Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 31

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Okay let us continue from the previous video where we left so we will now be talking about certain generic lema's of theorems about other delta network or in that sense it is actually true for any Baniyan network and.

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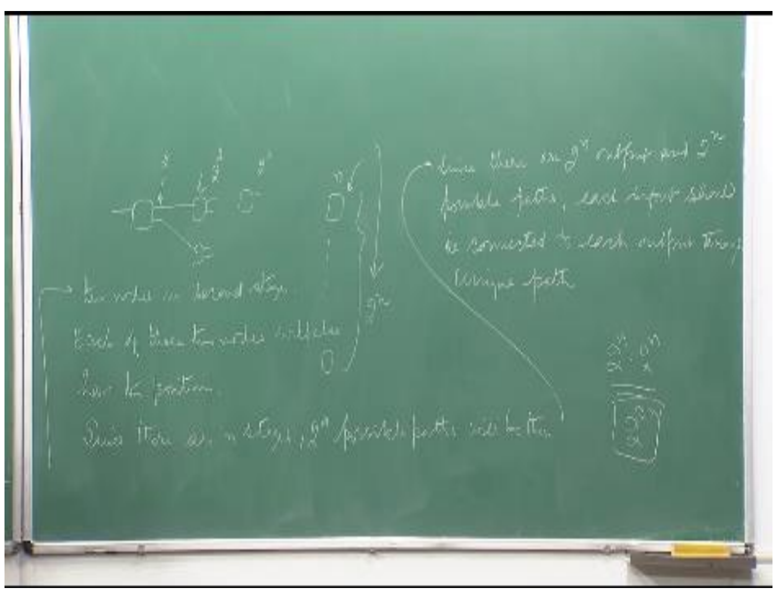

So we start with the Lema one so this says that in a 2/2 so remember I am actually not talking about the delta Banyan network constructed out of 2/2 cross bars okay so in a 2/2 network so outputs delta but it is also valid even for a Banyan configuration there is the unique path from each network input each so this is basically also a definition of a Banyan network so we can actually proof it also very easily in our construction so proof goes that there are 2 n paths from each input to all outputs.

So what is it actually mean is that you have one input and there are 2 n outputs it is a 2 n/2 2 n Δ so there has to be 2n paths from n and input to this then only there can be a connect which is there okay now in this case the first note actually has only two positions either it can be cross or bar so you can connect from here to here so this first stage it is only able to go to two ports next stage will have again two and I think if we will never ever be actually having a situation where these two output ports is going to be connected to the same switch.

They have to be connected two different switches so this will be connected to now two switches here which in turn will give me 2 22 number of ports which can be reached then if I go in this fashion the third stage will give 23 so by that time I reach the nth stage this will give 2 n okay so there only 2 n possible outgoing ports and there us going to be exactly one path from each input to each output okay so that is from this input there is going to be exactly one path to each one of the outputs because every time I can only create for count onto two things and I am creating a new path, so I am able to generate only 2^n paths from an input this is true for all other inputs also that is where this particular statement that na$2^{n/2}$ nΔ network.

There is a unique path actually holds true so whatever true for this input is also true for all other inputs and hence forth this lama is stands proof, okay. So whatever I listed that actually ned to be written here as a proof but I am not doing that because it is suppose to be explain, so you can write down that statement and then of course consequently this will get prove, or maybe I can write it down.

So we can write in this fashion so input is connected to only one node in first stage and this node has two positions so essentially it creates two paths and each of these positions can connect to two nodes in second stage and each one each of these two nodes will also have two position since there are n stages 2^n possible paths will be there and since there are 2^n outputs, $n2^n$ possible paths.

So outputs and each of the path should have unique 1 to 1 correspondence each input should be connected to each output through unique path this also as implication because from here there is 2^n paths the 2^n inputs so we would total number of possible paths which can exists will be $2^n.2^n$, okay so this trans out to 2^n number of paths which exist in total. So from here the level 1 is done, so let us go the theorem 1 this again is for the same $2^n x 2^n$ delta network. So whatever we have already understood I am just formally writing it actually.

