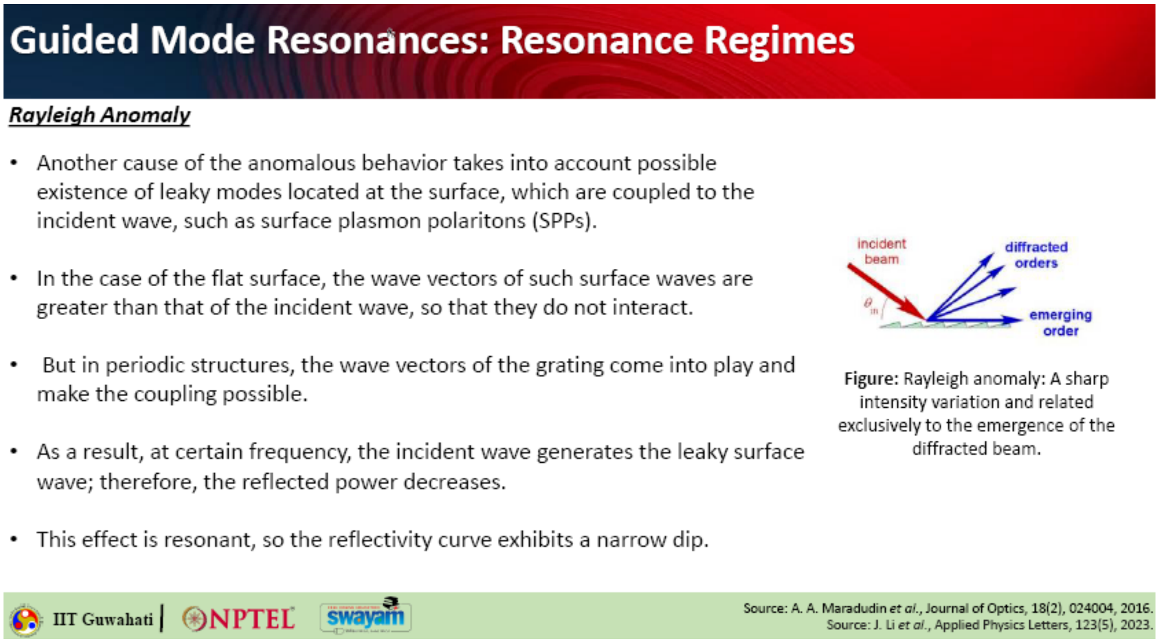
Source: A. A. Maradudin *et al.*, Journal of Optics, 18(2), 024004, 2016
Source: J. Li *et al.*, Applied Physics Letters, 123(5), 2023

So, the power has to be distributed among the remaining. So, that is how the anomaly disturbs the system. Another cause of the anomalous behavior takes into account the possible existence of leaky modes located at the surface. So, which are coupled to the incident wave such as in the case of surface plus one polar atoms. So, there also you can see in the case of grating it is mainly for those diffracted beam orders which are basically parallel to the grating. Now, in the case of the flat surface the wave guides of such surface waves are greater than the incident waves.



So, that they do not interact, but when you look for periodic structures the wave factors of the grating come into the play. So, because of that the grating or because of that the coupling becomes possible and as a result at certain frequency the incident wave will generate the leaky surface waves and in that case the reflected power will decrease. So, this effect becomes resonant and because of that the reflection curve will get a narrow dip very narrow dip and that will also give you a transmission maximum. So, for the light diffraction at the periodic structure which has got a grating number of g okay.

Guided Mode Resonances: Resonance Regimes

Rayleigh Anomaly

- For the light diffraction at the periodic structure with the grating number G, the Rayleigh's anomaly occurs if:

$$k_x \pm nG = k = \frac{\omega}{c} \quad (n = 1, 2, \ldots) \quad \text{(L29.10)}$$

where k is the wave number of the incident light and k_x is its component parallel to the grating plane.

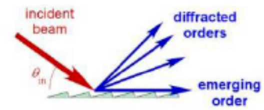


Figure: Rayleigh anomaly: A sharp intensity variation and related exclusively to the emergence of the diffracted beam.

Source: A. A. Maradudin *et al.*, Journal of Optics, 18(2), 024004, 2016.
Source: J. Li *et al.*, Applied Physics Letters, 123(5), 2023.

