Course Name- Nanophotonics, Plasmonics and Metamaterials

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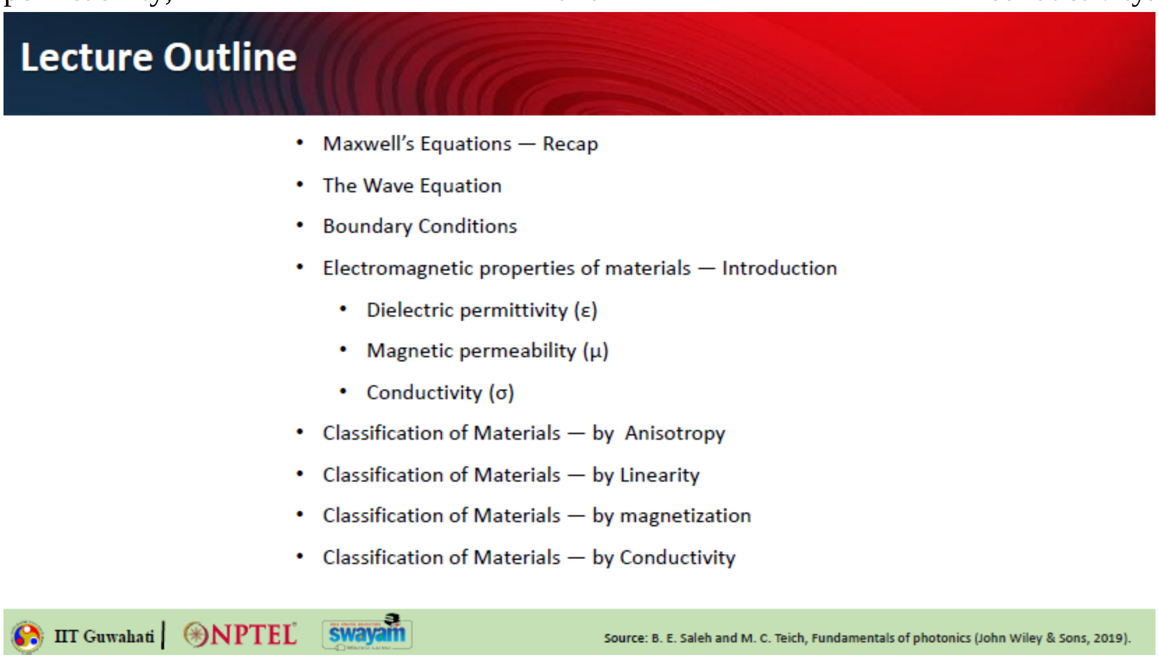
Week-02

Lecture -05

Hello students. Welcome to Lecture 5 of our online course on Nanophotonics, Plasmonics, and Metamaterials. Today's lecture is on Electromagnetic Properties of Material. In this lecture, we'll see a quick recap of Maxwell's equation, and then we'll see the derivation of wave equation and the boundary conditions. We'll then introduce the electromagnetic properties of materials, such as dielectric permittivity, magnetic permeability, and conductivity.



And then we'll look into the classification of materials by anisotropy, by linearity, magnetization, and conductivity. So, in the last lecture, we have seen the Maxwell's equation can be written in these two forms, the integral form and the differential form. Also, we have seen that in electrostatics or magnetostatics, the electric and magnetic fields are independent of each other, but in the dynamic or time varying nature, this electric and magnetic fields are getting coupled to each other. The first law that we have seen is basically the Gauss law.

Maxwell's Equations — Recap

Table: Comparison of Maxwell's equations for **static and time-varying** electromagnetic fields.

	Electrostatics / Magnetostatics	Time-Varying (Dynamic)
Electric & magnetic fields are...	independent	possibly coupled
Maxwell's eqns. (integral)	$\oint_S \mathbf{D} \cdot ds = Q_{encl}$ $\oint_C \mathbf{E} \cdot dl = 0$ $\oint_S \mathbf{B} \cdot ds = 0$ $\oint_C \mathbf{H} \cdot dl = I_{encl}$	$\oint_S \mathbf{D} \cdot ds = Q_{encl}$ $\oint_C \mathbf{E} \cdot dl = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot ds$ $\oint_S \mathbf{B} \cdot ds = 0$ $\oint_C \mathbf{H} \cdot dl = I_{encl} + \int_S \frac{\partial}{\partial t} \mathbf{D} \cdot ds$
Maxwell's eqns. (differential)	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = 0$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$

Note: Differences in the time-varying case relative to the static case are highlighted in blue.

The Gauss law tells us that the electric flux through any closed surface is equal to the charge enclosed by the surface. So, it actually describes the relationship between an electric charge and the electric field it produces. This is often pictured in terms of electric field lines originating from a positive charge and terminating on negative charges. And it also indicates the direction of the electric field at each point in the space. The second equation is this one, which is the Gauss law of magnetism.

So, the magnetic field flux through any closed surface is basically zero. And this is equivalent to the statement that magnetic fields are continuous and they have no beginning or end. Any magnetic field line entering the region enclosed by the surface must also leave the surface. It means that there is no magnetic monopole where the magnetic lines can terminate. And that is why we say that surface integral B.

ds is zero. The third law is the Faraday's law. So, you can see the difference in the Faraday's law in electrostatics and in electrodynamics. So, there it says that a changing magnetic field induces an electromotive force (EMF), and hence an electric field. The direction of the EMF opposes the change and that is why this negative sign comes into the picture, and this is called Lenz's law.

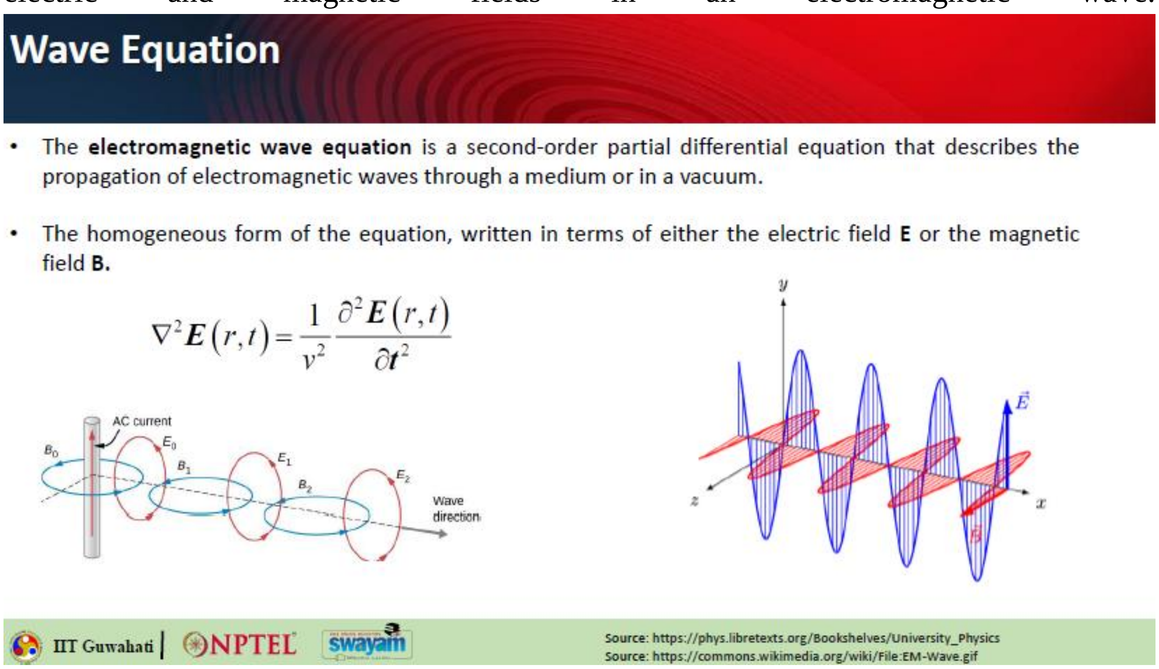
So, the whole thing is basically the Faraday's law of induction plus Lenz's law. Electric field from a changing magnetic field, so when you have the time derivative, it means the magnetic field is basically changing with time. So, it has field lines that can form closed loops without any beginning or end. And the last law is this one, which is also known as Ampere Maxwell law. So, here also you can see the difference between electrostatics and electrodynamics And you can see the blue terms which are basically the difference between the static and the dynamic fields, all these blue terms.

So, we are just focusing here at this moment. This is the integral equations. The same thing can also be written in terms of the differential equations which you have described in the previous lecture. So, what you see here is that the magnetic field, that is basically generated by either moving charges i.e-current or changing electric field d dt of displacement field is basically change in electric field.

So, these two can actually give you magnetic field. So, this is the fourth Maxwell equation, it actually encompasses the Ampere's law. So, only this part up to the I enclosured is basically the Ampere's law. And this is the contribution to this law done by James Maxwell. And this is where he has got all these electric and magnetic fields coupled to each other.

So, this adds this magnetic field term which is coming from change in the electric field lines. So, d is nothing but the electric displacement field. When you say d dt, it means time rate of change, means it's a time varying field. And that can also give you magnetic field. So, with that, let us look into how the wave equation is derived.

So, we understood that the electric and magnetic field gets coupled to each other and they can propagate through any region as electromagnetic wave. So, it can be described by wave equation. Now, the electromagnetic wave equation is basically a second order partial differential equation that describes the propagation of electromagnetic waves through a medium or in vacuum. Now, how to derive it? We'll see. So, this is how the electric and magnetic fields in an electromagnetic wave.

