Course Name- Nanophotonics, Plasmonics and Metamaterials

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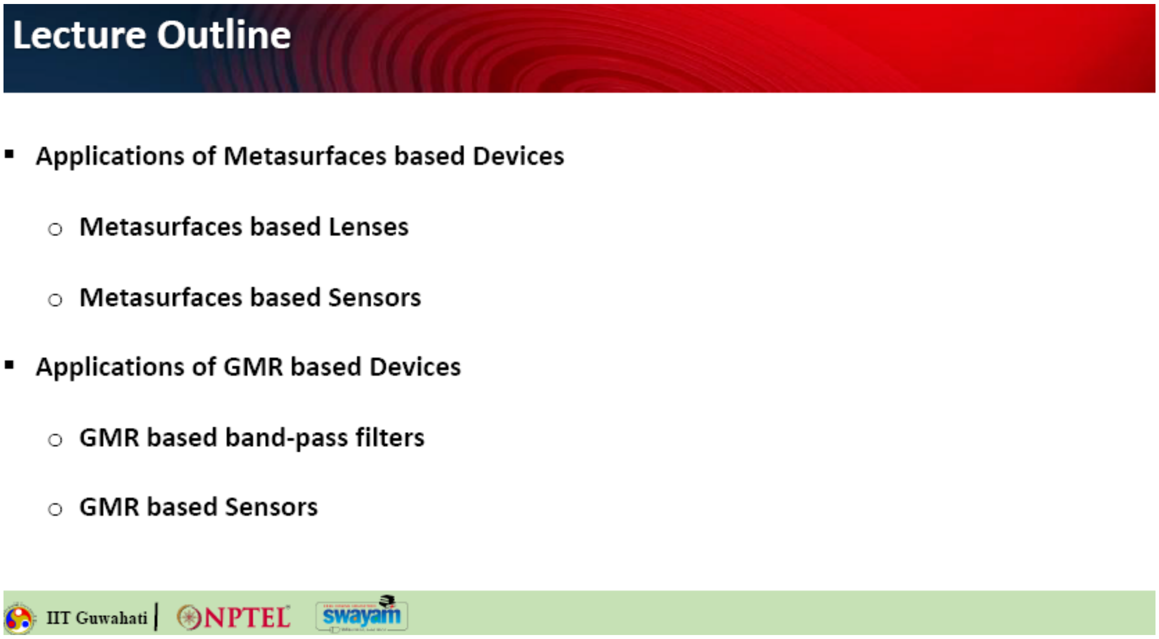
Institute Name- Indian Institute of Technology Guwahati

Week-10

Lecture -30

Hello students welcome to lecture 30 of the online course on Nanophotonics, Plasmonics and Metamaterials. Today's lecture will be on application of metasurfaces and GMR devices. So here is the lecture outline, first we will see some of the metasurface based devices such as metasurface based lens and sensor. We will go into the details of how these things are designed and then we will look into some of the applications of GMR based devices that is guided mode resonance based devices. We will discuss about some band pass filters as well as GMR based sensors. So metasurface I hope you all remember from the previous lectures that metasurface gives you the control on the amplitude, phase, polarization also on the frequency of the incident light you can modify those using metasurface.



Lecture Outline

- **Applications of Metasurfaces based Devices**
    - **Metasurfaces based Lenses**
    - **Metasurfaces based Sensors**
- **Applications of GMR based Devices**
    - **GMR based band-pass filters**
    - **GMR based Sensors**

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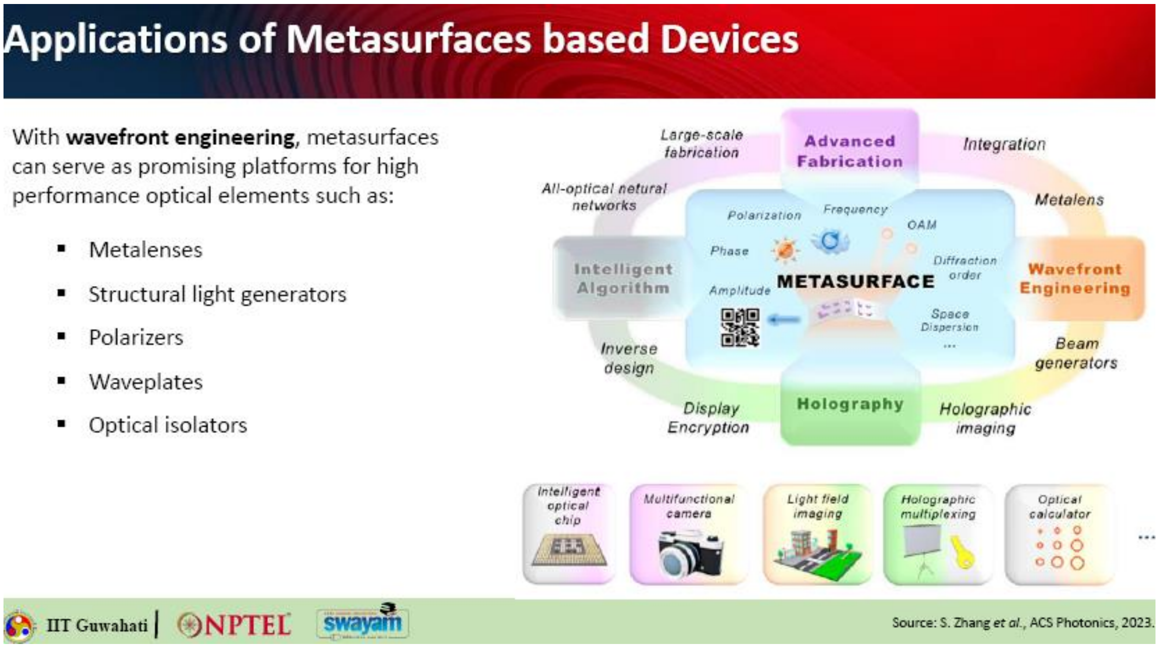
So the modification can be seen either on the reflected wave or the transmitted wave. You can also see some other applications something like OAM that is orbital angular momentum ok. We can have diffraction order, space dispersion and all these things. They also have application in wave front engineering these are called basically wave front engineering, holography, advanced manufacturing and intelligent surfaces.

So we will come into each of these briefly. So first let us focus on wave front engineering and we understood that metasurface can serve as promising platforms for high performance optical elements such as metal ends, then structural light generators, polarizers, wave plates, optical isolators and so on. So this is these are the applications of wave front engineering. You can also use metasurface for holography because they can act as excellent holographic recording medium and this is because of their large information capacity and sub wavelength units basically. You can take help of intelligent algorithm and in that case metasurface can significantly simplify the design, assembly and operation of optical systems.



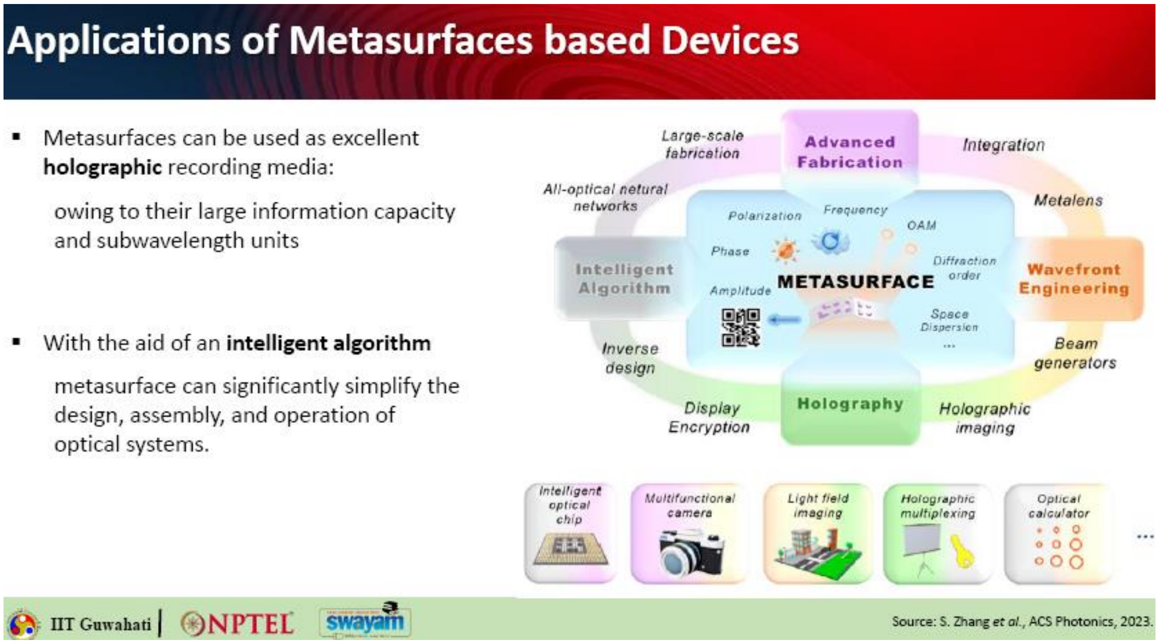
Applications of Metasurfaces based Devices

With **wavefront engineering**, metasurfaces can serve as promising platforms for high performance optical elements such as:

- Metalenses
- Structural light generators
- Polarizers
- Waveplates
- Optical isolators

Source: S. Zhang *et al.*, ACS Photonics, 2023.

You can also take help of inverse design algorithms to design metasurfaces for a particular application. So here in inverse design means you know the desired response and you ask the software to optimize the parameter space and give you the design which will be closely matching to the desired optical response. With the support of advanced fabrication metasurfaces with larger diameters and more complex structures can be made and that will allow you to achieve better performance and also complex functionalities. And the other important thing is that with the excellent properties of metasurfaces you can also develop advanced fabrication technologies such as nano imprint lithography and resonant laser printing. So metasurface can also help improving the standard fabrication technologies.



Applications of Metasurfaces based Devices

- Metasurfaces can be used as excellent **holographic** recording media:

  owing to their large information capacity and subwavelength units

- With the aid of an **intelligent algorithm**

  metasurface can significantly simplify the design, assembly, and operation of optical systems.

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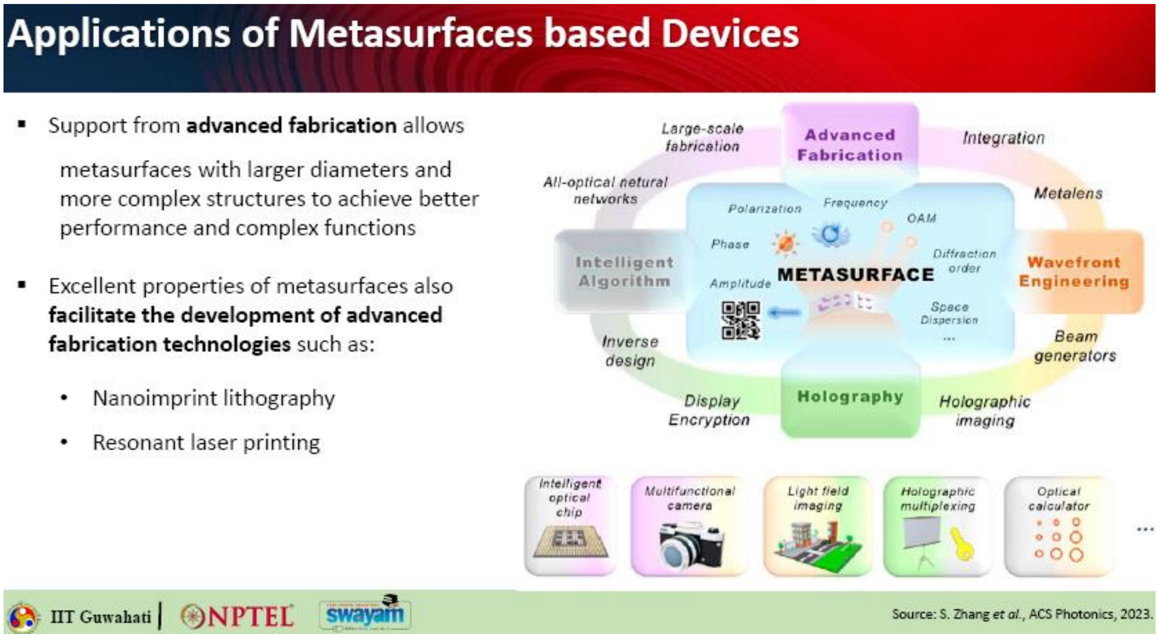
Source: S. Zhang *et al.*, ACS Photonics, 2023.

So, these are also some applications that we can foresee. You can think of intelligent optical chip based on metasurfaces, multifunctional camera, light field imaging, you have holographic multiplexing, optical calculators and so on. So all these things are possible using metasurfaces. Now let us take one particular example of a lens design based on metasurface. So here we are going to design a metasurface based broadband flat lens array.



Applications of Metasurfaces based Devices

- Support from **advanced fabrication** allows

  metasurfaces with larger diameters and more complex structures to achieve better performance and complex functions

- Excellent properties of metasurfaces also **facilitate the development of advanced fabrication technologies** such as:
  - Nanoimprint lithography
  - Resonant laser printing

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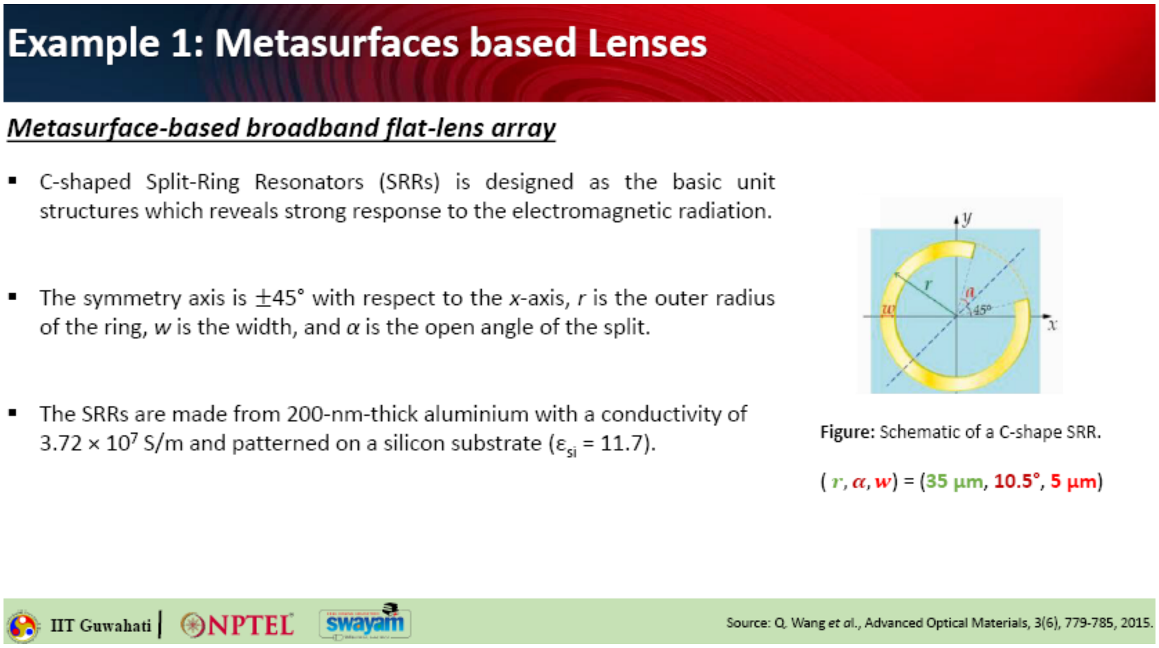
Source: S. Zhang *et al.*, ACS Photonics, 2023.