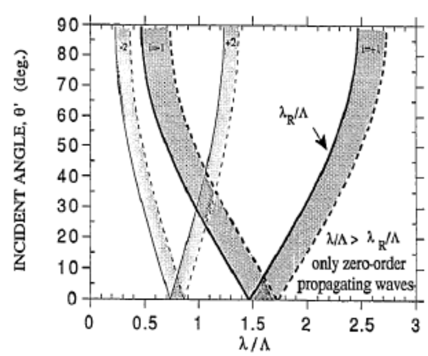
G can be written as 2 pi by the lattice period ok. You can say that Rayleigh anomaly will occur if k x plus n times G will be equal to the incident wave factor. So, if this matches so n is basically 1, 2, 3 this is happening because of the grating when there is a match ok. So, a Rayleigh anomaly will take place the wave will leak in ok and you will get a reflection dip. So, here k is the wave vector or wave number of the incident light and k x basically is the component of the component that is parallel to the grating plane of the wave vector right So, the 2 symmetric ah double Rayleigh anomalies can be seen here when the incident angle is 0 degree.

Guided Mode Resonances: Resonance Regimes

- Two symmetric double Rayleigh anomalies are shown in **Figure** at θ′ = 0°, one for $i = \pm 1$ and the other for $i = \pm 2$.
- An asymmetric double Rayleigh anomaly occurs at **θ′=** 30° for i = 2 and i **=** -1.
- During the interval designated as the resonance regime, the order i can, in accordance with inequality (L29.9), correspond to a guided mode.

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \leq |\sqrt{\epsilon_1}\sin\theta' - i\lambda/\Lambda| < \sqrt{\epsilon_g} \quad \text{(L29.9)}$$

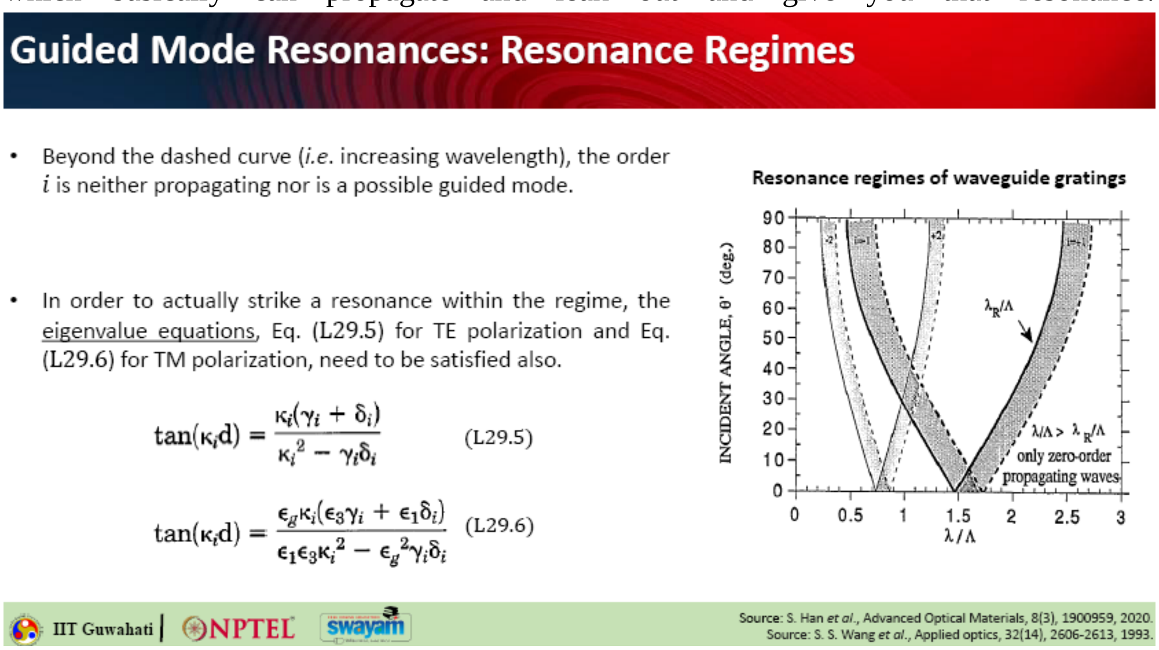
Resonance regimes of waveguide gratings



Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

You can see it for i plus minus 1 there is a degeneracy the other one is for i plus minus 2. So, these are basically symmetric double Rayleigh anomalies. There is also one asymmetric double Rayleigh anomaly that happens at theta equals 30 degree when i equals plus 2 and i equals minus 1 ok. Now, during the interval designated as this resonance regime the order i can actually correspond to the guided mode. So, you can say this is the first order, second order and so on and this is the equation that you have already seen. So, this theta is the incident angle and this limit tells you that which all modes including the order of the mode that can be allowed to propagate.

