Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 34

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Okay so let us continue from the previous video so we were actually discussing the transition matrix and for that we need to all fort in states so for reference I am actually making the corner so that.

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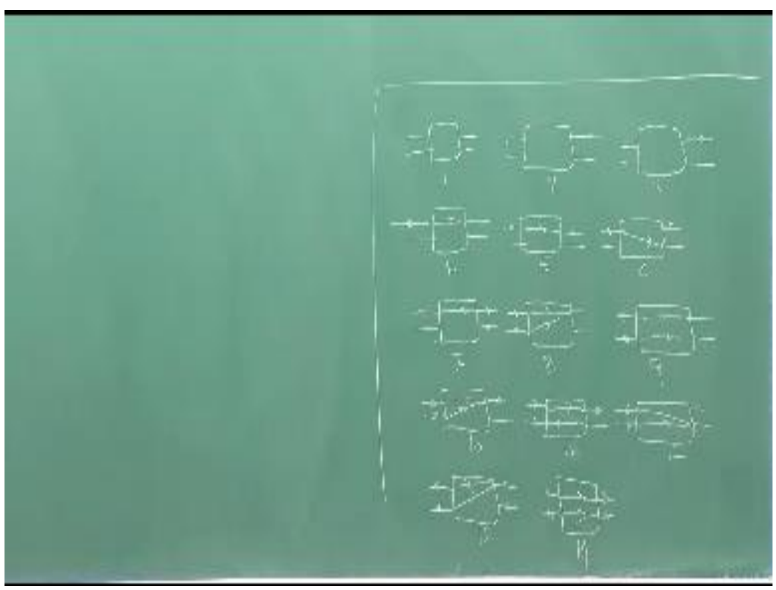

This can be referred Bacan forth again so quickly just writing it down it is a state one state two state four state five six seven this is state eight so these are the fortain states which where it mean referring to so coming back to the state transition table so.

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So I had so we will keep on filling so if you want to remain from a state one to state one and this particular transition table we call it at the gerthic stage transition table type one and this actually means going from m is the row id and i is the next state to which you are actually moving in the J^{th} stage this is the probability for that so if you want to reaming, from one to one if you are in state one you will surely will remain in 1 only is no other option if you are in a state two you will go to state two if there is no packet going out = QJ~ and if the packet goes out you come back to the state one.

So from three if you can want to go, you can actually if both the packets moves out you come to the state one if only one of them goes you come to state two and if both of the packets do not go from a state three you will come back to the state three only so from the 4^{th} one, so the 4^{th} state is this one there is no packet at the output so you are going to remain in the same state because there is no packet which can go out so in 4^{th} you will remain in the same state okay the 5^{th} state if you look there is one packet it can go it may not go.

So with probability PJ~ you will be coming to state 4 and with QA~ you will remain in go to the five okay so similarly I can keep on doing it so 6^{th} state again you can look there is only packet which is present it can go out you can come back to the 4^{th} state otherwise you will remain 6^{th} so in the 7^{th} state there are no their two packets so both of them goes out you come to four if of the upper one goes out in that case you will lend up in, six if the lower one goes out then you end up in five so and if none of them goes out you will end up in the same state which is seven okay similarly for eight.

I can write down so eight if you will see that there is no packet at the outgoing ports so it will always remain eight which surety with 9^{th} state again there is there are no packets it will remain in the 9^{th} state since this pars matrix rest all elements are 0 only certain elements so I am just filling up those if you are in 10^{th} state if the packet goes out you go back to eight if packet does not go out go to 10 okay so with 11^{th} state let us see what is going to happen there is only one packet out if the 11^{th} packet goes out then you will get into 9^{th} state packet does not go you remain in 11^{th} state and the 12^{th} see let us see what is going to happen the 12^{th} is only one packet which is at the top so if it goes out you will end up in 8.

Otherwise you remain in 12, 13^{th} if both of them goes out PJ $\sim^2$ else PJ~, QJ~ for 10^{th} or you will end up in 12^{th} state PJ~, QJ~ or you will end up in 13^{th} state PJ $\sim^2$ for the 14^{th} state you will be PJ $\sim^2$ with that you will remain 9^{th} state or you will come back to go to 2PJ ~, QJ~ for 11^{th} state or you will end up in the 14^{th} state itself OJ$\sim^2$ so that is the transition matrix for the when the packet will go out this is basically after a step one, okay in a state J.

