Adaptive Signal Processing
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Lecture - 11
Misadjustment & Excess MSE

So, let us take a quick recap of what we have been doing very quickly, I am just showing the previous slides.

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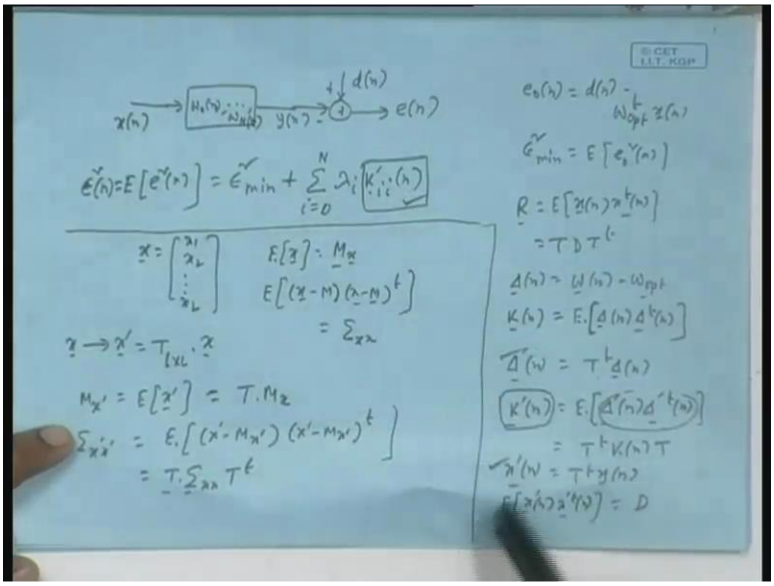

This is one of that you know filter structure which I began with everyday and we have calculating this en mean square error epsilon square, n is a variance of this mean square error. So, as I told you we are not using the optimal weight, because in the steepest descent procedure we need not keep R and p R matrix and p vector as it is. But we replaced by some approximation by some wear estimate, as a result the filter even in steady state will not be converging exactly on the optimal weights. But, will be dancing around that. So therefore e n will never be the optimal e n, so if you find out the mean square value of e n epsilon square n.

Now, we worked out its star equal to the minimum variance, I mean epsilon square mean plus something epsilon square mean comes out to what where you put the optimal weight the corresponding error e o n the variance of that there is epsilon square mean. But, plus some terms what are these terms. If you have the input autocorrelation matrix, this you decomposite like this T d T transpose T consist of the ortho normal Eigen vectors. So, D consist of the Eigen values which are all real and positive T is a unitary matrix, then this Eigen values lambdas they figure here and what are k I what is k I I prime.

Well, delta n we defined as the deviation of the weight vector from the optimal 1 this is the deviation, the deviation will only fluctuate around 0. So, mean 0 because the deviation I mean W n is fluctuating around W opt in the steady state, but it will not be equal exactly equal to 0 always. So, this delta n if you take the auto covariance matrix or autocorrelation matrix either way they are same, because mean is 0 you call it k n. Then from k n we get k prime n this way that from delta n. If you find delta prime n as T transpose, T transpose is coming from the autocorrelation matrix R.

So, T transpose delta n then delta prime n as a covariance matrix equal to k prime n which is related to k n by this. So, from delta n. We obtain delta prime n from the covariance matrix of delta prime is k prime n and diagonal entries of that figure in this expression. Simultaneously, we also derived x, x prime n from x n by T transpose x n, and x prime is then vector which has got autocorrelation matrix which is a diagonal matrix given by the Eigen values. So, these are very standard step discussed time and again, so no need to get into that then what I said is this that this quantity which depends on k I, I prime n.

So, we must make sure that with time n as n tends to infinity this term becomes remains under bound it does not grow because if it is grows then the error is unbounded and it will be the whole algorithm will be destroyed. So, it has to be some finite quantity under our control, in fact the quantity also can be then made lower by some appropriate parameters. But, this should be first bounded that is as n tends to infinity this quantity k I, I prime n must not go to infinity it must be bounded up to some finite value. So, that value should be under our control using some lambda or may be mu and all that with which will play with that.

So, therefore you have to see the behavior of this k I, I prime n which is a diagonal element of k prime n, but then I said that instead of taking only the diagonal elements of k prime n lets take k prime n let us see how this matrix evolves in time. So, we developed a, we are trying to develop a recursive equation for k prime n very quickly the steps were like this.

