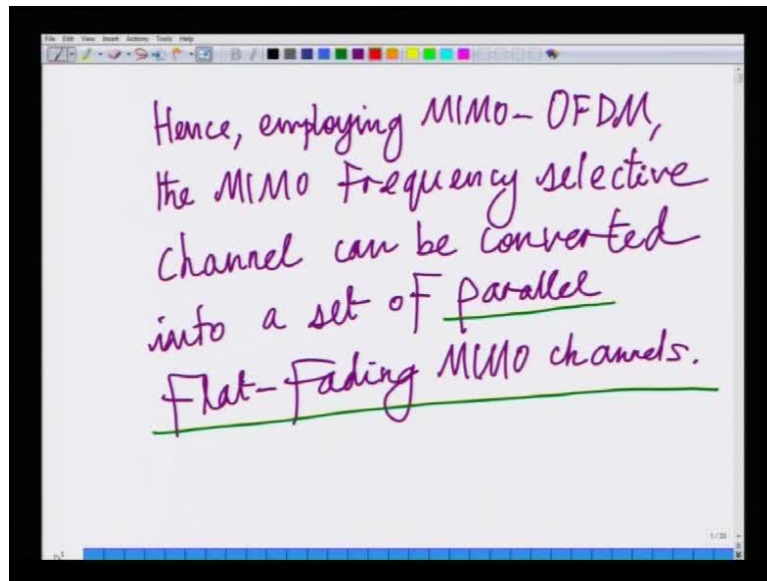


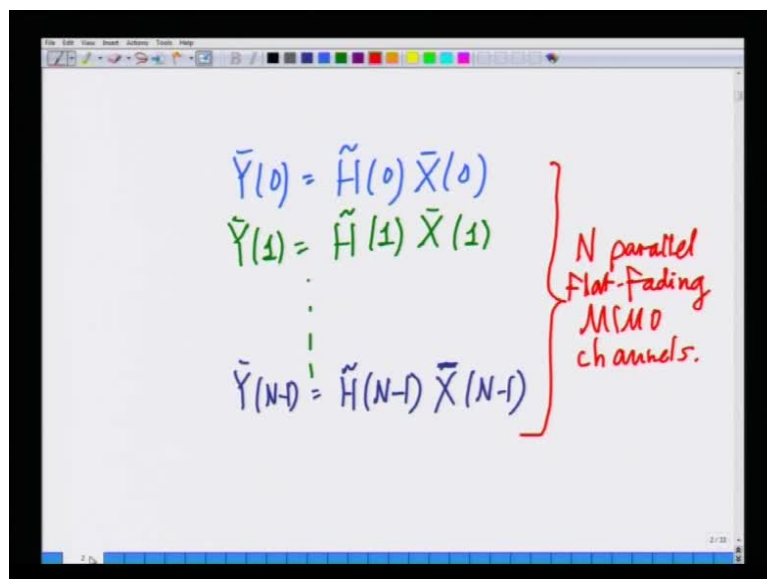
Advanced 3G & 4G Wireless Communication
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Lecture - 33
Impact of Carrier Frequency Offset (CFO) in OFDM

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Hello, welcome to another lecture in this course on 3 G 4 G wireless communication systems. In the last lecture we had talked about MIMO OFDM that is that is the use of OFDM

technology MIMO wireless communications. We said that MIMO OFDM converts a MIMO frequency selective channel into a set of parallel MIMO flat fading channels, such that across each sub carrier, the net wireless communication systems looks like a MIMO flat fading systems that is like vector \mathbf{Y}_0 is now \mathbf{H}_0 times \mathbf{X}_0 , where \mathbf{X}_0 is the transmit vector across the 0th sub carrier, \mathbf{H}_0 is the channel matrix and \mathbf{Y}_0 is the received vector that across the 0th sub carrier. Similarly, the first carrier so on so on so on till the $N-1$ sub carrier, where \mathbf{Y}_{N-1} is r dimensional received vector which is equal to \mathbf{H}_{N-1} times \mathbf{X}_{N-1} , where \mathbf{X}_{N-1} is the transmit vector across the $N-1$ sub carrier all right and.

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The image shows handwritten mathematical derivations for two types of MIMO receivers in a flat fading channel. The derivations are as follows:

ZF MIMO Zeroforcing Receiver:

$$\hat{\mathbf{X}}(k) = (\tilde{\mathbf{H}}(k))^{\dagger} \tilde{\mathbf{Y}}(k)$$

$$= (\tilde{\mathbf{H}}^H(k) \tilde{\mathbf{H}}(k))^{-1} \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{Y}}(k)$$

MIMO MMSE Receiver:

$$\hat{\mathbf{X}}_{\text{MMSE}}(k) = \underbrace{(\mathbf{P}_d)}_{\text{data power}} (\mathbf{P}_d \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{H}}(k) + \sigma_n^2 \mathbf{I})^{-1} \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{Y}}(k)$$

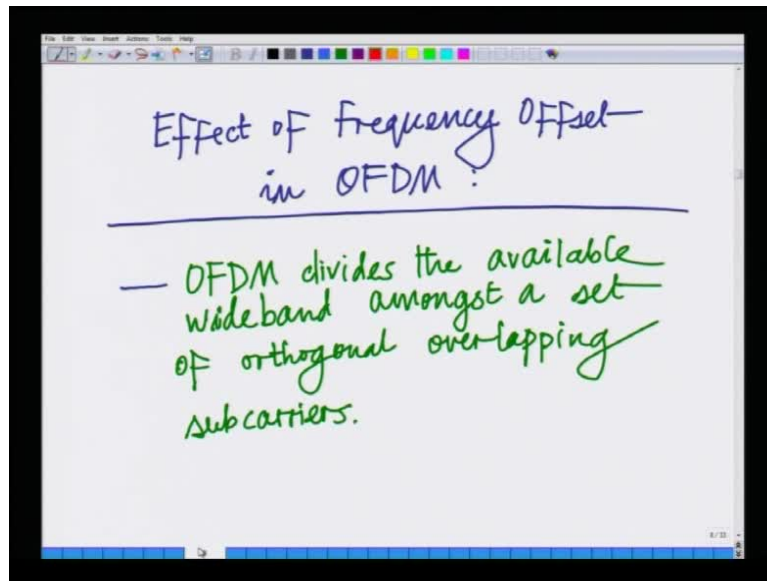
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$$\begin{aligned} & [\tilde{H}_{u,v}(0), \tilde{H}_{u,v}(1), \dots, \tilde{H}_{u,v}(N-1)] \\ &= N_{pt} \text{ FFT} \circ F \\ & \quad [H_{u,v}(0), H_{u,v}(1), \dots, H_{u,v}(L-1)] \\ & \quad \text{after zeropadding.} \end{aligned}$$