So once you have this you can actually find out after a step 1 the probability that very switch in J^{th} stage will be in state I after step 1 in time interval K will be given by summation of m goes from 1, 2, 14 this was the probability that you are in m^{th} state after 0^{th} step or at the beginning of the interval in J^{th} stage and of course your switch goes from m to I stage as per this partition matrix.

So you have to compute this transition matrix so PJ elements so these PJ ~ QJ ~ will be depending on the state probability of j plus first stage, okay. So in this case now 1< = I <= 14 this is how the I will be attracting, okay. Similarly we can actually build up a matrix which will be for stage 2, so we call this matrix as TJ 2, okay. So let me put it here and we will just fill up the entries again.

The matrix so this will also correspondingly will change so this is the matrix this should ne after step 2, okay. Transition matrix after step 2 so if you are in a state 1 so this transition matrix allows the packet to actually move to the second to the from input to the output in stages so if you are in a state 1 you will remain in a state obviously there is no other change if you are in state 2 you will remain in a state 2, okay.

You are in a state 3 you will remain in state 3 you are in a state 4 then your packet actually move to the output side and you will come to the state 2, okay. So from 4 you will be able to come to so this is the probability sorry this is the transition probability so which is one, so with probability 1 you will coming to state 2 from 4th state from 3rd from 5th state there is no possibility 5th to 5th it will remain.

From 6th state you will go to the state 3, okay and from state 7 you will remain in a state 7 there is no change, a state 8 if you look there is only one packet which will go on that side and you will end up in a state 5, okay. State 9 if you look both the packets will go to the output side and you will end up in a state 3, the state 10 so if you check from here you will remain in state 10 so 10 remains in 10.

State 11 if you look this packet will move to the output so you will end up in state which is 7 so 11 to 7 will be there transition, okay and then state 12 if you look there is output is empty so one of the packet will go there so any one of them actually can go on that side so you will end up in same state as 7 so 12 to 7 there will be a transition. 13 you will remain in state 13 only no change and state 14 will remain in state 14 only, so that will be happening after a step 2 so once you know this you can actually find out.

If you know what was your I cannot filled up a transitions based on this, this still P2 is the probability after step 2 what is the changes that what is the probability you will be in state 2 what is the chances that what is the probability you will being state 2 in case time step, this will be $\sum$ m=1 to 14 P_1 m_j, okay this is the probability after step 1, so once you know this you know the transition probability you know what is the chances of being in state i after step 2, so you actually write T_j^2 m_i and this is what is going to be use for this one again in this case i will go from 1 to 14. Now there is a third step which will be happening which is, when the packet will arriving at the input.

So we can actually do write a build up at transition matrix for that I call it T_j^3 and let me fill up, now we will writing P_j bar and Q_j bar as the elements here, again using same strategy, so if you again look at the state so you are in state 1.

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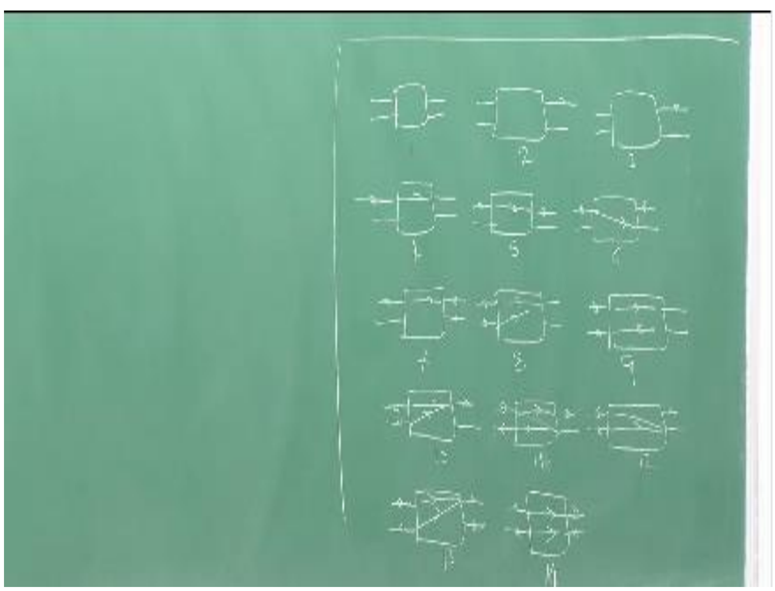