Now if you look into the dashed curve ok with the increasing wavelength now beyond the dashed curve that is we are looking for this one the dashed curves ok. The order i is neither propagating nor it is possible to beyond this it is not able to propagate or it is not even guided. So, in order to actually strike a resonance with this particular regime the eigenvalue equations need to be satisfied. So, these are the equations that need to be satisfied to fall within this particular regime right. So, these are the supported modes which basically can propagate and leak out and give you that resonance.



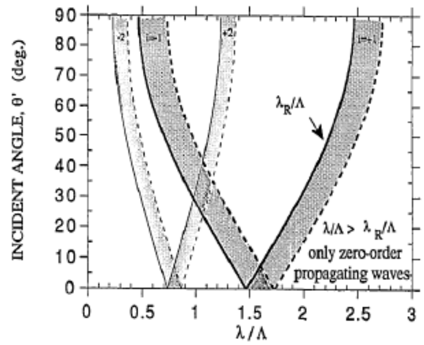
Now their solutions fall within the resonance regime with the location additionally dependent on the thickness of the filter that is the parameter d. So, here you can see at 0 degree the resonance of plus minus 1 is indistinguishable because they are overlapping here and at nonzero theta prime that is at nonzero degeneracy. So, you can put plus minus 1 and you can actually see that they actually do not have the degeneracy anymore. And the solid curve that you see here that is leveled as lambda r over capital lambda this indicates the last propagating ah higher order diffracted mode that is here in this case it is i equals plus 1 ok.

Guided Mode Resonances: Resonance Regimes

- Their solutions fall within the resonance regime with the location additionally dependent on the value of the filter thickness d.
- From inequality (L29.9) at $\theta' = 0°$, the resonances for $i = \pm 1$ are indistinguishable;
- a nonzero θ' removes the degeneracy.

$$\max\{\sqrt{\epsilon_1}, \sqrt{\epsilon_3}\} \leq |\sqrt{\epsilon_1}\sin\theta' - i\lambda/\Lambda| < \sqrt{\epsilon_g} \quad \text{(L29.9)}$$

Resonance regimes of waveguide gratings

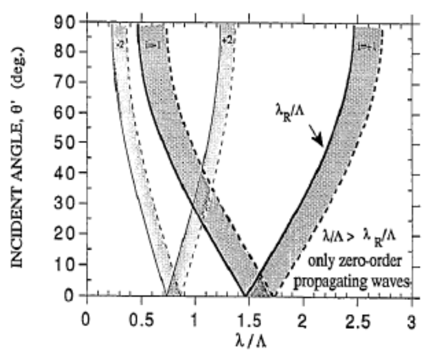


And it gets cut off at grazing angle okay, as the wavelength is permitted to increase. So, as you keep on increasing it actually does not increase further it will actually get cut off here. So, this R is basically the Rayleigh wavelength. So, you can actually call this as the Rayleigh wavelength ok and when lambda is greater than lambda R ok you can say only 0th order wave can propagate ok. So, in this regime only the 0th order mode will be propagating not the higher order modes. So, the diagram such as this figure are also useful in visualizing the resonance properties of diffraction gratings.