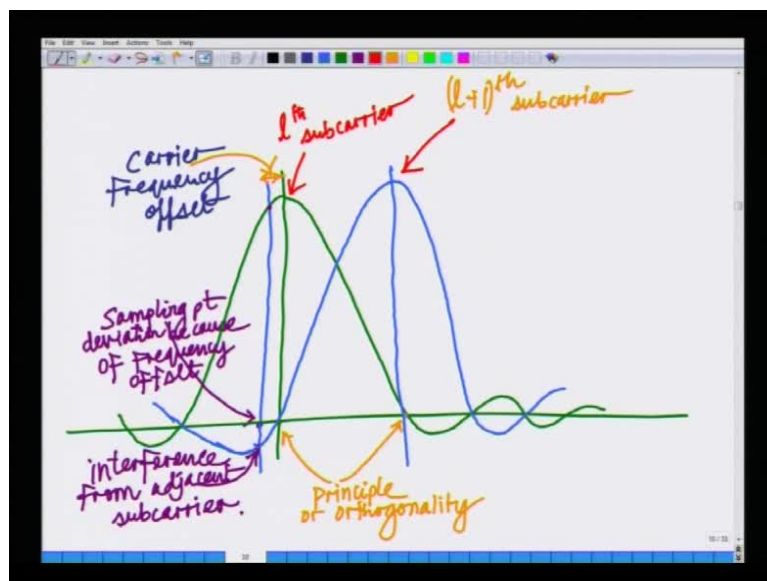
We are also seen that for the purposes of receiving across each sub carrier; similar to MIMO, we can either use 0 minus force thing or the MIMO MMSE equalizer that is we can use same similarly to what we had seen in the case of MIMO system, we can use both 0 forcing and MMSE equalizer, and with that we had also said that this channel matrix, how do you obtain channel matrix across each sub carrier? We said that if you consider the u v th element of all this channel matrices across each sub carrier; that is there are N such elements from 0 to N minus 1. They are nothing but then N point FFT of if you take all the channel taps matrices that is H_0, H_1, H_{L-1} ; take their u v th element their L such elements then you 0 pad them, and take the N point N point FFT of these elements then you will get the u v th element of the channel matrices across the sub carriers.

You can similarly you do that for all the elements and thus construct essentially the channel matrices for the sub carriers all right. So, that is how we construct these channel matrices in this MIMO OFDM system. It is fairly straight forward and its extension of the case for the single input single output OFDM system, but it is much more helpful in MIMO system, because MIMO equalization that is MIMO frequency selective channel selective channel poses much more bigger challenge for communication compare to single input single output frequency selective systems all right. So, that is the basic idea behind MIMO OFDM.

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And then with that, we had moved on the effect of frequency offset, what is the distorting effect of frequency offset in OFDM system? We said OFDM divides the frequency band into a set of orthogonal overlapping yet orthogonal frequency get us. The orthogonality is a very critical aspect actually of any OFDM system which means once you have an OFDM system as we had depicted in this figure last time. The orthogonality is lost which means now there will be ICI or inter carrier interface.

The carriers will start interfering with each other and the larger the frequency offset, the greater is this inter carrier interference, this is a very important aspect of any OFDM system and we had also started seeing them how to model the affect of this inter carrier interference as well as its frequency offset.

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Given, frequency offset ϵ , the baseband received samples are given as,

$$y_n = \frac{1}{N} \sum_{k=-N/2}^{N/2} X_k H_k e^{j2\pi n(k+\epsilon)/N} + w_n$$

Annotations in the image:

- y_n : n^{th} received sample
- X_k : data transmitted on the k^{th} subcarrier
- H_k : channel coefficient across k^{th} subcarrier
- ϵ : Normalized frequency offset
- w_n : noise

We had set for a frequency offset of epsilon, normalized with respect to sub carrier bandwidth. The received signal sample y_n is given as summation $X_k H_k e^{j2\pi n(k+\epsilon)/N}$ plus noise that is we said the system model for this.

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$$Y_l = \sum_n \sum_{k=-N/2}^{N/2} X_k H_k e^{j2\pi n(k-l+\epsilon)/N} + \sum_n w_n e^{-j2\pi nl/N}$$

$$=$$

And then, we had started looking at what is the affect when you take FFT of these received samples? Now with the presence carrier frequency offset what is the affect of this on the FFT that is what we will start looking at with that let us go on to today is lecture.

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$$Y_l = \frac{1}{N} \sum_n y_n e^{-j2\pi nl/N}$$

Received Symbol across (the) subcarrier

$$= \frac{1}{N} \sum_n \sum_{k=-N/2}^{N/2} X_k H_k e^{j2\pi n(k-l+\epsilon)/N} + \sum_n w_n e^{-j2\pi nl/N}$$

$$= \frac{1}{N} \sum_n X_l H_l e^{j2\pi n\epsilon/N} + \frac{1}{N} \sum_{k=-N/2}^{N/2} \sum_{k \neq l} X_k H_k e^{j2\pi n(k-l+\epsilon)/N} + \tilde{w}_l$$

So, we said that y_l that is what we said we said y_l is nothing but you consider that is the received symbol across l sub carrier is nothing but consider the FFT of the received samples as in this fashion, that is summation received samples $y_n e^{-j2\pi nl/N}$, let me remind you this is the l th FFT point which is the l th received symbol across the l th sub