There is no packet which is going to arrive at the input, okay so you have to look at the arrival probability. So both of them are free if there is no arrival, so Q_j bar square you will remain in the state 1, if only one packet arrives which is having probability of 2 P_j bar Q_j bar in that case this will end up in state 4, if both of them arrives then you will be actually in state either 8 or 9 one of the two states you will there. So correspondingly we can actually put up the values.

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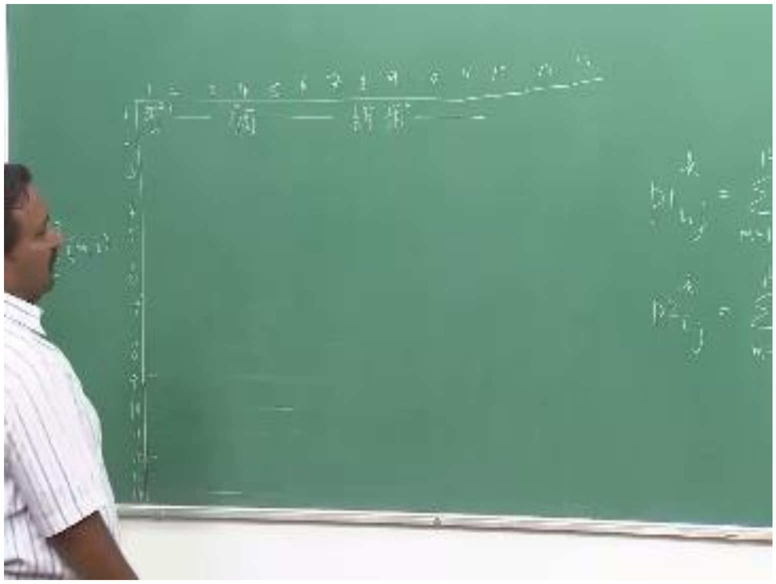

So it will Q_j bar square, remember it is a bar this time now for the third matrix and you will come to the fourth state if you have put P_j Q_j bar and of course with the 8th and 9th state you will going with half probability actually ½ P_j^2 bar ½ P_j^2 bar, okay.

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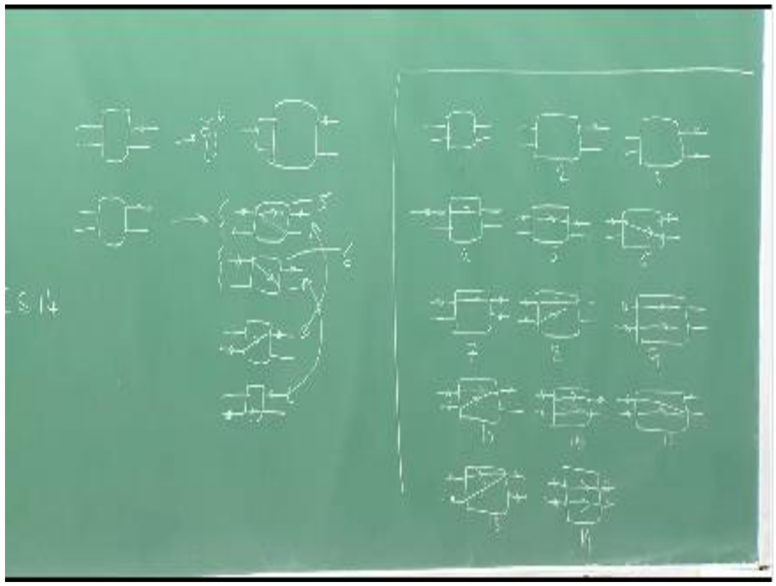

So if you are in state 2 then what is going to happen you can get in that case no packets so you will remain in state 2, so if you are here there is only no packets arriving so with Q_j^2 bar you will go back to the same state, okay if there is only one packet is coming so you may actually have two choices here, you may get a situation where the packet only one packet is coming so either packet comes here it does not come here, okay and this packet can be uniformly directed to either this case or this scenario these two.