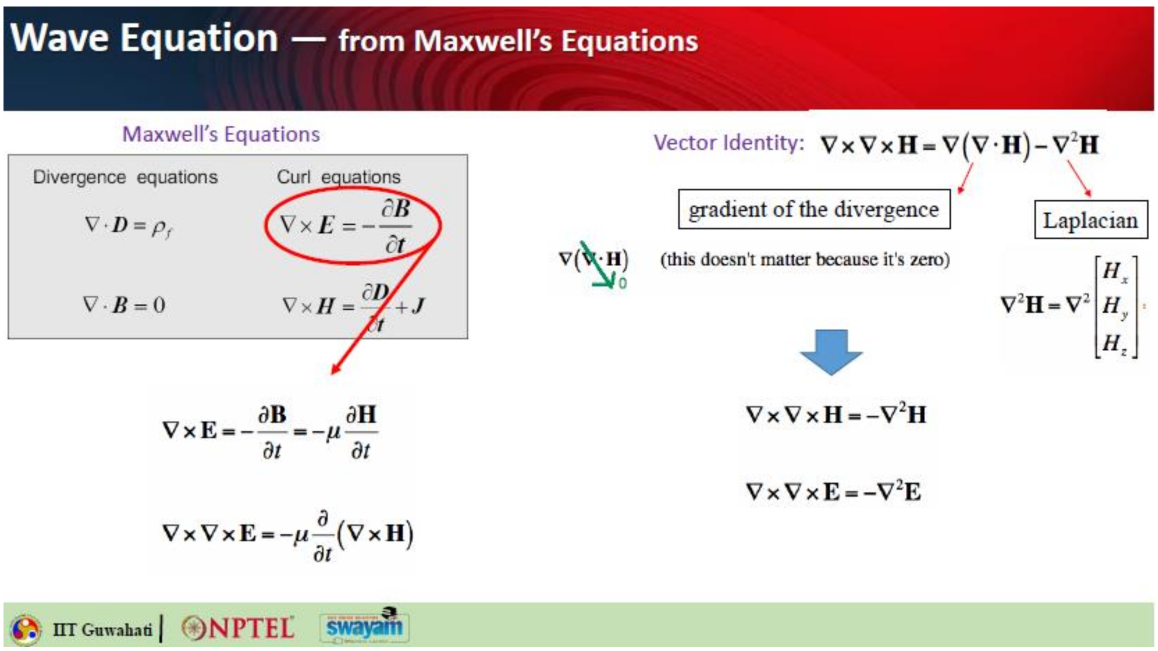


The wave is propagating along this x direction. And you can see the electric and magnetic field. They are basically oscillating along, electric field is along y and magnetic field is oscillating along z direction here. So, this is also another diagram that tells you that a current has got a magnetic field involved around it, magnetic fields generated, that also generates electric field and so on. So, this is how the electric and magnetic field lines are getting coupled and the electromagnetic wave is basically propagating in this particular direction.

So, you can see the homogeneous form of the equation. It is written as

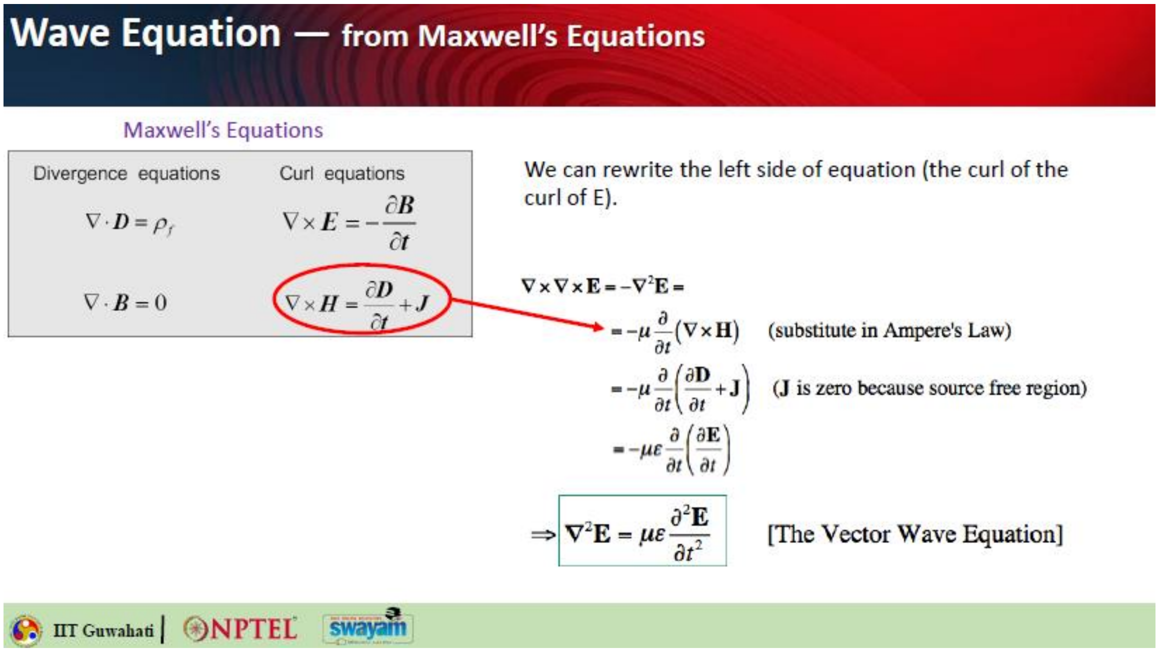
$$\nabla^2 \boldsymbol{E}(r,t) = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}(r,t)}{\partial t^2}$$

Now, our goal is here to determine how the wave equation is basically derived from Maxwell's equation. So to start with, let us look into this vector identity. That curl of a vector is nothing but the gradient of the divergence of that vector minus the Laplacian of the vector.



Now, with that, we can always say that the gradient of a vector, in case it's a source-free region, means there is no current or charge in that region. So, we can take the divergence to be zero. So, this particular curl of curl of a vector will be simply minus the Laplacian of that vector. And Laplacian operator we have seen in the previous lecture. So, you can write this for both magnetic field and electric field.

So, the equation for electric field looks like this. $\nabla \times \nabla \times \mathbf{E} = -\nabla^2\mathbf{E}$. Now, in the Maxwell's equation, do you have this particular term, curl of E? Yes, we do. So, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu\frac{\partial \mathbf{H}}{\partial t}$



And then, let us take curl on both sides. So, you have $\nabla \times \nabla \times \mathbf{E}$, that is this left-hand side you are able to get from here. And on the right side also, you do the curl. So, you have $\nabla \times \mathbf{H}$ coming here. Now, what next? You already know one identity for $\nabla \times \mathbf{H}$.

So, $\nabla \times \mathbf{H}$ can be given by $\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$. So, in the equation, if you go back, so $\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E}$. So, let us put it there. So, the left-hand side becomes $-\nabla^2 \mathbf{E}$. And the right side has got $-\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$.

$\nabla \times \mathbf{H}$ you can substitute from here. So, that is $\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$. Now, what is J? J is basically the current density. And we have assumed that it is a source free region, so the current density term can go to zero.

So, this term goes to zero. You simply have $-\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$ and D can be written as εE. So, once you do that, you have basically $-\nabla^2 \mathbf{E} = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$. Minus term you cancel out from both sides. So, you simply get this as your vector wave equation.

Wave Equation — speed of EM waves

- The connection between electromagnetic optics and wave optics is now evident.
- The **wave equation**, which is the basis of wave optics, is embedded in the structure of electromagnetic theory.
- The speed of electromagnetic wave is related to the electromagnetic constants ε and μ

Speed of the EM wave:

Compare $\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_r \dfrac{\partial^2 \boldsymbol{E}}{\partial t^2}$ and $\nabla^2 \boldsymbol{E} = \dfrac{1}{v^2}\dfrac{\partial^2 \boldsymbol{E}}{\partial t^2}$

$$\Rightarrow \quad v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_r} = \frac{c_0^2}{\varepsilon_r}$$

In Free Space (Vacuum):

$\mu_0 = 4\pi \cdot 10^{-7}$ [H/m]

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ [F/m]

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}795{,}638 \text{ [m/s]}$$

Here E is a vector. So, you can think of the three scalar wave equation in x, y and z direction. Now, it is evident from the wave equation that the connection between the electromagnetic optics and wave optics. We have seen that the wave equation is basically obtained from electromagnetic theory only. So, we can say that the speed of electromagnetic wave is hence related to the electromagnetic constants mu and epsilon. So, let us look into the speed of electromagnetic wave.

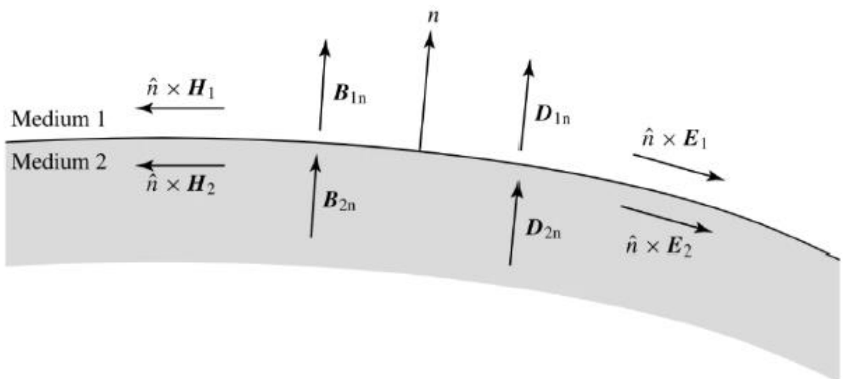
So, this is what we have got from the previous one. You have got $\nabla^2 \mathbf{E} = \mu\varepsilon \dfrac{\partial^2 \mathbf{E}}{\partial t^2}$. Here also you can write the same thing. Just that when we assume the light is in vacuum, we take μ_r as the permeability and μ_0 is the vacuum permeability.