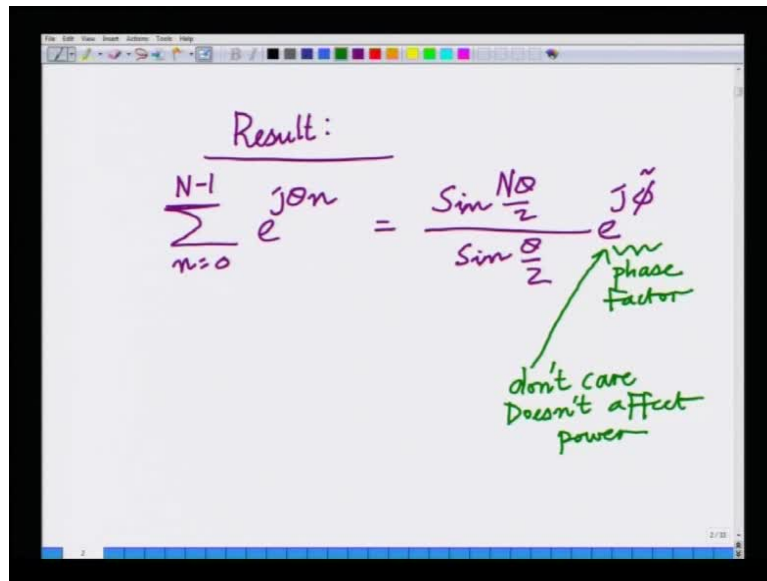
carrier. This is the received symbol across l th sub carrier this using the expression that we had earlier, this can be simplified as follows this is nothing but $\frac{1}{N} \sum_{k=-N/2}^{N/2} X_k H_k e^{j 2 \pi k \text{ minus } l \text{ plus } \epsilon \text{ over } N}$, that is I am doing nothing but I am simply substituting the earlier expression that we had for the N th sample y_n in this expression. That is $X_k H_k e^{j 2 \pi}$.

The expression that we had earlier I am simply substituting over here plus the noise part which is $\text{plus } \sum_n w_n e^{j 2 \pi n \text{ over } N}$. So, I have the expression for, I have the expression for, for y_n that is N th received sample, I am simply substituting that expression in this that gives me this, now what I am going to do is from this term over here, in this first term over here. I am going to isolate the term corresponding to k equals to 1; this is similar to what we had done earlier. Because I want to look at this as 2 terms; one is the desired part which is what I am suppose to receive on the l th sub carrier that corresponds to k equals to l and then the interference part that corresponds to all k not equals to l .

So, I am simply going to write this as summation of 2, 2 terms. Now, when k equals 1, this goes to 0; this simply reduces to $X_l H_l e^{j 2 \pi \epsilon \text{ over } 2 \pi \epsilon \text{ over } N}$, it should be N here; there is a N missing $2 \pi \epsilon \text{ over } N$. So, I am going to write this as summation $\frac{1}{N} \sum_n X_l X_l H_l e^{j 2 \pi N \epsilon \text{ over } l}$. Now, remember if there was no carrier frequency offset then this ϵ would have been 0; this would have been 1; this would have been summation 1 which would have been $N \text{ over } N$; so this is this would have yielded or $X_l H_l$ similarly to what we had earlier. Now, because of this carrier frequency offset, this is not exactly 1 all right that is the problem here, plus summation $\frac{1}{N}$, now k equals minus $N \text{ over } 2$ to $N \text{ over } 2$ however not equals 1, because we have isolated the term k equals 1, summation over N this is $X_k H_k e^{j 2 \pi N \text{ over } N \text{ into } k \text{ minus } l \text{ plus } \epsilon}$ plus some noise which is w tilde of l plus.

So, what we are doing is, I am taking the expression for the received N sample, I am substituting the FFT expression FFT at the FFT expression that is the l th FFT point and thus I receive I am formulating or I am deriving the expression for y_l which is nothing but the expression for received sub carrier received symbol across the l th sub carrier and that is given by this expression and that can be further simplified before I simplify that further.

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The image shows a digital whiteboard with a handwritten mathematical result. At the top, the word "Result:" is underlined. Below it, the equation is written as:

$$\sum_{n=0}^{N-1} e^{j\theta n} = \frac{\sin \frac{N\theta}{2}}{\sin \frac{\theta}{2}} e^{j\tilde{\phi}}$$

Below the equation, there are handwritten notes in green ink. An arrow points from the text "phase factor" to the term $e^{j\tilde{\phi}}$. Another arrow points from the text "don't care Doesn't affect power" to the same term.

I want to illustrate a result the result. I want to write here is the result that I will use in simplification that is as follows summation n equals 0 to N minus 1 e power j theta n equals, you can verify this and it is a simple problem of geometric progression and this is $\sin N$ theta over 2 divided by \sin theta over 2 into e power j phi tilde, where this phi tilde e power j phi tilde is some phase, so this is essentially some phase factor; we do not care about this phase factor, because it does not affect the power what I want to illustrate is what is the impact on the signal to noise power ratio.

So, magnitude e power j phi is 1, so it does not affect the phase however it does not affect the power, however this is the $\sin N$ theta by 2 \sin theta by 2. This is the critical factor for us all right, so this for us this is do not care as it does not affect power that is why I am not particular about this, this is do not care as it does not affect and it does not affect the net received signal power.

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$$Y_l = H_l X_l \frac{\sin \pi \epsilon}{\sin \frac{\pi \epsilon}{N}} \cdot \frac{1}{N} e^{j \tilde{\phi}_l} + \left[\sum_{\substack{k=-N/2 \\ k \neq l}}^{N/2} H_k X_k \left(\frac{\sin \pi \epsilon}{N \sin \left(\pi \frac{l-k+\epsilon}{N} \right)} \right) e^{j \tilde{\phi}_k} \right] + \tilde{w}_l$$

Annotations in the image:

- Desired Signal Part**: Points to the first term $H_l X_l \frac{\sin \pi \epsilon}{\sin \frac{\pi \epsilon}{N}} \cdot \frac{1}{N} e^{j \tilde{\phi}_l}$.
- Inter-carrier interference**: Points to the summation term $\sum_{k=-N/2, k \neq l}^{N/2} H_k X_k \left(\frac{\sin \pi \epsilon}{N \sin \left(\pi \frac{l-k+\epsilon}{N} \right)} \right) e^{j \tilde{\phi}_k}$.
- Gaussian Noise**: Points to the last term \tilde{w}_l .