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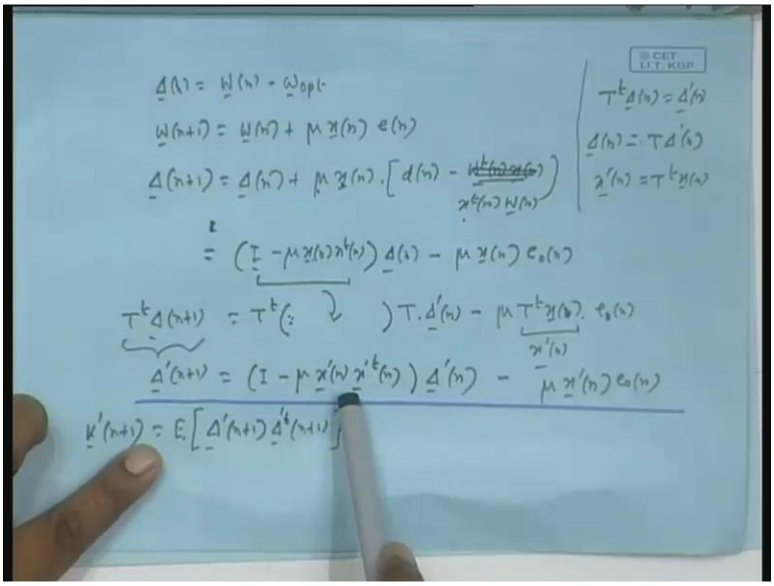

We know delta n equal to this, so we want to find out delta n plus 1 in terms of delta n delta n plus 1 was we know W n plus 1 this is the L M S equation. So, if you subtract W opt from this side and this side you get delta n plus 1 delta n here mu x n e n you expand d n minus x transpose W n and W n is again W opt plus delta put that back here. So, you get this, but we are not interested in delta we are interested in the prime, so you have to multiply both side by T transpose T transpose this you get this expression.

So, delta n T transpose delta n is delta prime, therefore delta n is T delta prime, so you put that here T delta prime into the delta T transpose of this side T transpose here. But, T transpose is a unitary matrix T transpose I, T was equal to I, T transpose x n was x prime n x transpose T that was x prime transpose. So, this is how you got it these are all standard step no question of explaining further I am just linking up that is all. So, this is the term, this is your k prime, so I now I want to do, this is your delta prime n plus 1, so what is k prime n plus 1 this is the covariance matrix for this vector. So, E of this matrix I want to, I want to develop recursive relation for this matrix, so k prime n plus 1 how it comes from k prime n, therefore what you do did then is this.

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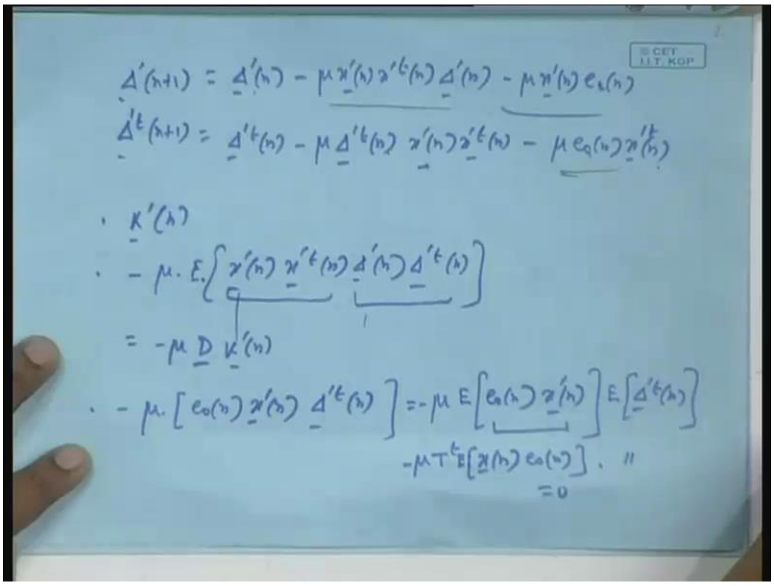

So, we took delta prime n plus 1 as it is as obtained by this equation, we wrote it entirely no parenthesis nothing broke it up. So, you take delta transpose of that, because after all what is k prime n plus 1 that is nothing but that is nothing but the this delta prime n plus 1 into its transpose then expected value. So, delta prime n plus 1 has got 3 terms transpose also will have 3 terms, so we will have 3 into 3 this multiplication will have nine terms that is very elaborate.

So, 3 terms, 3 terms we have all transposed and then we did cross multiplication and analyzed and then we found that we wrote down all the components. So, this and this was given next to k prime n you have to multiply and then take E operator on that this and this was given into k prime n this we worked out. So, this term was giving rise to 0 on this term was giving rise to mu D k prime n because this into, because of the independence assumption delta prime n k x prime they are independent. So, E will work on this E will work on this, so that is why D is the correlation matrix for x prime.

So, D will come delta prime delta prime transpose will give is to k prime n that was coming this was giving rise to 0. Now, again because of independence assumption this could be separated out from this and this is orthogonal to the elements of this that is why this is 0. So, this term was 0, so this is one term, this is one term wherever I get non zero term I have to collect them. So, I am just putting a tick not here one more term, so 3 terms done this, this, this next is this, this, this. So, this 1 again it is not very difficult delta prime delta prime transpose x prime x prime transpose again they are separable, so this will give rise to k prime this will gives rise to D.