And if there is a medium, we incorporate that medium also here. That medium relative permittivity, ε_r is the relative permittivity that is also included. So, this term is equivalent to $\dfrac{1}{v^2}$. So, if you equate these two things, you can simply write $v^2 = \dfrac{1}{\mu_0 \varepsilon_0}\dfrac{1}{\varepsilon_r}$. And this $\dfrac{1}{\mu_0 \varepsilon_0}$, that is basically c_0^2.

c naught is the speed of light in vacuum. And final relationship $v^2 = \dfrac{c_0^2}{\varepsilon_r}$. So, I think all of you know this constants μ_0 and ε_0 in a vacuum. And that also gives you c naught, that is basically the speed of light in vacuum. And that comes close to 3×10^8 m/s.

Boundary Conditions

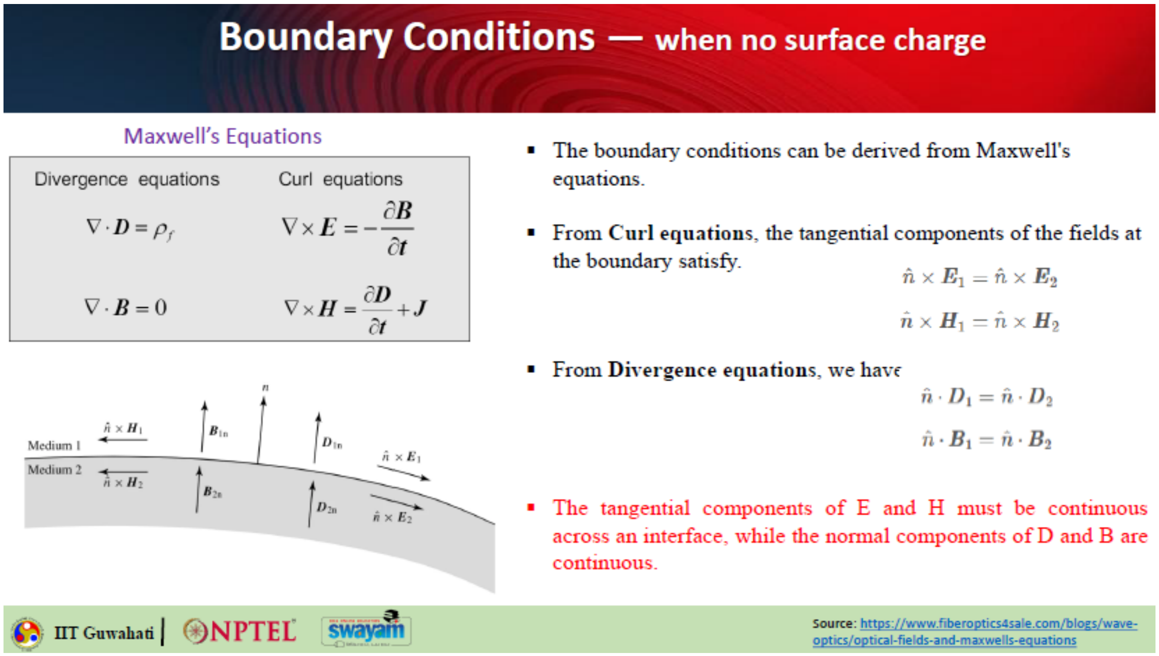
- At the interface of two media of different optical properties, the optical field components must satisfy certain boundary conditions.



Source: https://www.fiberoptics4sale.com/blogs/wave-optics/optical-fields-and-maxwells-equations

Now, let us look into boundary conditions. Now, at the interface of two medium of different optical properties, the optical field components must satisfy certain boundary conditions. And these boundary conditions become very important because they will tell us about the behavior of electromagnetic fields, such as electric field, electric displacement field, magnetic field at the interface of the two materials. Now, let us first consider the case when there is no source in the interface. So, now let us look into the boundary conditions.

So, at the interface of two medium of different optical properties, the optical field components must satisfy certain boundary conditions. So, as you can see, this is medium one and this is medium two. And this is the normal vector showing the normal of this interface. So, these are the normal components of the B and D fields, which is basically the magnetic flux density and electric flux density. And these are basically the tangential components of electric field at region one and region two, or you can say medium one and medium two.



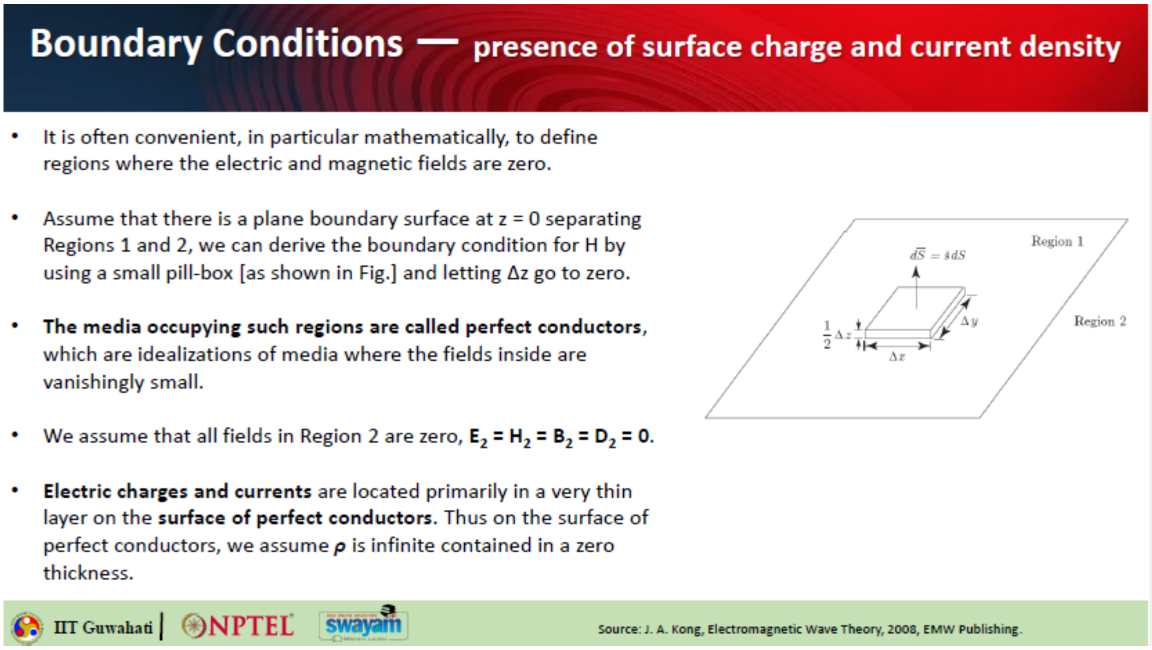
Here it is showing the tangential component of the magnetic field in medium one and medium two. So, these conditions are basically derived from Maxwell's equation. From the curl equations, so these are the two curl equations, we have seen them couple of times. So, from here we can say that the tangential component of the field at the boundaries must satisfy. So, you can actually calculate $\hat{n}$, which is nothing but the vector marking the normal to the interface, cross with E_1.

So, you have $\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2$. Similarly, $\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2$. In simple words, you can say that the tangential magnetic fields, so you can also say $H_{t1} = H_{t2}$ and $E_{t1} = E_{t2}$. And if you look into the divergence equation, one important thing that here no surface charge, so it's a charge-free region.

The conditions will slightly modify when there is some surface charge, we'll see in the subsequent slides. So, here you can see that these are basically charge-free region. And from the divergence equation, you can write that n, because they are in the same direction, you can write $\hat{n} \cdot D_1 = \hat{n} \cdot D_2$. It means the normal component of the electric displacement field should be continuous. Similarly, the normal component of the magnetic flux density should also be continuous.

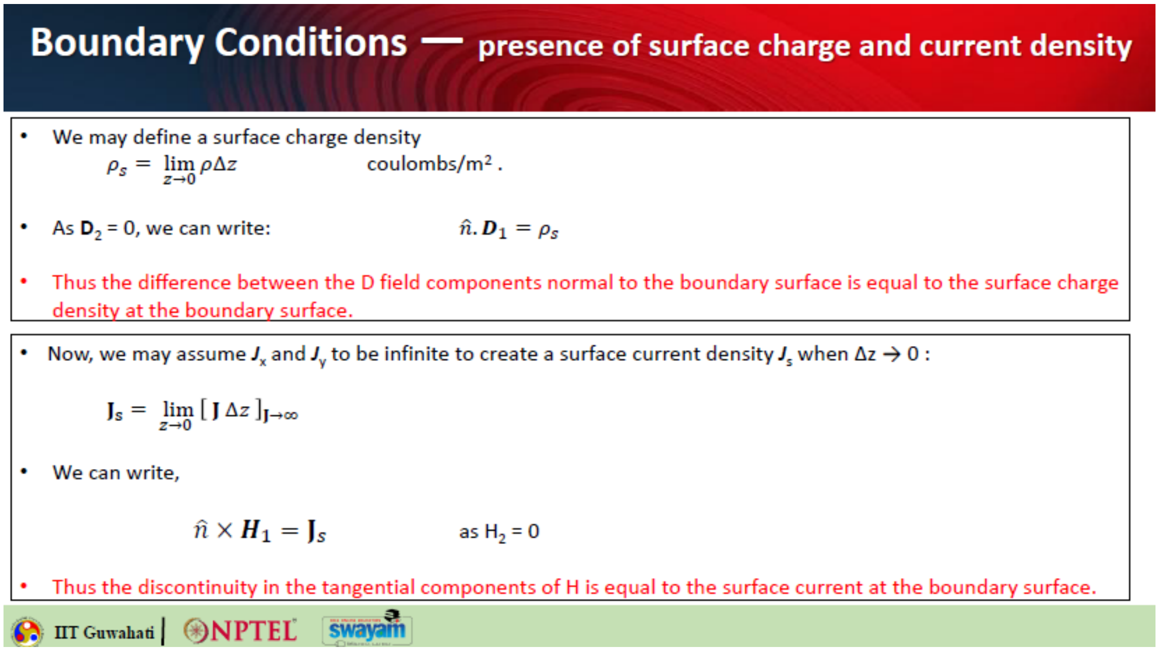
So, this is what is the summary of the boundary conditions that you can derive from Maxwell's equation, when there is no surface charge. Now, in the presence of surface charge or any current density, the boundary conditions will slightly modify. And the concept of surface charge density will have practical usefulness. So, here let us see how it can be obtained. So, it is convenient in particular mathematically, to define region where magnetic and electric fields are zero.