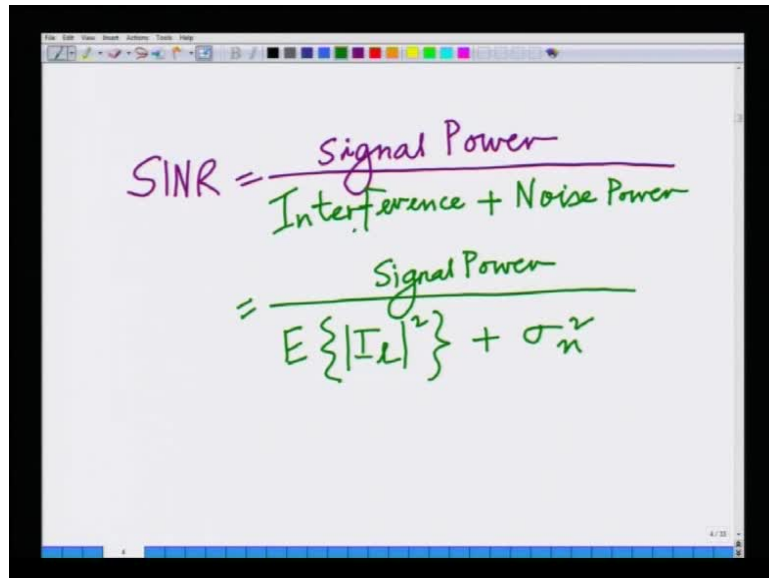
So, this now can be written as y_l , this can be written as y_l equals $H_l X_l \sin \pi \epsilon$ divided by $\sin \pi \epsilon$ divided by N into 1 over N into e power j some ϕ_l tilde all right. Now, again you can verify that if ϵ equals to 0 , limit ϵ tends to 0 ; this tends to N , N into 1 by N ; this tends to 1 ; this is $H_l X_l$, this is the same thing we had before plus what we have here is summation k equals minus N by 2 to N by 2 k not equals l $H_k X_k \sin \pi \epsilon$ divided by $N \sin \pi (l - k + \epsilon) / N$ into e power j ϕ_k tilde.

This is the interference term I_l , this is the interference term; this is the net interference now which is not equals to 0 , because of loss of orthogonality, because of the presence of this frequency offset, because you can see if this ϵ is 0 ; this whole interference this goes to 0 . Now, because of the presence of a carrier frequency offset, you end up having this inter carrier interference and this is nothing but the interference from all the other subcarriers other than the l th sub carrier plus w tilde all right.

This is the expression that we have. So, this I would like to say so this is the signal part for this, this here, this is the signal part that is the desired signal part, this part here this is the inter carrier interference part and this of course, this is our Gaussian noise. This is our thermal noise or Gaussian noise all right so let me just call it as Gaussian noise. So, there is three parts; one is the desired signal, other is the inter carrier interference and then again you have the Gaussian noise part all right. Now, similar to CDMA, now we said now, now we do not

not only have signal noise but we also have signal interference noise, hence in such scenario we use the signal not signal to noise ratio but signal to interference plus noise power ratio.

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The image shows a digital whiteboard with a toolbar at the top. The formula for SINR is written in two lines. The first line is $SINR = \frac{\text{Signal Power}}{\text{Interference} + \text{Noise Power}}$, where 'Signal Power' is in purple and the denominator is in green. The second line is $= \frac{\text{Signal Power}}{E\{|I_k|^2\} + \sigma_n^2}$, where the numerator is in green and the denominator is in green.

$$SINR = \frac{\text{Signal Power}}{\text{Interference} + \text{Noise Power}}$$
$$= \frac{\text{Signal Power}}{E\{|I_k|^2\} + \sigma_n^2}$$

So, now we want to characterize the SINR as SINR equals signal power divided by interference plus noise power equals signal power divided by expected interference power plus the noise signal power divided by interference power is nothing but expected $|I|^2$ plus sigma N squared which is the noise power. Now, let us start characterizing each term that is first I want to characterize signal power then the interference power then the noise power.

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$$\text{Signal Power} = E\{|H_l|^2\} E\{|X_l|^2\} \left(\frac{\sin \pi E}{N \sin \frac{\pi E}{N}} \right)^2$$

For large N ie # subcarriers

$$\lim_{N \rightarrow \infty} \sin \left(\frac{\pi E}{N} \right) \approx \frac{\pi E}{N}$$

$$\Rightarrow N \sin \frac{\pi E}{N} \rightarrow N \cdot \frac{\pi E}{N} = \pi E$$

So, let us start with the signal power aspect of this so there are three aspects, so first is so as we saw there are three aspects that is signal power, interference power, and the noise power. I am starting by characterizing the signal power. The signal power is nothing but we can see this is expected magnitude H_l square into expected magnitude X_l square into $\sin \pi \epsilon$ divided by $N \sin \pi \epsilon$ divided by N whole square all right, so this is the signal part that is expected magnitude H_l square that is the gain of the channel across the l sub carrier expected X_l square which is nothing but p_l which is the power transmitted across the l subcarrier or if you are using uniform power then this is simply p times $\sin \pi \epsilon$ divided by $N \sin \pi \epsilon$ by N whole square all right.

Now, we going to assume the limit in which there are large number of sub carriers that is I am going to simplify these expression further to get an idea of how this expressions look like when the number of sub carriers is typically the number of sub carriers is very large; it is the order of N equals 512 or N equals 1024. So, how does this expression look when N tends to infinity so for large N that is number of sub carriers, we have limit N tending to infinity $\sin \pi \epsilon$ over N as N tends to infinity $\pi \epsilon$ over N approximately tends to 0 that is limit $\sin \theta$ becomes very small and the limit θ becoming very small $\sin \theta$ approximately tends to θ . Hence this is approximately $\pi \epsilon$ over N for very large N implies $N \sin \pi \epsilon$ over N tends to N into $\pi \epsilon$ over N equals $\pi \epsilon$, hence for a large number of sub carrier number of sub carrier $N \sin \pi \epsilon$ over N approximately looks like $\pi \epsilon$. Hence this expression for signal power can be simplified as follows.

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Hence, the signal power for a large number of subcarriers N is given as,

$$\text{Signal Power} = E\{|H_l|^2\} P \left(\frac{\sin \pi \epsilon}{\pi \epsilon} \right)^2$$

where $E\{|H_l|^2\}$ is the expected magnitude squared of the channel gain, and P is the power in the signal. The term $E\{|H_l|^2\}$ is also labeled as $E\{|X_l|^2\}$ in the original image.

$$\text{Signal Power} = P |H|^2 \left(\frac{\sin \pi \epsilon}{\pi \epsilon} \right)^2$$

Hence, the signal power for a large number of sub carriers for N is given as expected mod H square power $\sin \pi \epsilon$ divided by $\pi \epsilon$ square, where p remember this p is the power in the signal, this is p is nothing but expected X 1 square which is the power in the symbol transmitted across the l th sub carrier. This is power this is nothing but the power the signal power or the symbol or the symbol power. Hence this is nothing but and I am going to assume that expected H 1 square is the same across all sub carriers as this is p magnitude H square, where magnitude H square is nothing but expected magnitude H 1 square that is the average gain across the sub carrier times $\sin \pi \epsilon$ divided by $\pi \epsilon$ whole square.