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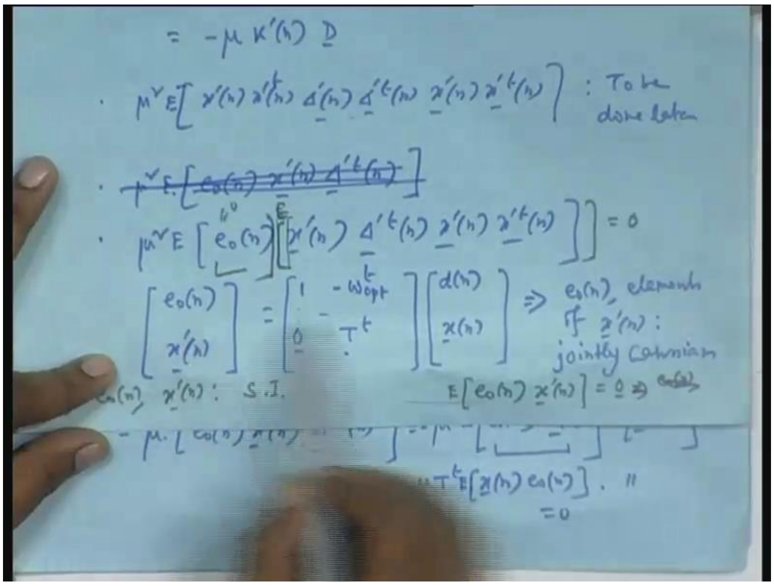

Now, that is why this and mu before that minus sign, so minus mu k prime n D previously it was D k prime n it is k prime n D, then this term minus minus plus mu square. So, this is a huge term that time I said I have to evaluate it later which I will do today, remember this term x prime, x prime transpose x prime, x prime transpose delta prime delta prime transpose.

But, again x prime, x prime transpose this I have to work out separately today to be done I said this cross term this cross term was x prime E 0, E 0 is scalar. So, you can put it behind, so in the front E 0 x prime and then delta prime transpose x prime x prime then I made the assumption that D n and all the elements of x n they are jointly Gaussian. So, how do you obtain e 0 n and x prime n E 0 n is nothing but D n minus W opt transpose x n and what is x prime n, T transpose x n 0 is here, 0 will take care of d n.

So, this is the matrix working on this giving you this, so any linear operator this matrix working on a Gaussian vector gives you a Gaussian vector, therefore these elements also are Gaussian. But, since they are Gaussian and E 0 n is orthogonal to all that is uncorrelated all the elements of x prime n that means this is also statistically independent because under Gaussian uncorrelated, means independent and vice versa. So, therefore E, e o n is statistically independent of this in this entire expression e o n can be separated out. So, because it was already by independence assumption independent of delta prime and now we have made it independent of x prime.

So, this is independent of the enter quantity on the right and now e of this and e of this is 0 because what is e o n after all T n minus W opt transpose x n, x n has 0 mean, D n has 0 mean. So, this was 0 mean, so this gives rise to 0, so no tick here and what are the other terms by the same logic this also this was becoming 0. So, you can see this cross term is 0 because after all this 2 can be separable from this and this orthogonal to the elements here. So, this and this also will give rise to 0, because e o n you can take out, it is independent of all the other quantities. So, this cross this cross terms will becomes 0 only this minus minus plus mu square E 0 square n which is scalar term x prime n, x prime transpose.

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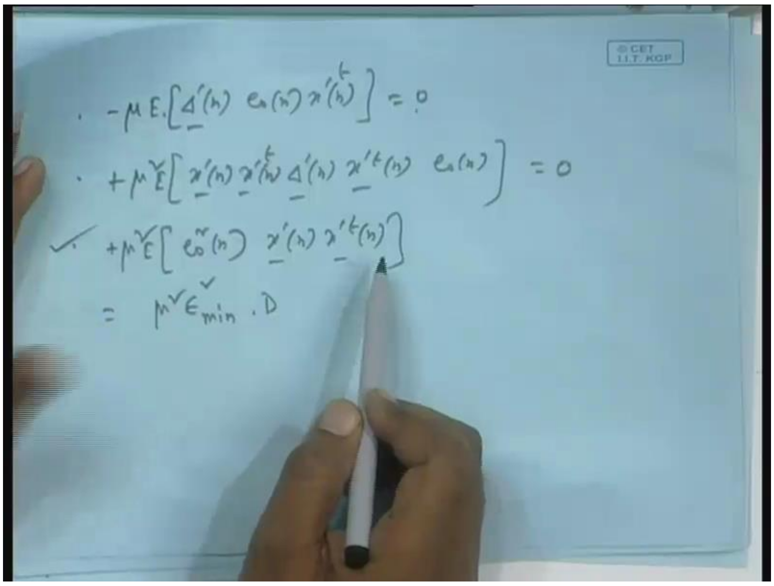

So, that is why I said those two terms are 0 and the third term is non zero E 0 square x prime, x prime transpose and e 0 is independent of this. Therefore, E 0 square is independent of these terms if x and y are statistically independent x square and y square are also statistical independent. So, this is separate which gives rise to epsilon square mean because the optimal error and this gives rise to the correlation matrix of that which is D. So, this is 1 term, so I have picked up the terms, now I have to evaluate the 1 which we did not evaluate last time, so let us start with that.

