Course Name- Nanophotonics, Plasmonics and Metamaterials

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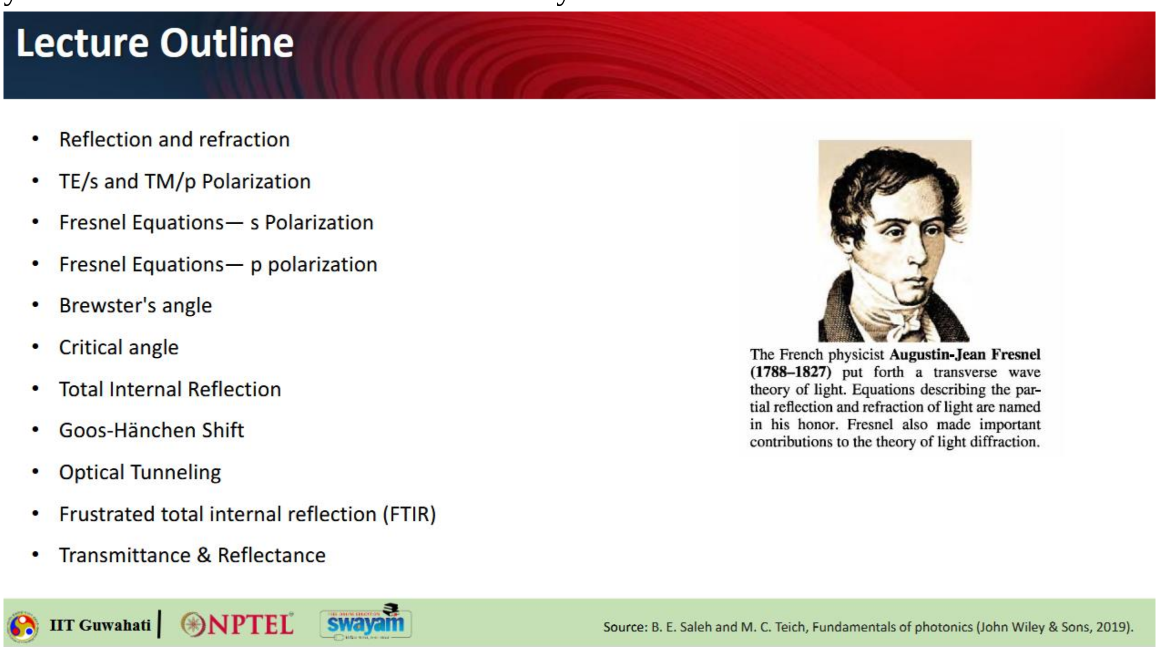
Week-03

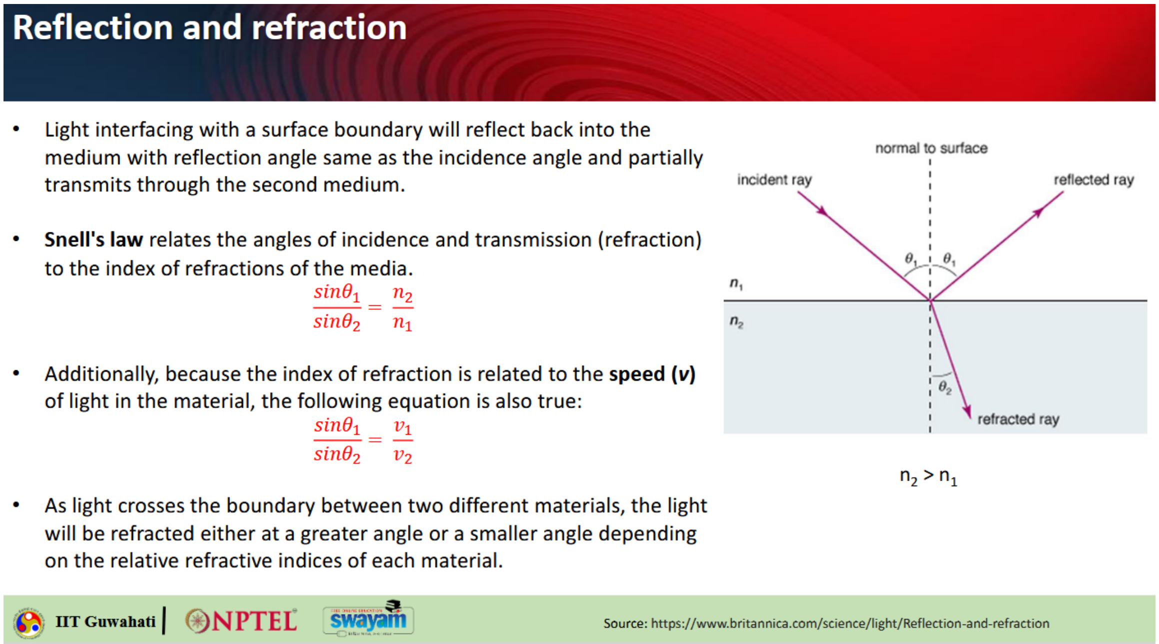
Lecture -08

Hello students, welcome to lecture 8 of the online course on Nanophotronics, Plasmonics and Metamaterials. Today's lecture will be on reflection and refraction, mainly the Fresnel equations. So here is a photograph of French physicist, Austin Jean Fresnel, who put forward a transverse wave theory of light. And the equations describing partial reflection and refraction are basically named in his honor. So Fresnel has made important contributions in the theory of light diffraction. So here is the lecture outline, as you can see we will discuss briefly what is reflection and refraction.



I am sure everybody knows about it, still a very brief overview. Then we will revise the basics of TE which is s polarization and also TM which is p polarization of light. And then we will see how to derive the Fresnel equations for s and p polarization. While doing that we will also introduce the concept of Brewster's angle, critical angle, total internal reflection, Gouge-Henshin shift, optical tunneling, frustrated total internal reflection, and finally we will see how to calculate transmittance and reflectance.

So let us look into what is reflection and refraction. So this particular diagram I am sure you have seen this many many times in your school days as well. So here is an interface between two dielectric medium, one has got a refractive index of n_1 and other has got refractive index of n_2. Let us assume that the incident ray is basically a plane electromagnetic wave which is traveling in medium n_1 and after traveling some distance So this is the interface of the boundary between two dielectric medium. And as soon as there is an interface or boundary where there is refractive index mismatch or there is impedance mismatch, we know that there will be reflection.



So some portion of this incident ray will be reflected back in the same medium. So how do you know the reflection angle? We have seen or we have proven that reflection angle is equal to the incidence angle. I am not going to derive that because that is very very basic that you have studied in school. And how do you measure this angle? This angle is measured with respect to the normal to the surface of the interface. Now some portion of the light is getting reflected.