So, this is nothing but the this is nothing but the this is nothing but the signal power. So, we have derived simplified the expression for the signal signal power in the presence of large number of sub carrier and the signal power in the presence of a carrier frequency offset of ϵ approximately looks as p , where p is the transmitted power magnitude H square is the average gain across the sub carrier times $\sin \pi \epsilon$ divided by $\pi \epsilon$ whole square this is the key factor.

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The image shows a whiteboard with the following handwritten text:

Interference Power:

$$E\{|I_L|^2\} = E\{|X|^2\} E\{|H_L|^2\}$$

$$\sum_{\substack{k=-N/2 \\ k \neq L}}^{k=N/2} \left(\frac{\sin \pi \epsilon}{N \sin \left(\pi \frac{l-k+\epsilon}{N} \right)} \right)^2$$

$k-l = u$ and assume $N \rightarrow \infty$

Now, let us characterize slightly difficult part which is the interference for the more elaborate part which is the interference. So, how does the interference power look like, how does the interference power look like? If you look at the interference power we said interference power is nothing but expected magnitude I square that is equal to expected magnitude X square expected magnitude H square into summation k equals minus N by 2 k not equals l k equals N by 2 times $\sin \pi \epsilon$ $N \sin \pi \frac{l-k+\epsilon}{N}$ divided by N this whole square. Remember when this simplification $\sin \pi \epsilon$ by $\sin \pi \frac{l-k+\epsilon}{N}$, this is arriving from this property that we looked at here all right. Remember we have introduced a special property over here and this simplification is simply arising from this property.

Now, I will have to do, now I will do a series of manipulation which are slightly, but I asked you to but fairly simple also I ask you to just pay attention, because there are more mathematical manipulation rather than anything else but the idea is essentially to simplify this expression. So, what I am going to do here as a first step to simplify this, I am going to set $k-l = u$, just to a simple substitution $k-l = u$ and also assume and assume n tends to infinity that is the number of sub carriers N is very large and hence this expression can be simplified as follows.

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$$E\{|I_k|^2\} = P|H|^2 \sum_{\substack{u=-\infty \\ u \neq 0}}^{\infty} \frac{(\sin \pi \epsilon)^2}{\left(N \sin \frac{\pi u}{N}\right)^2}$$

$u + \epsilon \approx u$

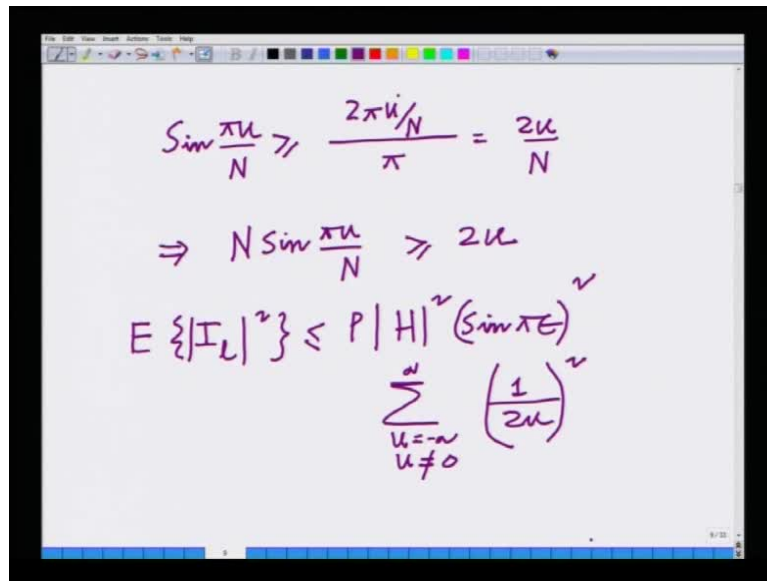
$$= P|H|^2 (\sin \pi \epsilon)^2 \sum_{\substack{u=-\infty \\ u \neq 0}}^{\infty} \left(\frac{1}{N \sin \frac{\pi u}{N}} \right)^2$$

$\sin \theta \geq \frac{2\theta}{\pi}$

Expected magnitude I_k square equals p magnitude H square summation u equals minus infinity. Remember I have now substituted k minus 1 equals u and at the same time k not equals 1 which means u not equals 0, $\sin \pi \epsilon$ whole square divided by $N \sin \pi u$ divided by N whole square, where in the denominator I have used the approximation, u plus ϵ approximately equal to u , because remember that ϵ is a very small quantity and u is varying from minus infinity to infinity. So, for large u u plus ϵ is approximately the same as u u is small fraction.

So, this is the net expression that we that we get and hence this can be further simplifies as follows that is the interference power which is p times magnitude H square also $\sin \pi \epsilon$ come $\pi \epsilon$ square. This comes out of the summation summation what do we have here u equals minus infinity to infinity, u not equals 0, 1 over $\sin \pi u$ divided by N whole square. Now, $\sin \theta$ now I want to use some approximation here, the first approximation I am going to use is that $\sin \theta$ is greater than equal to 2θ over π . So, $\sin \theta$ is greater than or equal to 2θ over π .

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The image shows a digital whiteboard with handwritten mathematical derivations in purple ink. The first line shows the inequality $\sin \frac{\pi u}{N} \geq \frac{2\pi u/N}{\pi} = \frac{2u}{N}$. The second line shows $\Rightarrow N \sin \frac{\pi u}{N} \geq 2u$. The third line shows the final bound: $E \{|I_L|^2\} \leq P |H|^2 (\sin \pi \epsilon)^2 \sum_{\substack{u=-\infty \\ u \neq 0}}^{\infty} \left(\frac{1}{2u}\right)^2$.

I want to use this for this term πu by n which means I can write this as $\sin \pi u$ divided by N is greater than or equal to 2π that is 2θ , where θ is πu divided by N , $2 \pi \pi u$ divided by 2θ divided by π equals $2 u$ divided by N implies $N \sin \pi u$ divided by N greater than or equal to $2 u$, and hence now I can simplify this expression here. by simply approximating bounding this expression as follows. I can bound this expression as expected I_L^2 as since $N \sin \pi u$ by N greater than equal to $2 u$, expected I_L^2 is in fact less than or equal to because this term is in the denominator. This is p magnitude H square $\sin \pi \epsilon$ square summation u equals minus infinity to infinity, u not equals 0 1 over $2 u$ whole square all right.