So this is the unit cell design to start with. So it is basically a C shaped split ring resonator as you can see and this dashed line is basically the symmetry axis. And you can see the symmetry axis makes 45 degree angle with x axis. And why do we choose this? I will come to that. The schematic of this C shaped SRR where the parameters are basically this.



Example 1: Metasurfaces based Lenses

Metasurface-based broadband flat-lens array

- C-shaped Split-Ring Resonators (SRRs) is designed as the basic unit structures which reveals strong response to the electromagnetic radiation.
- The symmetry axis is ±45° with respect to the *x*-axis, *r* is the outer radius of the ring, *w* is the width, and α is the open angle of the split.
- The SRRs are made from 200-nm-thick aluminium with a conductivity of 3.72×10^7 S/m and patterned on a silicon substrate ($\varepsilon_{si} = 11.7$).

Figure: Schematic of a C-shape SRR.

(r, α, w) = (**35 μm**, **10.5°**, **5 μm**)

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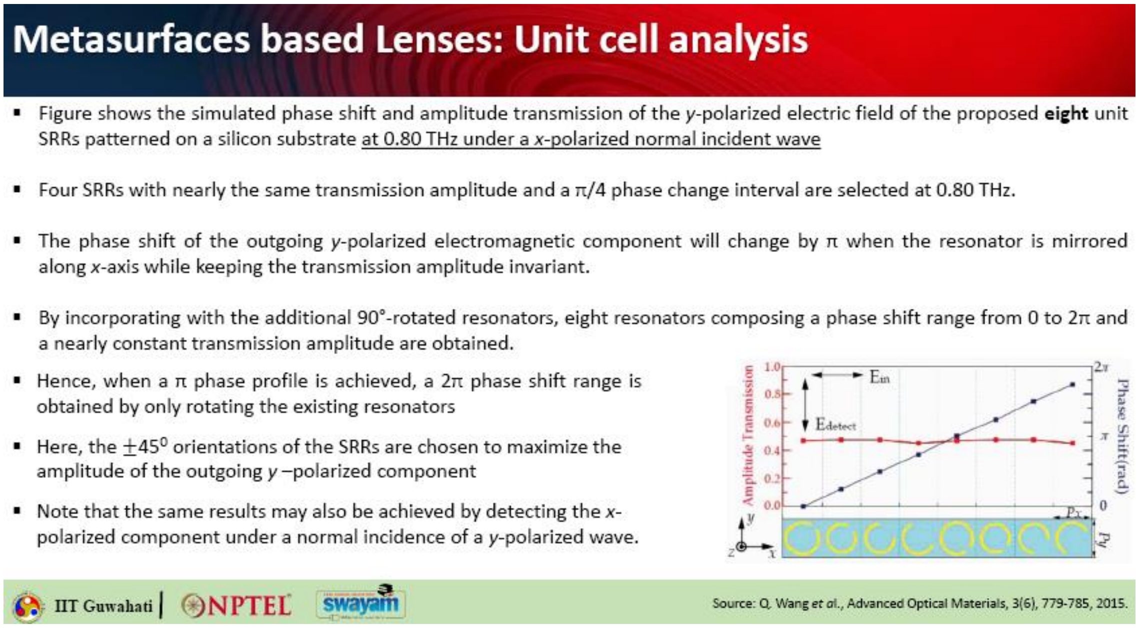
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

So the radius, the outer radius of the ring is 35 micron. Alpha is this opening. You can see there is a cut. So, this cut opening makes an angle of 10.5 degree. Here it is definitely more but this is just a schematic. And then you have w that is the width of this ring to be 5 micron. Now how it works? So, we position the symmetry axis as plus minus 45 degree with respect to X axis and then this SRR are basically made of 200 nanometer thick aluminum which has got a conductivity of $3.72 \times 10^7 \mathrm{S/m}$. And you can pattern this on top of a silicon substrate which has got a permittivity of 11.7. So, you can consider epsilon silicon as 11.7.



Metasurfaces based Lenses: Unit cell analysis

- Figure shows the simulated phase shift and amplitude transmission of the *y*-polarized electric field of the proposed **eight** unit SRRs patterned on a silicon substrate at 0.80 THz under a *x*-polarized normal incident wave
- Four SRRs with nearly the same transmission amplitude and a π/4 phase change interval are selected at 0.80 THz.
- The phase shift of the outgoing *y*-polarized electromagnetic component will change by π when the resonator is mirrored along *x*-axis while keeping the transmission amplitude invariant.
- By incorporating with the additional 90°-rotated resonators, eight resonators composing a phase shift range from 0 to 2π and a nearly constant transmission amplitude are obtained.
- Hence, when a π phase profile is achieved, a 2π phase shift range is obtained by only rotating the existing resonators
- Here, the $\pm 45^0$ orientations of the SRRs are chosen to maximize the amplitude of the outgoing y−polarized component
- Note that the same results may also be achieved by detecting the *x*-polarized component under a normal incidence of a *y*-polarized wave.

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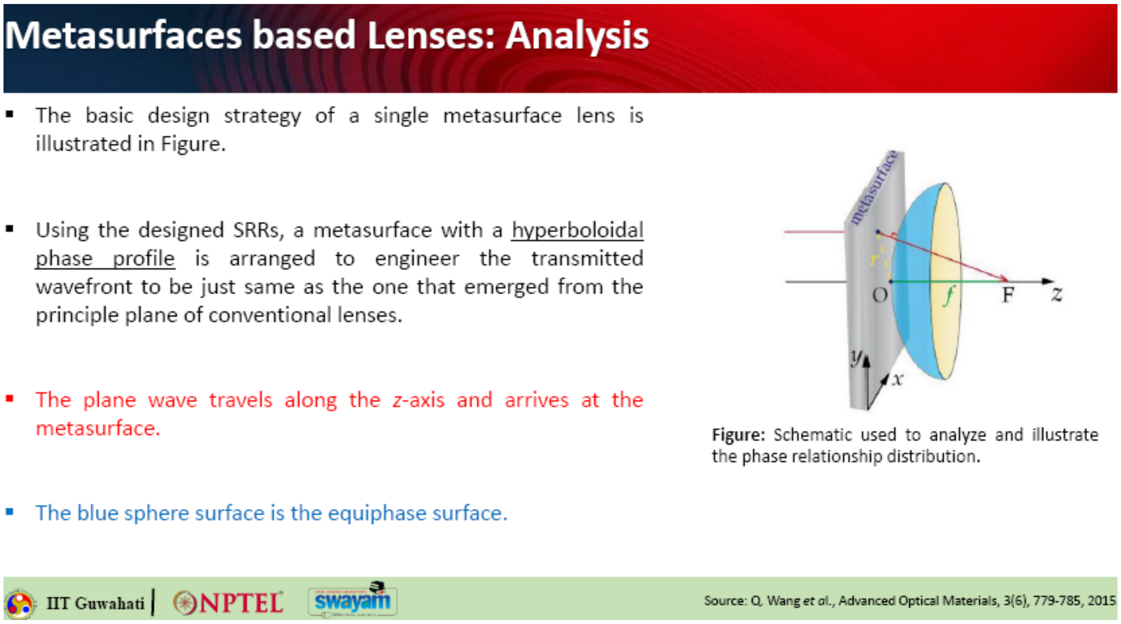
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

Now what is the good thing about this? You can see here, so you have taken different different shapes of this unit cell and you can actually calculate the amplitude transmittance through an infinite array of this kind of unit cells when repeated periodically in both dimensions.



Metasurfaces based Lenses: Analysis

- The basic design strategy of a single metasurface lens is illustrated in Figure.
- Using the designed SRRs, a metasurface with a hyperboloidal phase profile is arranged to engineer the transmitted wavefront to be just same as the one that emerged from the principle plane of conventional lenses.
- The plane wave travels along the *z*-axis and arrives at the metasurface.
- The blue sphere surface is the equiphase surface.

Figure: Schematic used to analyze and illustrate the phase relationship distribution.

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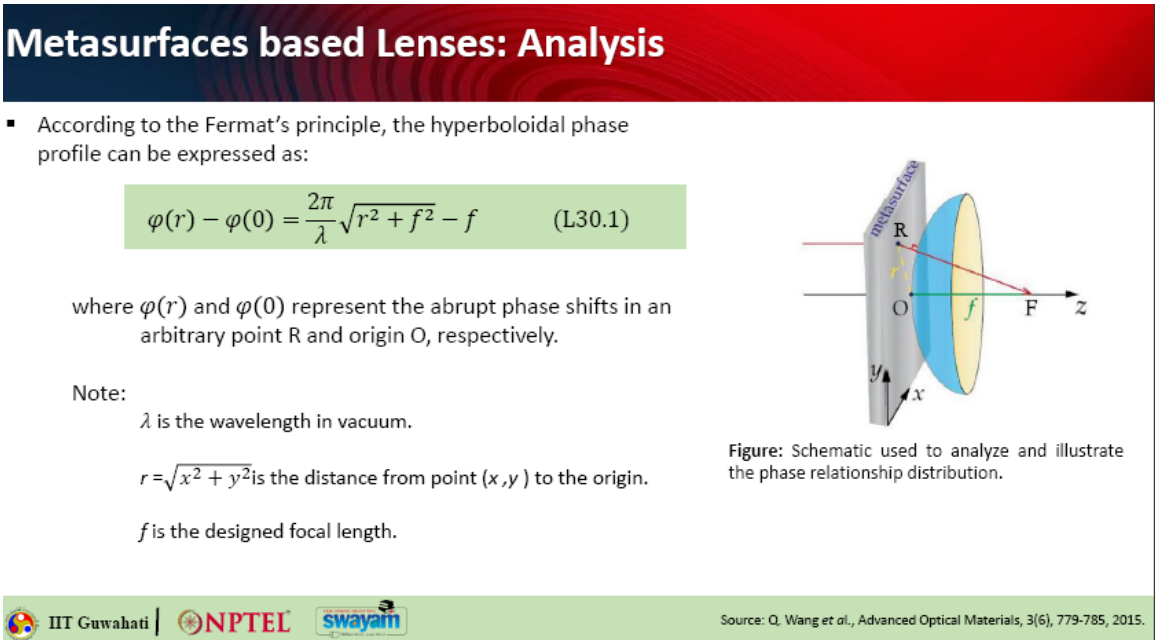
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015

And also, you calculate the phase shift that is plotted here in this particular right axis. And these are all done for the case where your incident light is X polarized and you have chosen the frequency to be 0.8 terahertz. And these are the 8 different unit cell designs of the split ring resonator as you can see. And the output light is basically Y polarized. So here you can see this is X and Y. So this is the detected light that is Y polarized and the incident light is X polarized. So here you see that the first four if you look into the first four designs they have slightly different split ring. So somewhere the opening angle is changing, somewhere the position is changing and so on. So here it is basically the opening is getting bigger and bigger. So that actually helped us to do one thing.



Metasurfaces based Lenses: Analysis

- According to the Fermat's principle, the hyperboloidal phase profile can be expressed as:

$$\varphi(r)-\varphi(0)=\frac{2\pi}{\lambda}\sqrt{r^2+f^2}-f \qquad \text{(L30.1)}$$

where $\varphi(r)$ and $\varphi(0)$ represent the abrupt phase shifts in an arbitrary point R and origin O, respectively.

Note:

λ is the wavelength in vacuum.

$r=\sqrt{x^2+y^2}$ is the distance from point (x ,y) to the origin.

f is the designed focal length.

Figure: Schematic used to analyze and illustrate the phase relationship distribution.

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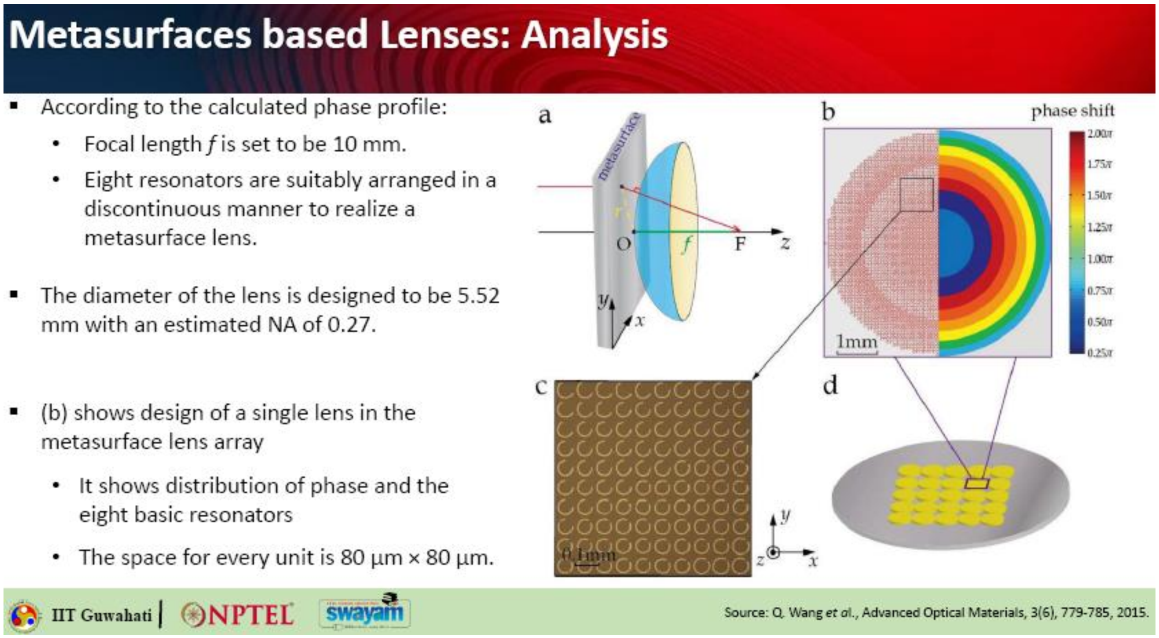
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

So, you have actually calculated the transmittance in these four cases and you can see they are more or less equal transmission amplitude. However, the phase if you see the first one has got a phase of 0 degree, then 45 degree, then 90 degree and another 48 degree, so it is 135 degree and so on. So, these are allowing you to add a phase difference of pi by 4, pi by 4, pi by 4 and so on. So, this opening is allow you to add the phase. Now the phase shift of the outgoing Y polarized light can also change by pi when the resonator is mirrored by 40 degree. The resonator is mirrored along the X axis while keeping the transmission amplitude in variant. So that is how you actually got this additional four resonators.