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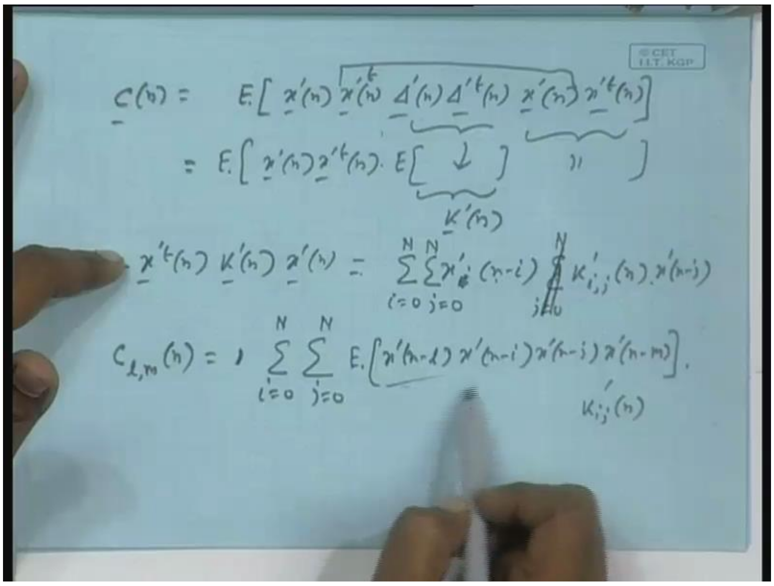

So, what was the term that was, you all agree I can apply E over this inside because this is separable from this and this side after all this will be a matrix this will be a matrix. So, this is a matrix all cross terms will it will be a product matrix, on that if you apply E in those product terms there will be terms coming from x prime n and this side and this side.

So, E can be when E works on that see if you take the entire matrix to resulting product matrix each element of that will be a summation of many product terms scalar products. Now, there if you apply E on each you can separate out components coming from this and components coming from them, can you see this. So, I can apply e inside or we have to talk on this further you have one matrix, see we will have some two terms say some terms multiplied here and some term some terms.

So, actually the product matrix in each term will be each term of the resulting product matrix will be summation of terms elements summation because after all if you take this product. So, any matrix into matrix is what this into this plus this into this and plus is there summation again another matrix. So, when you do this matrix product each term of the resulting matrix is a summation of many terms, but each term in that summation is again a product of several scalar terms. So, scalar once coming from this side from this side, from this side there are on each scalar if you apply E you can separate out.

So, you can apply E over the terms coming from the separately and E over the terms coming from the separately and you get a resulting expression that is following this method. But, if I beforehand apply E over this only I still get a matrix of those terms which I would have got by applying E after this entire expression and then you multiply and then apply reapply e again you will get the same thing. So, I think you can now you can realize it otherwise try a 2 by 2 matrix, if you want if you apply E over this entire quantity and this remains as it is what is this quantity.

So, this is equal to not D this is k prime n this entire expression suppose I call the matrix c n is after all not random because e has been applied already. So, what is C n, C n is E of this, this transpose k prime n this, this transpose, now look at this x prime transpose is a row vector k prime n is a matrix and x prime n this much is a column vector. So, matrix into column vector is a column vector pre-multiplied by a row vector, so that will be a scalar this much is a scalar. But, I am not taking transpose this much is a scalar, so scalar can be pulled out also suppose a scalar value is 2, so the 2 can be pulled out not out means not out of E.

So, I mean can be kept in the front then x prime x prime transpose is a scalar term, so please identity this is a scalar, so I can, so if you take this element only this is a scalar. So, the scalar term, let us evaluate the scalar term what is the scalar term I will apply E later what is the scalar term. So, this is a matrix, this matrix will multiply this vector, so we will get a vector i-th element of this vector i-th element of that vector multiplied and summed for all i-th element of this product vector is what. So, if you take the i-th row of k prime n first column first element second column, second element third column.

So, third element like that go on multiplying add that will give rise, the i-th element of this product vector that times i-th element of this row vector multiply and go on summing it for all the I is. So, that means i-th element of this guy is n minus I, you remember the definition of x n vector, x n, x n minus 1, x n minus 2 up to x n minus capital n. So, is x prime n, so x prime n minus I and then you have got this guy k prime I, j this is a matrix i-th element of this is denoted by this times j-th entry, i-th row j-th entry into j th entry here this much is very simple.