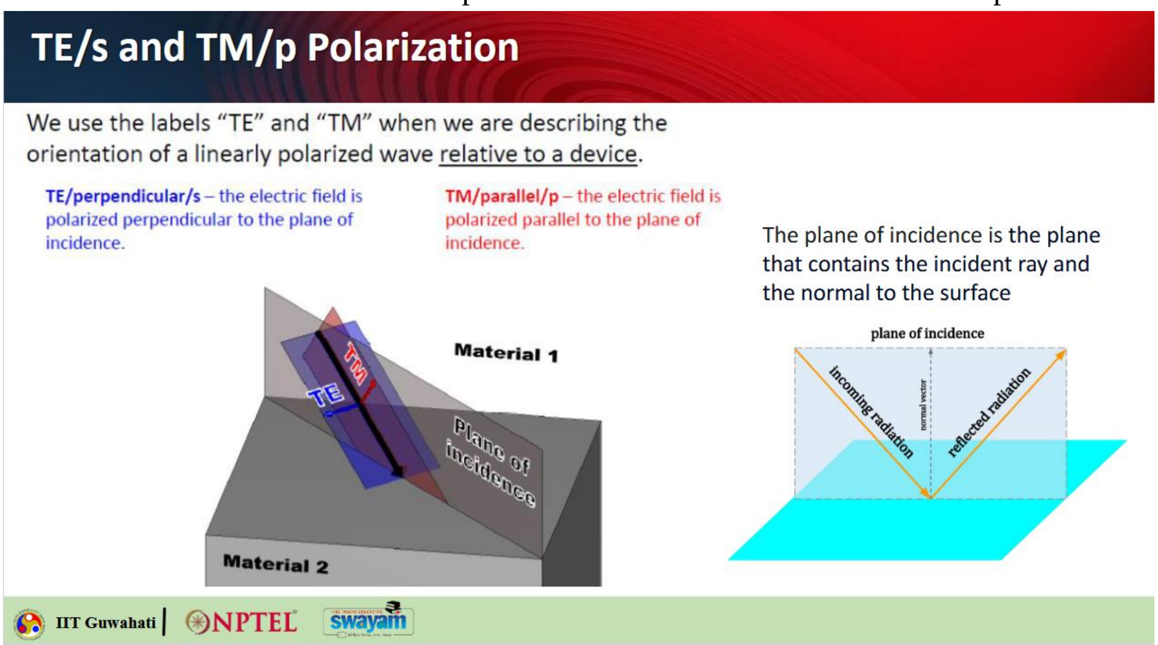
What is happening to the remaining light? Remaining light actually enters the second media that is n_2. So n_2 medium or you can say medium 2 which has got a refractive index of n_2. Same thing. Now depending on whether n_2 is greater than n_1 either close to the normal or it will be away from the normal. So, in this case let us assume this particular case where n_2 is greater than n_1.

In that case the light has actually entered a denser medium from a rarer medium. In that case the refracted ray will be closer to the normal. So this particular law of refraction is also known as Snell's law. So Snell's law basically relates this angle theta 1 that is the incidence angle with the refraction angle that is theta 2. And how they are related?

They are related by this equality which is $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$ or you can write $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

Now we have seen refractive index is nothing but the ratio of the speed of light in vacuum over the velocity of light in that particular medium. So, if you write n_2 as c by v_2 similarly n_1 can be written as c by v_1. It means you can actually replace this ratio with the ratio of the speed of light in each of this medium n_1 and n_2. So, if you consider those speed as v you can write that $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$. So, I am also calling them like v_1 by v_2 that is also same thing as v_1 over v_2.

So they are the same thing. So this is how you can also take Snell's law. You can understand the meaning of Snell's law that sin theta 1 over sin theta 2 is nothing but v_1 over v_2 the ratio of the velocity of light or plane wave in these two particular mediums. Now let us move on to the concept of TE and TM polarization. In the previous lecture we have seen that TE polarization is also called s polarization.



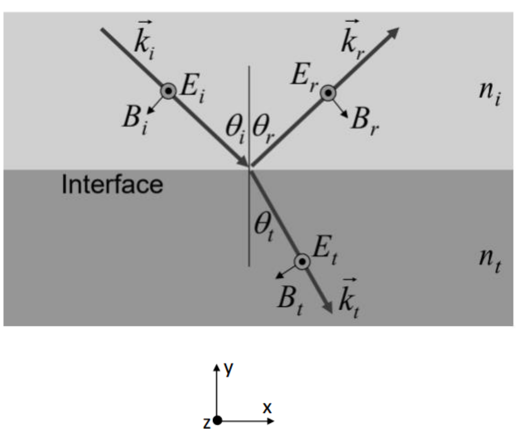
S standing for strike out. It means we will see that in such cases where you have two different materials, the material 1 and material 2 and this particular is the interface of the boundary and there is an incident ray there will be a reflected ray something like this and there is a normal to the surface. So you can draw a plane that contains all these three vectors that is the incident or incoming radiation, reflected radiation and the normal. So this particular plane is called the plane of incidence. Now in this plane of incidence there are two possibilities of the electric field vector to lie that is what the polarization is.

So you can either have the electric field vector that shown as red arrow here, that is basically parallel to the plane of incidence. So we can say this is parallel polarization or p polarization. There is other case also possible that is this blue case. In blue case what happens the electric field vector is basically out of the or it is basically striking out of the plane of incidence. In that case you can call as s polarization.

So this is s (TE). So, you can say TE or perpendicular or s they are all carrying the same meaning. So these are the two polarization. Now why we are studying this because when a plane wave with a particular polarization is incident at the interface the way the polarization will interact with the interface will be slightly different for S and P polarization. That is why we have to analyze the reflection and refraction for these two polarization cases differently.

Here we are not bothered about the polarization we have not discussed about polarization but then we understood that with polarization the reflection of the light will be different and that is what the contribution of Fresnel. So Fresnel equations tells us to calculate or teaches us how to calculate the reflection and refraction or transmission. Refraction can be taken as transmission also. So it can tell us how to calculate the coefficient of reflection and transmission. So, let us assume this particular case we are assuming the case of S polarization first.

Fresnel Equations— S-Polarization



- Beam geometry for light with its electric field **sticking up out of the plane of incidence** (i.e., out of the page)
- Boundary Condition for the Electric Field at an Interface:
  The Tangential Electric Field is Continuous
- The component of the E-field that lies in the xz plane is continuous as you move across the plane of the interface.
- Here, all E-fields are in the z-direction, which is in the plane of the interface.
- Thus, $E_i(y = 0) + E_r(y = 0) = E_t(y = 0)$

Now this is everything in the plane of incidence as you can see this is a 2D diagram. So this is how the coordinates are marked. This is x axis this is y axis and z axis is basically coming out of the screen. Now here you can see clearly that this is the wave vector ki. i stands for incident.

So this is the incident ray. Then there is the reflection that is kr that is a vector and then you have this transmitted one that is called kt. This is ni the incident medium refractive index nt is the transmitted medium refractive index. This is the interface. Angles can be marked very easily this is theta i this angle is theta r and this angle is theta t clear all these things are marked.

Now we are assuming that this particular case is for S polarization. Now we know that for S polarization the electric field is actually striking out. So this is how you put striking out if the field used to go in you will show it like a cross. So this is where the field is basically coming out. So when you have this field coming out upon reflection also it remains same.