So if you take a mirror of this particular design with respect to x axis you come up with this one. So this will give a pi phase shift as corresponding to this one. So this was 0, so this guy has got a phase shift of pi, okay. Similarly the one this design had a phase shift of it is actually 135 degree, so you can actually see what it will be. So, 135 plus pi, so that will be the phase here.



Metasurfaces based Lenses: Analysis

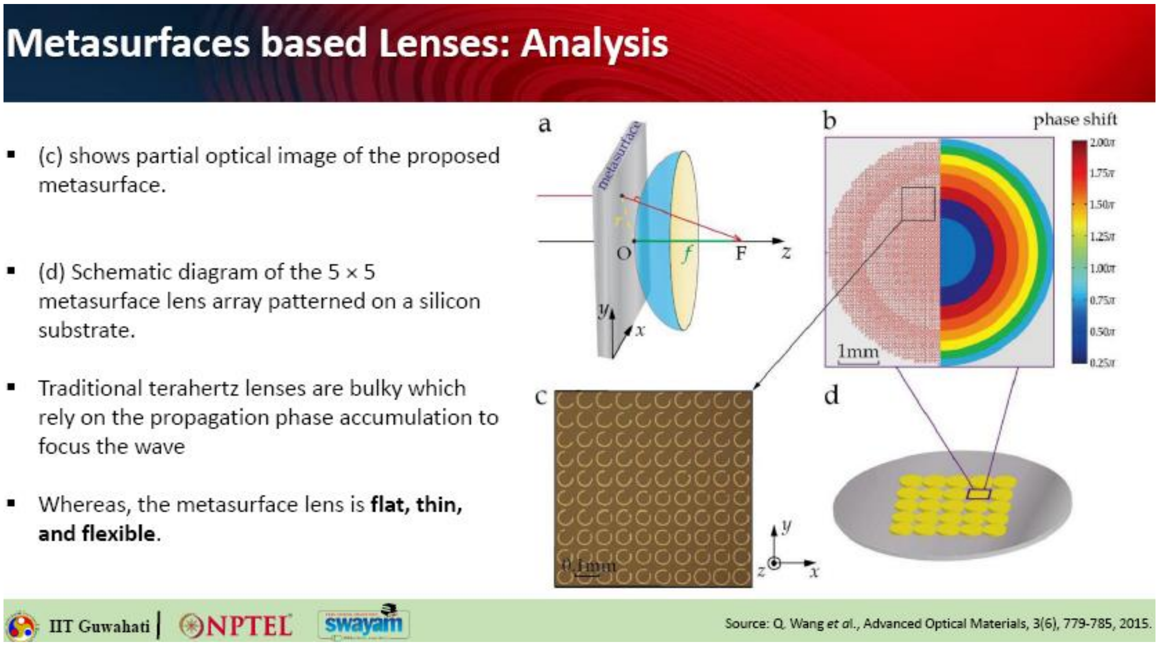
- According to the calculated phase profile:
  - Focal length *f* is set to be 10 mm.
  - Eight resonators are suitably arranged in a discontinuous manner to realize a metasurface lens.
- The diameter of the lens is designed to be 5.52 mm with an estimated NA of 0.27.
- (b) shows design of a single lens in the metasurface lens array
  - It shows distribution of phase and the eight basic resonators
  - The space for every unit is 80 µm × 80 µm.

Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

So, you can actually go up to, so you can start from here, so you can say this is pi and then you can go up to 2 pi, okay. So that way you are able to get you know starting from 0 to 2 pi all the phase you can actually see you are able to achieve for all these different designs of the unit cell and you are keeping the amplitude more or less constant, amplitude of the transmission. So this is the beauty of this particular design. Now if you do the similar kind of thing, okay for say the opposite orientation that means if you consider incident light to be y polarized you will get similar results for transmitted or detect, yeah transmitted wave which will be in that case x polarized, okay. So same thing can be repeated once you exchange the polarization.

So it will work perfectly fine in that case you have to also take the mirror across the Y axis, okay. So this is how the different designs of the unit cell allows you to achieve more or less similar transmission but you are able to manipulate phase at particular interval. Now with that you can design a flat metasurface lens. So this one the slab that you see here is basically that particular metasurface lens which is a flat lens and when you are designing this flat lens using those splitting resonators we have just seen you are able to achieve a hyperbolahedral phase profile like this, okay and this will allow you to engineer the transmitted wave front, okay in such a way as if it is coming from a conventional lens and that is how a flat metasurface will also be able to focus light from here to this particular point. So, this is what metasurface will be able to achieve.

Now here we have assumed that the plane wave travels along z direction and it will come to the metasurface like this from the metasurface it will get focused at a particular focal point f and small f is the distance focal length and here you see the blue sphere surface is basically the equiphase surface, okay. Now according to Fermat's principle the hyperboloidal phase profile can be expressed using this formula where phi r and phi 0 basically represent the abrupt phase shift at an arbitrary point r so this is any arbitrary point r and the origin is basically o. So this is at that r point and this is at o point. What is lambda? Lambda is basically the wavelength of the light in vacuum, okay small r is basically the distance from of this point which is having coordinate xy from the origin so it is square root of x square plus y square and f is the focal length that you are designing. So, based on that you can actually see you can alter the focal length you have to choose that particular element in that particular way, okay.



Metasurfaces based Lenses: Analysis

- (c) shows partial optical image of the proposed metasurface.
- (d) Schematic diagram of the 5 × 5 metasurface lens array patterned on a silicon substrate.
- Traditional terahertz lenses are bulky which rely on the propagation phase accumulation to focus the wave
- Whereas, the metasurface lens is **flat, thin, and flexible.**

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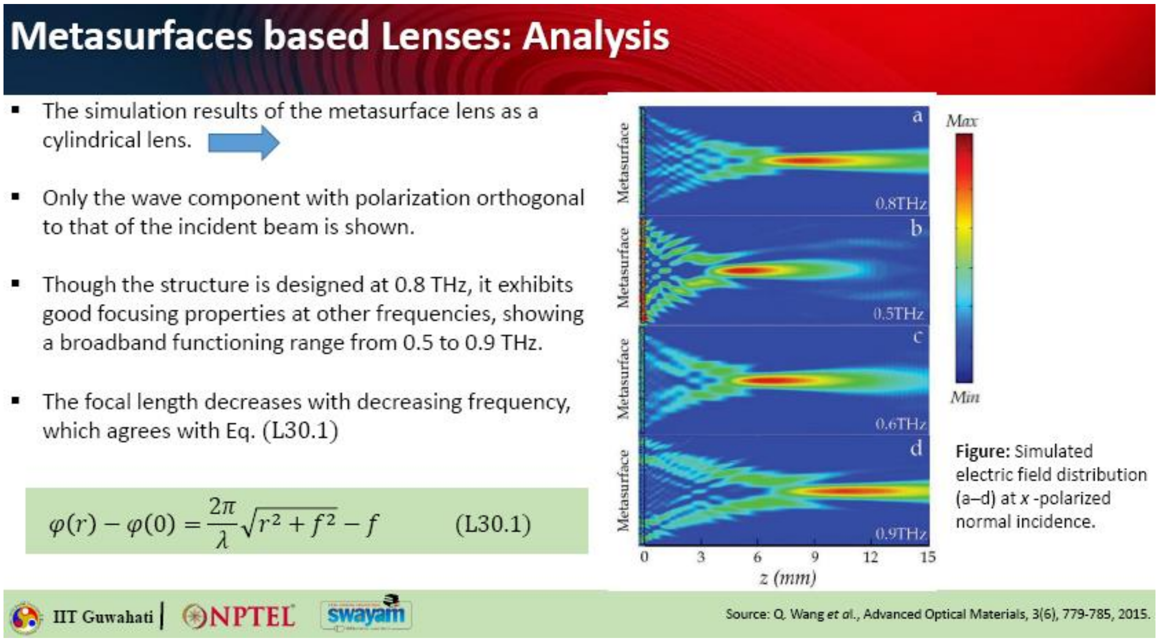
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

So here the placement of those different elements that you have seen that will decide what should be the focal length. So you have got complete control on designing this kind of metal lens. Now let us take one example so let us fix that the focal length should be around 10 mm, okay and you are using those 8 resonators in a particular pattern to see realize this particular metal lens, okay. So here you see the diameter of the lens is designed to be of so here each of them are basically one of those metal lens. So, the diameter is taken to be 5.52 mm and it has got a numerical aperture of 0.27, okay. And here you see this is basically the profile, okay. So this one is how the phase shift changes, okay. So, as you go inside the darker colors they look like having higher so this sorry this red colors they look like higher phase 2 pi, okay and here the blue one is having very low phase shift.

So that way it is not pattern in a particular fashion it is it should have a hyperbola kind of pattern so that you are able to focus that light beam. So you have to introduce the phase to the incoming light so that it can bend and focus at a particular point. So this shows design of a single lens in the metal surface, okay. It also shows the distribution of the phase and the 8 different resonators and the space for every unit is basically 80 micron by 80 micron. And here the length scale is 1 mm.

Now this if you take this particular portion and zoom it you will see this is how you have placed your metal lens. So you see metal lens design will require help of numerical simulation tools to optimize the design so that you can have a focal length of 10 mm or whatever is your requirement. So it depends on the placement of this elements you will be able to change the phase shift profile. And this is the schematic of 5 by 5 metasurface lens array which is patterned on a silicon substrate. So, what is the advantage as you can see here if you think of traditional terahertz lenses they are basically bulky and they are bulky because they rely on the phase accumulation of the wave while it is traveling through the lens and that gives you the focusing of the wave.



Metasurfaces based Lenses: Analysis

- The simulation results of the metasurface lens as a cylindrical lens. →
- Only the wave component with polarization orthogonal to that of the incident beam is shown.
- Though the structure is designed at 0.8 THz, it exhibits good focusing properties at other frequencies, showing a broadband functioning range from 0.5 to 0.9 THz.
- The focal length decreases with decreasing frequency, which agrees with Eq. (L30.1)

$$\varphi(r) - \varphi(0) = \frac{2\pi}{\lambda}\sqrt{r^2 + f^2} - f \qquad \text{(L30.1)}$$

Figure: Simulated electric field distribution (a–d) at *x* -polarized normal incidence.

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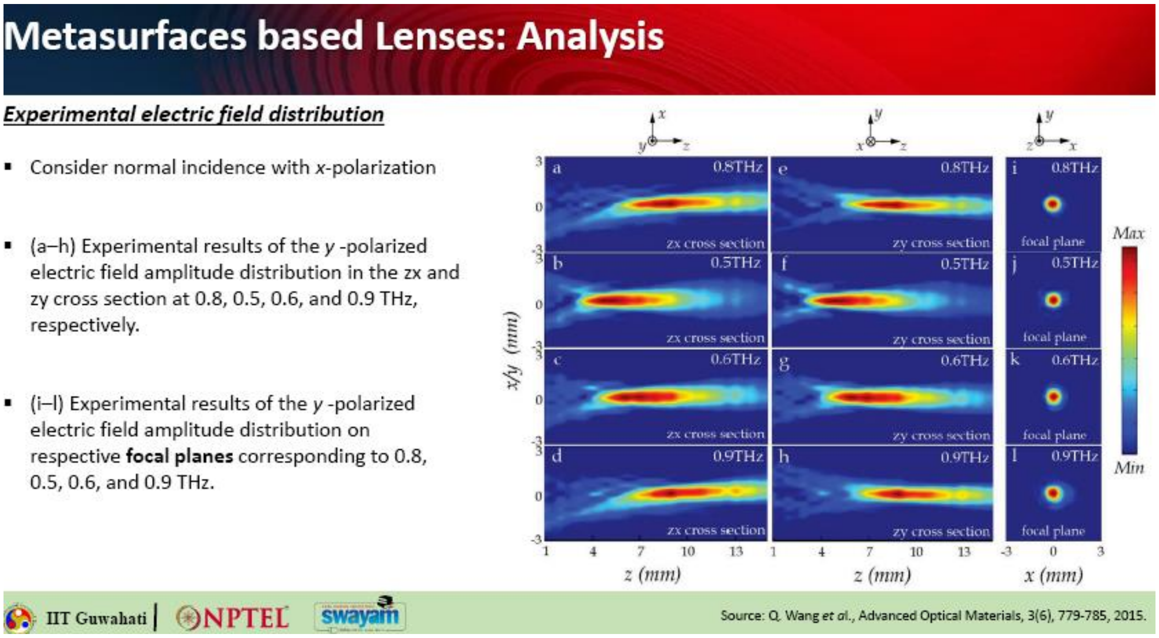
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

Here it is different. Here in metal surface your surface is introducing the phase. So metal surface lens can be flat, thin and flexible. So this is why metal surface lens has got a lot of application in the future. When you do the analysis of this let us first look into the simulation results. So, these are the simulation results of the metal surface lens.

It is used as a cylindrical lens. So these are the simulated electric field distribution for x polarized light incident normally. So here only the normal components are basically only the wave components with polarization orthogonal to the incident beam is shown and the structure was designed for 0.8 terahertz. So, you can see the focusing over here but you can also see that it is working fine for other wavelengths something like 0.5 terahertz as well as 0.6 terahertz or 0.9 terahertz. So it actually has a very broad band functioning range. And the focal length you can actually see that the focal length is decreasing with frequency.



Metasurfaces based Lenses: Analysis

Experimental electric field distribution

- Consider normal incidence with *x*-polarization
- (a–h) Experimental results of the *y* -polarized electric field amplitude distribution in the zx and zy cross section at 0.8, 0.5, 0.6, and 0.9 THz, respectively.
- (i–l) Experimental results of the *y* -polarized electric field amplitude distribution on respective **focal planes** corresponding to 0.8, 0.5, 0.6, and 0.9 THz.