Now, you hold I fixed means i-th row go on swiping j as j moves j equal to 0, j equal all the columns of i-th row multiplying the different elements of the vector. So, the row into column, so this product vector you get i-th element, i-th element and i-th element here multiply and sum it for all the I. So, this is the scalar term instead of having the sum here I can put the sum here also instead of it here I can now together. But, normally D S P we do the other way double summation with an interchange the summation and all that, but here I am doing moving backward just took if like this fine.

But, this was only this part with e here of course, but before that I have got a column vector a row vector. So, this is a scalar this entire thing is scalar you can call it k or alphas you can call it even alpha, alpha n say. So, this is the matrix what is the typical element say l m-th element of this matrix that is E column vector row vector column vector x prime row vector x prime transpose and multiplied by. Now, whatever is the matrix that is the multiplied by a scalar alpha, alpha you forget for the time being what is l m-th element.

So, l th entry of this vector times m th entry of this you understand this is a column vector this is a column vector column vector versus row vector this is this is the thing. So, first this into this, this into this, this into like that first row, second element into first second into second, second like the second row, so l-th element into 0, 1, 2 like this. So, I am interest in m-th, so m-th, so m-th guy, l-th guy, so here will be x prime n minus l, l-th guy of this vector and on the right side will be x prime n minus m. But, now this is a scalar term after all this is scalar element, so whether you put the term before or inside summation all it does not matter.

So, I apply E, E also has to be there, but E can pushed inside the summation these were already present from this side another term this x prime n minus l is coming l-th guy of this vector that I am pushing in then these two. So, remember k prime is not a random quantity k prime is correlation matrix, but already E was applied to obtain the correlation matrix. So, this is not random it will goes out n minus j and the n-th fellow, here that also will come in all this multiplied by this k I, j prime n this is the thing [FL].

So, this quantity pre multiplied by l-th fellow here that is x prime n minus l post multiplied by x prime n minus, but what is pre and post multiplication these are all scalars. So, scalar term scalar term scalar term you can push the two scalars inside here and here and then apply E over it. So, 4 x prime terms these are the random numbers random variables I mean E over that this is not random, so it goes outside e that is and summed as before. So, please see that you understand some statistical analysis again I tell you one purpose of this course, one main purpose is to make you conversion with this kind of statistical analysis procedure.

So, that you yourself can study other literature and do analyze on your own because these are the, these are the things which come very common come often in communication control and signal processing this kind of analysis. Now, I have got this terrible quantity, so if it is only a product of 2, I know what it is if it is 1, I know, but it is a product of 4. So, you know that is a big problem it is called a fourth order moment correlation is called second order moment mean first order this is the fourth order.

So, again see I will make use of that Gaussian assumption and under Gaussian adjoin, we know already that they are jointly Gaussian. So, under jointly Gaussian thing you know there are some results on this kind of I mean fourth order movements I will not prove that. So, I mean if you are interested you can see my lectures NPTEL lectures on probability random variables these are derived, but very difficult. So, I will only coat you a result first from probability and statistics on expected value of this kind of product of 4 terms which are jointly Gaussian.

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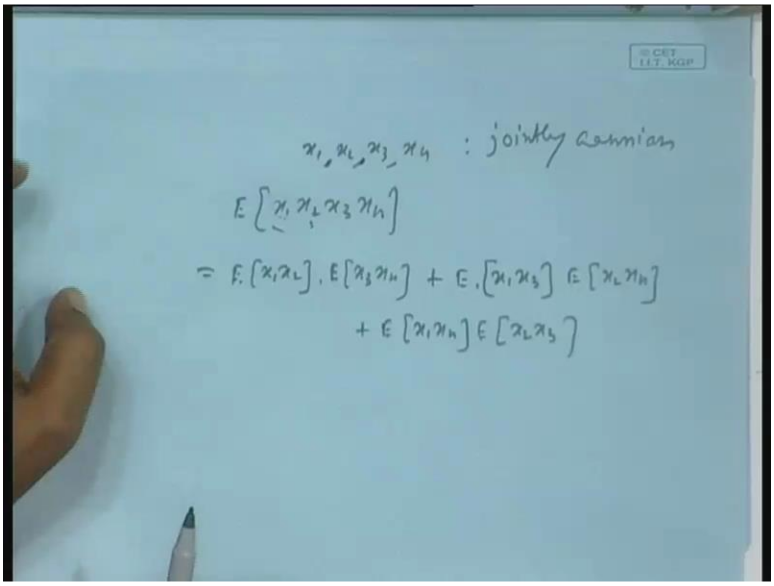

So, that is like this you know if suppose you have got four terms x 1, x 2, x 3, x 4 forget all those just new variables x 1, x 2, x 3, x 4 and suppose they are jointly Gaussian, jointly Gaussian. Then if you have I do not remember whether they are required to be have 0 mean or not, but to be required right, but to be on the safe side let us assume 0 mean because that does do any harm to us we already are assuming 0 mean cases for all the data.