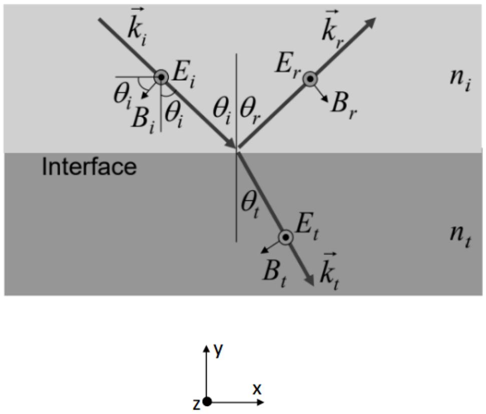
And while transmission also it remains in. So the electric field is throughout in the S polarization that is out of this page. Now accordingly you can also mark what will be the magnetic field direction. So how you do that? So always remember E cross B will give you the direction of wave propagation. So here the wave propagation direction is known E we have assumed that we are considering the one that is striking out.

So you can take E cross B and that should come in this particular direction that is in the direction of k vector. So that way we will find that the *Bi* that is the magnetic field vector is basically in this direction for the incident wave. For the reflected wave it is in this direction and for the transmitted wave it is in this direction. You can easily do E is up E cross B the thumb is showing the direction of wave propagation. So that way you can always verify that whether these directions are marked correctly or not.

Now let us apply the boundary conditions. So the boundary is basically here where Y equals 0. And we have known from Maxwell's equation that in the boundary the tangential fields, the tangential electric field and tangential magnetic field they are continuous. So let us find out the component of the electric field that is lying in this particular interface. So the component of the electric field that lies in the xz plane which is this particular interface xz plane is the interface is continuous as you move along the across the plane of interface make sense.

And here all these electric fields are all only in Z direction. So, when you write at this particular interface you can write $E_i(y=0)+E_r(y=0)=E_t(y=0)$. So these are the two fields on the one side of the interface and this is the field that is on the other side of the interface. So exactly at the boundary you can write that these two fields will add up and give you this field.

Fresnel Equations— S-Polarization



- Boundary Condition for the Magnetic Field at an Interface:

  The Tangential Magnetic Field is Continuous

- The total B-field in the plane of the interface is continuous.

- Here, all B-fields are in the xy-plane, so we take the x-components:

$$-B_i(y = 0)\cos\theta_i + B_r(y = 0)\cos\theta_r = -B_t(y = 0)\cos\theta_t$$

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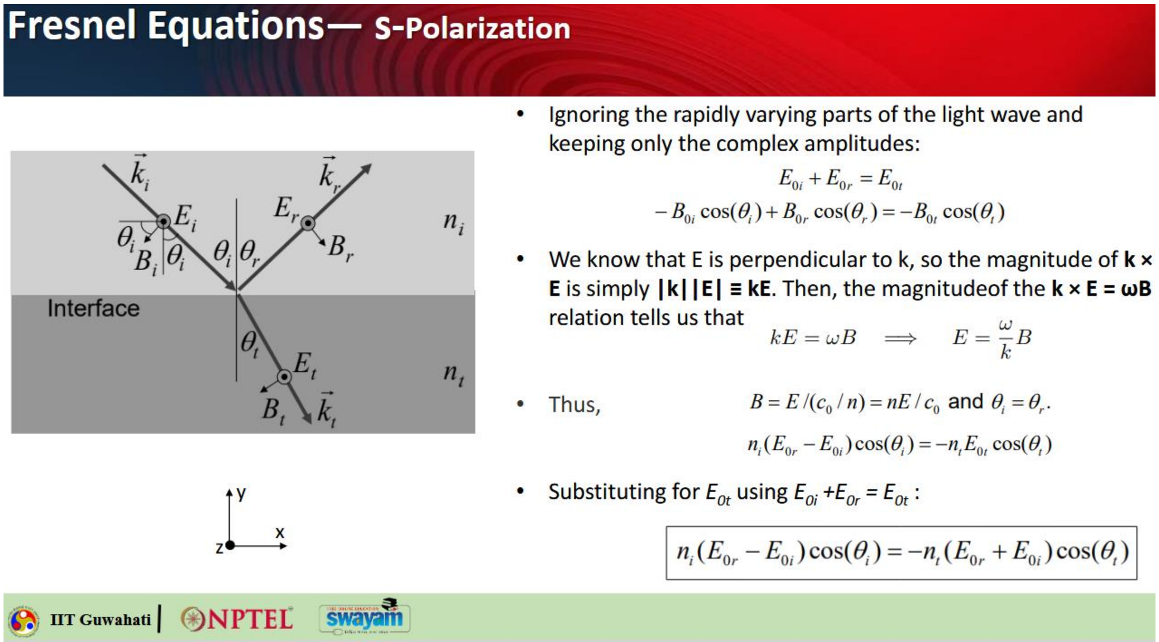
Okay. Understood. So this is the boundary condition for electric field. Now let us apply the boundary condition for magnetic field. So, we know that the tangential magnetic field should also be continuous. So, in that case we have to find out what is the tangential component of the magnetic field because the magnetic field vector is making an angle with the interface.

Right. So if this angle is theta i we can find out that this angle is also theta i and again we can say that this angle is also theta i. So Bi will have a x component it is going in the minus x direction. Okay. So, the x component which is along this will be $-B_{0i}\cos(\theta_i)$.

Right. So minus *Bi* cos theta i and we are equating it at the interface. So Y equals zero. Similarly, if you take this one you can find out the component is nothing but this will be along plus so there is no question of putting a minus here. So, you can simply write $-B_i(y=0)\cos\theta_i + B_r(y=0)\cos\theta_r = -B_r(y=0)\cos\theta_t$. Again, this one this angle will be theta t, I am not sure just do the same thing in all the cases here also you will see that this component is minus B_t cos theta t and B_t should be evaluated at Y equals zero.

So you have got the two equations. Right. One is this one that tells you about the continuity of the magnetic field or the tangential magnetic field at the interface. You also have the other equation that is this one $E_i(y=0)+E_r(y=0)=E_t(y=0)$. Okay that the tangential component of the electric field are continuous. This is the tangential magnetic field across the interface is continuous.

So you have got these two equations. Now one equation is in the form of ***E*** another is in the form of ***B***. So we need to now convert everything to electric field because that is how we will be able to find out the ratio of the electric field vector that is reflected over the incident reflected field that will give us the reflection coefficient. That is our final goal. And also, we have to find out the ratio of E_t over E_i that will give you the amount or the ratio of the electric field that is being transmitted. So, with that objective let us see how we can proceed further.



So, we know that E is perpendicular to k that we have seen. So, you can also write that ***E*** cross k can be simply written as so the cross product so they can be written as $|k||E|$. So, you can simply write ***k E*** fine. The other thing would be that this vector $k \times E$ can also be written as omega ***B***.

Okay. So, this is the relation we know that $k \times E = \omega B$. Okay. And you can write $E = \frac{\omega}{k} B$. So, this is how you know electric field and magnetic field are related. So, if you take that you can actually convert all the magnetic field here into electric field.

Okay. So, ***B*** can be written as E divided by this factor $\frac{\omega}{k}$ and $\frac{\omega}{k}$ can be also written as (c_0/n). What is n? n is the refractive index of the medium. So, if you simplify this equation it becomes nE/c_0. So that's much simpler form. So, ***B*** the magnetic field can be written as nE/c_0 .