Other cases the packet actually comes on the other side and it can be directed on this side or other one so those that possibility is this actually. So you can actually see these two cases are same equivalent, these two cases are also equivalent so with half probability this will be happening with half this will be happening so this corresponds to state 5, and this corresponds to state 6, okay. So we can now correspondingly put the values.

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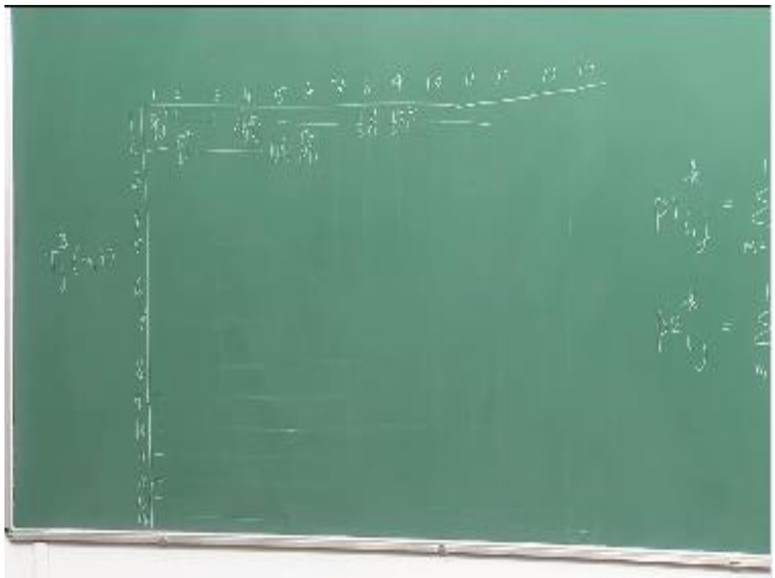

So Q_j bar will happen you will remain in same state with fifth and sixth you get P_j bar Q_j bar, now there is a problem and two packets will come so then slightly tricky so let see what is going to happen if there are two packets arriving.

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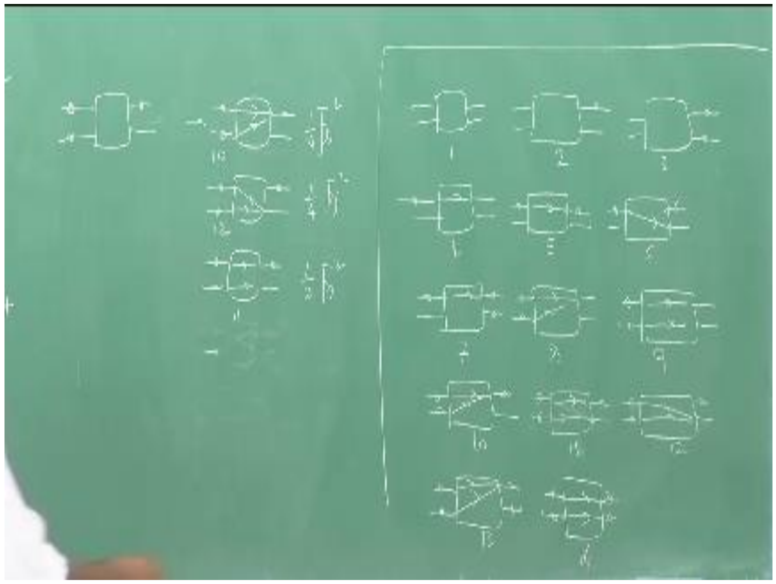

So we are talking about the state 2, state 2 is this so the two packets which are arriving so it is a P_j^2 bar the probability but in which all state I can move. So these both can be directed to the same port the upper one so this can go to this scenario, this can go to this scenario and this can happen this one and you can have a scenario like this, okay. So these all four can happen with the equal probability so this will be happening with P_j^2 bar 1/4, this will be happening with P_j^2 bar ¼ now these two are technically same.