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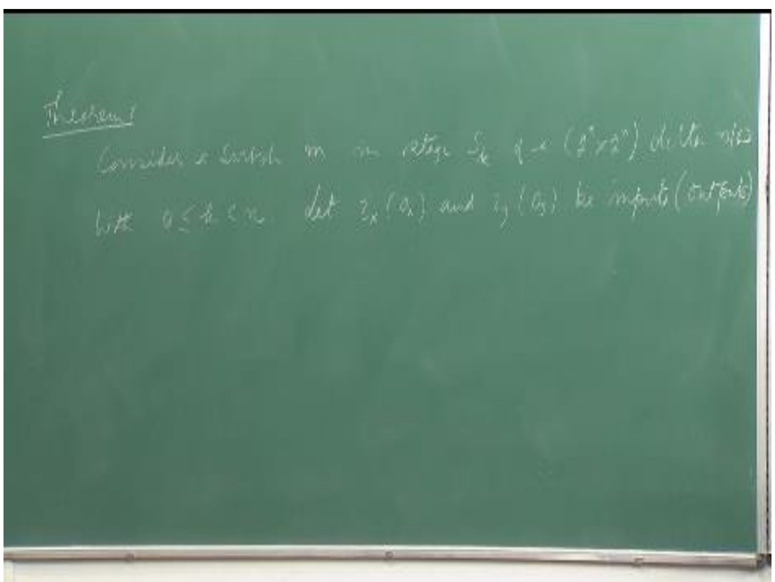

So theorem 1, says that you consider a switch m in a stage $S_k(2^n x 2^n)$ delta network with 0 less than k less than n, okay so stage is going to change from stage 0 S_0 to S_{n-1} total n stages., okay so it is there is no equality being put here because of that, now let $i_x(o_x)$ so i stands for input o stands for the output, so i_x and i_y and o_y be input so whenever I talk about o_x o_y these are output when I talk about i_x and i_y these are inputs, so what is actually happening is the middle stage any switch if this one is S_k stage.

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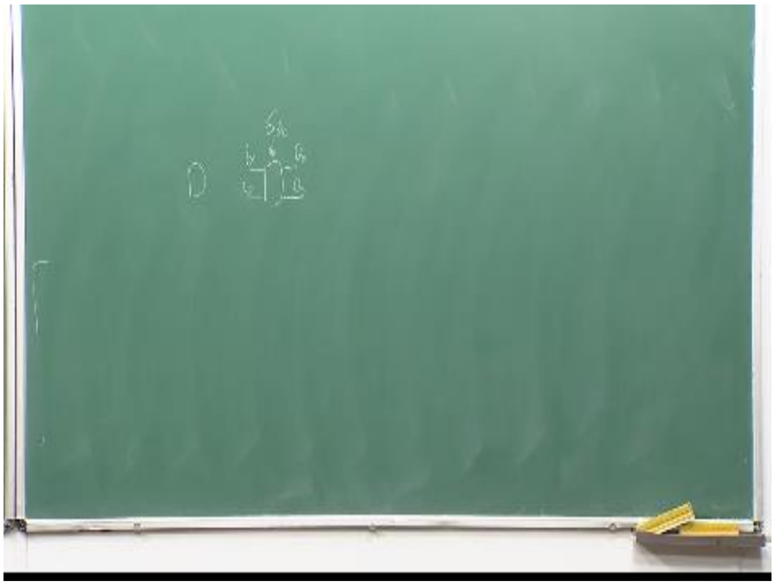

If this one is S_k stage so there are two inputs and the two outputs so I call this one as i_x I call this as an i_y, okay I can talk about o_x and o_y so these are the inputs so you can take any switch, so this one is the S_k stage and some switch so anyone of these can be taken, so because whatever assumptions we will be taking is actually going to be true because of that.

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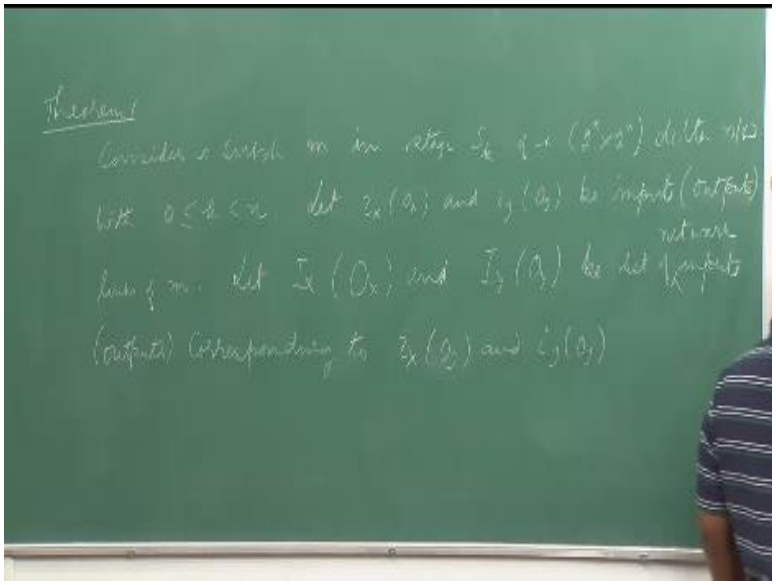

So input and output links of m so this has to be, then we define let I_x this is set actually now O_x with this output set I will define, I will explain what is it set of input in fact set of network input or outputs corresponding to ITNOY what is it actually mean is that if I look at this actually has been connected to some switch which must be having to inputs which is connected to further switches and so on so ultimately this will be now mapping on to certain inputs of the switching network delta network.