And we already know that θ_i equals θ_r. Okay. So, with that if you put it here so θ_i and θ_t are same they can be written as theta i altogether. So, you take cos theta i common. So, what you are left with this term and this term they are both converted into electric field now.

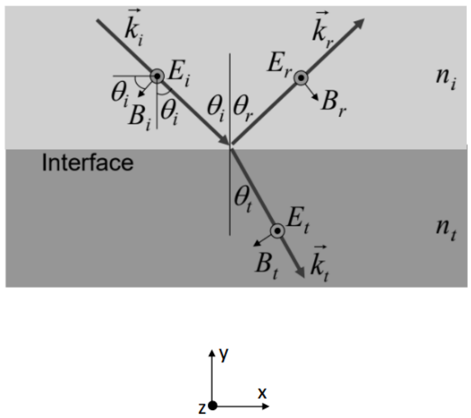
So you get n_i common. Okay. And then you have ***E***. Why n_i for both? Because both are in the same medium n_i. Right. Reflection and incidence are in the same medium. So, you have got n_i that this one $\left(E_{0r} - E_{0i}\right)\cos\left(\theta_i\right)$.

Right. And then you will have this term which is minus n_t because n_t is the refractive index of the medium where the transmission is taking place. So, you have $n_t \cos(\theta_t)$ and then you will have also E_{0t}. So, finally you see you have got everything in terms of the electric field. Now you can substitute E_{0t} using E_{0i} plus E_{0r} from the first equation. And once you do that you get everything in terms of E_{0i} and E_{0r}.

So E_{0t} is also removed. So, this is the thing. So, now if you take this equation and rearrange the terms and put everything like E_{0r} on one side and E_{0r} on another side you get this is what you are getting. This is simply you know side changing and finally you get this. And from this equation it is very simple. You can simply do E_{0r} over E_{0i} and that is your reflection coefficient.

So you can call it as small r. And why it is given perpendicular because you are doing it for s polarization. So, you can get this one. So, r perpendicular is nothing but E_{0r} over E_{0i} and that is nothing but the ratio of this over this. So, you get $\left[n_i \cos\left(\theta_i\right) - n_i \cos\left(\theta_i\right)\right] / \left[n_i \cos\left(\theta_i\right) + n_i \cos\left(\theta_i\right)\right]$. So, this is the equation for reflection coefficient for s polarized light.

s-Polarization— Reflection & Transmission Coefficients



Rearranging $n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0r} + E_{0i})\cos(\theta_t)$ yields:
$E_{0r}\left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right] = E_{0i}\left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right]$

- Solving for E_{0r} / E_{0i} yields the reflection coefficient:

$$r_\perp = E_{0r} / E_{0i} = \left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right] / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

- Analogously, the transmission coefficient, E_{0t} / E_{0i} is

$$t_\perp = E_{0t} / E_{0i} = 2n_i\cos(\theta_i) / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

- These equations are called the Fresnel Equations for perpendicularly polarized (s-polarized) light.

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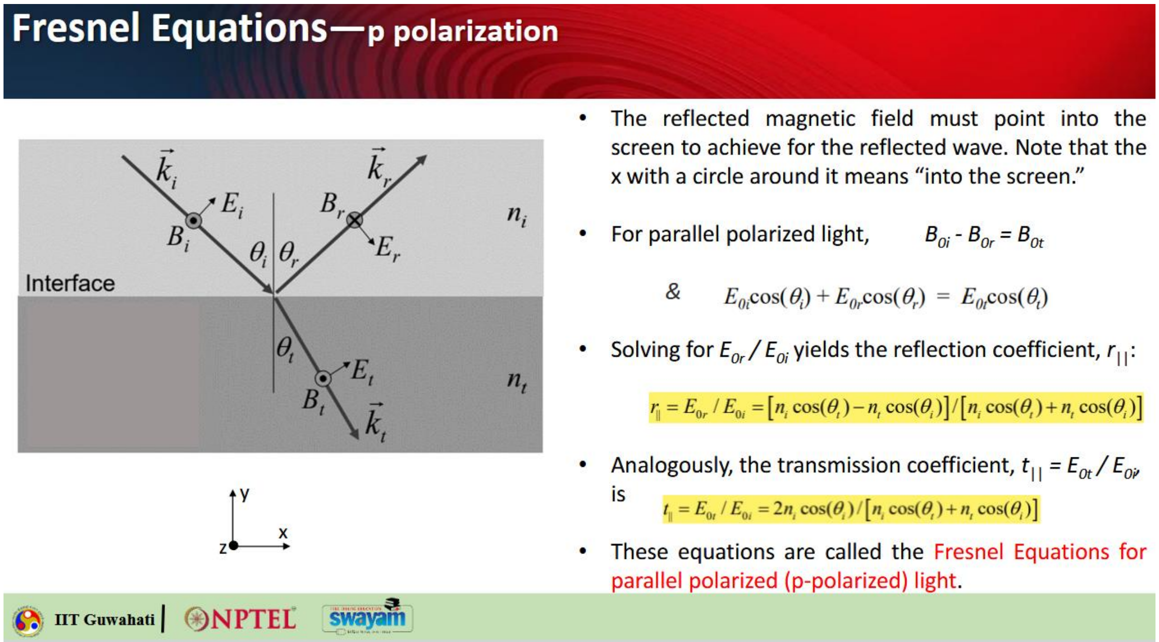
Similarly, you can calculate what is it for E_{0t} over E_{0i} and you will see that you are getting this particular equation that is $2n_1\cos(\theta_i)/\left[n_i\cos(\theta_i)+n_t\cos(\theta_t)\right]$. You see the denominator remains same only the numerator is changing. So, these equations are called the Fresnel equations for perpendicular polarized light or s polarization or TE polarization. So, the same exercise needs to be also done for p polarization. So, in p polarization the electric field is basically parallel.

So we take this particular direction of the electric field. So, this is this wave directions are fixed. So k_i, k_r, k_t they are all known. So, this is how the reflection and transmission will look like. Now if you assume that the electric field is along this direction again using that you know E cross B should be in the direction of k you can find out the direction of B_i that is the magnetic field.

In this case it is actually coming out of the page. Given reflection this upward electric field will become downward like this. So, in that case the magnetic field will also change the orientation it will go into the page. But transmission will have more or less similar feature that of the incident one just that the transmitted ray will be shifted more towards the normal if we assume that this is a higher refractive index material than this one. So, analysis remains very much similar. So first as we have actually done the calculation for the previous case it was straightforward for electric fields.

Here the magnetic fields are only along z direction. So, the tangential component of the magnetic field at the interface will be easier to find out. Let us assume the direction upward as positive and inside it is going this direction we can take as negative. So here you can write that B_{0i} minus B_{0r} will be equal to B_{0t}. So, this is positive, this is positive, this is negative.



So this is the equation that you get. And for the electric fields you can see that this is the tangential component that is E_i cos theta i. This one will have E_r or you can say E_{0r} cos theta r and this one will have E_t cos theta t. So, this is how the components will look like. So, this is the equation that tangential electric field is also continuous. Now what next? We need to convert this magnetic fields into electric fields.