So I can simply write it as 1/2 P_j^2 bar so this one is straight, this one state 11 you can see from here this one is state 12 and this one is state 10, so 10, 12 and 12 so we have to write the corresponding values so it will 10, it will be 1/4, 11 be ½ P_j^2 and 12 will be 1/4 again, okay. So similarly now moving over to the third one, so the third one we are talking about this particular state and let us see the scenarios which happen in this case.

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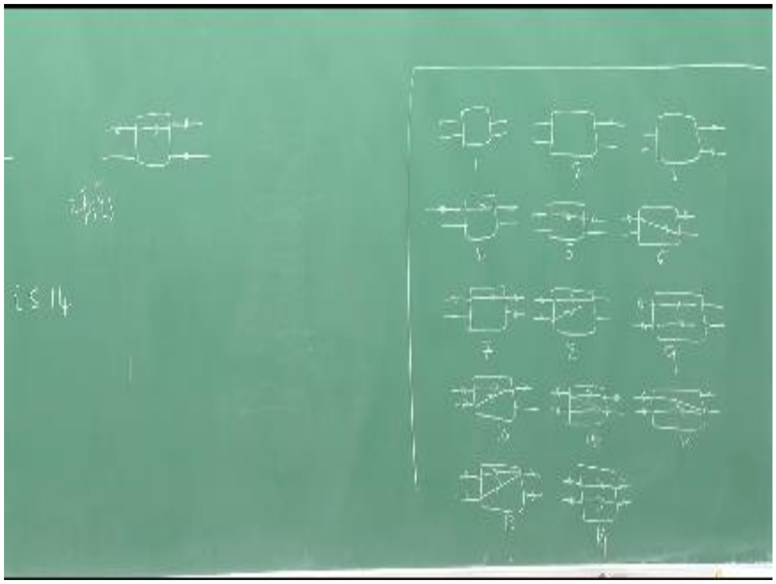

So they are two packets which are there at the buffer, so I can actually write may not get any packet so with Q_j^2 bar I will remain in the same third state, okay. If there is only one packet which is coming so this will be directed to any one of the ports so with 2 P_j bar Q_j bar you will end up in a state which is 7.

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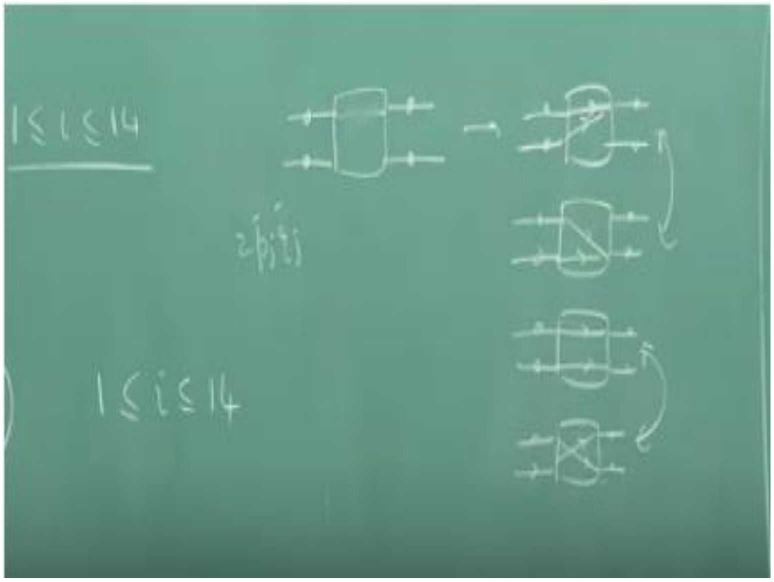

And when 2 packets are coming then what is going to happen if y2 packets will come you will have a situation when both of these are directed to one okay they can be directed to this scenario they can be directed to parallel or they can be directed in cress cross manner so these two are same states these two are the same sates so it is half of okay so 13 and 14 they actually belong to 13 and 14.