So, let's assume in this particular figure, there is a plane boundary surface. So, this is the boundary surface. This is exactly at z =0, separating region one and region two. And we can derive the boundary conditions for H by using a small pillbox, which is having a height of Δz.



So, here you see in region one, it is $\Delta z/2$. So, in the other side of this particular boundary, this pillbox is also having a height of $\Delta z/2$. So, overall the height is Δz and we will see that Δz can go to zero. So, the media that is occupying such region are called perfect conductors. And these are idealization for any media where the fields inside are vanishingly small.

In conductors, the field will lie on the surface. So, inside there will be no field. So, we can assume that all fields in region two are basically zero. So, $E_2 = H_2 = B_2 = D_2 = 0$. So, now the electric charges and the currents are primarily located on the thin layer on the surface of the perfect conductors. Thus, on the surface of the conductors, we can assume that, rho is at basically infinite contained in a zero thickness.



Okay, because we will make this thickness to be almost zero. So, your charge density can go to infinite. So, if that is the case, you can also write that surface charge density $\rho_s = \lim_{z\to 0} \rho\Delta z$. It is basically $\rho \times \Delta z$.

And the unit will be C/m^2. So, we have seen here our assumption tells us that there is nothing in the second layer region, so D_2 =0. So, we can write, $\hat{n}.\boldsymbol{D}_1 = \rho_s$. Okay, that is the charge towards the region one. So, this is the difference that you see in the presence of a surface charge density. So, here the difference in D components normal to the boundary surface is basically equal to the surface charge density at the boundary surface.

So, here the normal components of displacement field are not same. There is a difference and the difference is basically the surface charge density. Similarly, when you assume that the surface current density along x and y are infinite. To create a surface charge density Js when the Δz , the thickness of this field box goes to zero. So, again, you can write $\mathbf{J}_s = \lim_{z\to 0} [\mathbf{J}\Delta z]_{\mathbf{J}\to\infty}$

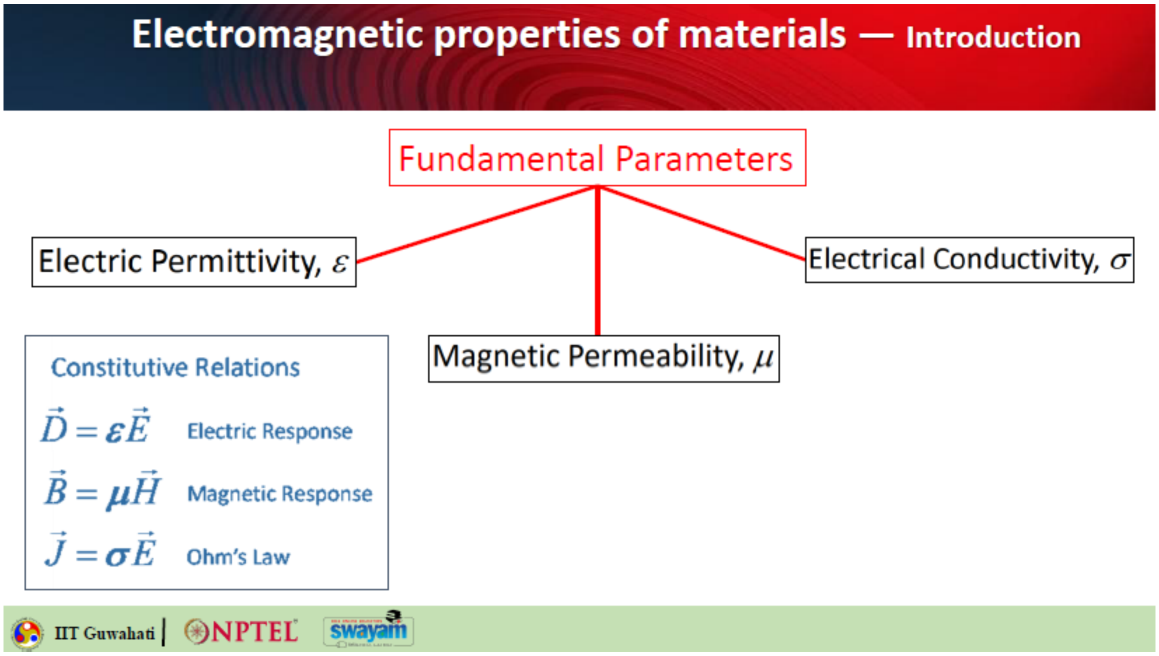
And that tells you that the tangential component of the magnetic field in region one, that is $\hat{n} \times H_1 = \mathbf{J}_s$. Whereas H2 in the region two, it is basically zero. So, there is a difference between the tangential component of the magnetic field in region one and region two. And that the difference is given by this surface current density. So, in a tabular form, if I want to show this, you can see this column shows the vector form and this writes the scalar form of the same equation.

	Vector form	Scalar form	Description
1.	$\hat{e}_n \times (\vec{E}_1 - \vec{E}_2) = 0$	$E_{t1} - E_{t2} = 0$	Tangential electric field, $\vec{E}$, is continuous.
2.	$\hat{e}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{j}_s$	$H_{t1} - H_{t2} = j_s$	The discontinuity of the tangential H field equals the surface current.
3.	$\hat{e}_n \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$	$D_{n1} - D_{n2} = \rho_s$	The discontinuity of the normal $\vec{D}$ equals the surface charge density.
4.	$\hat{e}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0$	$B_{n1} - B_{n2} = 0$	The normal component of $\vec{B}$ is continuous.

So, as you can see, the difference in the tangential components of the electric field along the interface is zero. However, the difference in the normal component of the displacement field is equal to the surface charge density, ρ_s. And you can also see the difference in the tangential component of the magnetic field is J_s, that is the surface current density. However, the normal components of the B, that is the magnetic flux density, is also continuous. So, you can simply remember these equations that when there is surface current density or surface charge density, only these two factors are getting disturbed.

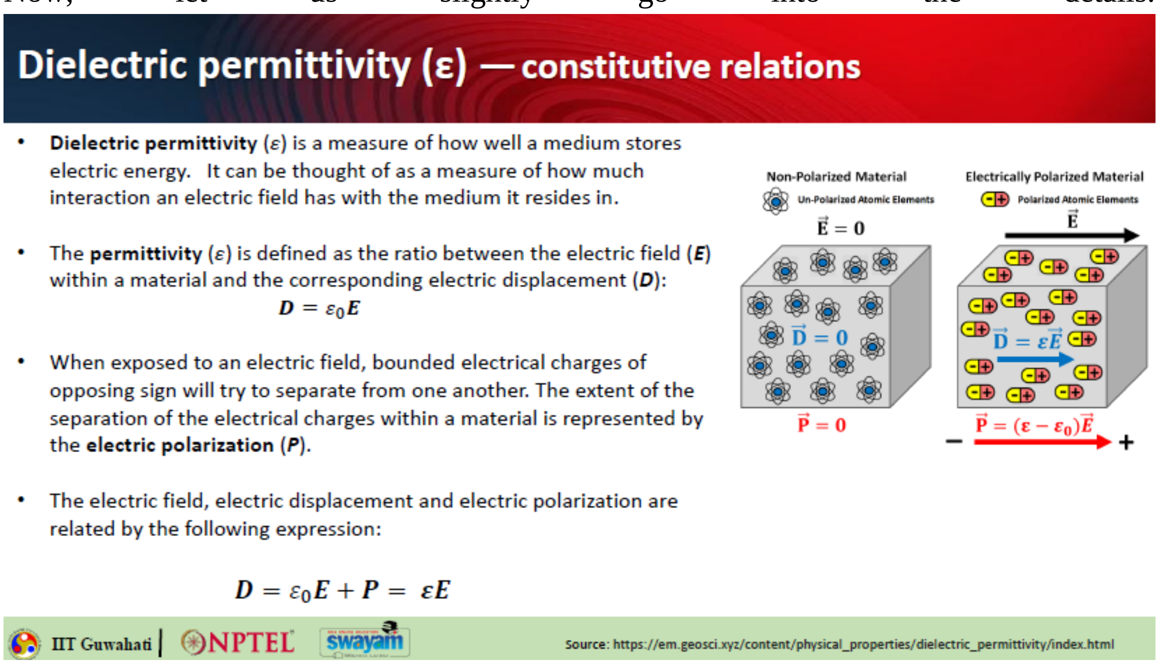
So this will be the boundary conditions in that case. So the normal component of the electric displacement field will be altered and the tangential component of the magnetic field will be altered. So, with that, let us now move ahead and discuss about the electromagnetic properties of material. So, why we need that? Because when any material interacts with an electromagnetic field, there are certain parameters in that material that quantifies that interaction, and that can be given as the constitutive relations. When the field actually interacts with any material, there are a couple of fundamental parameters.