So, if it is required I am not sure to change that you remember then under this joint Gaussian assumption this can be broken. So, first you have one product x 1, x 2 it is like this you know E x 1 x 2 multiplied E x 3 x 4 just simple permutation combination that is x 1, x 2, x 3, x 4. Then there will be another term involving x 1, x 2, x 3, x 4 and another term x 1 x 4 multiplied by x 2 x 3 that is E, these things you know you should always remember because they help you in you just. So, you are suppose finding what should be the difficult place spot in this thing you know in this analysis you.

Then if you know this you immediately assume that Gaussian thing and things become simple these are common tricks techniques applied in all this analysis procedures. So, now I have got, I have got this quantity you see this it is very easy to remember first two, second two and then first and third second and fourth and then first and fourth second and third.

But, what is e is working on only a product second order second order is no problem for you this works for Gaussian case I do not know with 0 mean. But, let us assume that since I mean we are dealing with 0 mean cases it will in this case work, so this I will apply here after all these 4 terms are jointly Gaussian.

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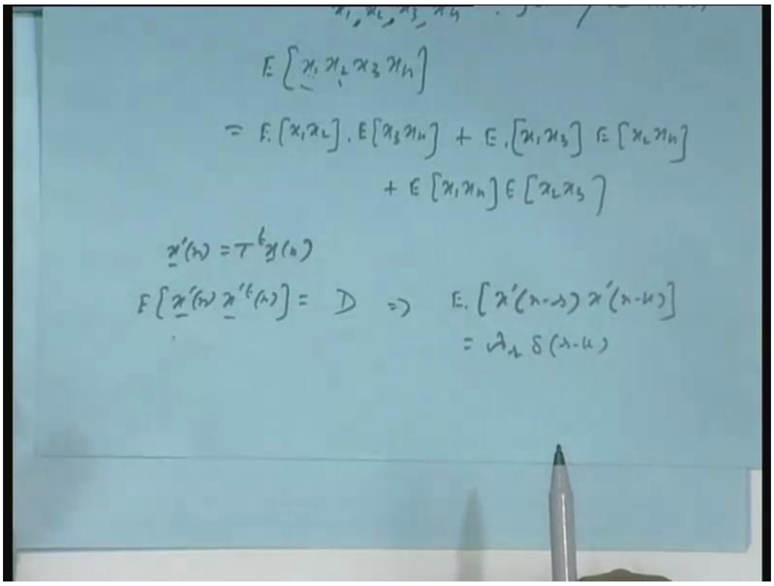

Now, remember x prime n please see one thing x prime n was what T transpose x n and what was the implication. So, if you take E of vector into this correlation matrix what was that thing that was diagonal D that is this vector is such that any two term. So, they are uncorrelated there cross terms there correlation is 0 only individual terms of the variance is given by the Eigen values you understand this product this is the diagonal matrix. So, cross terms i-th guy, l-th guy from here there correlation is 0 each guy has got some variance given by the Eigen values right.

So, that means E of in general if I take say any i-th fellow here or say any r-th fellow n minus R and say I if take anybody say k n minus k r-th fellow k th fellow. So, that will be what if R n k equal to 1, same R n k are same then only it will have some value otherwise 0. So, firstly there will be a delta R minus k or k minus R, it does not matter and when R equal to k it should be the variance of x prime n minus r or n minus k they are same. So, that will be given be lambda r th fellow r th lambda r or lambda k both are same, so you have to keep once lambda R, lambda R are positive and real. Now, please remember this, this will help us in this analysis very much and I have got only those cross terms.

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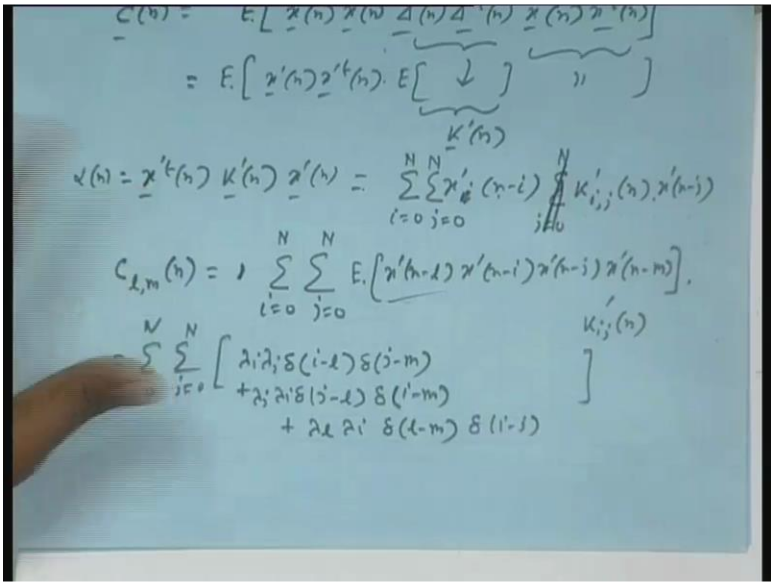