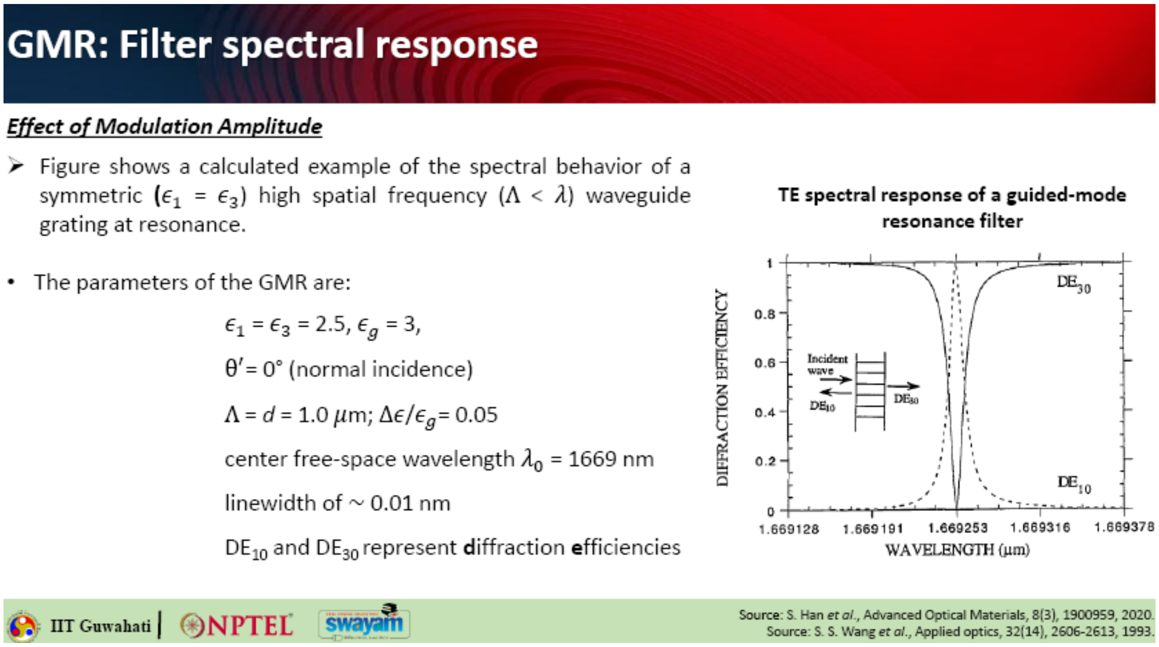
Guided Mode Resonances: Resonance Regimes

- The solid curve labeled λ_R/Λ indicates where the last propagating higher-order diffracted wave (i = + 1) gets cut off at the grazing angle as the wavelength is permitted to increase.
- λ_R is thus a Rayleigh wavelength.
- For $\lambda > \lambda_R$, only the zero-order waves propagate.
- Diagrams such as this Figure can be useful in visualizing the resonance properties of waveguide gratings.

Resonance regimes of waveguide gratings



Now let us look at the effect of modulation amplitude ok. So, this particular figure calculates an example of the spectral behavior of the symmetric high symmetric filter or symmetric waveguide symmetric means epsilon 1 is considered to be same as epsilon 3 ok.



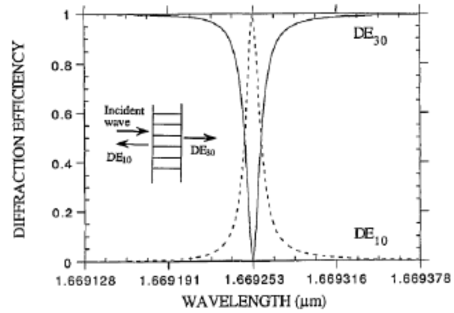
And it has high spatial frequency. So, lambda is considered to be greater than the grating period ok. And what are the parameters $\varepsilon_1 = \varepsilon_3 = 2.5$ and $\varepsilon_g = 3$ this normal incidence these are the parameters we have considered the center free space wavelength to be 1.669 and the line width we got is 0.01 nanometer and this DE10 these are basically the diffraction efficiencies ok. So, 1 0 is reflected 3 0 is the transmitted one. So, you can only see that the zero forward and backward diffracted waves propagate and in this case all other modes are basically cut off right. And the diffraction efficiency represents the intensity of the various diffracted wave.

Guided Mode Resonances: Filter spectral response

Effect of Modulation Amplitude

- Only the zero forward- and backward-diffracted waves propagate with all higher-order waves cut off.
- The diffraction efficiency represents the intensity of the various diffracted waves.
- Note the 100% energy exchange and the smooth lines that are obtainable.

TE spectral response of a guided-mode resonance filter



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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

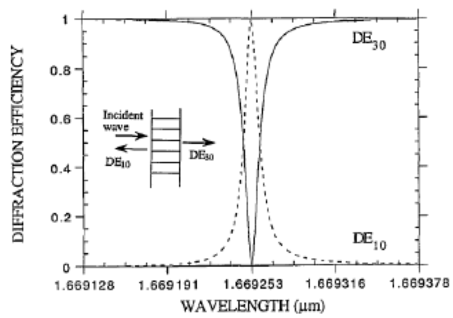
So, here you can see that this is 3 0 the transmitted one is getting a dip and the reflection 10 is getting a peak ok. And there is a 100% energy exchange between these 2 modes and the smooth lines can be obtained.