One is electric permittivity, that is ε. then you have magnetic permeability, that is μ and then there is electric conductivity, σ. These are the three parameters that more or less, quantifies the interaction of any material with an electromagnetic wave. So, if you look into the constitutive relations, the first one is basically describing the electric response. So, by electric response, I would like to mean here is that the permittivity is basically giving you the relationship between the applied electric field and the displacement field that is being generated.



So, it also tells you how much is the polarization the material is going to experience under the influence of this applied electric field. Similarly, magnetic response is characterized by the permeability. So, here also it tells you that what will be the magnetic flux density in the presence of a magnetic field given by H. The last one is also known as Ohm's law, where J =σE. It tells you that, when in the presence of an applied electric field, how much will be the current density in that particular material.

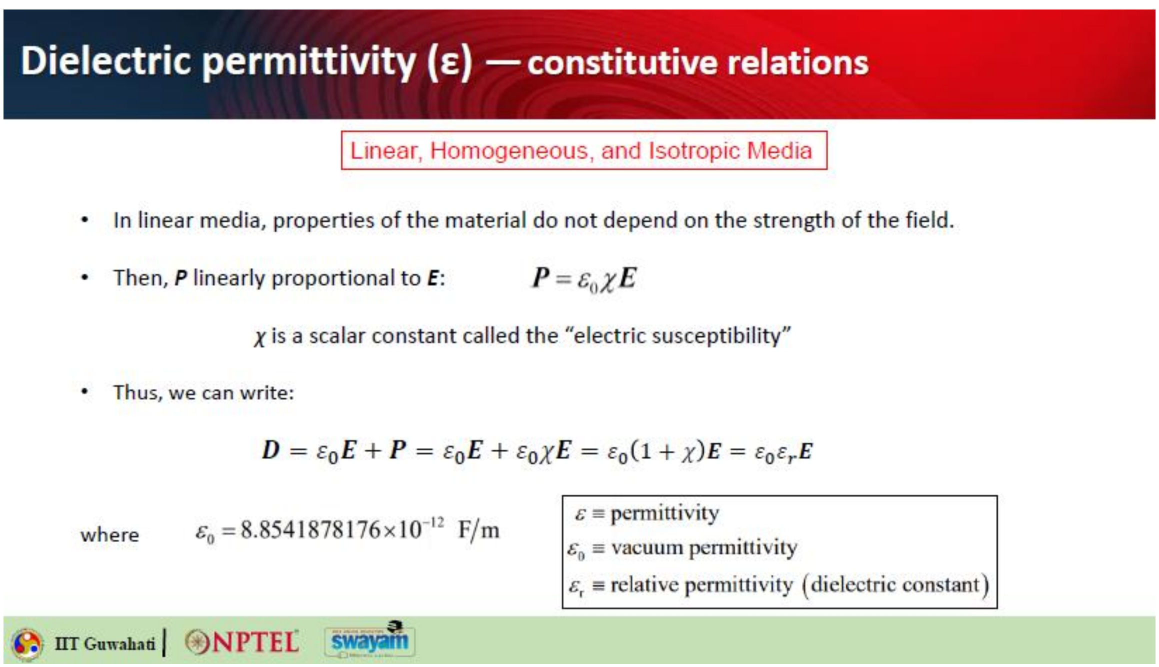
So, that actually tells you about the electrical conductivity in that material. So, σ stands for this conductivity. So it describes how much this material is able to conduct electricity. Now, let us slightly go into the details.



We have seen this couple of times, but just that to give you a quick recap. Permittivity, epsilon is basically the ratio of the displacement field over the applied electric field. And it tells us that, how much interaction an electric field has with the medium it resides in. So, this is a particular diagram. I have shown this before as well. So, this is for an unpolarized material, where you can see all the atoms and the electron clouds are in the same place.

So, when there is no electric field, there is no polarization, there is no displacement field as well. As soon as there is an applied electric field, this electric field will try to push the electron cloud away. So, the nucleus and the electron cloud will get slightly separated. So, that will create the polarization. So, you can see the polarization is basically proportional to the applied electric field, of course, and it can be given as epsilon minus epsilon naught times E.

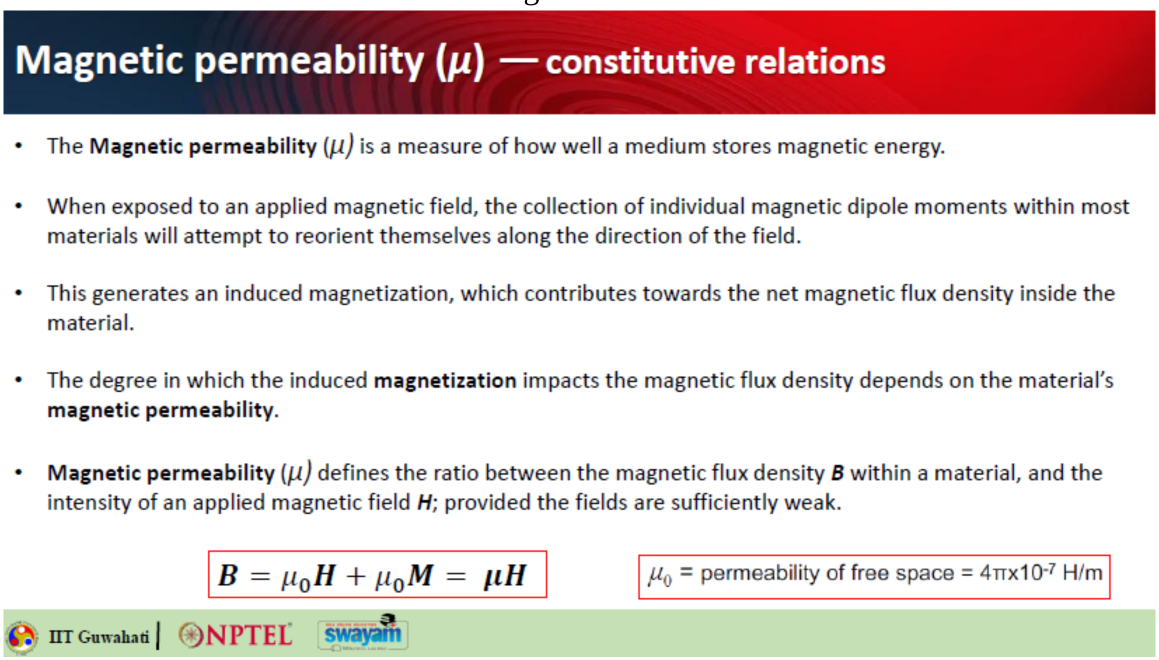
So overall, you can find out the displacement field $D = \varepsilon_0 E + P$. I think it's given here. So, the displacement field or electric displacement can be written as epsilon naught E plus P. And if you take this equation for P, you will get D equals epsilon E. So, ε_0 is basically the vacuum permittivity, but epsilon is basically the permittivity of this material.



Now, we'll define the constitutive relationships for linear, homogeneous and isotropic media. So, whenever I say linear media, it means the properties of the material do not depend on the strength of the field. And something like here that, polarization P is basically linearly proportional to the electric field applied.

So, $\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$. What is χ, χ is a scalar constant. It is called also electric susceptibility. So, when you write the equation of electric displacement field, $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \boldsymbol{E} + \varepsilon_0 \chi \boldsymbol{E}$. And you take ε_0E common, you will see 1+χ. And this one plus chi is nothing but your ε_r, that is the relative permittivity. So, simply you can say relative permittivity is nothing but1+χ, and all other parameters are already known.

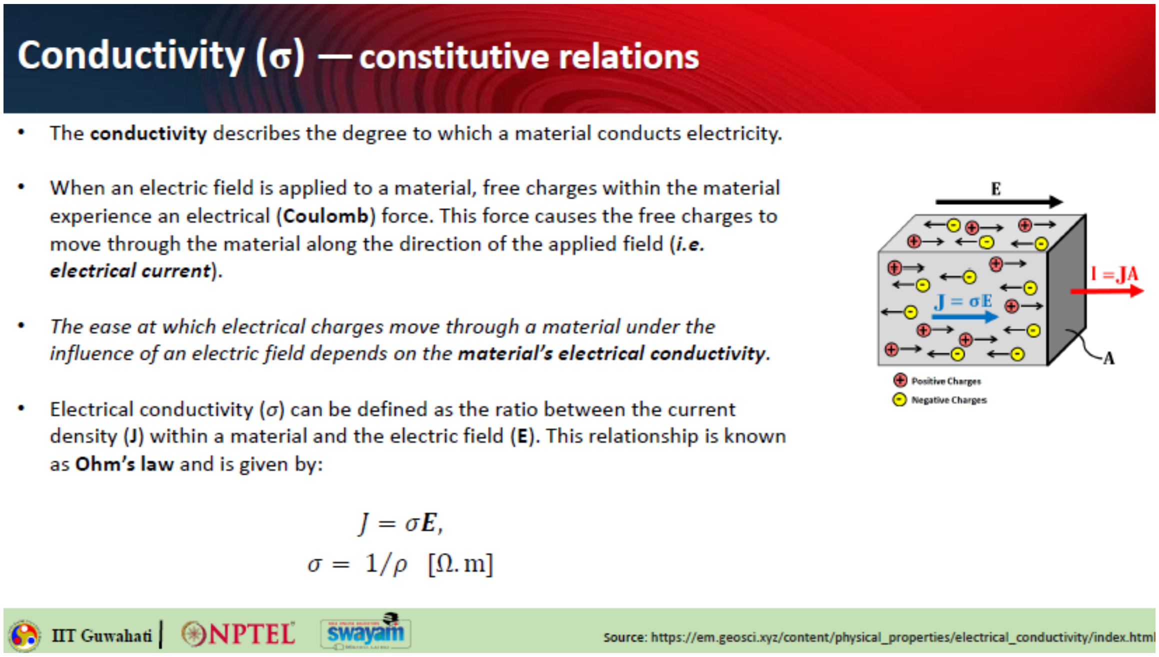
$\varepsilon_0 = 8.8541878176 \times 10^{-12}\,\mathrm{F/m}$. Now, inside a material medium, the permittivity is determined by its electrical properties. and permeability is determined by the magnetic properties. So, whenever you say permeability, that is μ, it is basically a measure of how well a medium stores magnetic energy. So, when exposed to an applied magnetic field, the collection of individual magnetic dipole moments within most materials will attempt to reorient themselves along the direction of the field.