Okay so same I can do now with 4 the fourth state is this so fourth state let us see what is going to happen there is only one packet which can arrive and that packet can be now directed to there is no packet arrival you remain in 4 states so that is q okay and the other packet which arrives so there is only one packet which arrives this can be directed here or this can directed palely so it is half of so I have to write here this a 10, 9 okay.

So now similarly I can do it for 5 so in fact more or less now it is straight forward the fifth one you will remain in state five there is no packets which arrives if the packets arrive in the fifth state this can directed to this or this with equal probability if it is directed to this it is half actually so this turns out to be case number 10 and if it is not this case number 11 so it is between divided between 10, 11 so it is half of pj bar okay.

So sixth if you look sixth state is this one there is only one pack\et which can arrive this can get directed to this or this with equal probability if they are both directed to this you actually have got 12 if they and if this is directed to this one then you end up in 11 so 11 and 12 basically will be there so 6 will remain in 6j bar with the 7th similarly I will remain in 7th state with qj bar and 13 and 14 I will do half of pj bar actually.

Okay then we look at 8 so there is no possibility of packet coming in you will remaining in a state 8,9 this same 10, 11,12,13, 14, all of them will be unity so 8 always remaining the state 8,9, will remain in 9 and so on and this is the last stage which will also be unity okay, so these are last matrix and based on this I can actually estimate what is 3p ij this incidentally also is equal to p0k+1 ij as per the way I have define.

So this I can write down the equation is m goes to 1 to 14 p2 mj in step k tj 3 mi I goes from 1 to 14 okay so once you have this I can actually keep on I can actually use this p0 which are there to estimate at any stage what is he probability that a packet will be available at that port okay so packet availability can be written now will be requiring this again so I can erase this and write down the equation.

So probability that the packet will be existing is define by pj at a state j and this probability will remain same for all output links of the state j all also chase so this is probability that I packet exist at an output link of a switch in stage j at time in fact tk that is a when the interval will start so this value will be nothing but so basically I will just look at what are the state probabilities at time in a step 0 basically so I will be using p0 and the various states of when you are in state one there is no packet at the output link.

So this cannot be there with ½ it is going to be there if you are in state 2 , so I can write down this ½ (P0 ,2jk) the third one there are 2 packets ,so with there will be actually two terms ,so one which is with the half other one without ½ so this will be P0(3jk+..) the fourth one will not be there ,fifth one will be there with the ½ and similarly the sixth one will be there with the ½ ,the seventh one will be there with full ,and eighth will not be there and ninth

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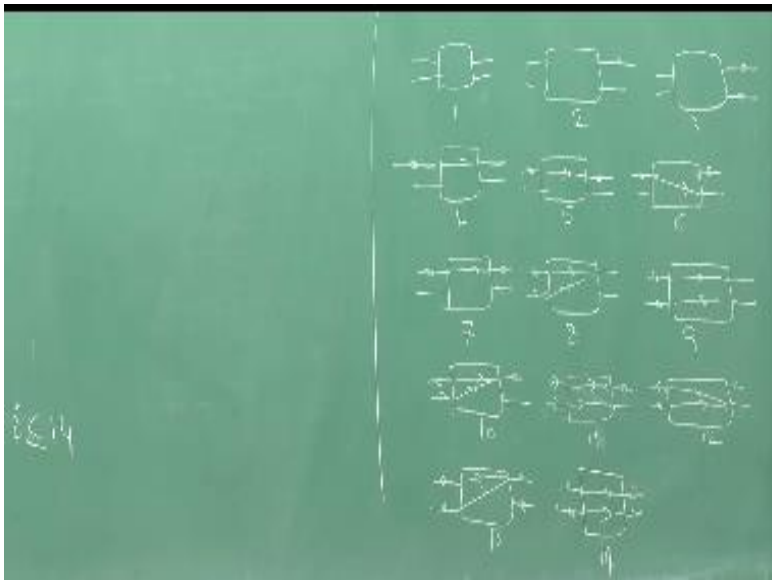

It will not be there tenth will be there with ½ ,eleventh will be there with ½ ,twelfth will be there with ½ thirteen and fourteen will be there with full ,so that's the probability that a packet will be existing so this will be required to essentially estimate the sequence. So we will be using two boundary conditions once we have all these equations I can now erase these equations ,you have to remember it for computational purpose, so our boundary conditions which will be there will be,