How do you do that? $B = nE / c_0$. Go back. nE / c_0. So, this equation you can use here and you get everything in terms of electric field and then when you take the ratio of E_{0r} over E_{0i}, so you will get this is the ratio $\left[n_i \cos(\theta_t) - n_t \cos(\theta_i)\right] / \left[n_i \cos(\theta_t) + n_t \cos(\theta_i)\right]$. So, they look similar but they are not same. If you put both the equations side by side you will be able to see the difference.

So just a quick look here it is $n_i \cos(\theta_i)$. Here also it is $n_i \cos(\theta_i)$ but in the case of p polarization it is basically $n_i \cos(\theta_t)$. Here also $n_i \cos(\theta_t)$. So that is how they are different. Similarly, you can also find out what is the transmission coefficient that is basically E_{0t} over E_{0i} and this is the ratio. Always remember that reflection coefficient, transmission coefficient that denominators are same, the numerators are different.

So this is the equation called Fresnel coefficients or Fresnel equation for parallel polarized or p polarized or TM polarized light. So, this is the summary. Here I am putting the equation side by side you will be able to appreciate it more. So, this is the reflection coefficient for perpendicular or s polarized.

This is for t perpendicular. This is t parallel. This is r parallel. So, all these are the four equations that will tell you the reflection and transmission for s or p polarized light. Well and good. Now if the light is normally incident that is your theta i equals 0. So, when you put theta i equals 0 you will see that the reflection coefficient for parallel and perpendicular polarization they become same.

Fresnel Equations— Summarize

s-polarized light:

$$r_\perp = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_\perp = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

p-polarized light:

$$r_\parallel = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

$$t_\parallel = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

Putting $\theta_i = 0$, $n_i = n_1$, and $n_t = n_2$

$$r_{//} = r_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$

and $$t_{//} = t_\perp = \frac{2n_1}{n_1 + n_2}$$

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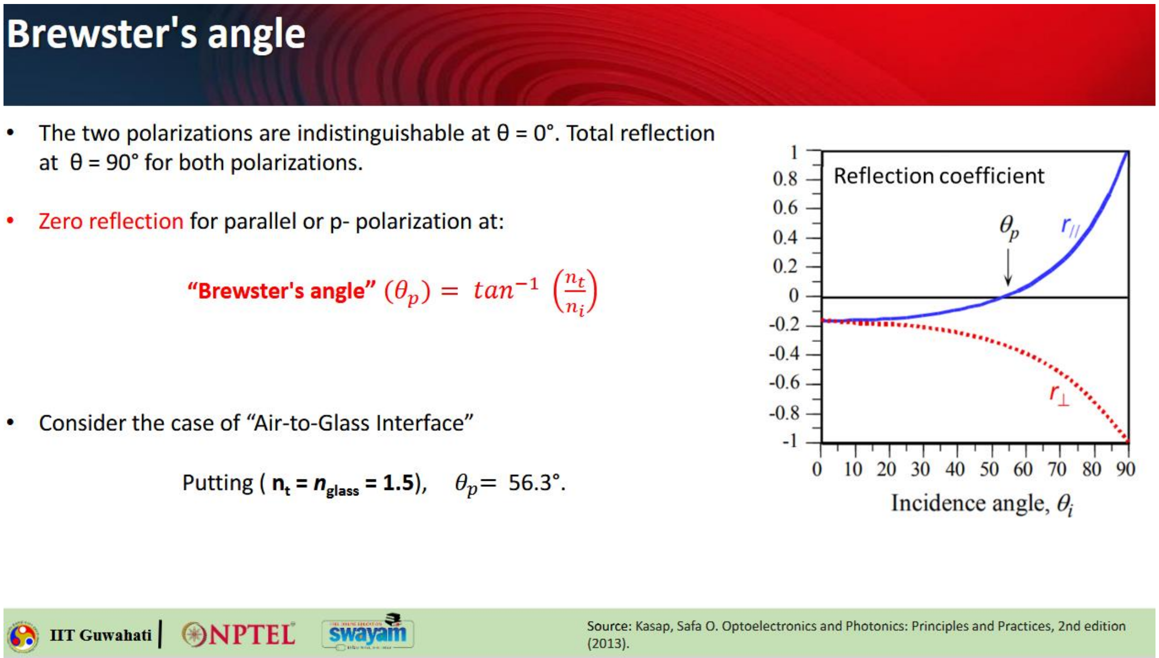
It turns out to be $\frac{n_1 - n_2}{n_1 + n_2}$. Similarly, the transmission coefficients t parallel and t perpendicular also become same. They become $\frac{2n_1}{n_1 + n_2}$. It's very simple. You put theta i equals 0 everywhere.

And theta 2 you can also find out using Snell's law. That is your theta t because you know n_1 and n_2. So that way this equation gets simplified. It is also very clear from this particular case if you go back to the definition that if you go here and you see that when the incident angle is 0 that is you are actually having light incident like this. In that case this plane and that plane they are also identical. Because in that case you will not have a unique plane you can have plane either this way or that way because incident beam and reflected beam are now in the same direction or like incident beam coming up and up from top to bottom reflection will go from bottom to top but they are along the same line because you are having normal incidence.

In short in normal incidence your both polarization become same or interchangeable. That is why the equations also show the same. So, you will get similar reflection and transmission coefficient for s and p polarization when you have normal incidence. Now there is a concept called Brewster's angle that is very important. So, we have understood that the two polarizations become indistinguishable when theta is 0.



And you will also see that there is total reflection when theta equals 90. It means when incident angle is 90 that you are actually having a light that is incident along the interface what you will see, you will see the transmission is basically 0. So total reflection is possible when angle incident angle is 90. Now there is a possibility of 0 reflection based on the equations that if you look for p polarization when you look for p polarization in this case there is a possibility that reflection can be 0 when this term and this term becomes 0. And that is the case or that is the particular angle which is called the Brewster's angle.

So you can have $\left(\theta_p\right) = \tan^{-1}\left(\frac{n_t}{n_i}\right)$ Now if you consider air to glass interface and air if you take as 1 and n_t that is the transmitted refractive index can be taken as n glass which is 1.5. In that case you can calculate and find out that theta p is equal to 56.3 degree.

So this is a plot that shows the reflection coefficient versus the incident angle. So, as you can see when the incident angle is 0 both the polarization this is for parallel polarization this is for s polarization or perpendicular polarization. So, both polarizations are having the same reflection coefficient when the angle is 0 but as the angle increases they diverge both are having different values. But when the angle strikes 56.3 degree for the case of parallel polarization that is the case when the reflection coefficient will be 0. For parallel polarization it means the light will be only available in one particular polarization.

So Brewster's angle can be used to make a unpolarized light polarized by means of reflection only. So, if you actually change your incident angle to this particular angle 56.3 degree at an air glass interface means one side is air another side is glass you are shining light on the glass. If you shine light at a 53.

6 degree angle the reflection light will be only perpendicular polarized. That means it will be only s polarized the p component is not there. So, this is Brewster's angle. There is another important concept called critical angle which you must have studied in those school days also. So, these are all coming from the Snell's law on different kind of possibilities that if you assume that this is a denser medium and this is a lighter medium and that is written here n1 is greater than n_2 and we see that the angle of refraction is larger. So, what happens when you have critical angle the transmitted ray is basically goes along the interface.