Now, this reorientation will induce magnetization, which contributes towards the net magnetic flux inside the material. And the degree to which this induced magnetism impacts the magnetic flux density depends on the material's magnetic permeability. So, we'll see that soon that magnetic permeability is also defined as the ratio of the magnetic flux density within the material and the applied magnetic field. Provided that both the fields are sufficiently weak and we are talking in terms of a non-magnetic material here.

So, if you take this M=0, $B = \mu H$. So, this is a generic formula. You can write $\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M}$. But usually in optical field, we talk about materials which do not contain magnetic properties.

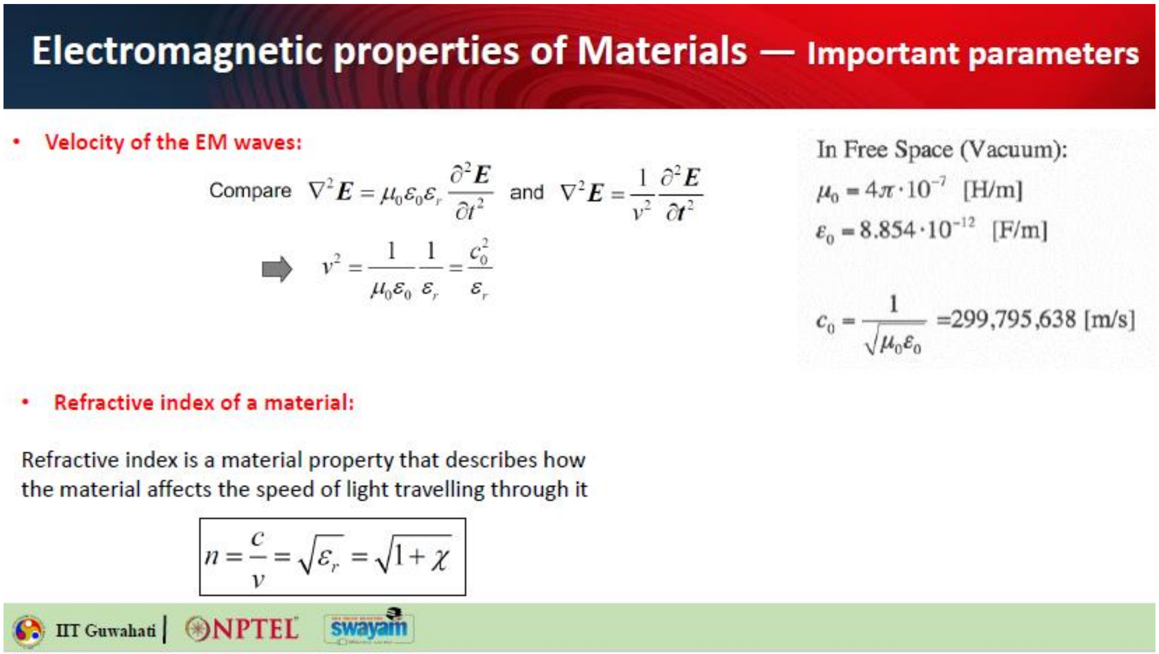
So, you can take magnetization as one or zero. You can remove this. Rather, μ_r=1. So, you can remove this and you can simply write B= μH. What is μ? μ is the permeability of the free space and that is given by $4\pi \times 10^{-7}\,\mathrm{H/m}$. And the third one we have seen that is conductivity.



So, conductivity describes the degree to which a material can conduct electricity. And what happens when an electric field is applied to a material? The free charges which are inside, these are not bound charges, the free charges inside the material, they will experience an electric force that is basically the Coulomb force. And this force will cause the free charge to move through the material in the direction of the applied electric field. So, electric field is applied in this direction. So, all the positive charges will move from left to right.

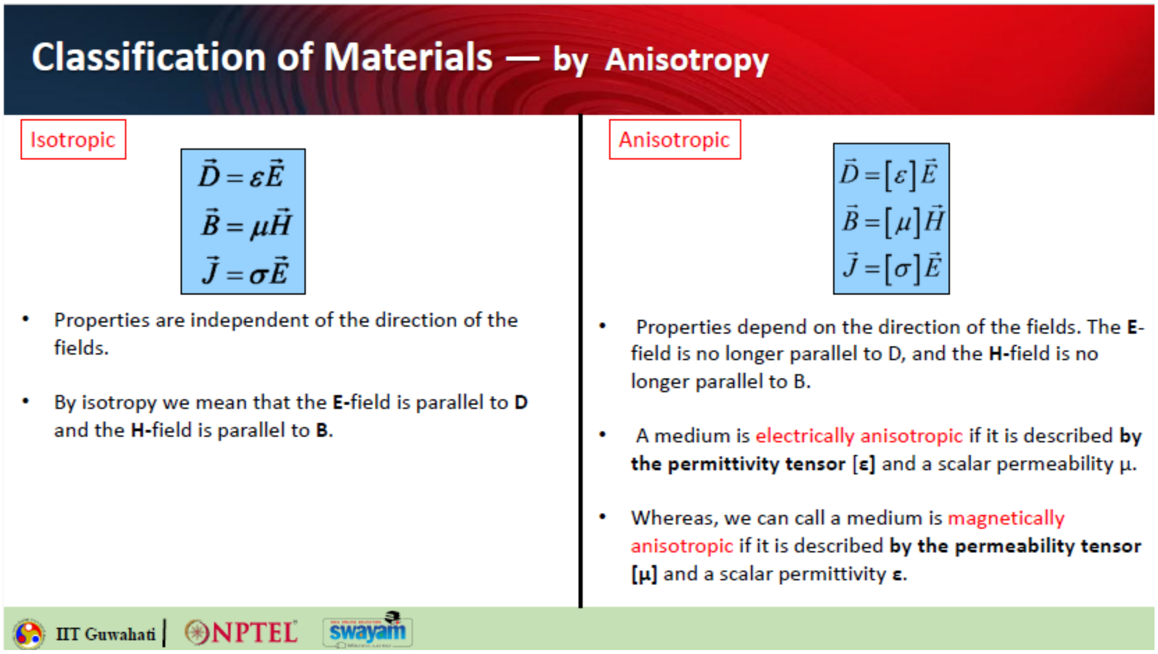
Whereas, the electrons will move on the other side. So, the ease at which an electric current or electrical charges can move through a material under the influence of an electric field, that depends on the material's electrical conductivity. So, if you denote this by sigma, electrical conductivity, it is basically J equals σE. So, what is σ? σ is basically J/E, that is the current density over the applied electric field. If you take that ratio, you will get electrical conductivity.

And inverse of sigma is also known as resistivity. So, sigma can be written as one over ρ, and the unit is ohm meter. Now, let us try to find out the velocity of electromagnetic waves. To do that, we can go back and revisit the wave equation. So, this was the wave equation we derived. And in these two formats, if you equate these two formats, you will see that this particular term, $v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_r}$.



From that you can find out what is v^2, $v^2 = \frac{c_0^2}{\varepsilon_r}$. Now, why I am actually showing this slide again? Because this slide actually gives a very important parameter in optics and photonics domain, which is called the refractive index of material. So, if you take the square root on both sides, you will get $v^2 = \frac{c_0^2}{\varepsilon_r}$.And you will see that that is basically nothing but square root of epsilon r is nothing but n, that is the refractive index of the material. In other words, you can say n=c/v. So, from that, you can also define refractive index as a ratio of the speed of light in vacuum over the speed of light in that particular medium, which has got the refractive index of n.

What are the other relationship? $n = \sqrt{\varepsilon_r}$. You can write ε_r as 1 +χ, so you can write $n = \sqrt{1+\chi}$. So, refractive index is a very, very important optical property of any material and it is defined as the ratio of the speed of light in vacuum over the speed of light in that particular medium. So, now let us do a quick classification of materials by its anisotropy. So, first, if we see if any material is isotropic, it means the properties of that material does not depend on direction of the fields.