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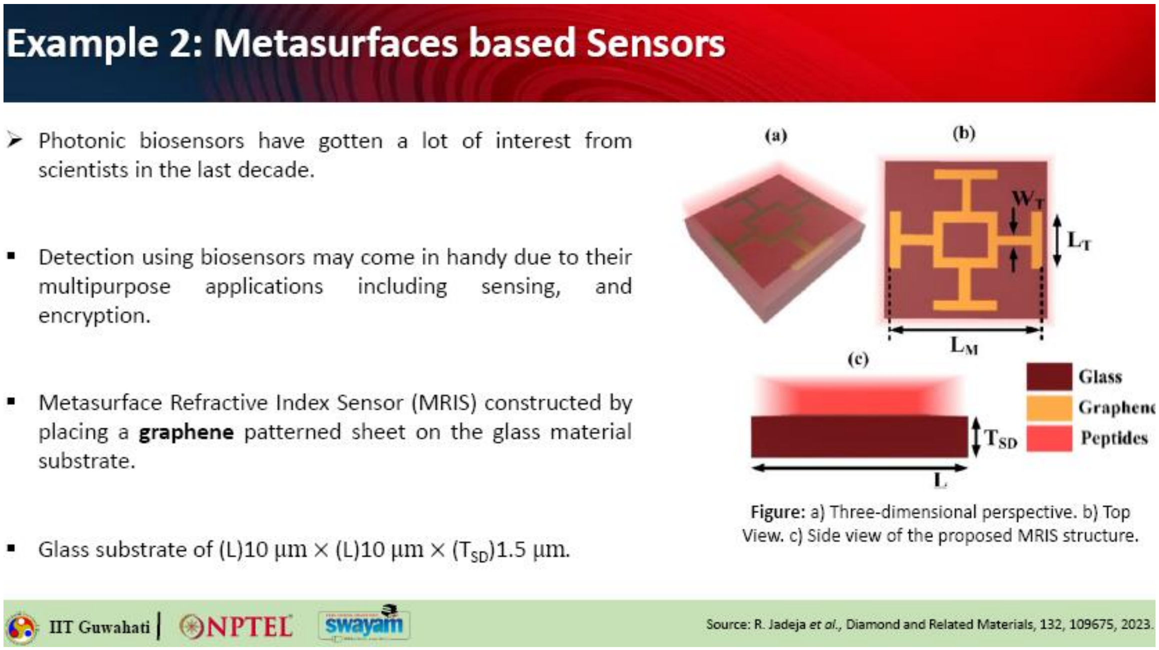
Source: Q. Wang *et al.*, Advanced Optical Materials, 3(6), 779-785, 2015.

So 0.5 has got a smaller or shorter focal length as compared to 0.8 whereas 0.9 will have a larger one as compared to 0.8 terahertz and this is pretty good because here also from the equation you can see that is happening. If you look at the experimental results so these are the experimental results.

So here also you can see the focusing is taking place. So here again we are considering x polarization normal incident light. So a to h are basically showing you the y polarized electric field amplitude distribution in zx so this entire column is for zx cross section and this entire column is for zy cross section and this particular column shows you from the focal plane. So whatever is so if you actually take a cut at this particular point you see this nice focusing happening. So, this actually gives you evidence that the lens you have designed using this kind of c-shaped splitting resonators work pretty well and that not only works for the frequency that you have centered it around you can also work on the frequencies adjacent to that.



Example 2: Metasurfaces based Sensors

- Photonic biosensors have gotten a lot of interest from scientists in the last decade.
- Detection using biosensors may come in handy due to their multipurpose applications including sensing, and encryption.
- Metasurface Refractive Index Sensor (MRIS) constructed by placing a **graphene** patterned sheet on the glass material substrate.
- Glass substrate of (L)10 μm × (L)10 μm × (T_{SD})1.5 μm.

Figure: a) Three-dimensional perspective. b) Top View. c) Side view of the proposed MRIS structure.

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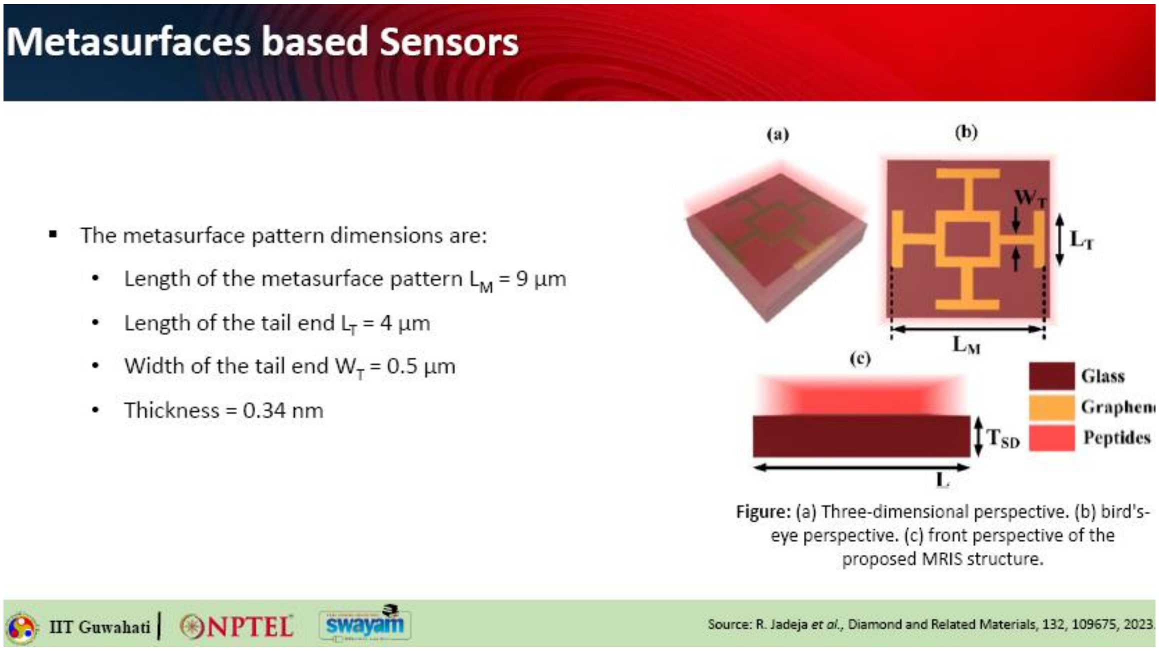
Source: R. Jadeja *et al.*, Diamond and Related Materials, 132, 109675, 2023.

So let us now look into another design based on metasurfaces. Let us look into a sensor made of metasurface. Now photonic biosensors have recently got a lot of interest and detection using biosensors may come handy due to the multipurpose application something like sensing encryption etc. So you can think of making a sensor or you can say metasurface refractive index sensor that is constructed by placing a graphene pattern sheet. So here you can see this particular structure is made up of graphene so you have taken a graphene sheet and patterned it and then you put it on top of a glass material.

So this is how the structure is so if you see this is the three-dimensional perspective this is the top view and this is the side view so here you can clearly see you have graphene you have peptides okay which is the surrounding medium okay and then you for detection that is how it will work as a biosensor whenever there is a change in the refractive index the resonant wavelength will show some shift and you can detect that shift and that will be the detection. So this is the structure so here the glass substrate is 10 micron by 10 micron by 1.5 micron this is the thickness and when you pattern the graphene in the metasurface so this is the metasurface which has got a unit cell dimensions of this one so this length is 9 micron, LT which is the end of the length of the tail end that is 4 micron and you have width of the tail end that is 0.5 micron and the overall thing has got a single layer graphene thickness that is 0.34 nanometer. Now first thing is to know what will be the permittivity of this graphene material so you can actually estimate that from the graphene's conductivity. So for a single layer graphene sheet the graphene conductivity can be split into two parts that also you can do for multi layers not only necessary for single layer so it has got two parts like intra band and inter band conductivity so you can write the permittivity in terms of conductivity then so epsilon omega can be written as 1 plus sigma over epsilon naught omega delta ok from that you this sigma s has got two parts this is intra band contribution and this is inter band contribution as you can see here and you can observe that omega is the angular frequency that you are using and this delta as I told you this is the thickness of the sheet ok graphene sheet.



Metasurfaces based Sensors

- The metasurface pattern dimensions are:
  - Length of the metasurface pattern L_M = 9 µm
  - Length of the tail end L_T = 4 µm
  - Width of the tail end W_T = 0.5 µm
  - Thickness = 0.34 nm

Figure: (a) Three-dimensional perspective. (b) bird's-eye perspective. (c) front perspective of the proposed MRIS structure.

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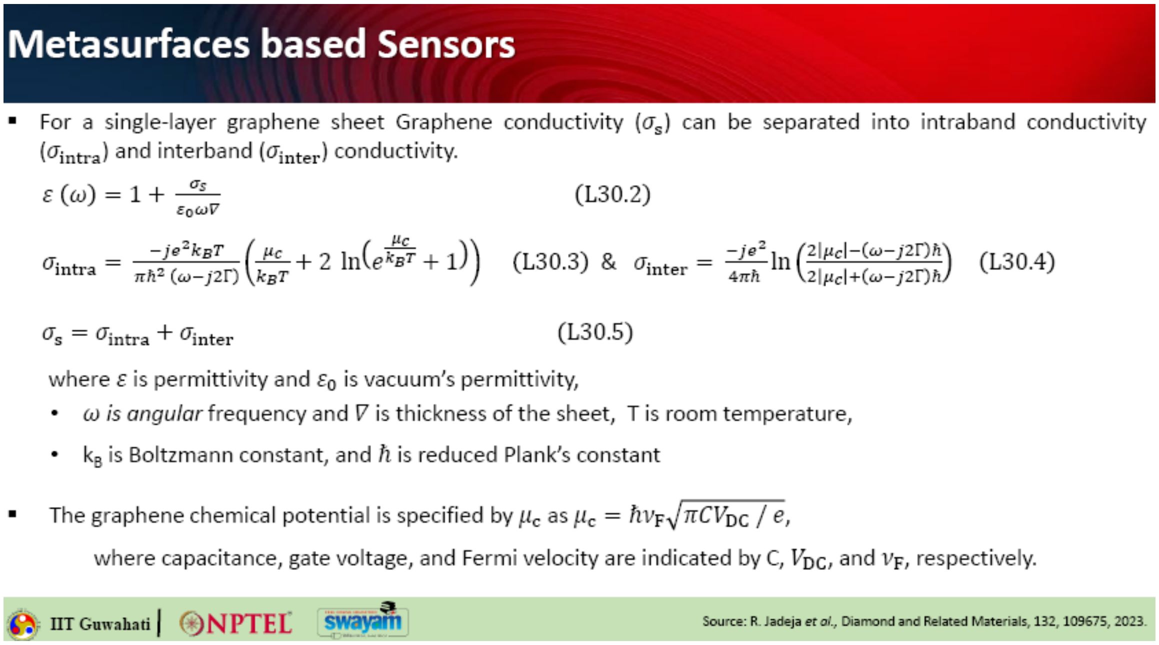
Source: R. Jadeja *et al.*, Diamond and Related Materials, 132, 109675, 2023.

So, for single layer it is 0.34 nanometer you can also put multi layer over here whatever is thickness that will change the permittivity ok and in this formula T is the room temperature k_B is the Boltzmann's constant and h cross is basically the reduced Planck's constant. So, using this formula you can find out what is your σ_s from σ_s you can estimate what is the permittivity ok.



Metasurfaces based Sensors

- For a single-layer graphene sheet Graphene conductivity (σ_s) can be separated into intraband conductivity (σ_{intra}) and interband (σ_{inter}) conductivity.

$$\varepsilon(\omega) = 1 + \frac{\sigma_S}{\varepsilon_0 \omega \nabla} \qquad \text{(L30.2)}$$

$$\sigma_{\text{intra}} = \frac{-je^2 k_B T}{\pi \hbar^2 (\omega - j2\Gamma)} \left(\frac{\mu_c}{k_B T} + 2\ \ln\left(e^{\frac{\mu_c}{k_B T}} + 1 \right) \right) \quad \text{(L30.3)} \ \ \& \ \ \sigma_{\text{inter}} = \frac{-je^2}{4\pi\hbar} \ln\left(\frac{2|\mu_c| - (\omega - j2\Gamma)\hbar}{2|\mu_c| + (\omega - j2\Gamma)\hbar} \right) \quad \text{(L30.4)}$$

$$\sigma_s = \sigma_{\text{intra}} + \sigma_{\text{inter}} \qquad \text{(L30.5)}$$

where ε is permittivity and ε_0 is vacuum's permittivity,

- ω *is angular* frequency and ∇ is thickness of the sheet, T is room temperature,
- k_B is Boltzmann constant, and $\hbar$ is reduced Plank's constant

- The graphene chemical potential is specified by μ_c as $\mu_c = \hbar v_F \sqrt{\pi C V_{DC} / e}$,

  where capacitance, gate voltage, and Fermi velocity are indicated by C, V_{DC}, and v_F, respectively.

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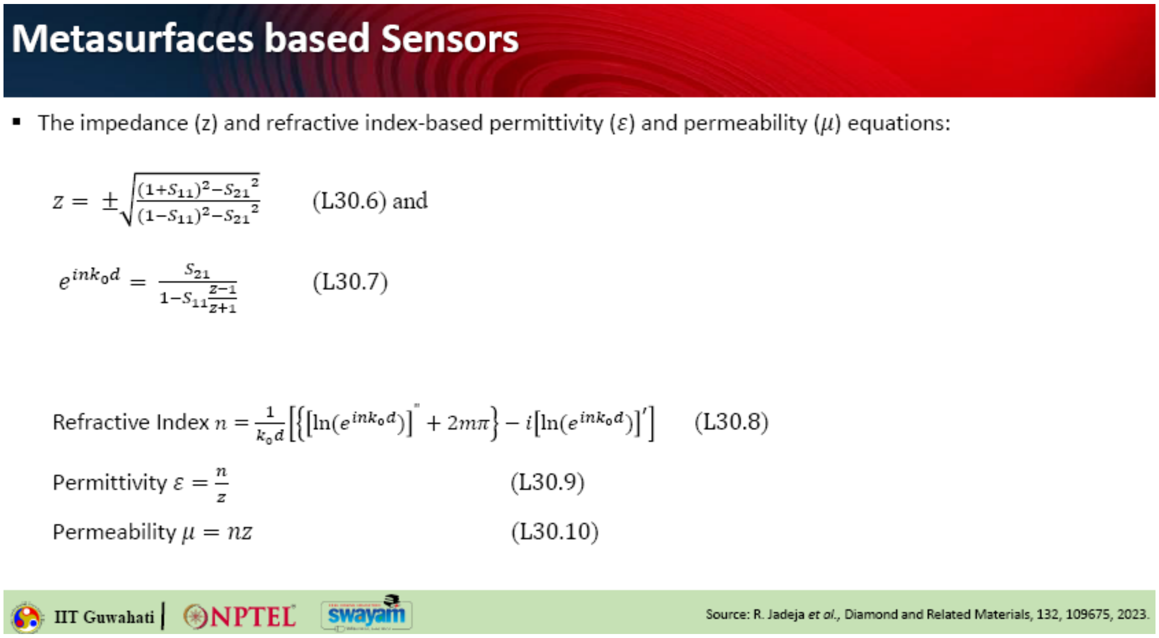
Source: R. Jadeja *et al.*, Diamond and Related Materials, 132, 109675, 2023.

You can also correlate the chemical potential of the graphene ok with the capacitance voltage and the Fermi velocity ok. So this is also a way to find out what is mu c that is the graphene's chemical potential and they will be applied into this particular equation of conductivity. So with all the parameters known here ok you can obtain the permittivity and that allows you to also find out what is the impedance and refractive index based permittivity and permeability. So, you can see that the impedance can be calculated from the s_{11}. So, you if you take this kind of unit cell and then periodically extend it in both x and y direction and you put two ports one along z plus and one along z minus ok.