Critical Angle

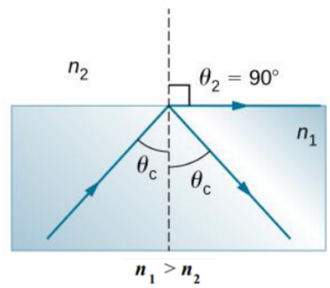
- **From Snell's law:**

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

- When the incident angle equals the **critical angle** ($\theta_1 = \theta_c$), the angle of refraction is 90° (θ_2=90°).

- Noting that sin (90°) =1 , Snell's law in this case becomes

$$\sin\theta_c = \frac{n_2}{n_1} \rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$



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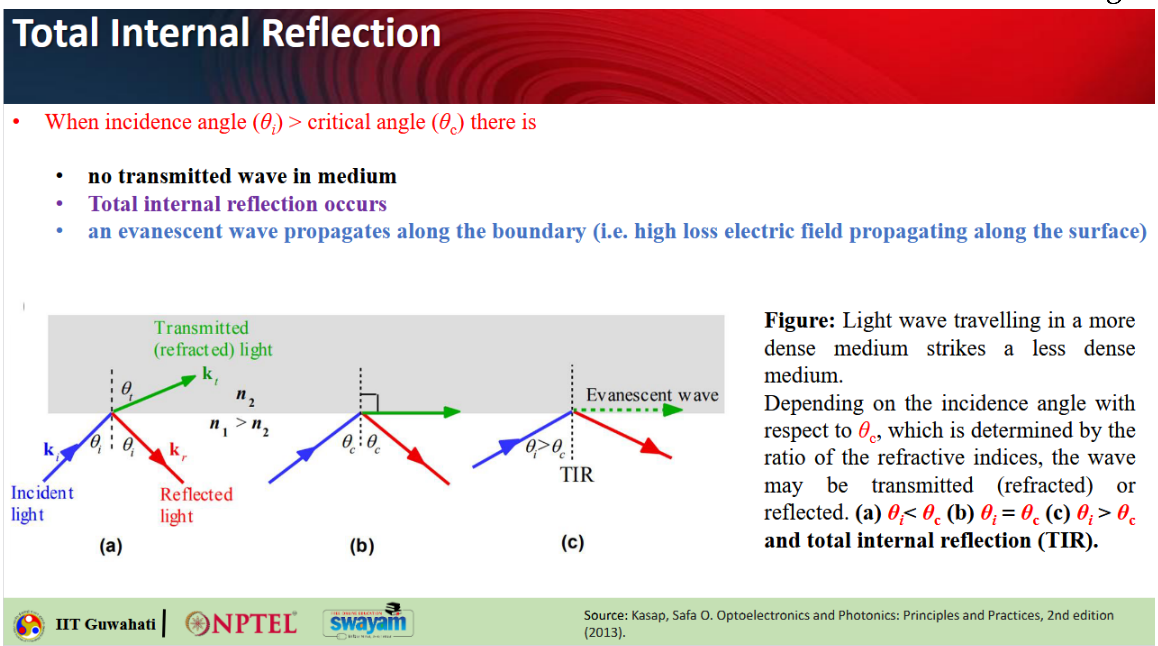
That means the angle of transmission is 90 degree. So that is the definition of critical angle. So, you get theta 2 equals 90 degree. Now in that case if theta 2 becomes 90 degree so this is 1. So, you can simply write theta 1 is nothing but your critical angle that is theta c.

So theta c can be written as n_2 over n_1. So, you can write what is $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$. And this is only applicable when the light ray is propagating from denser to rarer medium. So, this is the concept which also brings in what happens when light is incident at an angle larger than critical angle. So that will bring you total internal reflection. Now when the incident angle theta i is greater than the critical angle theta c you will get no transmission in the second medium.

All the light that is incident will be completely reflected back. So, there is no partial transmission complete reflection. That is why it is called total internal reflection. However, an evanescent wave actually propagates along the boundary. So that is basically high loss electric field propagating along the surface.

Evanescent fields do not propagate too long because they are very lossy. So, these are the three situations as you can understand. So here theta i is much smaller than critical angle. So, you get the normal reflection and transmission. Transmission is nothing but the refracted light.



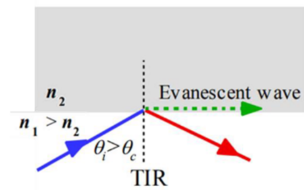
This is the case where your incident angle is exactly same as the critical angle. So, you get this is the incident light you get some reflection and the transmitted light is exactly along the interface. And when you have incident angle larger than critical angle you will see that there is some reflection and this light is also now coming back in the same direction. So, you are getting everything reflected just that a bit of electric field will be propagating as a lossy wave that is evanescent wave along the interface. So, this is the case of total internal reflection.

So total internal reflection gives rise to evanescent waves. And this allows you to satisfy the boundary condition at the interface in case of total internal reflection. So, if you look into the field in medium 2 so this is the field in medium 2 that propagates along the boundary edge at the same speed as the incident wave and dissipates into the second medium. So, you can take E_t perpendicular the tangential transmitted one for the perpendicular one polarization.

You can write it as (y,z,t). So, $e^{-\alpha_2 y}$ so this is the y direction. So, this is how the field decays that is how the amplitude of the evanescent field will decay exponentially into the medium. And then it will also propagate as $e^{j(\omega t)}$ so it will oscillate and propagate. And the direction of propagation is along z so this direction is z we can take. So, the evanescent wave vector is given as $k_{iz} = k_i \sin\theta_i$

Total Internal Reflection — Evanescent wave

- When $\theta_i > \theta_c$, there must still be an electric field in medium 2 or the boundary conditions will not be satisfied
- The field in medium 2 is an evanescent wave that travels along the boundary edge at the same speed as the incident wave and dissipates into the 2nd medium



$$E_{t\perp}(y,z,t) = e^{-\alpha_2 y} e^{j(\omega t - k_{iz} \cdot z)}$$

$$k_{iz} = k_i \sin\theta_i \qquad \text{evanescent wave vector}$$

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} \qquad \text{attenuation coefficient}$$

The penetration depth of the electric field into medium 2 is

$$\delta = 1/\alpha_2 \quad \rightarrow \quad E_{t\perp} = e^{-1}$$

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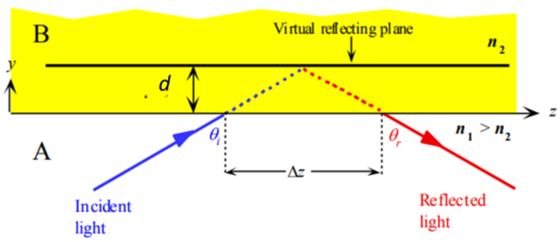
Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So $k_{iz} = k_i \sin\theta_i$ that is basically the sin component of the incident wave vector. And alpha 2 this alpha 2 that you see as I mentioned this is basically the attenuation coefficient that can be calculated as $2\pi n_2$. So n_2 is the refractive index of this medium over lambda what is the wavelength in vacuum into $\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$. So, this is how you get the attenuation coefficient. So, we can consider penetration depth in this particular medium 2 as a depth where delta penetration depth is defined as delta, delta can be taken as 1 over alpha 2. So, if you take y equal delta that is 1 over alpha 2 what do you find? You find this field comes down to be $= e^{-1}$.