So, this outer sum remain as it is permit me to skip some step because you already know this result I will write directly E over this product into E over this product. So, E over this product will be what delta l minus I or delta I minus l delta I minus l and lambda you can say I or l it does not matter. So, lambda i, lambda i delta i minus l from these 2 multiplied by E of these two you remember that expansion for Gaussian, E of this first and second into E of second third and fourth. So, third and fourth will give rise to lambda j delta j minus m, so lambda j delta lambda j or m lambda j delta j minus m.

Then next is first and third second and fourth first and the now you can very easily see first and third would give rise to what lambda j delta j minus l and delta i minus m lambda I between third and fourth. So, there is one more term first and fourth second and third, first and fourth will gives rise to lambda l delta l minus m and first second and third will give rise to say lambda I delta I minus j. But, this entire thing summed over I equal 0 to n j equal to 0 to n and this multiplied by this quantity k i j prime n this is what I have. So, let us handle this term separately it is not very difficult looks clumsy, because of the del beauty of delta you know things will become very simple very soon.

Now, start with the first term just look at l and m are fixed from you fixed by you from outside l and m are your choice they are not variable you have fixed up we want to find out this particular entry for a particular choice of l and m. Now, I and j are running in this summation, so only when I equal to l and j equal to m it will have a non zero value. So, that means this will give rise to what lambda l lambda m k prime l m n that means one term will be can you see this, no you cannot see this line right.

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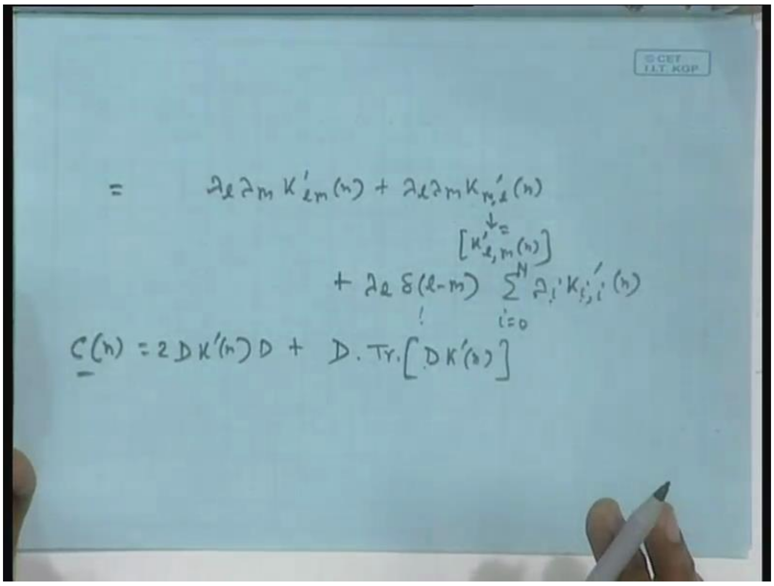

So, one term will be lambda l lambda m k prime l m n, i equal to l j equal to m here also l and m are your choice, but j and I are changing. So, only where j equal l and I equal to m you get nonzero value, so that will give rise to again lambda l lambda m k prime m l. But, k prime is symmetric first let me write out m l, but this is equal to, now that let me write also that k prime this is equal to k prime essentially these two quantities are same.

So, lambda l lambda m and k prime l m n because k prime is a symmetric matrix correlation matrix after all third guy is more interesting third guy third guy has this problem l and m are your choice only I and j are. So, l and m can be brought outside the summation they are not they are for your particular choice of l and m it will have some value either 0 or 1 if you have chosen l equal to m 1. So, you understand that this will be this term will give rise to I mean it will amount to a diagonal matrix in some way. So, if these other terms were not there only this then this would have become a diagonal matrix and only for l equal to m this will have nonzero value otherwise.

But, I have got other terms also, so this will contribute to a diagonal matrix, so anyway delta l minus m, I bring outside lambda l I can bring outside summation lambda i not summation. So, there was double summation you see I equal to 0 to n j equal to 0 n lambda I delta I minus j for each i j is changing. But, in this summation I will take only those values for which j equal to i, so you pick up 1 i and then as j moves from 0 to n only j equal to that i has to be taken. Then it becomes 1 and how many such cases will come n cases only, so it simply becomes lambda i k i i not i j do not forget this guy.