Metasurfaces based Sensors

- The impedance (z) and refractive index-based permittivity (ε) and permeability (μ) equations:

$$z = \pm\sqrt{\frac{(1+S_{11})^2-S_{21}{}^2}{(1-S_{11})^2-S_{21}{}^2}} \qquad \text{(L30.6) and}$$

$$e^{ink_0d} = \frac{S_{21}}{1-S_{11}\frac{z-1}{z+1}} \qquad \text{(L30.7)}$$

$$\text{Refractive Index } n = \frac{1}{k_0d}\left[\left\{\left[\ln\left(e^{ink_0d}\right)\right]'' + 2m\pi\right\} - i\left[\ln\left(e^{ink_0d}\right)\right]'\right] \qquad \text{(L30.8)}$$

$$\text{Permittivity } \varepsilon = \frac{n}{z} \qquad \text{(L30.9)}$$

$$\text{Permeability } \mu = nz \qquad \text{(L30.10)}$$

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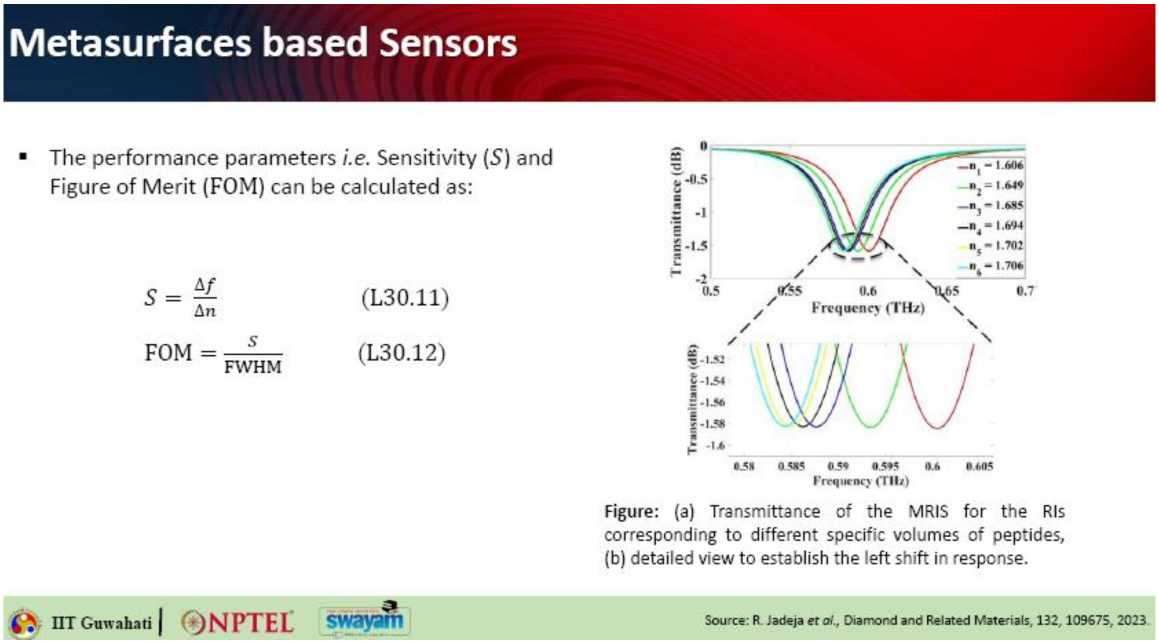
Source: R. Jadeja *et al.*, Diamond and Related Materials, 132, 109675, 2023.

So you can obtain what is the s_{11} parameter there is the reflection parameter. So reflection so port 1 say light is coming from port 1 which is on the top of your sheet ok. So s_{11} will be the reflectance or reflection coefficient and s_{21} will be the transmission coefficient ok. So, from that you can also obtain this parameter e^{ink_0d} can be related to this s parameter and the impedance that allows you to calculate what is the refractive index ok. So refractive index can be correlated with the permittivity using this formula which is epsilon equals n over z and permeability can be written as $\mu = nz$ ok.

So this is how you can obtain the parameters for this particular graphene based structure. And then you look into the performance parameter for this particular sensor. So you can understand that there will be a transmission dip for this particular sensor and when you change the surrounding media refractive index the transmission dip will change ok. So in that case you can find out what is the sensitivity of your device when it is working as a sensor. How it is sensing? You are changing the surrounding medium permittivity accordingly the transmission dip wavelength is basically changing right.



Metasurfaces based Sensors

- The performance parameters *i.e.* Sensitivity (S) and Figure of Merit (FOM) can be calculated as:

$$S = \frac{\Delta f}{\Delta n} \qquad (L30.11)$$

$$\text{FOM} = \frac{S}{\text{FWHM}} \qquad (L30.12)$$

Figure: (a) Transmittance of the MRIS for the RIs corresponding to different specific volumes of peptides, (b) detailed view to establish the left shift in response.

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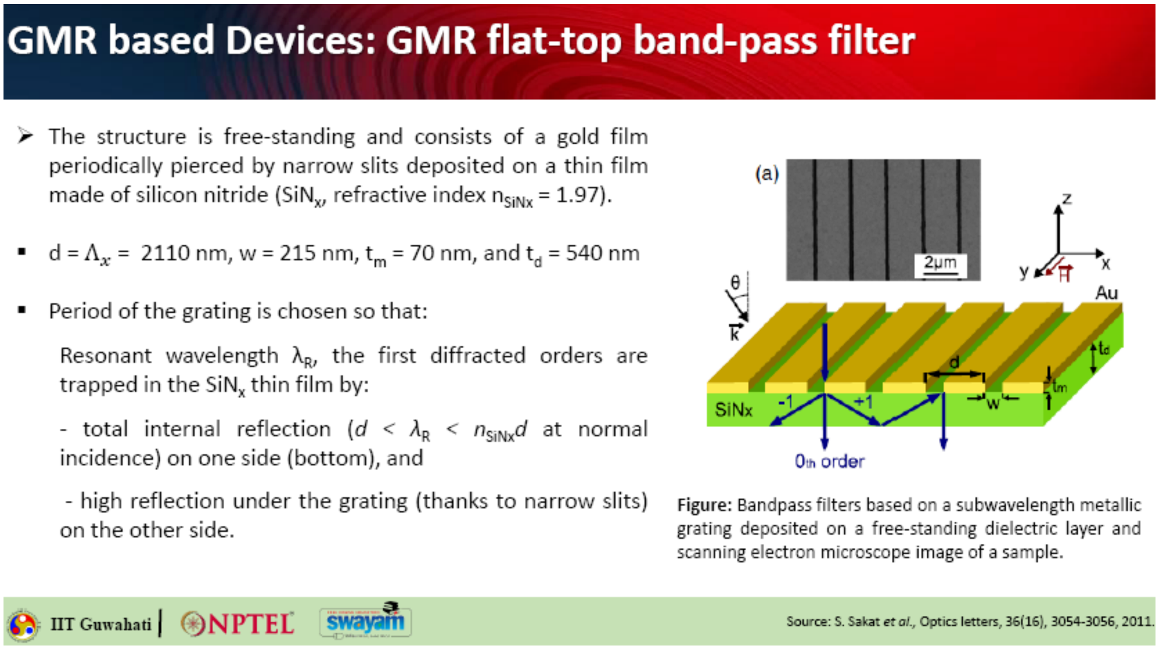
Source: R. Jadeja *et al.*, Diamond and Related Materials, 132, 109675, 2023.

So you can find out sensitivity equals delta f that is the change in the frequency over the change in the refractive index of the surrounding medium. You can also find out the figure of merit of this sensor as S divided by the FWHM of this that is full width half maxima of one of this dip okay. So this is how you can use metasurface based devices for lens for sensing and all this thing and they are they are pretty sensitive devices that you can make using metasurfaces. So now let us look into some examples for GMR based devices. So GMR if you remember these are basically guided mode resonance based devices and as I mentioned in the previous lectures that GMR gives you excellent filters ok very high quality filters.



GMR based Devices: GMR flat-top band-pass filter

- The structure is free-standing and consists of a gold film periodically pierced by narrow slits deposited on a thin film made of silicon nitride (SiN_x, refractive index $n_{SiNx} = 1.97$).
- $d = \Lambda_x = 2110$ nm, $w = 215$ nm, $t_m = 70$ nm, and $t_d = 540$ nm
- Period of the grating is chosen so that:

  Resonant wavelength λ_R, the first diffracted orders are trapped in the SiN_x thin film by:

  - total internal reflection ($d < \lambda_R < n_{SiNx} d$ at normal incidence) on one side (bottom), and
  - high reflection under the grating (thanks to narrow slits) on the other side.

Figure: Bandpass filters based on a subwavelength metallic grating deposited on a free-standing dielectric layer and scanning electron microscope image of a sample.

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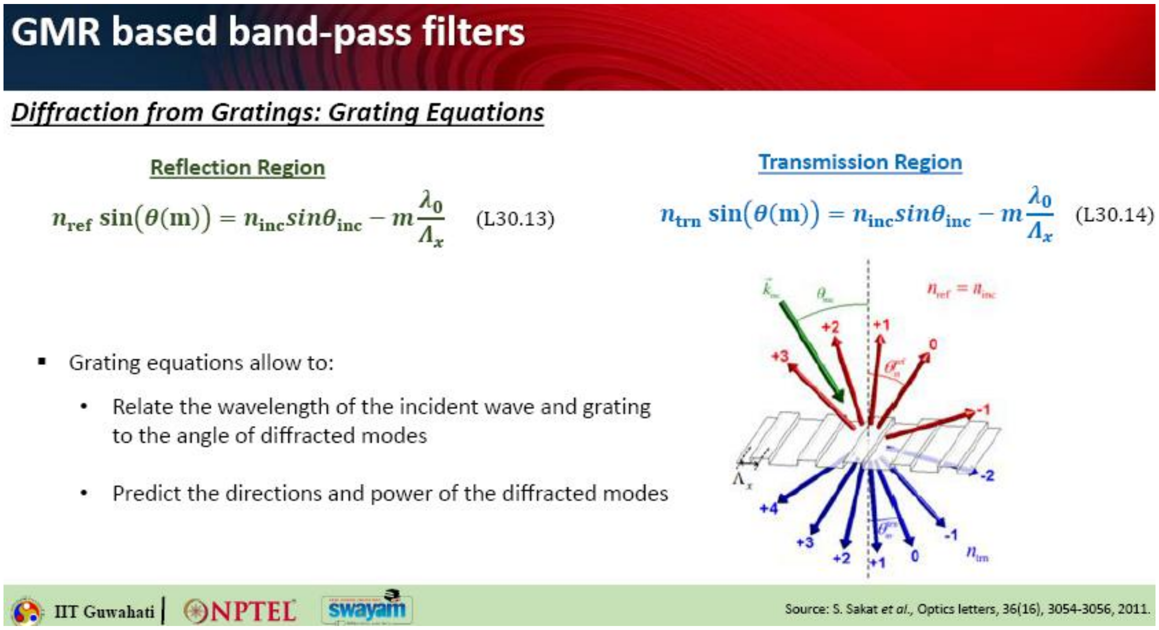
Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

So let us look into a design of flat top band pass filter for terahertz telecommunication application. So here is a structure that is freestanding and it consists of gold film which is periodically pierced as you can see these are cuts in the gold film as you can see this actual image and this is the schematic and it is basically sitting on a thin film of silicon nitride ok. So, we are calling it as SiN_x and the permittivity is 1.97 ok and this is a gold film and here are the parameters for this one. So you have the periodicity of this grating that you have created this is basically a 1D grating ok it is this grating period ok w is basically the width of the slit 215 nanometer, t_m and t_d are basically the height or the thickness of the metal layer thickness of the dielectric layer that those are 70 nanometer and 540 nanometer respectively.



GMR based band-pass filters

Diffraction from Gratings: Grating Equations

Reflection Region

$$n_{\mathrm{ref}}\sin(\theta(\mathrm{m})) = n_{\mathrm{inc}}\sin\theta_{\mathrm{inc}} - m\frac{\lambda_0}{\Lambda_x} \quad \text{(L30.13)}$$

Transmission Region

$$n_{\mathrm{trn}}\sin(\theta(\mathrm{m})) = n_{\mathrm{inc}}\sin\theta_{\mathrm{inc}} - m\frac{\lambda_0}{\Lambda_x} \quad \text{(L30.14)}$$

- Grating equations allow to:
  - Relate the wavelength of the incident wave and grating to the angle of diffracted modes
  - Predict the directions and power of the diffracted modes

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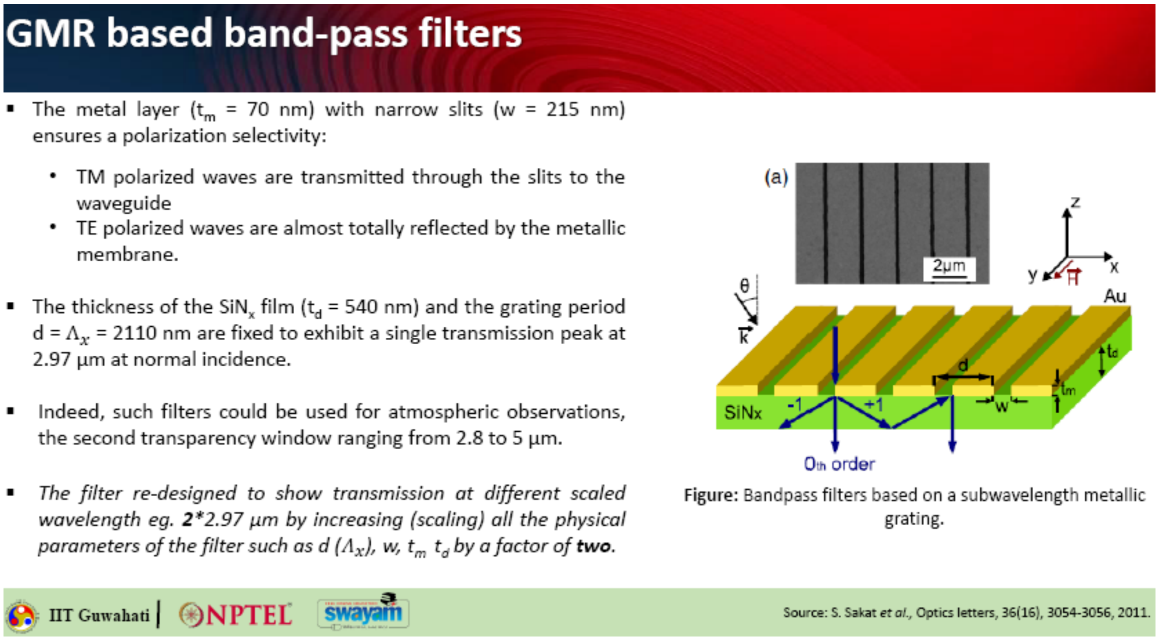
Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

Now there is a way to choose this period of the grating it should be chosen such that the resonant wavelength lambda R ok the first diffracted orders are trapped in the in this silicon nitride film. So when you first diffracted order means you are talking about plus 1 and minus 1 so you are letting the 0th order to escape ok. So you have to make sure that there is total internal reflection ok and total internal reflection you can think of the condition here this is the range of lambda R ok when it is normally incident. So you will get total internal reflection from this bottom interface and you should get high reflectance from the top interface which is basically the narrow slits in the gold film ok. So, this is what should happen so you are having incident light falling from this side on the grating ok which has got sub wavelength metallic grating deposited on freestanding dielectric layer in this case it is silicon nitride ok and you want the first order diffracted waves to be trapped inside and the 0th order to come out ok.