So that is where the definition of penetration depth also comes in. So, where penetration depth is the depth where the field goes to 1 over e of the initial value and that is how it is obtained. So, I hope it is clear that when total internal deflection takes place there is an evanescent field that is excited at the interface but the evanescent field decays exponentially in the medium and it tries to propagate but it is a high because it is decaying it is a very lossy one it will decay down and it will not be able to propagate any further. So, it only decays or it stays there for a small distance short distance.

Goos-Hänchen Shift

- The reflected light beam in **total internal reflection** appears to have been laterally shifted by an amount Δz at the interface.
- It appears as though it is reflected from a virtual plane at a depth *d* in the second medium from the interface. The lateral shift is known as the Goos-Haenchen shift.
- The lateral shift depends on the angle of incidence and the penetration depth.
- We can represent the reflection as if it were occurring from a virtual plane placed at some distance d from the interface, then from simple geometric considerations, $\Delta z = 2d \tan\theta_i$.
- Here, d ≈ δ (penetration depth).



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Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

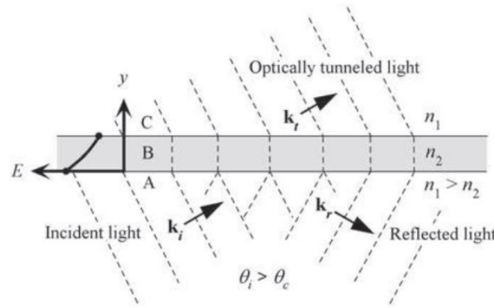
There is another concept called Goos-Hanchen shift. So, that is also related to the total internal deflection. So, if you see the total internal reflection the reflected beam in total internal reflection appears to be laterally shifted. It does not come from the exactly same amount the same point where the incident field is. It is laterally shifted by some amount called delta z. As if it is getting reflected from a virtual reflecting plane that is somewhere inside. So as mentioned it appears that it is reflected from a virtual plane at a depth d in the second medium from the interface and this lateral shift is known as the Goos-Hanchen shift.

And this shift also depends on the angle of incidence and the penetration depth. So, you can see the formula here. So, $\Delta z = 2d \tan\theta_i$. So, it depends on the angle of incidence and d can be roughly taken as the penetration depth. So, it depends on both penetration depth as well as the incidence angle. And when this second media becomes very thin in that case the field will be able to penetrate this interface and go into this medium B and then it will also be able to reach the other side of the interface.

So you will see that what you are supposed to be a total internal reflection but as the field is able to penetrate this thin layer B it is also able to reach somewhat outside. So, this light was initially not allowed but because of this thin layer light has now tunneled through this different medium although it is having that kind of condition. So, this particular light is called optically tunneled light. So, let's see how do we describe this. The phenomena in which an incident light is partially transmitted through a medium where it is forbidden in terms of simple geometrical optics.

Optical Tunneling

- When medium B is thin, the field penetrates from the AB interface into medium B and reaches BC interface, and gives rise to a transmitted wave in medium C.
- This phenomenon in which an incident wave is partially transmitted through a medium where it is forbidden in terms of simple geometrical optics is called **optical tunneling**.
- It is due to the fact that the field of the evanescent wave penetrates into medium B and reaches the interface BC before it vanishes.



Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So if you take a simple geometrical optics and you make sure that your $\theta_i > \theta_c$ that means you are supposed to get a total internal reflection. It means no light should be able to go on the other side of the medium. But what you have seen that because this thickness is so thin but or you can say this layer is so thin that the field is actually able to penetrate and go to this interface and from there it gives rise to a transmitted wave in the medium C. So, it is basically optical tunneling. And this is happening all because of the evanescent field that is able to penetrate into medium B and then it could reach the interface of B and C before it vanishes and that is where this optical tunneling will come into picture.

Frustrated total internal reflection (FTIR)

- **Frustrated total internal reflection** is utilized in **beam splitters**.
- In the beam splitter cube in Figure (b), two prisms, A and C, are separated by a thin film, B, of low refractive index. Some of the light energy is now tunneled through this thin film and transmitted into C and out from the cube.

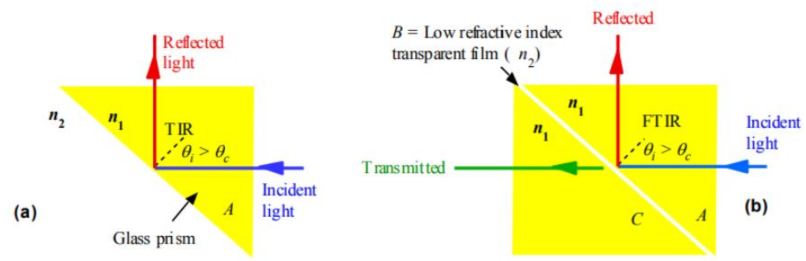


Figure (a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by **FTIR**.

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Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

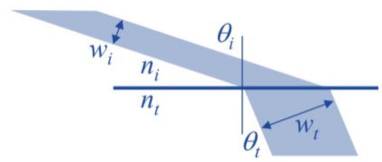
And this has given rise to a very interesting effect called frustrated total internal reflection or FTIR and using this effect people are able to make beam splitter. Now let us look into this particular prism this yellow one is a prism. So here you see you are actually having an incident light and when the incident light angle is larger than the critical angle you are getting total internal reflection. So, this is one glass prism. Now if you bring another glass prism next to this first prism but introduce a small gap of air in between that means you have a low refractive index transfer in film say n_2 in between.

In that case what is happening that this particular effect will take place that this is the air region where the evanescent field will be able to penetrate and come to the third region where it will be able to transmit again. Though it was forbidden but because of this thin layer it is now allowing. So, what is happening the incident light will now get split into two ways one is the reflected one, one will be the transmitted one and that is how a beam splitter works. So, some portion of the light energy is now able to tunnel through here this through this thin film and it can get transmitted into C and finally out of the cube. So, these are very very important device people have used and continuously using beam splitter that is based on this frustrated total internal reflection effect.