Something like that, we can write it like this,

$$\vec{D} = \varepsilon\vec{E}$$
$$\vec{B} = \mu\vec{H}\,.$$
$$\vec{J} = \sigma\vec{E}$$

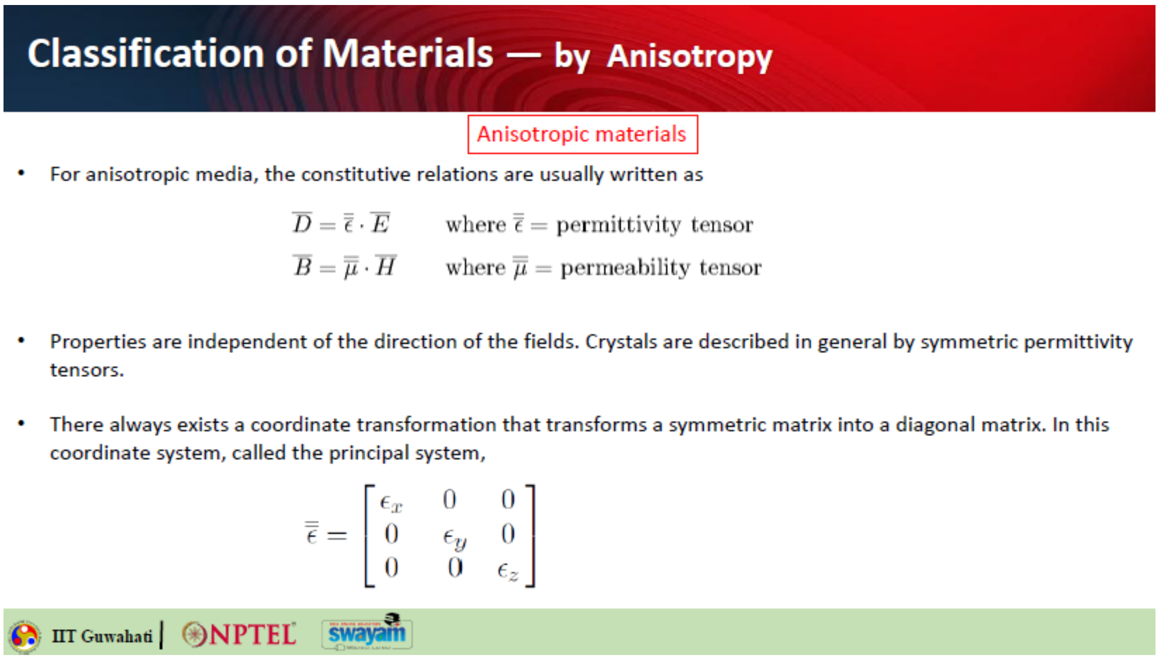
So, they are not ε, μ and σ are not dependent on the direction of the field, they are scalar quantity, so these are isotropic. And here you can simply mean that because they are not having any direction, so E field and D field, they are parallel, H and B will be parallel, E and J will be also parallel. So, that is isotropic case. Now, coming to the anisotropic case, obviously it means that the properties here, epsilon, mu and sigma, depend on the direction of the fields.

It means you can no longer say that E is parallel to D or H is parallel to B and so on. You have to compute whatever is the actual direction for each of these fields. So, in that case, when a material becomes electrically anisotropic, it has to be described by a tensor and we call this as permeability tensor and it will also have a scalar permeability. Whereas, when a medium becomes magnetically anisotropic, it is described by a permeability tensor, that is this one, and it will have a scalar permittivity. So, these are like a different notation of writing the same thing.

Permittivity tensor can be written as, this can be the tensor notation. So, this is how in anisotropic media, the constitutive relationships look like.

So,

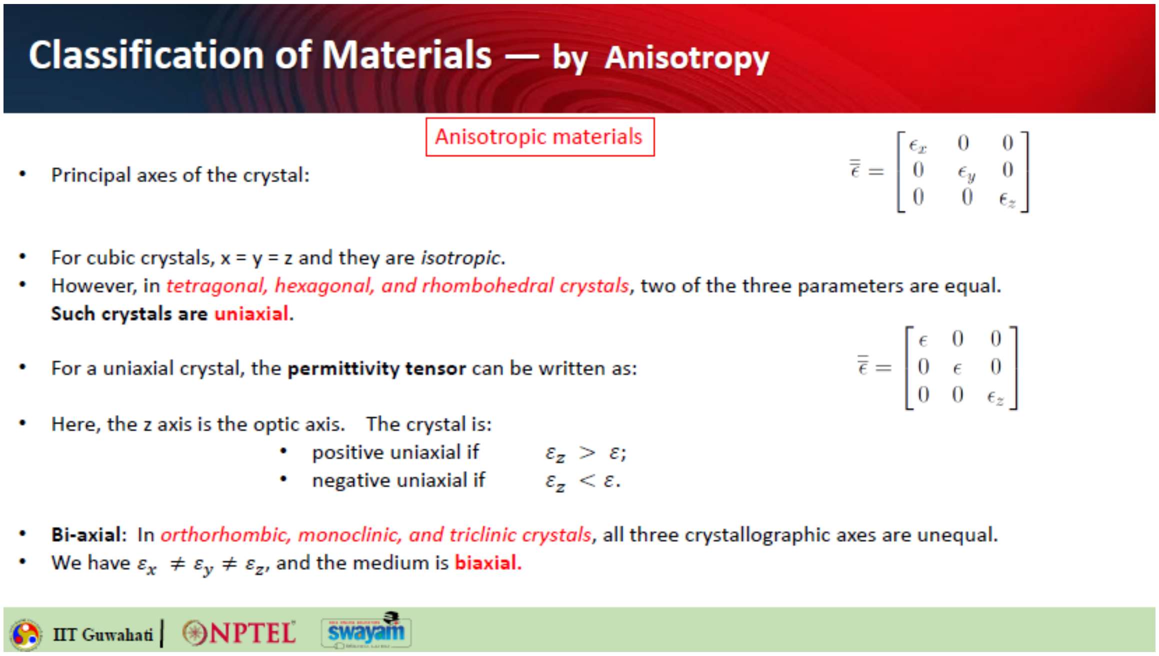
$$\vec{D} = [\varepsilon]\vec{E}$$
$$\vec{B} = [\mu]\vec{H} \ .$$
$$\vec{J} = [\sigma]\vec{E}$$



Now, the properties are independent of the direction of the field and the crystals are described in general by symmetric permittivity tensors. There always exists a coordinate transformation that would like to transform a symmetric matrix to a diagonal matrix. And in this coordinate system called the principal axis, the permittivity tensor will typically look like this. So, this becomes a diagonal matrix. From a symmetric matrix, you can get a diagonal matrix by doing some coordinate transformation.

So, here you can see, there is ε_x, ε_y and ε_z . It means there is a permittivity along x direction, a different one along y direction and different one along z direction. So, three direction, you have three different permittivity. So, this is how the principal axis for any anisotropic medium will look like. So, this is how the permittivity tensor looks like, it is a diagonal one.

So, all non-diagonal elements are zero. So, this helps in doing the computations. And if you take example of a cubic crystal where x, y and z are all equal, then this tensor also becomes isotropic. But in the case of other crystal types, something like tetragonal, hexagonal, rhombohedral, two of the three parameters are equal. Say, any two, let's assume that epsilon x and epsilon y are equal. So, these kinds of crystals are called uniaxial crystal.

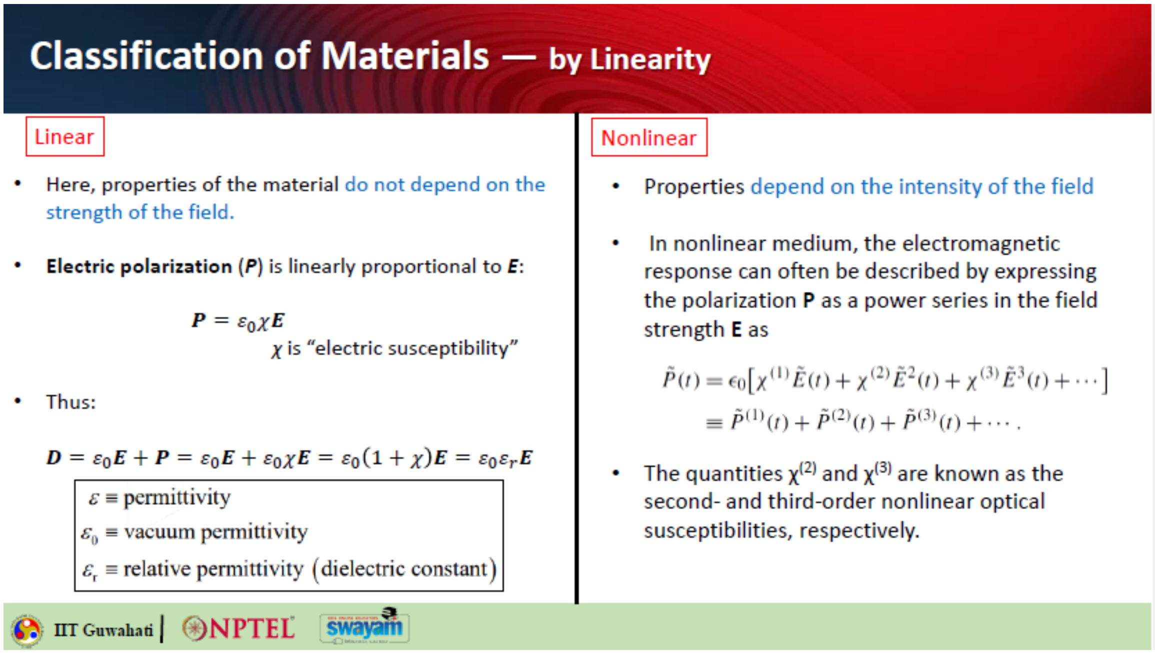


For the case of uniaxial crystal, the permittivity tensor can be written as this. So, you have the tensor where $\varepsilon_x = \varepsilon_y$. So, you can simply write them using epsilon. And this is different, ε_z is different. So, it means that along x and y, they have the same permittivity, but along z, they have a different one.

So, you can take z as the optic axis. And you can call this particular crystal, a positive uniaxial crystal, if the permittivity along z is larger than the permittivity along x and y. And you can call it negative uniaxial if it is other way, that epsilon z is less than epsilon. So, as such, you can actually think of an index ellipsoid kind of situation where ε_x, ε_y and ε_z are basically giving you an ellipsoid.