Guided Mode Resonances: Filter spectral response

Effect of Modulation Amplitude

- Thus the **symmetric** waveguide-grating filter can produce a **symmetric spectral response** in reflection with nulls on both sides of the peak.
- A corresponding notch filter response appears in transmission.
- Note: when higher-order waves propagate also, pure nulls may not be obtained.

TE spectral response of a guided-mode resonance filter



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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

And what is the good thing about this kind of symmetric filter the symmetric waveguide grating filter they also produce very symmetric spectral response.

Guided Mode Resonances: Filter spectral response

Effect of Modulation Amplitude

- The spectral response of an **asymmetric** ($\epsilon_1 \neq \epsilon_3$) high spatial frequency waveguide-grating filter is shown to illustrate the **asymmetric** response typically obtained.

The parameters are:

$\epsilon_1 = 1$, $\epsilon_3 = 2.161$, $\epsilon_g = 3$,

$\theta' = 0°$ (normal incidence)

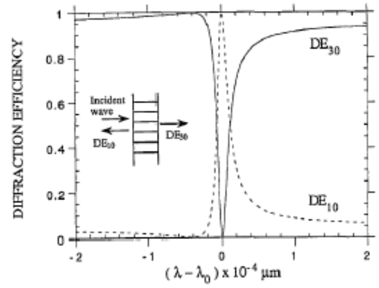
$\Lambda = d = 0.33\ \mu m$

$\Delta\epsilon/\epsilon_g = 0.05$

center free-space wavelength $\lambda_0 = 547$ nm linewidth of ~ 0.02 nm

DE_{10} and DE_{30} represent diffraction efficiencies.

TE spectral response of an guided-mode resonance filter.



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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, you have 0 nulls on both side of the peak and that is a perfectly symmetrical filter which is usually not the case with any other resonant kind of structure. And if you see the transmission you get a very beautiful notch filter ok. And if you are allowing the higher order waves to propagate you will see that pure nulls may not be obtained.

Guided Mode Resonances: Filter spectral response

Effect of Modulation Amplitude

- **Figure** shows the calculated normalized filter linewidth $\Delta\lambda/\lambda$ as a function of the normalized modulation amplitude $\Delta\epsilon/\epsilon_g$ of the waveguide-grating.
- The parameters are $\epsilon_1 = 1$, $\epsilon_3 = 2.161$, $\epsilon_g = 3$, $d / \Lambda = 1$ at normal incidence.
- If we take, for example, $\Lambda = 1\ \mu m$, the calculated linewidth in this case lies in the range 20 fm < $\Delta\lambda$ < 200 pm for the modulation range used.
- The modulation amplitude $\Delta\epsilon/\epsilon_g$ is therefore seen to be an effective way to control the linewidths of the guided-mode resonance.

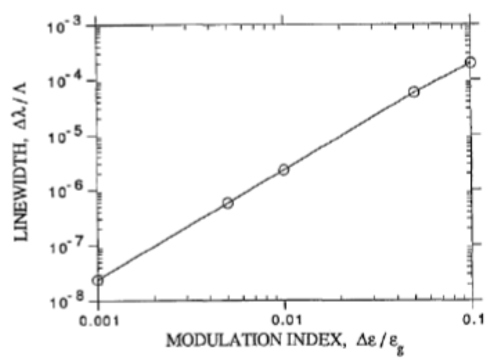

Figure: Calculated relation between the modulation index and the linewidth of a guided-mode resonance filter for TE polarization.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993

So, one of these legs will be kind of asymmetrical and it will be different. So, if you look into asymmetrical filter where you have ε_1 not equal to ε_3 and these are the parameters. So, in that case surely your spectral response is not symmetrical and you do not have null on both sides perfect null on both sides. Let us also look into the effect of amplitude modulation ok. So, here it shows the normalized filter line width as a function of the modulation amplitude of the waveguide grating.