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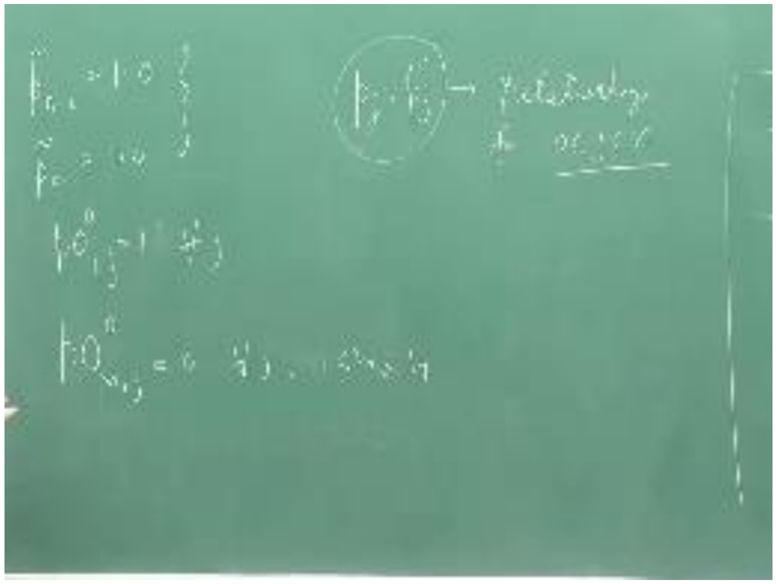

Pn-2~=1.0 packet is immediately emptied the moment it goes to the output link of the n minus second switch, arrival is the maximum loading condition packet is always available these two has to be used and initially you assume that that there are no packets anywhere , zero packets in the network, so n of course you also assume the initial condition P0 and the 0^{th} time step in the beginning 1j so you are always going to be in state 1 ,okay, for all stage is for all “j” this will be equal to 1 ,to begin with and of course P0 in time step 0 ,all other states for all j and m>1 it is not equal to 1 ,so for all other states it will be made 0 that’s the initial condition ,when you have to do the computation .

So what you do is using this actually you will find out ,what is p1 mj0 ,you will be doing it for all well use of M so you already have after 0^{th} step so based on this you compute what’s going to be thereafter the first step ,and you will be doing it for all j ,okay, so for all j you will be doing so from there you will be now computing p(20mj) again this will be done for all, mj’s from there you will be now doing ,p30 mj for again all ,well use of m and well use of j.

And from there this is nothing but this will become your P(0) for step 1 now mj this will be fed here and then the next iteration ,next step will be happening ,you will iteratively keep on computing ,so ultimately all your state probabilities ,they will actually taken to be 0 in the beginning ,this will start building up and ultimately you will get steady states situation in your computation.

And once you have that you can actually from their get that these particular probability pj which will give you the throughput so if you know Pj you can compute Pn-1 and that will give you the throughput on that particular line, okay, so but when you have should stop your iteration that is still remains the question.

So normally the way it should be done is this is the probability that a packet will be there on a line and this the probability that packet is going to, go out this is the conditional probability so packet is there and its going to go out, when this value becomes relatively constant, so your computation would have been stabilize that point of time, and this should become stable for all j.

Okay, so fouler stages same average number of packets going out this will remains constant irrespective of j so p1 so this actually means among various stages, so if you look at the first stage your value of Pj of p1 P0 in this case will be higher, okay and what will happen because this is higher pj~ will be lower actually in this case.

As you go to the next stage p(1) will be actually lower but p1~ will become higher ,so this will actually start growing and this will be reducing across the ages but their product will become same and that's what should be observed in your computation ,and that's when you should you can stop your computation and then you use that estimation to estimate your throughput.

Okay so once you have this so you can actually find out what is your p(n-2),okay so that's will give you the throughput and total number of ports 2^n you can find out what's the total delay , normalised delay this will give you the total throughput of the switch. So that's how this actually computation this whole model can be used to estimate the performance of the buffered delta.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL, IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production