So this is how you will be having band pass filter. So they too will block ok and this will only pass. So let us see how we do this so to do this we need to understand the diffractions from the grating so we should understand the grating equations. So here you see a schematic shows only a grating and then this is the incident wave vector and there is this is the transmitted wave vector that is the 0th the fundamental one and the 0th one shows the reflected fundamental wave vector ok. And there are $\pm 1, \pm 2$ ok plus 3 minus these are not shown ok. So ± 1 is shown for the reflectance and for transmittance ± 1 is shown ± 2 is also shown and beyond that it is not shown ok.



GMR based band-pass filters

- The metal layer (t_m = 70 nm) with narrow slits (w = 215 nm) ensures a polarization selectivity:
    - TM polarized waves are transmitted through the slits to the waveguide
    - TE polarized waves are almost totally reflected by the metallic membrane.
- The thickness of the SiN_x film (t_d = 540 nm) and the grating period d = Λ_x = 2110 nm are fixed to exhibit a single transmission peak at 2.97 µm at normal incidence.
- Indeed, such filters could be used for atmospheric observations, the second transparency window ranging from 2.8 to 5 µm.
- *The filter re-designed to show transmission at different scaled wavelength eg.* ***2*****2.97 µm by increasing (scaling) all the physical parameters of the filter such as d (Λ_x), w, t_m t_d by a factor of* ***two****.*

Figure: Bandpass filters based on a subwavelength metallic grating.

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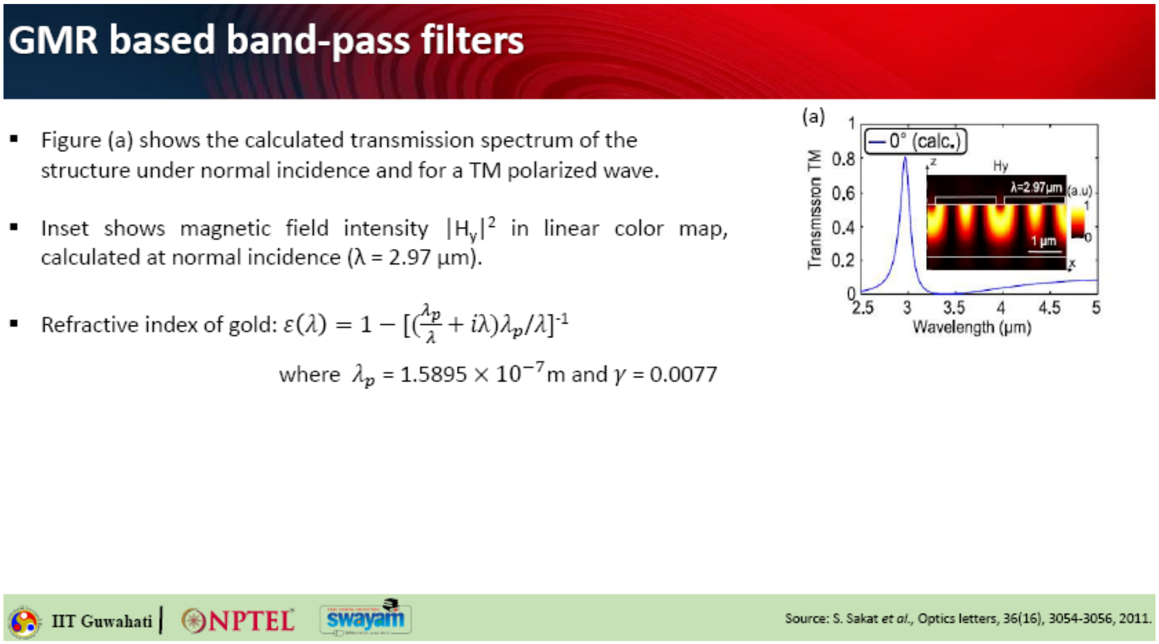
Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

So what are the equations for the reflection region this is the equation. So n ref is the refractive index of the this particular medium sin theta m ok will be equal to n incident sin theta incident minus m lambda naught over capital lambda x. So capital lambda x is basically your grating period m is the order ok. So here you can find out all those different angles in which now the reflection will take place. Similarly, you can also find out for different m equals 0, ± 1 what will be the condition that is satisfied and you can obtain what are those angles through which the transmission of 0th order or $\pm 1, \pm 2$ order will take place.

Now this grating equations they allow us to relate the wavelength of the incident wave and grating to the angle of the diffracted modes. So that is that is very good. So this is how the angles are related to this wavelengths and you can also predict the directions and the power of the diffracted modes right. The directions means the angles will only tell you the direction.

So this information you can obtain from this grating equations. Now coming back to this structure we considered the metal layer to be 70 nanometer and the slits are very narrow which are 215 nanometer and this ensures a polarization sensitivity. What does it mean? It is designed in a way that the TM polarized wave are basically transmitted through the slits ok in which case electric fields are like this but the T that is proportional or parallel where the electric field is parallel to x will be basically reflected from this metallic membrane. They will not be able to pass through ok. And this particular thickness of the dielectric layer and the grating period that is d ok is selected such that you only get single transmission peak ok and here the design is meant for 2.97 micrometer and you have chosen normal incidence. So this basically the incident angle. So here you have considered theta to be 0 ok. So such filters this kind of filters can be very useful for atmospheric observation which has got a transparency range from 2.8 to 5 micron. So here you can only allow single transmission peak so you can use it for this kind of atmospheric observations.



GMR based band-pass filters

- Figure (a) shows the calculated transmission spectrum of the structure under normal incidence and for a TM polarized wave.
- Inset shows magnetic field intensity $|H_y|^2$ in linear color map, calculated at normal incidence ($\lambda = 2.97$ μm).
- Refractive index of gold: $\varepsilon(\lambda) = 1 - [(\frac{\lambda_p}{\lambda} + i\lambda)\lambda_p/\lambda]^{-1}$

  where $\lambda_p = 1.5895 \times 10^{-7}$ m and $\gamma = 0.0077$

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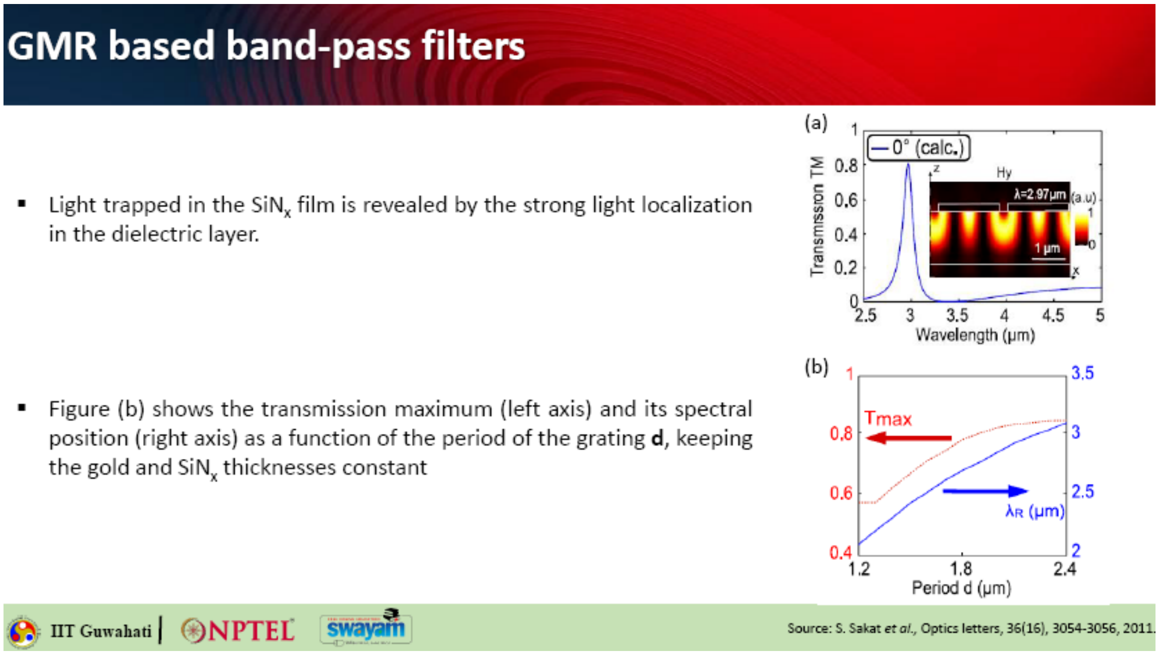
Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

And how the design will change this this particular method of designing redesigning a filter easily is called also called frequency scaling. So how it works? So say instead of 2.97 center wavelength if you want to double the wavelength ok that means you want to scale your design ok. In that case all the physical parameters of the design will also get doubled ok something like the periodicity the width and the thickness and all those things will also get doubled ok.

So you can scale it up so that will allow you to get that ok. Now let us first focus on this particular design which was meant for 2.97 so it is you can see here it is speaking at 2.97. So this is the transmission spectrum of the TM polarized wave for this particular design ok. And the inset shows ah the magnetic field intensity which is modulus of Hy square ok for this particular incident on.

And gold has been assumed to have a refractive index which is given by this particular equation ok. So you can actually obtain the gold permittivity using these equations. Here two important things are seen so this is basically the gold membrane and this layer is basically your silicon nitrate ok. And figure b shows ah the transmission maximum in the left axis and its spectral position so where it happens.

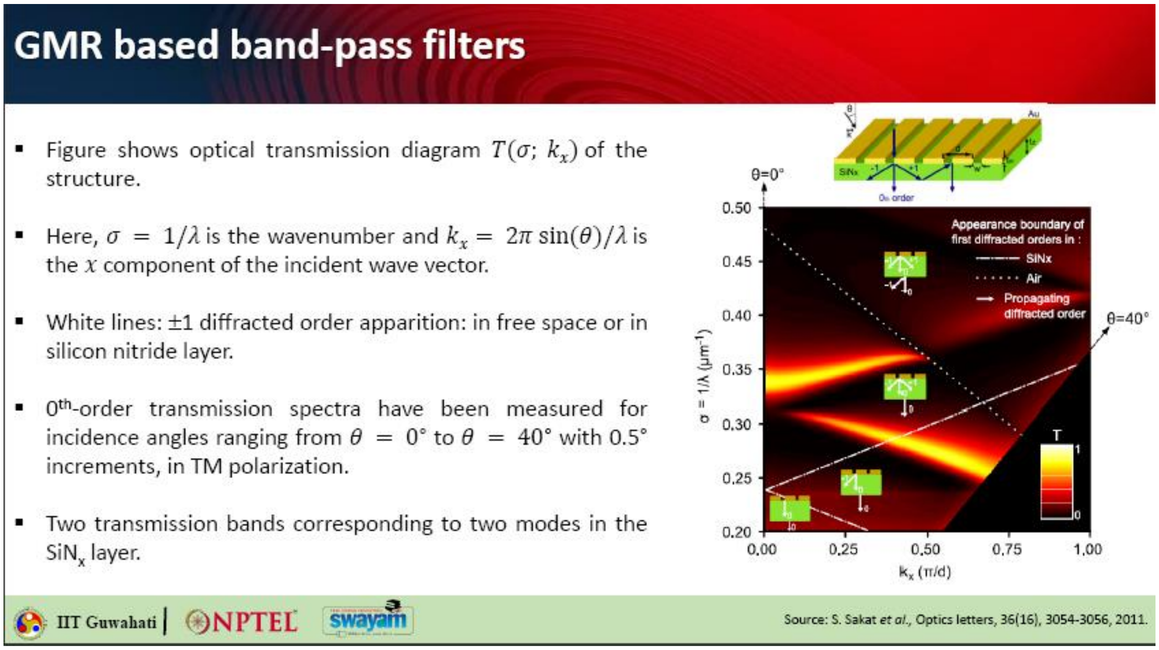


GMR based band-pass filters

- Light trapped in the SiN_x film is revealed by the strong light localization in the dielectric layer.
- Figure (b) shows the transmission maximum (left axis) and its spectral position (right axis) as a function of the period of the grating **d**, keeping the gold and SiN_x thicknesses constant

Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

So everything is changing when you change d ok. So you can change the periodicity and you can tune the resonance position as well as the transmission maximum. So here you are just keeping gold and the silver layer silver nitrate silicon nitrate layer thickness constant ok. So that brings me to case where the scaling actually takes place over this latest period ok and the width of the slit ok. So this factors the thickness factors will not play a role ok. Now if you look into the optical transmittance diagram which is T as a function of sigma and k_x ok this is how it looks like it is a parametric plot.



GMR based band-pass filters

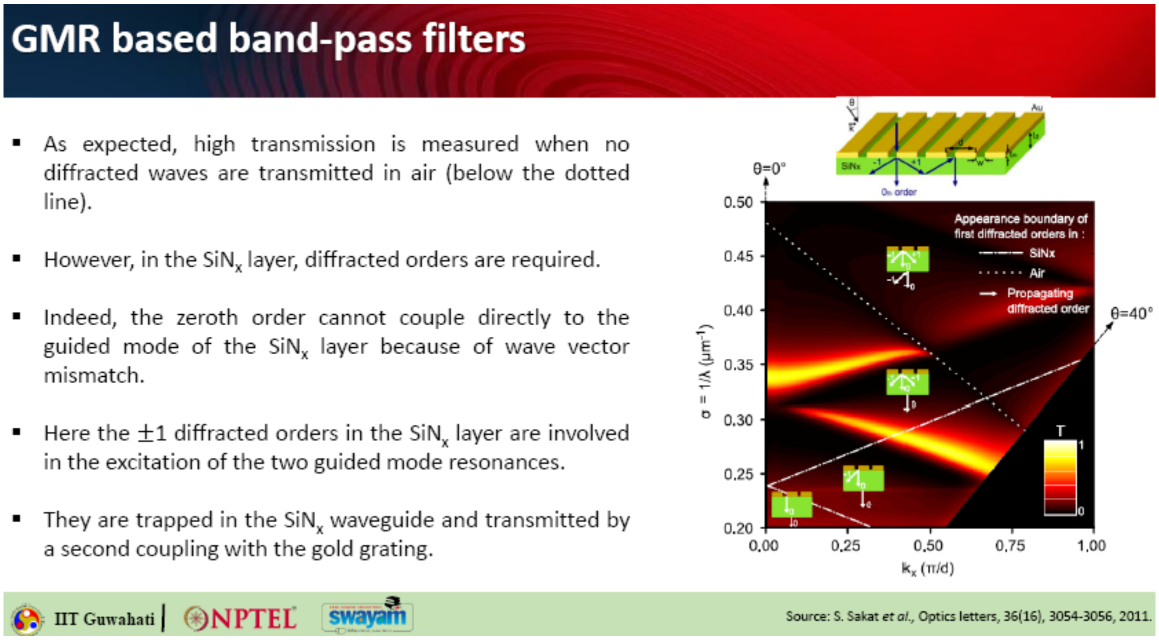
- Figure shows optical transmission diagram $T(\sigma;\, k_x)$ of the structure.
- Here, $\sigma = 1/\lambda$ is the wavenumber and $k_x = 2\pi\sin(\theta)/\lambda$ is the x component of the incident wave vector.
- White lines: ±1 diffracted order apparition: in free space or in silicon nitride layer.
- 0^{th}-order transmission spectra have been measured for incidence angles ranging from $\theta = 0°$ to $\theta = 40°$ with 0.5° increments, in TM polarization.
- Two transmission bands corresponding to two modes in the SiN_x layer.

Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

Here sigma is basically 1 over lambda that is the wave number and k x is the x component of the incident wave vector and k x can be calculated as 2 pi sin theta over lambda. Now here couple of important things are there. So this diagram as you can see is divided into areas corresponding to the existence of propagative diffractive orders in air. So here you can see only 0th order is coming out, but plus minus 1 modes are excited here 0th order is coming out, but only plus 1 is excited here 0th order is coming out and no diffracted modes are excited. Here it is 2 modes are coming out so 0 and minus 1 is coming out and plus 1 and minus 1 are also getting excited it is ok.

So from that 2 modes are getting coming out only plus 1 is getting trapped ok. So basically there are different regions and which are separated by this different white lines as you can see. So, if you see the plus minus 1 diffracted orders in free space and silicon nitrate so this is in air ok and this one is in the silicon nitrate ok and 0th order transition spectra have been measured for the incident angle ranging from 0 to 40 degree. So, this is the case where you have theta equals 40 degree ok and that has been done with 0.5 degree increment for TM polarization and those transmission plots are actually here. So the bright color shows highest transmission and darkest color shows almost 0 transmission ok and there are 2 transmission bands corresponding the corresponding for 2 modes.

So you can see there are 2 transmission bands here for the 2 modes of silicon nitrate layer. What are the 2 modes? One is basically the ± 1 ok. Now as expected high transmission is measured when no diffracted waves are transmitted in air. So that is below this particular dotted line.



GMR based band-pass filters

- As expected, high transmission is measured when no diffracted waves are transmitted in air (below the dotted line).
- However, in the SiN_x layer, diffracted orders are required.
- Indeed, the zeroth order cannot couple directly to the guided mode of the SiN_x layer because of wave vector mismatch.
- Here the ± 1 diffracted orders in the SiN_x layer are involved in the excitation of the two guided mode resonances.
- They are trapped in the SiN_x waveguide and transmitted by a second coupling with the gold grating.

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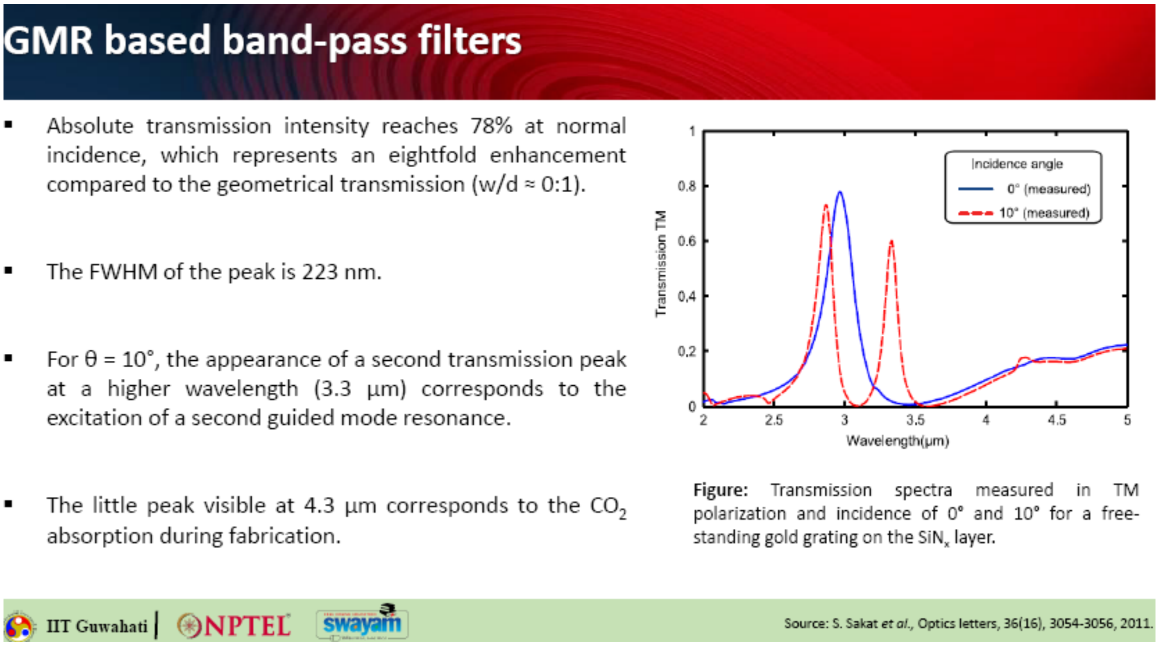
Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

Above this dotted line there is a diffracted mode is also transmitted. So the transmission drops a bit here no diffracted modes are there so you get very strong transmission. However in the silicon nitrate layer that is here ok diffracted orders are required. So the

0th order cannot couple directly to the guided mode of silicon nitrate that you can understand. So 0th order cannot directly get coupled to silicon nitrate and that is why also they are simply coming out because in order to do that you need to have wave vector matching or you can say that there is wave vector mismatch between the 0th order and that is why 0th order will not get coupled directly as guided mode.



GMR based band-pass filters

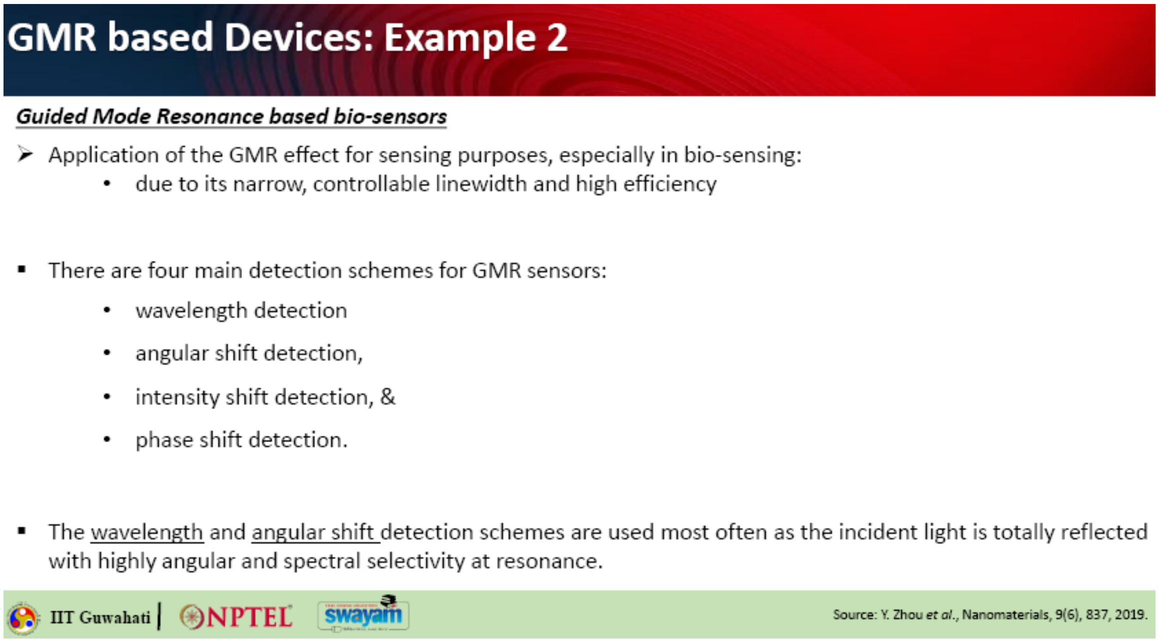
- Absolute transmission intensity reaches 78% at normal incidence, which represents an eightfold enhancement compared to the geometrical transmission (w/d ≈ 0:1).
- The FWHM of the peak is 223 nm.
- For θ = 10°, the appearance of a second transmission peak at a higher wavelength (3.3 µm) corresponds to the excitation of a second guided mode resonance.
- The little peak visible at 4.3 µm corresponds to the CO_2 absorption during fabrication.

Figure: Transmission spectra measured in TM polarization and incidence of 0° and 10° for a free-standing gold grating on the SiN_x layer.

Source: S. Sakat *et al.*, Optics letters, 36(16), 3054-3056, 2011.

However, for plus minus 1 diffracted mode in the silicon nitrate SINx layer ok it can give you those 2 guided mode resonances which we are able to see that when there are 2 guided modes resonances the transmitter become bright ok. So, this guided mode resonances are basically in the silicon nitrate waveguide and they actually are transmitted by a second coupling with the gold grid. So when you measure them ok so you can measure the absolute transmission intensity and you can see it reaches 78 percent at normal incidence and that is basically almost 8 fold enhancement compared to the geometrical transmission and that is significant. So without the guided mode resonance effect you will hardly see any significant transmission here ok you will only see like 10 percent or something like that. So, you can calculate the FWHM that is 2 to 3 nanometer and when you do it for 10 degree you are seeing 2 different transmission peaks.

So you are also getting a second peak at 3.3 micrometer which corresponds to the excitation of second guided mode resonance ok. So that is happening so when you go towards this so this is the axis where theta is 0 this is where theta goes to ah 40 degree. So you are actually able to see more modes coming in and there is also another little peak visible at 4.3 micrometer and but this is corresponding to carbon dioxide absorption during the fabrication. So, this is a fabrication issue other than that these are the 2 things that completely matches our prediction.



GMR based Devices: Example 2

Guided Mode Resonance based bio-sensors

- Application of the GMR effect for sensing purposes, especially in bio-sensing:
  - due to its narrow, controllable linewidth and high efficiency

- There are four main detection schemes for GMR sensors:
  - wavelength detection
  - angular shift detection,
  - intensity shift detection, &
  - phase shift detection.

- The wavelength and angular shift detection schemes are used most often as the incident light is totally reflected with highly angular and spectral selectivity at resonance.

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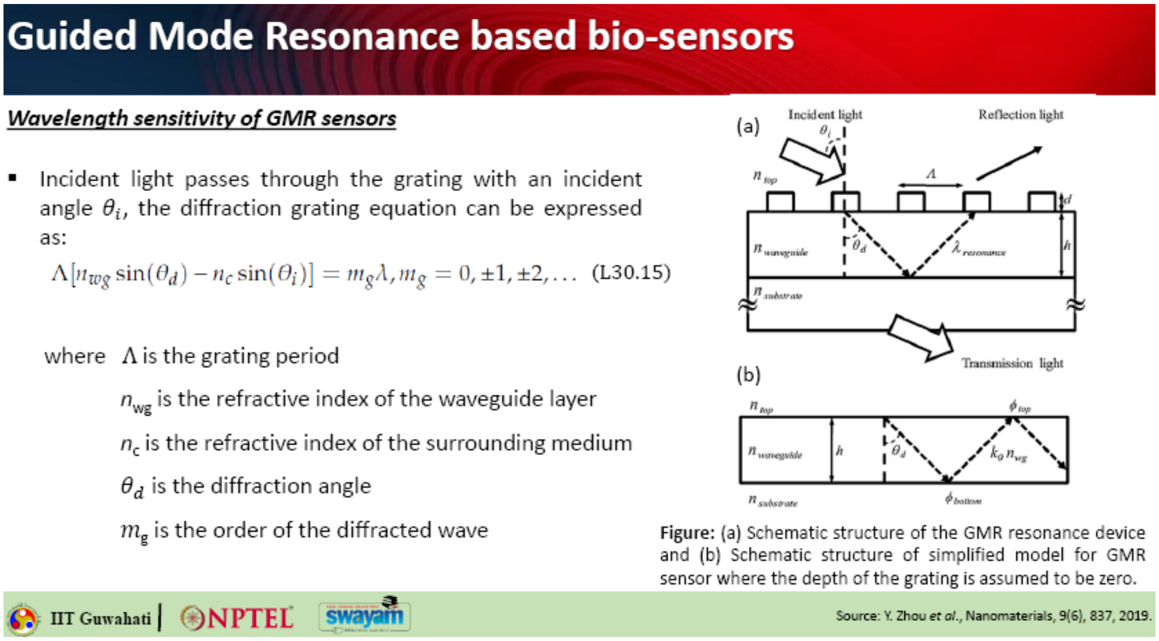
Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019.

So next thing is as I mentioned GMR based devices have very high Q they can also be used for sensing application. So let us look into GMR based ah biosensors and it is very popular to use GMR effect for sensing especially in biosensing because they have very narrow controllable line width and they make very high efficient filters. And there are 4 main detection schemes for GMR based sensors, wavelength detection, angular shift detection, intensity shift detection and phase shift detection. Among this the top 2 that is wavelength detection and angular shift detection they are the most commonly used one as incident light is totally reflected with highly angular and spectral selectivity at resonance. So, you can see the schematic here the schematic shows so this is where you have the grating this is your waveguide and this is the substrate.



Guided Mode Resonance based bio-sensors

Wavelength sensitivity of GMR sensors

- Incident light passes through the grating with an incident angle θ_i, the diffraction grating equation can be expressed as:

$$\Lambda[n_{wg}\sin(\theta_d) - n_c\sin(\theta_i)] = m_g\lambda, m_g = 0, \pm1, \pm2, \ldots \quad \text{(L30.15)}$$

where Λ is the grating period

n_{wg} is the refractive index of the waveguide layer

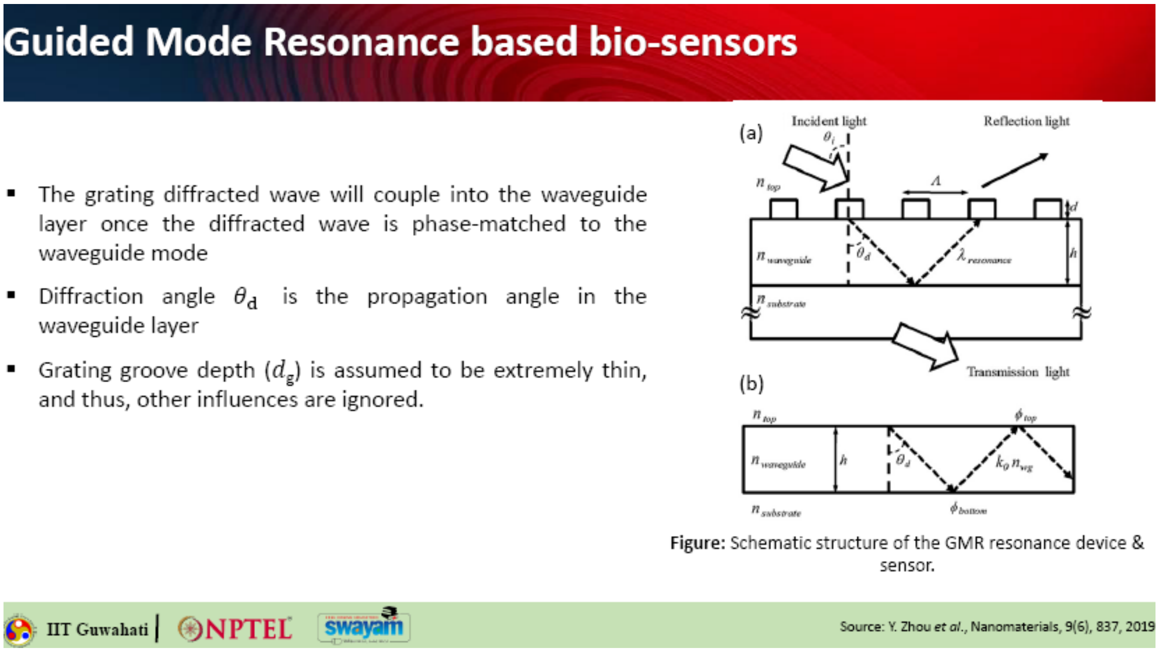
n_c is the refractive index of the surrounding medium

θ_d is the diffraction angle

m_g is the order of the diffracted wave

Figure: (a) Schematic structure of the GMR resonance device and (b) Schematic structure of simplified model for GMR sensor where the depth of the grating is assumed to be zero.

Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019.



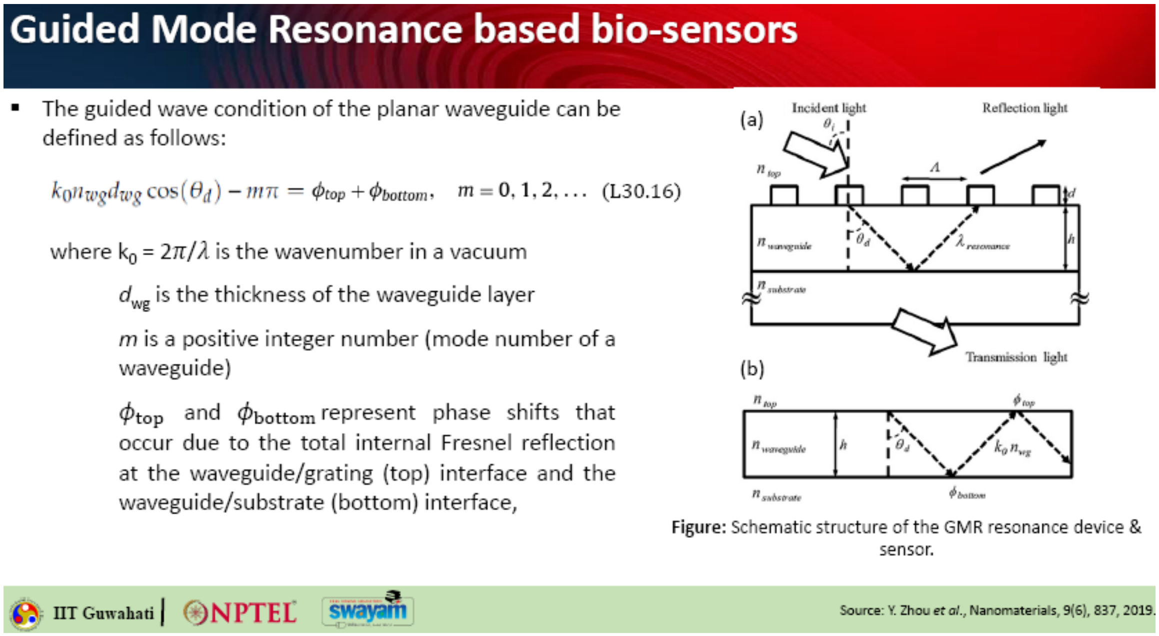
Guided Mode Resonance based bio-sensors

- The grating diffracted wave will couple into the waveguide layer once the diffracted wave is phase-matched to the waveguide mode
- Diffraction angle θ_d is the propagation angle in the waveguide layer
- Grating groove depth (d_g) is assumed to be extremely thin, and thus, other influences are ignored.

Figure: Schematic structure of the GMR resonance device & sensor.

Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019

So, this is the GMR resonance device. So incident light passes through the grating with an incident angle. So here the incident angle is θ_i ok and the grating equation will be the gamma capital gamma that is a grating period n_{wg} that is the refractive index of the waveguide sin θ_d that is the diffracted angle ok minus n_c, n_c is the sliding refractive index sin θ_i that will be equal to mg that is the order of the diffracted wave times lambda. So what is mg? It can be 0 , ±1, ±2 and so on. So this particular diagram can also be written ok we will discuss this later. So, it is like this can be seen also like this where the so this is basically a simplified model of your GMR sensor where the height or the depth of the grating is assumed to be 0.



Guided Mode Resonance based bio-sensors

- The guided wave condition of the planar waveguide can be defined as follows:

$$k_0 n_{wg} d_{wg} \cos(\theta_d) - m\pi = \phi_{top} + \phi_{bottom}, \quad m = 0, 1, 2, \ldots \quad \text{(L30.16)}$$

where $k_0 = 2\pi/\lambda$ is the wavenumber in a vacuum

d_{wg} is the thickness of the waveguide layer

m is a positive integer number (mode number of a waveguide)

ϕ_{top} and ϕ_{bottom} represent phase shifts that occur due to the total internal Fresnel reflection at the waveguide/grating (top) interface and the waveguide/substrate (bottom) interface,

Figure: Schematic structure of the GMR resonance device & sensor.

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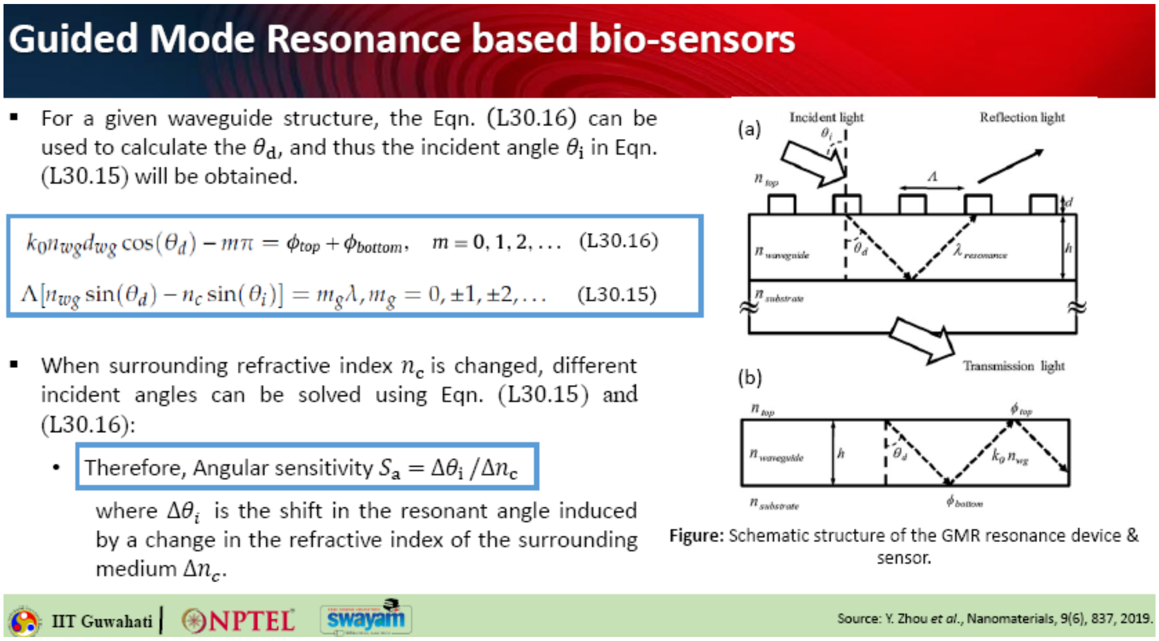
Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019.

So here you just show top layer waveguide and substrate layer and you are just showing what is happening ok. So, this is the diffracted wave and then it gets totally internally reflected and so on it this is how it propagates inside this waveguide. Now the grating diffracted wave will couple into the waveguide layer once the diffracted wave is phase matched that we all know that only so there is diffraction but when it is phase matched it will be supported in this waveguide and it can propagate. So, θ_d is the propagation angle in the waveguide layer and we for simplicity we can think that this d_g the height of the grating or the groove ok should be can be extremely thin or small and they can be ignored.



Guided Mode Resonance based bio-sensors

- For a given waveguide structure, the Eqn. (L30.16) can be used to calculate the θ_{d}, and thus the incident angle θ_{i} in Eqn. (L30.15) will be obtained.

$$k_0 n_{wg} d_{wg} \cos(\theta_d) - m\pi = \phi_{top} + \phi_{bottom}, \quad m = 0, 1, 2, \ldots \quad \text{(L30.16)}$$

$$\Lambda[n_{wg}\sin(\theta_d) - n_c \sin(\theta_i)] = m_g \lambda, m_g = 0, \pm 1, \pm 2, \ldots \quad \text{(L30.15)}$$

- When surrounding refractive index n_{c} is changed, different incident angles can be solved using Eqn. (L30.15) and (L30.16):
  - Therefore, Angular sensitivity $S_{\mathrm{a}} = \Delta\theta_{\mathrm{i}} / \Delta n_{\mathrm{c}}$

    where $\Delta\theta_i$ is the shift in the resonant angle induced by a change in the refractive index of the surrounding medium Δn_c.

Figure: Schematic structure of the GMR resonance device & sensor.

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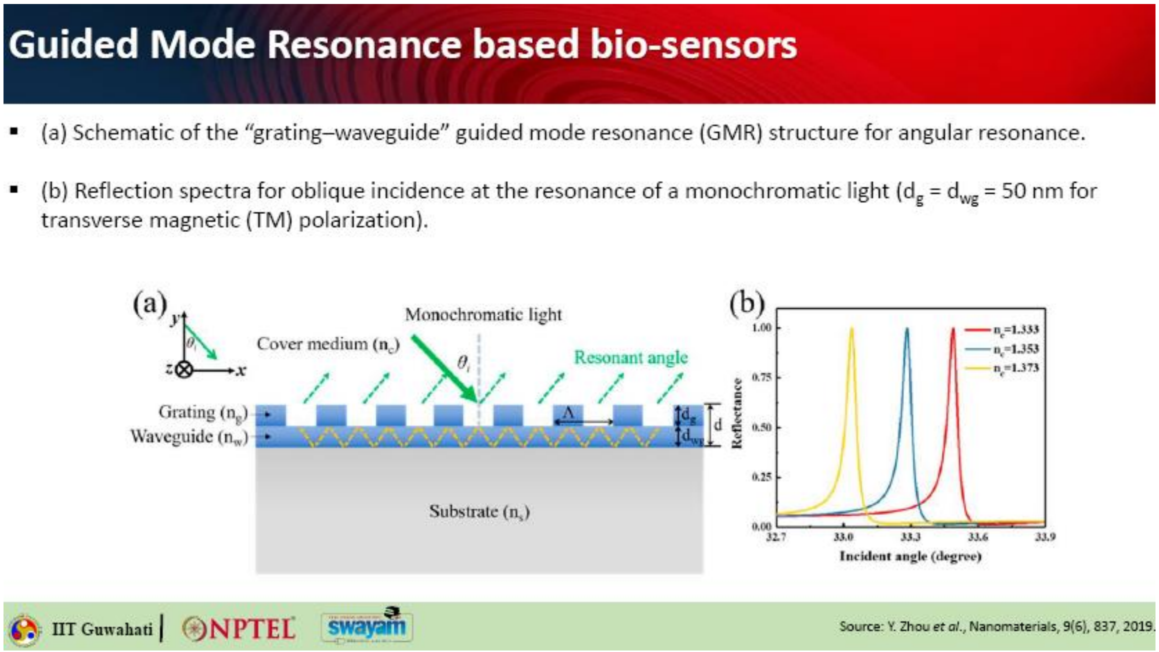
Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019.

So this allows us to look for the conditions of phase matching. So, the guided wave condition of the planar waveguide can be defined from this one. So, if this is θ_d this angle is θ_d ok and this wave vector is k_0 okay, k_0 is basically the incident k_0 is the wave vector in the in air or in vacuum you multiply this by the refractive index of n_g layer so you get the wave vector in this one ok and then you can take the cos theta component ok. So you will get this one basically. So, this 1-mπ okay so this cos θ_d ok will give you the vertical this this particular one ok the component of the wave vector and when you take -mπ that should be equal to the total phase that is there in the top and the bottom of the two interfaces ok. So you can write k_0 is basically 2 pi by lambda which is the wave number in vacuum d wg is basically the thickness of the waveguide or that is the height here okay, m is a positive integer it is basically the mode number of the waveguide can be 0 it can be , ±1, ±2 and phi top and phi bottom as you can see these are basically the phase shifts that occur due to total internal the final reflection at the top and the bottom interfaces.

So top interface is basically the waveguide grating interface and the bottom one is basically waveguide substrate interface right. So, for a given waveguide structure you can use this particular equation to calculate what will be θ_d ok and then you can use this equation to find out what is the θ_i that will give you that θ_d ok. So that way the incident angle that will allow you to get this particular modes and then when the surrounding refractive index n_c change what will happen different incident angles can be resolved ok. So you can actually obtain the angular sensitivity like this. So, you can define angular sensitivity as S a which is delta theta i that will be the change in the incident angle whenever there is a change in the surrounding refractive index surrounding medium refractive index.



Guided Mode Resonance based bio-sensors

- (a) Schematic of the “grating–waveguide” guided mode resonance (GMR) structure for angular resonance.
- (b) Reflection spectra for oblique incidence at the resonance of a monochromatic light ($d_g = d_{wg}$ = 50 nm for transverse magnetic (TM) polarization).

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Source: Y. Zhou *et al.*, Nanomaterials, 9(6), 837, 2019.

So here you can say that delta theta i is basically the shift in the resonant angle that is introduced by the change in the refractive index of the surrounding media that is delta n_c. So here you see this the first one it shows the schematic of the grating waveguide guided mode resonance. So, you see there is guided mode resonance for this structure and that helps for angular is resonance and this also shows the reflection spectrum ok in this case ok for the refractive index values of 1.333 ok that is the red one then nc equals 1.353 and n_c equals 1.373 ok. So you can see they are color coded and what is important here to notice is that with this kind of change minor change in the refractive index the the incident angle needs to change ok. So that is how it is angular resolved or you can actually use this as a sensor right. So with that we stop here today and this ends our discussion on metasurface and GMR based devices. In the next lecture we will discuss about another interesting topic that will be on transformation optics and we will see how to make invisibility clocks. Thank you if you have got any query you can drop an email to this email address mentioning MOOC on the subject line.