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$$\begin{aligned}
 &= P|H|^2 (\sin \pi \epsilon)^2 \sum_{u=1}^{\infty} \left(\frac{1}{2u}\right)^2 \\
 &= \frac{1}{2} P|H|^2 (\sin \pi \epsilon)^2 \sum_{u=1}^{\infty} \frac{1}{u^2} \\
 &\quad \underbrace{\sum_{u=1}^{\infty} \frac{1}{u^2}}_{\pi^2/6} \\
 &= \frac{\pi^2}{12} P|H|^2 \sin^2 \pi \epsilon \\
 &= 0.822 P|H|^2 \sin^2 \pi \epsilon
 \end{aligned}$$

So, that is the net approximation that we have over here and this can be further simplified as follows, this is equal to I will do a series of manipulations p magnitude H square $\sin \pi$ epsilon square times 2. I will write the integral from u equals minus infinity to infinity as 2 times the integral u equals 1 to infinity 1 over $2u$ square. This is nothing but half p magnitude H square $\sin \pi$ epsilon square summation u equals 1 to infinity 1 over u square all right. And this is simply the sum 1 over u square, where I use the set of all positive integers and this has a standard this.

We know the result of this summation; this is simply π square over 6. Hence this is nothing but π square over 12 into p into magnitude H square \sin square π ϵ which is essentially nothing but 0.822 times p magnitude H square \sin square π into epsilon all right. So, this is the net expression for the interference power. Hence now, I can write the SINR, hence what we have done is we have, we have again derive the power the entire expression for the interference power is 0.822 times p times norm H square \sin square π epsilon, and also you can observe as a if epsilon equal to 0, this is 0 that there is no inter carrier interference, and hence interference power is 0 which is what we expect and hence this is nothing but the SINR or hence the net SINR.

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Hence, SINR in the presence of carrier frequency offset of ϵ is given as,

$$\text{SINR} = \frac{P|H|^2 \left(\frac{\sin(\pi\epsilon)}{\pi\epsilon} \right)^2}{0.822 P|H|^2 \left(\frac{\sin(\pi\epsilon)}{\pi\epsilon} \right)^2 + \sigma_n^2}$$

Annotations:

- signal power (points to the numerator)
- Interference power inter-carrier interference (points to the first term of the denominator)
- noise power (points to the second term of the denominator)
- approximation under $N \rightarrow \infty$ is large number of subcarriers (points to the 0.822 factor)

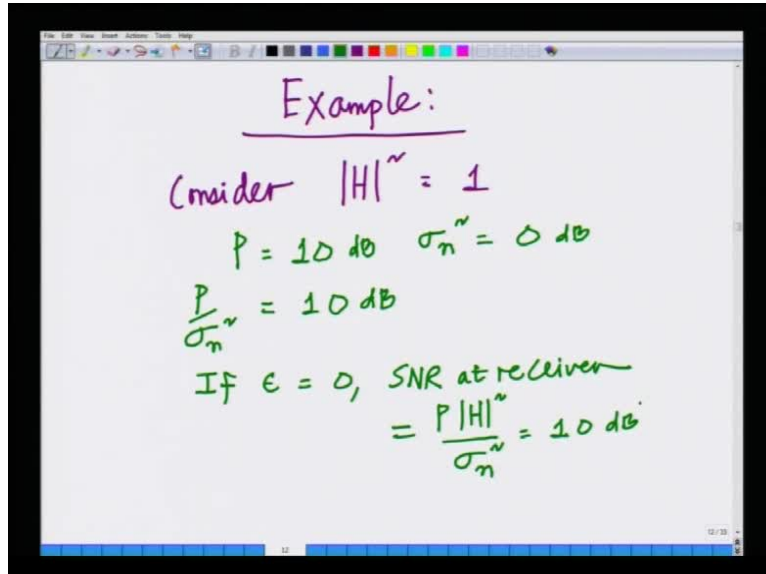
Hence, SINR in the presence of carrier frequency offset of epsilon is is given as, SINR equals p magnitude H squares in π epsilon divided by π epsilon whole square divided by $0.822 p$ magnitude H square $\sin \pi$ epsilon whole square plus σ_n^2 square, where σ_n^2 square is noise power and this is the expression for the SINR in the presence of carrier frequency offset. Let me remind you again, this is the signal power; this is the interference power from inter carrier interference; this is the noise power; this is the noise power.

Now, we have derived the signal to interference noise ratio in the presence of carrier frequency offset of epsilon all right which is p magnitude H square, where magnitude H square; remember is the average gain across each sub carrier that is expected $H^2 \sin \pi$ epsilon by π epsilon whole square divided by $0.822 p$ magnitude H square plus $\sin \pi$ epsilon square plus σ_n^2 square, which is the thermal noise. So, this is the net expression and you should verify that in absence of carrier frequency offset that is the epsilon equals to 0.

This reduces to simply p magnitude H square divided by σ_n^2 square which is essentially the expression for the original OFDM signal to power ratio without any inter carrier interference all right. You should verify that in fact it is straight forward. If you look at this its directly visible, if you look at this expression; this tends to 1; the numerator $\sin \pi$ epsilon by π epsilon tends to 1; if epsilon is 0 and this denominator the interference tends to 0. Hence, this is the expression all right, and also remember this expression is an approximation. It is valid under reasonably good approximation in which the number of sub carrier N is tending to

infinity. The approximation one of the approximations is that is large number of that is large number of sub carriers.

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Example:

Consider $|H|^2 = 1$

$P = 10 \text{ dB}$ $\sigma_n^2 = 0 \text{ dB}$

$\frac{P}{\sigma_n^2} = 10 \text{ dB}$

If $\epsilon = 0$, SNR at receiver

$$= \frac{P |H|^2}{\sigma_n^2} = 10 \text{ dB}$$

So, that is what we have here, now let us do a small example. So, let us illustrate a small example to illustrate, illustrate the performance of the SINR under the presence of carrier frequency offset. Consider a system with average sub carrier power gain as unity. Let us consider consider an OFDM system such that magnitude H square equals unity that is average gain of, and we also consider the scenario with signal power or transmitted symbol power p equals 10 d B and sigma n square equals 0 d B.