Guided Mode Resonances: Filter spectral response

Effect of Mode confinement

- The spectral linewidth can also be controlled by the degree of confinement of the modes in the associated unmodulated waveguide.
- Figure shows a numerically calculated plot of the normalized linewidth of a symmetric ($\epsilon_1 = \epsilon_3$) high spatial frequency waveguide grating filter as a function of the refractive-index difference $\Delta n = \sqrt{\varepsilon_g} - \sqrt{\varepsilon_1}$.
- The parameters are $\epsilon_g = 3$, $\Delta\epsilon/\epsilon_g = 0.05$, and $\Lambda = d = 1\ \mu$m, with $\epsilon_1 = \epsilon_3$ varying from 1.0 to 2.95.
- Strongly confined modes ($\Delta n \rightarrow 1$ in this example) exhibit the largest linewidth.

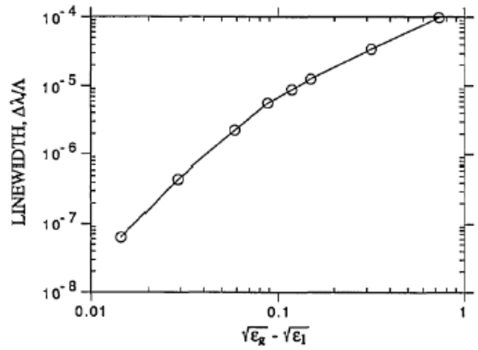

Figure: Calculated relation between the refractive-index difference $\sqrt{\varepsilon_g} - \sqrt{\varepsilon_1}$ and the linewidth of a symmetric guided-mode resonance filter for TE polarization.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, these are all kind of normalized. So, delta epsilon is the modulation index and delta lambda is basically the line width and if you take these are the parameters. So, this is in a symmetric parameter and you can see that if you consider grating period to be 1 micrometer the line width that you calculate falls in the range of 20 femtometer to 200 picometer. So, that is basically the range. So, you can understand how narrow these resonance filters are and they are very very high quality resonance.

GMR: Electro-Optic Switching

- Figure illustrates the resonance line under variation of the average grating permittivity for an asymmetric waveguide-grating structure.
- In this example, the Bragg condition is satisfied, but this is not a requirement for the resonance to occur.
- The Bragg condition $\lambda/\Lambda = 2\sin\theta$ is satisfied.
- The parameters for this example are $\Delta\epsilon/\epsilon_g = 0.005$, $d = 0.32\ \mu$m, $\epsilon_1 = 1$, $\epsilon_3 = 2.161$, $\Lambda = 1\ \mu$m, and $\theta' = 31°$; DE_{10}, DE_{11}, DE_{30}, and DE_{31} are diffraction efficiencies.

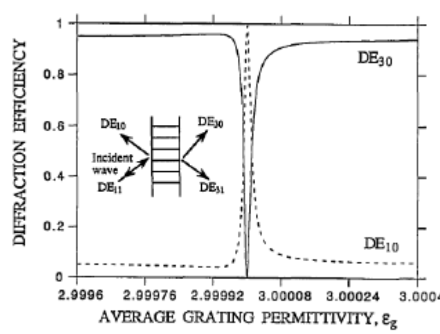

Figure: Resonance line in terms of the average relative permittivity of the waveguide grating for TE polarization.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, this is a graph that will tell you that for what kind of modulation you should have what kind of line width. You can also think about the mode confinement from this particular graph it allows you to see the spectral line width can also be controlled by the degree of confinement of the mode in an associated unmodulated waveguide.

GMR: Electro-Optic Switching

- The point is that the resonance can also occur under Bragg conditions.
- With nonzero-order diffracted waves also propagating, as indicated by the inset, pure nulls are not observed next to the resonance.
- Note that a variation in the average relative permittivity of the grating can induce a resonance.
- Hence, electro-optic effect can be used to switch (linewidth of ~10^{-5} for this example) energy from the transmitted wave to the reflected wave (and vice versa) with low applied voltage.

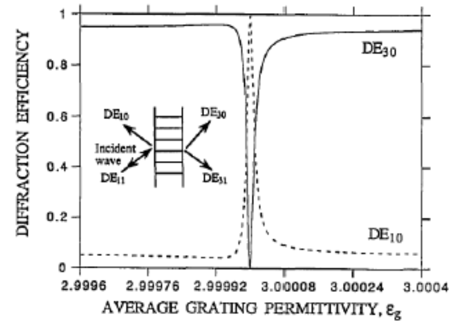

Figure: Resonance line in terms of the average relative permittivity of the waveguide grating for TE polarization.