So, when is ε_x and ε_y are same, that means the cross section is same. So, this is becoming a circle. And then on top, you can think of, this is ε_z. So, if ε_z is larger than this one, then it is called positive uniaxial. If epsilon z is smaller than epsilon, it is negative uniaxial crystal. Now, there are other types of crystal as well which are called biaxial crystals, something like orthorhombic, monoclinic and triclinic, where all three crystallographic axes are unequal.

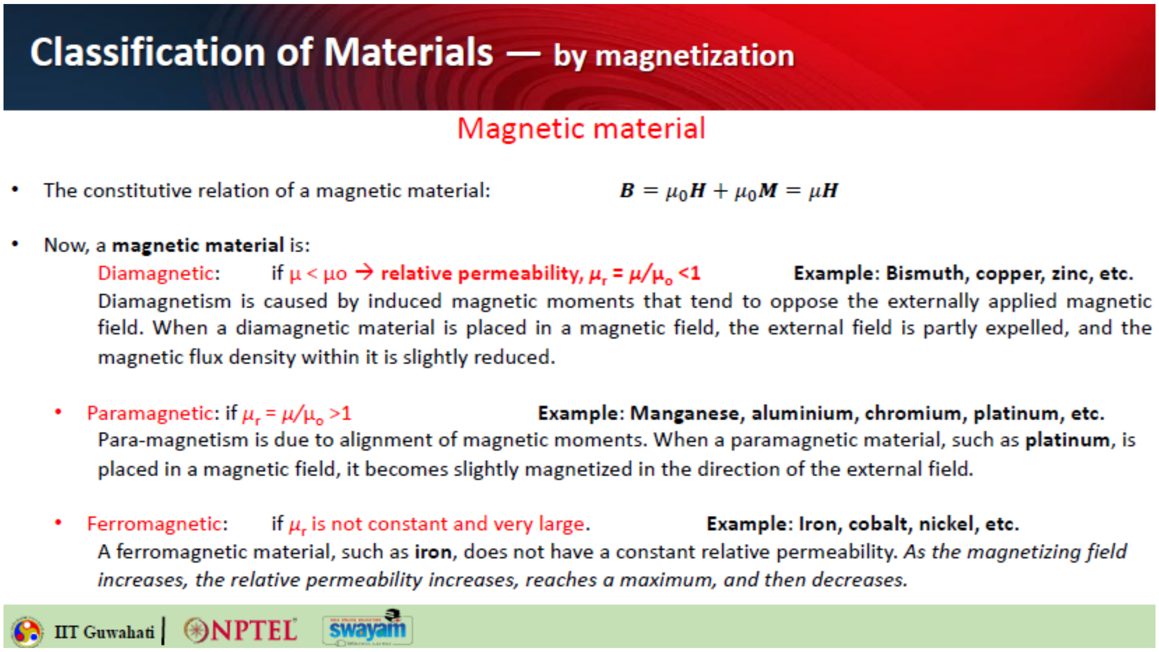
In that case, is ε_x, ε_y and ε_z, all are different. And this kind of medium is also called biaxial medium, okay. Understood. So, now we can also classify materials by linearity. So, when you say linearity, it means the property of the material does not depend on the strength of the field.



So, we can take like electric polarization, which is P and that is linearly proportional to the electric field E. There is a mistake in the equation, $\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$, this term is not there. okay. So, this χ is the electric susceptibility. So, from that, we can, we have seen this equation couple of times, that $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$.

So, when you put P, P is only this term, not this term, this one, you simply cross, I will just cut it here. This term is not there, okay. So, you can simply take, $\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}$ this is a typo. So, when you put it back here, you get $\varepsilon_0 \varepsilon_r \boldsymbol{E}$ So, you can write $\boldsymbol{D} = \varepsilon_0 \varepsilon_r \boldsymbol{E}$. So, this is for the linear materials. Now, obviously, nonlinear materials, the property depends on the intensity of the field. That means, in nonlinear medium, the electromagnetic response can often be described by expressing the polarization as a power series of the field strength E. That is, you can write the polarization in terms of $\chi^{(1)}\tilde{E}(t)$, $\chi^{(2)}\tilde{E}^2(t) +$ $\chi^{(3)}\tilde{E}^3(t)$ and so on. You can look for other higher order terms as well. So, here, it was only the first order, but here you can see, you have got second order, third order and all other orders possible.

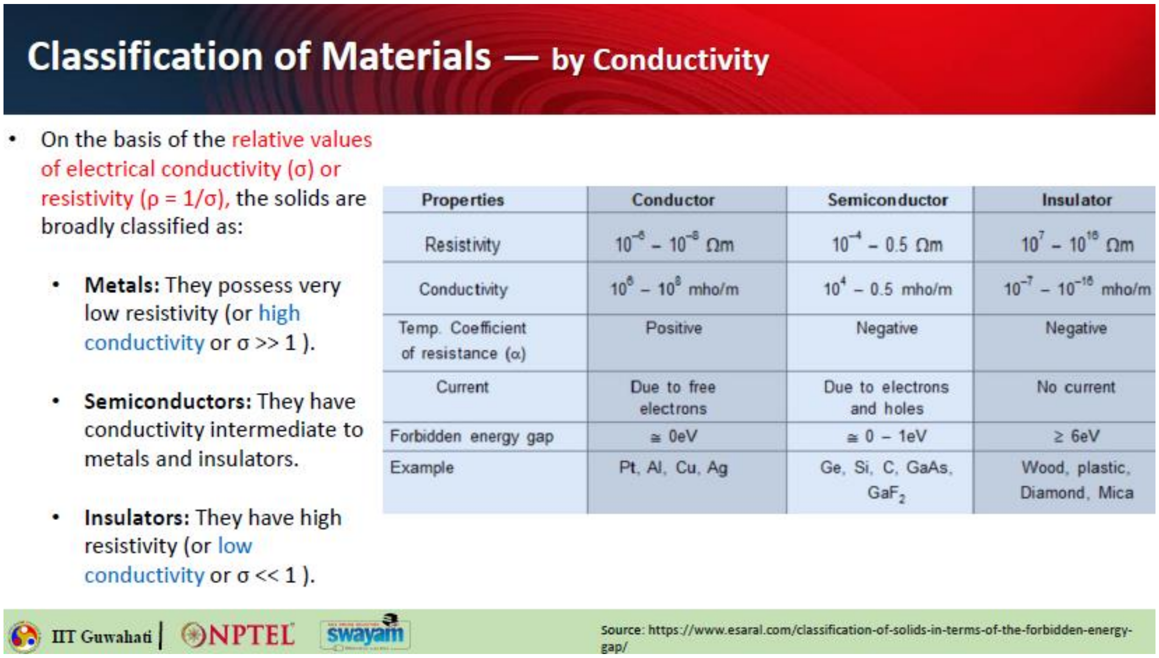
So, the chi square and chi cube, these are called the second and third order nonlinear optical susceptibilities respectively. So, there are materials which show this nonlinear susceptibilities as well. Now, materials can be also be classified by magnetization properties. So, we can have magnetic properties where, the constitute relationship becomes B =μ H. Now, a magnetic material can be of roughly three types.



One is diamagnetic, that is, when the permeability $\mu < \mu o$, that is, the relative permeability is less than one. For example, bismuth, copper, zinc, etc. So, in this kind of material, diamagnetic material, it is caused by the induced magnetic moments. They tend to, oppose the externally applied magnetic field. So, when a diamagnetic material is placed in an external magnetic field, the external magnetic field is partly repelled and the magnetic flux density inside the material slightly reduces.

And that is how they reduce then the permeability in the surrounding or vacuum. So, that is how it is, $\mu_r = \mu / \mu_0 < 1$. So, on the other hand, there are paramagnetic materials. So, in that case, paramagnetism is basically due to alignment of the magnetic moments. So, when a paramagnetic material such as platinum, chromium, aluminum, manganese, these are placed in magnetic field, they become slightly magnetized in the direction of the external field.

In that case, you will have μ_r, that is $\mu_r = \mu / \mu_o > 1$. You can also have material which are called ferromagnetic. So, if the relative permeability is not constant and it is very large, such as in the case of iron, cobalt, nickel, these are called ferromagnetic material and they do not have any constant relative permeability. As the magnetization field increases, the relative permeability also increases, it reaches the maximum and then it decreases. So, further we can classify materials based on conductivity. This we all know that this is basically on the basis of the relative values of electrical conductivity or you can classify based on resistivity, which is the reciprocal of conductivity.



So, there are three types of materials or solid can be described as metals, which have got very high conductivity or very low resistivity. You have semiconductors, which have intermediate conductivity between metals and insulators and insulators are those which are having very high resistivity or very low conductivity. So, this is typically the scale of the properties. So, for conductor, semiconductor and insulator, you can see resistivity is in the order of 10^{-8} to 10^{-6} ohm cm. Whereas in the case of insulator, it is somewhere 10^{7} to10^{18} that's huge difference. Similarly, the conductivity, you see the conductivity for conductors are basically 10^{6} to 10^{8} mho/m. mho is basically the opposite of ohm. And whereas in insulator, it is 10^{-8} to 10^{-6}. You can think of the currents, in conductor, it is mainly because of the free electrons.

In semiconductor, you all know it is due to both electrons and holes. Whereas in insulator, there is no current. There is no band gap in conductors. There is a band gap from 0 to 1 electron volt for semiconductors. And for insulators, it is mostly more than 6 electron volts. And these are the examples of the common semiconductors, conductors and insulators, you all know.

So common insulators are wood, plastic, diamond, mica and all this. You have metals, conductors and germanium, silicon, gallium arsenide, these are the common semiconductors. So with that, we will stop here and in the next lecture, we will see the propagation of electromagnetic waves in dielectric medium. And in case you have got any query, as I mentioned before, you can drop me email at this particular email address. Thank you.