That is the signal the symbol to noise power ratio p over sigma N square equals 10 d B. Now, in absence of a carrier frequency offset, if carrier frequency offset; if epsilon equals 0 then we have SNR receive is nothing but SNR at receiver equals p magnitude H square divided by sigma N square which is equal to 10 d B. So, we have a 10 d B SNR in the absence of any carrier frequency offset that is the receiver end transmitter that is carrier oscillator is perfectly synchronized to the incoming carrier wave then we have SNR of 10 d B. There is no inter carrier interference however if the carrier frequency offset. Consider now a slight distortion.

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Consider a carrier frequency offset $\epsilon = 5\% = 0.05$

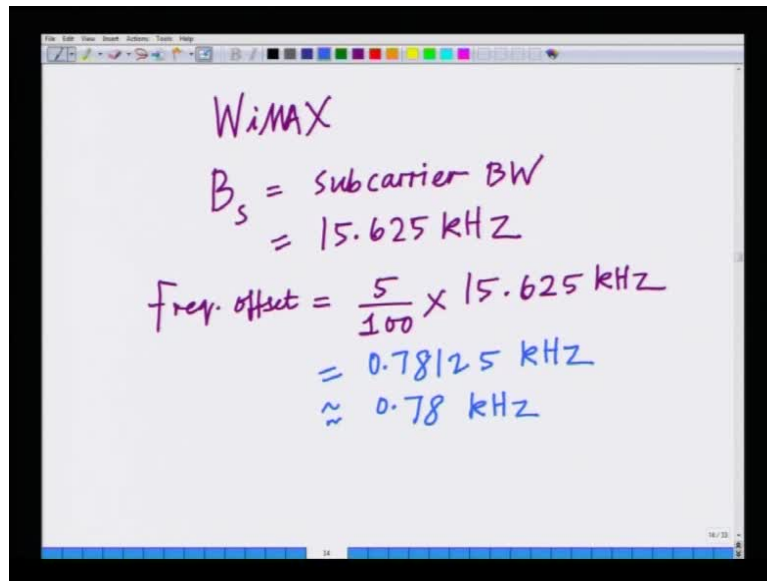
$$SINR = \frac{10 \times \left(\frac{\sin \pi 0.05}{\pi 0.05} \right)^2}{0.822 \times 10 \times (\sin \pi 0.05)^2 + 1}$$
$$= 8.25$$

Reduction in SINR = 1.75
= 17.5 %

Consider a carrier frequency offset epsilon approximately equal to 5 percent. Consider a carrier frequency offset epsilon equals 5 percent equals 0.05. Consider a carrier frequency offset epsilon equals 5 percent equal 0.05, then we have the SINR equals remember SINR equals p that is 10 into magnitude H square into sin pi epsilon divided by pi epsilon whole square; that is sin pi into 0.05 divided by pi into 0.05 whole square divided by 0.822 into p that is 10 into sin pi epsilon which is 0.05 sin pi of 0.05 whole square plus sigma n square, which is the noise power; which is one and this can be shown to be you can verify this by computing this is 8.25.

Previously in the absence of carrier frequency offset the SINR was 10 dB which is 10. Now, as carrier frequency offset has increased to that is we have a carrier frequency offset of 5 percent, the SINR has dropped to 8.25. Hence, the reduction in SINR equals 10 minus 8.25, which is 1.75 which is essentially 17 percent, which is equal to 17 percent, because out of 10 the degree is 1.75 which is essentially corresponds to 17.5 percent reduction in SINR. So, because of the carrier frequency offset of 5 percent, epsilon equals 5 percent then there is a 17.5 reduction SINR.

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The image shows a digital whiteboard with handwritten text in purple and blue ink. At the top, 'WiMAX' is written in purple. Below it, the subcarrier bandwidth is defined: $B_s = \text{subcarrier BW}$ followed by $= 15.625 \text{ kHz}$. Then, the frequency offset is calculated: $\text{freq. offset} = \frac{5}{100} \times 15.625 \text{ kHz}$. This is followed by the result $= 0.78125 \text{ kHz}$ and an approximation $\approx 0.78 \text{ kHz}$.

Let us look at what it means to have a carrier frequency offset of 5 percent. Now, we said 5 percent is the net is normalized to the bandwidth of the sub carrier. Let us go back to our WiMAX example WiMAX; we said that is fixed profile WiMAX. It has roughly, it has let me remind you. It has 256 sub carriers all right each of bandwidth; the sub carrier bandwidth B_s equals sub carrier bandwidth equals 15.625 kilo hertz.

We said the bandwidth per sub carrier is 15.625 kilo hertz. So, 5 percent of 15.625 kilo hertz that is the frequency offset. So, the frequency offset net absolute terms is equals 5 by 100 into 15.625 kilo hertz that is equal to 0.78125 kilo hertz that is approximately 0.78 kilo hertz all right. So, the net frequency the absolute frequency offset of 5 percent in this WiMAX system it means that it is about 0.78 kilo hertz, but WiMAX we said operates at 2.4 Giga hertz byte. So, if you look at this absolute frequency offset as if percentage of net carrier frequency. So, what is the percentage of this frequency offset at 2.4 Giga hertz?

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The image shows a digital whiteboard with handwritten mathematical calculations. The text is as follows:

$$\therefore @ 2.4 \text{ GHz carrier}$$
$$= \frac{0.78 \times 10^3}{2.4 \times 10^9}$$
$$= \frac{0.78}{2.4 \times 10^6} \left\} \leftarrow \text{extremely small fraction}\right.$$
$$\approx \frac{1}{3} \times 10^{-6}$$

So, the percentage at 2.4 Giga hertz carrier equal 0.78 divided by 0.78 kilo hertz divided by 2.4 Giga hertz which is essentially 2.4 into 10 to the power of, 10 to the power of 9, and if you look at this is approximately this is equal to essentially 0.78 divided by 2.4 into 10 power 6. So, if you can look, if you look at if you look at this, this is an extremely small fraction; if you look at this, this is 0.78 kilo hertz and the 2.4 Giga hertz. This is roughly the fraction, so if you look at this this is an extremely small fraction. In fact this is approximately equal to one-third into 10 power minus 6 that is it is of the order of 10 power minus 6.