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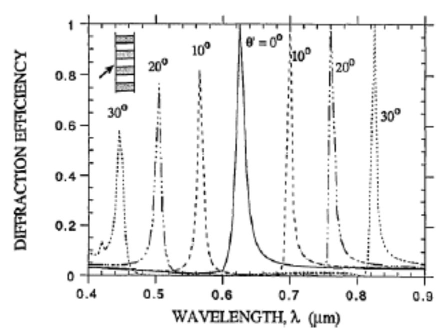
Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, if you do not have modulation and you can think of an effective unmodulated waveguide. So, this particular figure shows the numerically calculated plot of normalized line width. So, here you are taking a symmetric case, but you are considering the difference in the refractive index to be your x axis and you are varying the line width.

GMR: Variation with angle of incidence

- Figure shows the calculated TE filter reflectivity as a function of the wavelength with the angle of incidence as a parameter.
- The peak efficiencies are carefully determined by numerical calculations on a fine grid.



$\epsilon_1 = \epsilon_3 = 2.25$
$\epsilon_g = 3.125$
$\Lambda = 0.4$ μm
$d = 0.15$ μm

Figure: TE filter reflectivity for a **waveguide grating with square wave grating shape.**
The angle θ' is the external (air) angle of incidence.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, here you will see that you are varying the parameters from 1 to 2.95 right. This is the epsilon 1 that you are varying and this is how you actually get this variation okay. So, strongly confined states something like delta n will be tending to 0 exhibit the largest linewidth. You can also use this for electro optic switching in some applications something like this particular filter illustrates the resonance line under the variation of the average grating permittivity of an asymmetric filter or asymmetric waveguide grating structure. So, here you can see there are many other modes which are getting diffracted. So, here in this particular example the break grating is satisfied, but this is not a condition requirement for the resonance to occur.

Guided Mode Resonances

- **Resonance Regime**: one peak is permitted at θ′ = 0 with linewidth of ~100 nm. As θ′ increases, the separation of the resonances increases also. At θ′ = 30°, at the shorter wavelengths, the guided-mode resonances corresponding to i = -1 and i = +2 can both be excited in close proximity.
- The large peak at θ′ = 30° corresponds to i = -1 with the smaller peak caused by the wave with i = +2, as shown clearly on the far left-hand side of Figure on left.

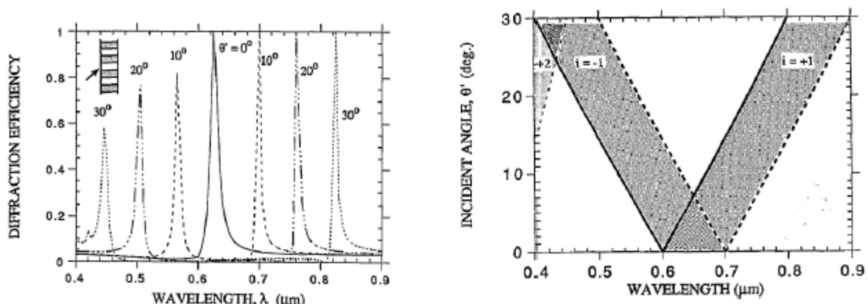

Figure: Resonance regime for the waveguide grating.

Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, the break grating condition lambda over capital lambda equals 2 sin theta is satisfied here and these are the parameters that we considered and you see that 1 0, 1 1, 3 0 3 1 these modes are also present ok and that is why you you are not seeing perfect null in this resonance.

Guided Mode Resonances

- Here the results for the TM case are demonstrated.
- The resonances are narrower than those for the TE case.
- The resonances (three of them very narrow) for $\lambda > 0.6$ μm in Figure are not shown on the TM plot

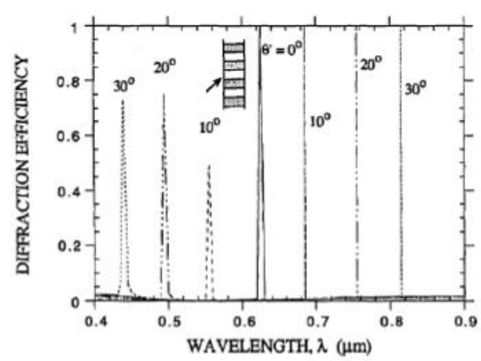

Figure: TM filter reflectivity for the waveguide grating.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

The point is that here the resonance can also occur under break condition and with non-zero diffracted wave also propagating as you can see here in this inset pure nulls are not obtained next to the resonance and a variation in the relative average relative permittivity can then introduce a resonance. So, in that case a electro optic effect can be used to switch energy from the transmitted to the refracted because you can switch change slightly the effective relative permittivity of this grating and that can introduce the resonance or it can move out the resonance. So, you can actually use a electro optic effect for getting this resonance or not ok. This particular figure shows the calculated TE filter reflectivity as a function of wavelength with different incident angle.