Transmittance (T)

- T = Transmitted Power / Incident Power = $\frac{I_t A_t}{I_i A_i}$

where, $I = \left(n\frac{\varepsilon_0 c_0}{2}\right)|E_0|^2$, $\frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{cos(\theta_t)}{cos(\theta_i)}$, $A = area$



Note: the beam has width w_i

- The beam expands (or contracts) in one dimension on refraction

$$T = \frac{I_t A_t}{I_i A_i} = \frac{\left(n_t \frac{\varepsilon_0 c_0}{2}\right)|E_{0t}|^2}{\left(n_i \frac{\varepsilon_0 c_0}{2}\right)|E_{0i}|^2}\left[\frac{w_t}{w_i}\right] = \frac{n_t |E_{0t}|^2 w_t}{n_i |E_{0i}|^2 w_i} = \frac{n_t w_t}{n_i w_i} t^2 \quad \text{since } \frac{|E_{0t}|^2}{|E_{0i}|^2} = t^2$$

$$\Rightarrow \boxed{T = \left[\frac{(n_t \cos(\theta_t))}{(n_i \cos(\theta_i))}\right] t^2}$$

s-polarized light:

$$r_\perp = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_\perp = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

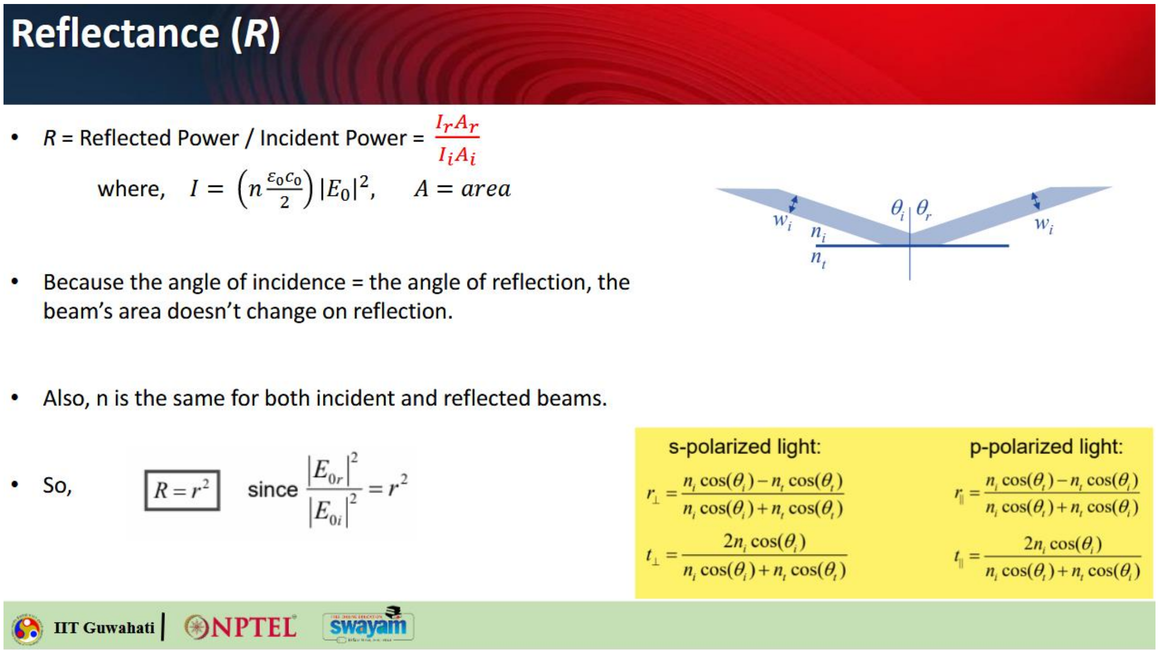
p-polarized light:

$$r_\parallel = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)}$$

$$t_\parallel = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

So understand with overall understanding of all these conditions of reflection and refraction or transmission. Now let us finally calculate that transmittance capital T. All the previous parameters were reflection coefficient transmission coefficient they were given with small letters small r small t this is transmittance this is basically the ratio of the transmitted power over the incident power. So, if you talk in terms of intensity so It is the transmitted intensity multiplied by the transmitted beam area A_t is the area of the transmitted beam over I_i into A_i. So, what is I intensity it can be written as roughly we say that intensity is proportional to amplitude square modulus or amplitude square or exact relation is $\left(n\frac{\varepsilon_0 c_0}{2}\right)|E_0|^2$ that is electric field amplitude square. And what is At over Ai so upon transmittance we have seen that because the beam is going from one refractive index medium to another refractive index medium there is a change in the beam width.

So the beam width can change to w_t from w_i and that can be written in terms of $\frac{\cos(\theta_t)}{\cos(\theta_i)}$ that means if you try to match the width at the boundary they have to be same and that is how you can actually get this ratio. So, you can find this ratio as $\frac{\cos(\theta_t)}{\cos(\theta_i)}$.



So finally you can find out what is transmittance t. So, it is $\frac{I_r A_r}{I_i A_i}$. So, this is how you do it. So $\frac{A_t}{A_i} = \frac{\cos(\theta_t)}{\cos(\theta_i)}$. So, when you do that you can find that you are actually getting a ratio of $\frac{|E_{0t}|^2}{|E_{0i}|^2}$ and $\frac{|E_{0r}|}{|E_{0t}|} = t$. Now depending on which polarization you are talking about you are talking about s or p polarization you can put the formula for t perpendicular or t parallel. So, this will be your t square right. So, this can be t parallel square or t perpendicular square and then you have to multiply it by this factor that is $\frac{(n_t\cos(\theta_t))}{(n_i\cos(\theta_i))}$.

So, this is the formula of transmittance. So, transmittance capital T is basically $T = \left[\frac{(n_t\cos(\theta_t))}{(n_i\cos(\theta_i))}\right]t^2$. Now small t you will pick from either this or this depending on s or p polarization. Similarly, in the case of reflectance life is slightly easier because here the beam width will be similar because the beam is basically in the same medium. So, you do not need to worry about the area because the A_r and A_i will be same. Intensity you have seen they are basically proportional to n and also E naught square again here n remains same so only E naught square will be coming in the ratio right.

So reflectance R will be simply you can put it as $\frac{|E_{0r}|^2}{|E_{0i}|^2}$ and the ratio of the electric field is basically small r right. So, when you take square of it is basically r^2. So that becomes capital R that is reflectance is basically nothing but reflection coefficient square. And again, depending on the polarization of the electric field whether it is s or p polarized you can choose either this formula for r perpendicular and or you choose this one r parallel.

R & *T* - Summary

- **Reflectance:** Relative (%) intensity of the reflected light traveling through the media

$$R = R_{\perp} = R_{\parallel} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Here, $\theta_i = 0$ for normal incidence, $n_i = n_1$, and $n_t = n_2$

- **Transmittance:** Relative (%) intensity of the transmitted light traveling through the media

$$T = T_{\perp} = T_{\parallel} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

- Sum of the transmittance and reflectance in any conserved system must equal 1

R + T = 1

s-polarized light:

$$r_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\perp} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

p-polarized light:

$$r_{\parallel} = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)}$$

$$t_{\parallel} = \frac{2n_i \cos(\theta_i)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)}$$

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So this is the summary of this r and t polarization that at normal incidence the reflectance will be same. You will have r perpendicular will be equal to r parallel and this is given by $\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$ and you can also find out what is the transmittance in that case. So, transmittance will be $\frac{4n_1 n_2}{(n_1 + n_2)^2}$. So, always remember that reflection coefficient and transmission coefficient they may not add up to 1 but because the power will be maintained ok. So, whatever is the incident power some part of it is getting reflected some part is getting transmitted. So, the reflection power and transmission power will be equal to the incident power or you can say reflectance plus transmittance will be equal to 1 which is not true for small r and small t this is only true for capital R and capital T ok.

So, this is the summary of this particular reflection and transmission coefficients using Fresnel equation. So, with that I will stop here in case you have any queries you can drop an email to this particular email address mentioning MOOC on the subject line. So, in the next lecture we will start discussion about absorption dispersion and scattering of light. Thank you.