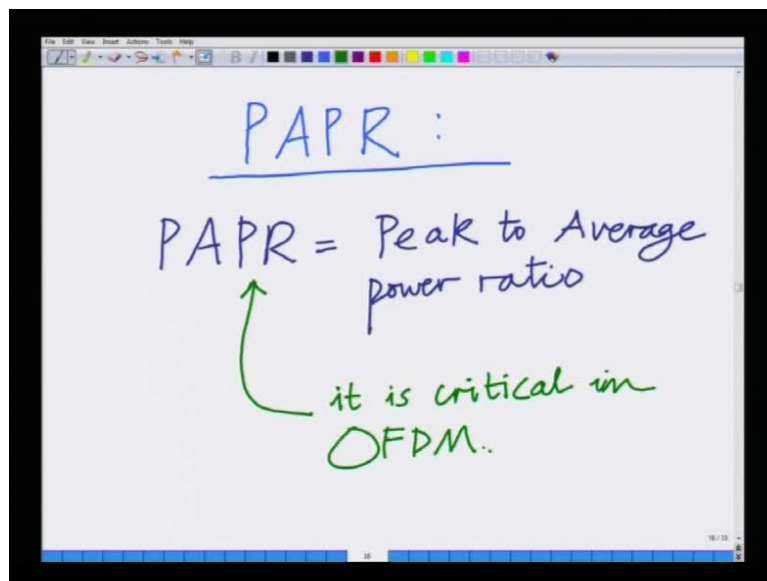
So, what this means is at the Giga hertz frequency when your carrier at 4 G wireless system is at Giga hertz, even if you have a carrier frequency offset that is roughly 10 power minus 6 fraction of that carrier frequency or roughly 10 power minus 4 percent that is 0.0001 percent of that carrier frequency that can result in huge S/N in fact 17.5 or 20 percent decrease in SINR the receiver end; which means that carrier synchronization at the receiver end. This 4 G OFDM system is a very critical aspect.

Because if the number of sub if the number of sub carriers is of course, typically large that is 256 and as you progress to higher and higher bandwidth. The number of sub carriers is going to increase that is 256, 512, 1024 which means that you synchronization also has to improve. Because, even if you have a small carrier frequency offset at the receiver that means inter carrier interference which means interference from a large number of sub carriers and the tremendous decrease in the SINR at the receiver that is what this seems to imply.

So, carrier synchronization is a very, very critical aspect of any OFDM system. Because, if the carriers at transmitter and receiver are not synchronized then we have serious interference inter carrier interference which results in a tremendous decrease in the SINR at the receiver. So, with this we finish the topic of the impact of distorting effects on OFDM; that is the impact of carrier frequency offset again to go through this material again to enhance your understanding of the various expression that we have derived for the impact of the SINR affects the signal power, the interference power and what is the single to interference noise ratio and what is for some reasonable carrier frequency offsets in fact.

We have looked at the examples, what is the impact in a practical scenario? What is the impact of having a carrier frequency offset of 5 percent at the receiver and so on all right, and next I want to address one more practical issue. Another equally important if not more significant practical issue in OFDM which is related to the PAPR that is peak to average power ratios.

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So, I want next issue I want to address in this context is the PAPR issue in OFDM. This is the PAPR issue in an OFDM system. What does PAPR stand for? PAPR stands for PAPR equals peak; it requires peak to average. PAPR stands for simply stands for peak to average power ratio and simply put this is a critical this is a critical factor in OFDM. This is critical, this is critical in an OFDM system. What does this PAPR issue for instance? Consider a typical non

OFDM transmission system. Let us go back for a moment to understand. Let us go back to a single carrier system.

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Consider a non-OFDM or single carrier system with BPSK modulated symbols

$$\begin{array}{ccccccc}
 x(0) & x(1) & x(2) & \dots & \dots & \dots & \dots \\
 +a & -a & +a & \dots & \dots & \dots & \dots
 \end{array}$$

Power in each symbol = a^2
 $= \text{Peak power}$

Average power = $E\{|x(k)|^2\} = a^2$

Consider a non OFDM or single carrier system OFDM is a multi carrier system. So, I am saying just let's go back and consider our single, let us go back and consider our single carrier system for instance. Consider a typical BPSK with BPSK modulated symbols. Consider a single carrier system with BPSK modulated systems. We have let us consider its transmitted symbols sequence of x_0, x_1, x_2, \dots . Now, each of this is plus or minus. Let us say the amplitude level is a and the information depending on the phase is plus a or minus a that is for a plus one you transmit plus a ; for minus you transmit minus a . So, we have plus a let us say the next symbol is minus a , plus a and so on.

Now, if you look at this sequence, what is the power in each symbol? The power in each symbol is each symbol is either plus or minus a in both cases the power is simply a^2 which is expected a^2 which is simply a^2 . So, the power in each symbol is a^2 which is also the peak power. Because, if you look at power in each symbol is the same; so the peak power is a^2 as well as the average power is also a^2 . So, this is also the peak power, how about the average power equals expected magnitude a^2 ; the average power is also a^2 .

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Hence, in this single carrier system, both peak and average power = a^2

Ratio $\frac{\text{Peak Power}}{\text{Average Power}} = 1 = 0 \text{ dB}$

Hence, there is no significant deviation from the mean power level:

Hence, both peak power and average power in this single carrier system is a square when a is the transmitted amplitude. Hence, both peak and average power equal to a square. Hence the ratio of peak power to average power equals 1. In practice it is not exactly 1. It is close to 1, because of pulse shaping and so on in the base band; but for all practical purposes, I can say the peak to average power in a single carrier system is close to 1, which is essentially you can write this also in dB terms as 0 dB that is there is no.

So, if you look at the mean power level that is there is no significant fluctuation of the power over the mean, that is both the mean and the peak power that is; if you look at the power instantaneously, there is it is it is closely tight to the mean that is there is no significant deviation about that mean. Sometimes, you can have a signal which has some mean but there can be significant positive and negative variation about the mean so that the net average is the same but the deviation is high.

What we are saying here is that both the mean and peak is also same which means the deviation about this mean are also controlled essentially that is what this mean that is what this says. Hence, there is no significant deviation from the mean power level. Hence, this ratio close to 1 also means, hence there is no significant deviation from the mean power. Hence there is no significant deviation from the mean power level.

So, we looked at the PAPR the peak to average power ratio, in the traditional conventional single carrier system and we said that it is close to 1 that is the mean power is a square. The

peak power is also a square. Hence, there is no significant fluctuation of the instantaneous power over the average. So, because of lack of time, we are going to stop here and we will start from this point and continue this discussion in the next lecture when will look at what is how does the PAPR look like in an OFDM system.

Thank you very much.