So this set I define as ix the second input also be again connecting to multiple switches so this set of inputs I define is i_y similarly when I look at this, this is connecting some switches this must be further connecting to some switches and so on so ultimately I will have at some place the outputs I call it o of x and some output o of y of course you can notice that I have drawn them to be separate there is no common agreement between them that is one of the statements of the theorem actually.

That is the way I have the reason I have done it this way so once this is true then I_x or o_x remember this is a set okay this is the small o and these are the capital O I think I make it bigger and r disjoined okay now this is true because when you whatever is true in the forward direction these connecting to two different switches okay this is like a de multiplex tree so ultimately it is going to go to one set it is going to go to another set.

And these two sets have to be disjoined this also true in the reverse direction okay same way i_x and i_y have to be disjoined so this follows from the lama which we have done further the b statement goes that number of elements inside I x and I y should be same and this has to be equal to 2 is power k and o x o y that’s cordiality of the two sets so y should be equal to 2 is power – n – k -1 so this actually can be understood in this session that this the 3 stage okay from where I go I get two inputs here when I go to this stage I get this one will become 2 this 2 will become 4

And so on ok so you count from here so there has to be s 0 to s 0 – 1 s k – 1 so the number of stages present here are k stages so this one input will forget in two and ultimately 2 is power k inputs here so I x cordiality has to be 2 is power k using same logic cordiality of I y I has to be also 2 is to power by k going into forward direction the last stage will be s of n – 1 ok that we have to define k can go maximum of n – 1 so this one will fork out into two so how many stages

Are there after s k so this is n – 1 I reduce this k out of it n – 1 n – k so those many not two responsible count theses stages after this and 2 is power that what is been return 2 is power n – k so cordiality of the outgoing sets output sets this is two is to power n – 1 n – k so both will be equal sets so that is the second statement this also follows from the lemma 1 now the next statement let me write it here itself a path from any input network any network input.

So when I talk about network input I am talking about, whatever the input coming to the network basically for the s0 stage, whenever I talk about network output, I'm talking about sn-1 stage, output coming out of those switches, when a part from many input network, in union of ix union of iy to network output, in ox union oy must pass through switch m.

So this also comes from the level because, this is connecting to the switches, so no other switch here will be connecting to these ones. This particular set is only reachable from here, from nowhere else, every this are de multiplied and technically which we are creating and if you see another switch is going to go to a different set, another switch going to a different switch. SO they will not be mapping onto a same set actually.

They cannot map to uniquely to the same set, so the moment if you look at this, and this anything any member from here want to connect here, it has to go to this switch only, there is no other option, so that also comes from level one and the d part which is the exactly 2 $^{n+1}$ paths from network inputs to network outputs, will pass through, switch n. So you can see total number of elements, which are here are 2^{2k}, total number of elements which are here is 2 $^{2n-1-k}$.

So from here to here how many total number of connections which you can setup ,you have to just simply multiply ,which will give me 2^{2k}(2*n-1-k) sorry this numbers have to multiply so this will be added actually ,so this will give me $2^{2(n-1)}$ okay, so this has to be multiplied by 2 ,so this is one set ,this is the total set I have these two ,so once I have to add these two ,so once I have so I am technically multiplying by 2 .

So this will turn out to be 2^n same I have to do here so I have to multiply by 2 this thing has to be 2, so this will become 2^{k+1}so when I will do it here when I am actually taking this into this there will be total number of possible paths which can exists it will be 2^{k+1}+n-k, which will give me 2^{n+1} so these many number of paths are possible from a single switch.

And how many switches are there these will be 2^{n-1}these many switches are present in this stage, so total number of paths which can actually exists is simply nothing but multiplication of these two, so this will give me 2^{2n}, okay, right $2^{2n}$and of course what we have computed as total number of possible number switches is will be, possible number of paths which can exists.

That depends mapping from input to output will be given by this total possible paths, which are there, so that’s the theorem 2 and all this is directly as a consequences of lemma1 so using de multiplex-try concept the proof actually comes from there.

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