Guided Mode Resonances: Summary

- Thus, RWGs or guided mode resonances (GMRs) use the periodicity of a grating to couple light into a thin waveguide.
- They have been therefore used extensively as waveguide couplers for optical communication and signal processing for the in-coupling and out-coupling of thin waveguide modes with strong wavelength, polarization, and angular dependences.
- Their in-coupled quasi-guided modes can interfere dramatically with the incident illumination depending on the phase delay accumulated in the in-coupling in the waveguide which create anomalous reflection and transmission features, creating unique zeroth-order properties.
- This mechanism makes them highly efficient narrowband or broadband reflectors, as well as transmission filters with applications as laser mirrors, advanced detection systems, or spectrometers.

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Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020.
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993.

So, you have got theta equals 0 plus 10 minus 10 plus 20 minus 20 plus 30 minus 30 and so on. So, the peak efficiencies they are all dependent on the incident angle as well. So, here are the parameters which are used for calculating this and for this particular case you have also obtained what are the resonance regime as we have seen. So, here you can actually see that we have calculated for i equals plus 1 and minus 1 ok and there is a large peak corresponds to i equals minus 1 with smaller peaks happening at i equals plus 2 ok because there is a case here corresponding to 30 ok. So, here you can see that at yeah this is the case for theta prime equals 30 degree which corresponds to your plus 2 ok.

The large peak at theta prime equals 30 degree corresponds to i equals minus 1. So, this one with a smaller peak caused by the wave with i equals plus 2. So, i equals plus 2 gives this particular smaller width ok. So, this is basically seen from this one. So, the same results can also be demonstrated for that other polarization case TM polarization case. So, in this case you can see the resonances are even more narrower ok and with that we will conclude what we understood in the GMR.

Guided Mode Resonances: Summary

- In another direction, cost-efficient fabrication processes and unique appearance have enabled their applications in optical authentication and document security.
- Their polarization-dependent behavior has led to the development of polarizers, polarization rotators, and waveplates.
- The control of the optical near-field has found widespread applications in biological refractive index sensing, fluorescence sensing, nonlinear effects, and optical switching, as well has enhancement of solar light harvesting.
- We will discuss the features of GMR based devices in next lecture.

Source: S. Han *et al.*, Advanced Optical Materials, 8(3), 1900959, 2020
Source: S. S. Wang *et al.*, Applied optics, 32(14), 2606-2613, 1993

So, RWG or GMR basically they use the periodicity of a grating to couple light into a thin waveguide. They have been therefore, used very extensively as waveguide couplers for optical communication and signal processing for both in coupling and out coupling of thin waveguide modes with strong wavelength polarization and angular dependence. Their in coupled quasi guided mode can interfere dramatically with the incident illumination depending on the phase delay that is accumulated in the in coupling of the waveguide which can create anomalous reflection or transmission features creating unique zeroth order properties. So, you can actually get those very sharp reflection or transmission peak or dip. This mechanism makes them highly efficient narrow band or broadband reflectors as well as transmission filters with applications as laser mirror, advanced detection systems, spectrometers etc. So, in other direction you can think of cost efficient fabrication processes and unique appearance have enabled their applications in optical authentication and document security.

Their polarization dependent behavior can help them to be used as polarizer, polarization rotator, wave plates etc. The control of the optical near field has found widespread application in biological refractive index sensing, fluorescence sensing, non-linear effects, optical switching etc as well as enhancement of the solar light harvesting. So, in the next lecture we will discuss some of these devices based on this effect. So, with that we conclude. Thank you for your attention.

In the next lecture we will consider and discuss the applications of matter surfaces and this GMR based devices. If you have got any queries you can drop an email to this email address mentioning MOOC in the subject line. Thank you.