But, this is very important function my botheration is about this fellow so you cannot throw this is this fine. So, let us this is the particular element of the resulting matrix C looking at this can I construct the entire matrix these are only scalar terms first look at this guy, I have got this and this same. So, twice this no point in holding handling this separately twice this, what is this, for your sake let me work out on another piece of paper say, what is this quantity.

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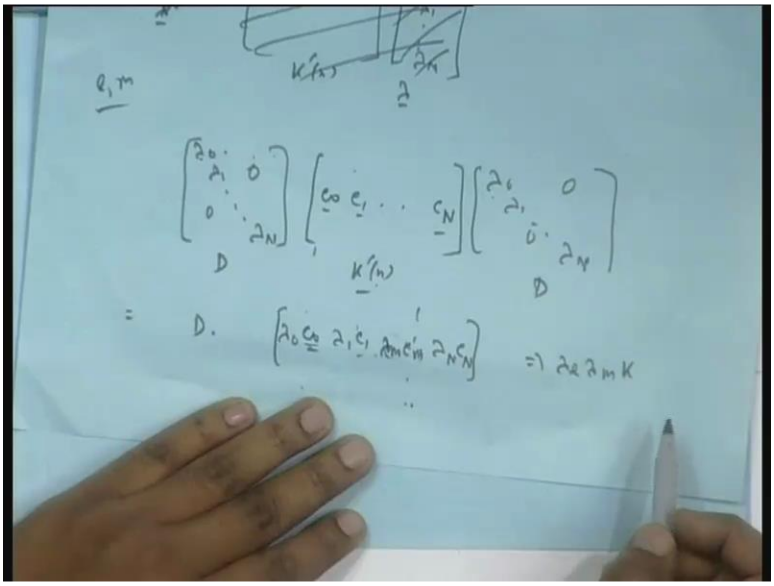

Now, you have got suppose k prime n if you have a vector lambda 0 lambda 1 dot, dot, dot lambda n I call it lambda vector transpose and if you have got lambda vector lambda 0 lambda 1 dot, dot lambda n, sorry just a minute no not this just a minute. So, suppose you have got k prime n you have got the vector D, D you know the diagonal matrix consist of the Eigen values and here also. So, you get a matrix only 0, 0, 0, 0 D matrix into matrix, into matrix what is the l m-th element of that matrix you can see easily.

So, first consider this multiplying two matrix means I told you do not multiply matrices like a school boy multiply by linearly combining the columns. Now, lambda 0 and all zeros lambda 0 times first column that will come up lambda 1 times because other zeros will multiply other columns no contribution. So, wherever you do this kind of multiplication lambda 0 times first column lambda 1 times second column dot, dot, dot lambda n times last column. So, I am interested in suppose the l m-th element, anyway this will give rise to again the same D into let me write out lambda 0 times say first column 0-th column lambda 1 second column dot, dot, dot lambda n.

So, these are column vectors of this matrix see here and, now multiplying by D if you multiply by D pre multiplication that time what I told you have to linearly combine the rows by the elements of the like you are combining the columns by the elements of this. So, if you have this row lambda 0 into first row then next element into second row next element into third row and combine, so what will that give rise to lambda 0 times. Now, this is a matrix, various rows are there lambda 0 times first row lambda 1 times second row lambda 2 that will be the final matrix.

So, what is the l for the l-th row of the final matrix lambda l will come from this side lambda l times l th lambda l times l-th row here this is these are columns. So, these are column vectors and then you are forming rows first row times lambda 0 other rows will multiply by the 0, 0, 0 no contribution. So, lambda 0 times first row will be the final first row lambda 1 times the second row will be the final, second row I am interested in the l-th row, m-th element in the final matrix. So, l-th row will be what lambda l times l-th row here that will be the final l-th row and in that l-th row I am interested in the m-th element and the m-th element will be what, m-th column.

So, lambda m lambda n times whatever value it had, so that means this will give rise to lambda l lambda m times whatever value it had this matrix had at the lm th position. So, what is lambda m, C m lambda l lambda m gone this is the column vector you are now looking at the l-th thing. So, l m-th element of this matrix after all the columns are of this matrix only C 0, C 1 dot, dot, here from only they come lambda l lambda m done. So, I am looking at this column because it is m-th and l th row of that, so l-th row and m-th column of what does it mean there is lm th element of this matrix is.

Now, I do not know I can, I mean just by say practice you know you should reach this reach a stage when you can see I mean you can directly write down you do not have to do this inspection like I can do it today. So, it is not very difficult I mean this things come very easily you can easily quickly write down the matrices I just a practice. But, there is actually nothing conceptual here it is just a practice, I can easily see which is post multiplying what is post multiplying. So, how will row effect how will the column be effected, since you are not use to you have you have to do this visual inspection and all that, so this is your k prime l that is what you had here lambda l lambda m this.

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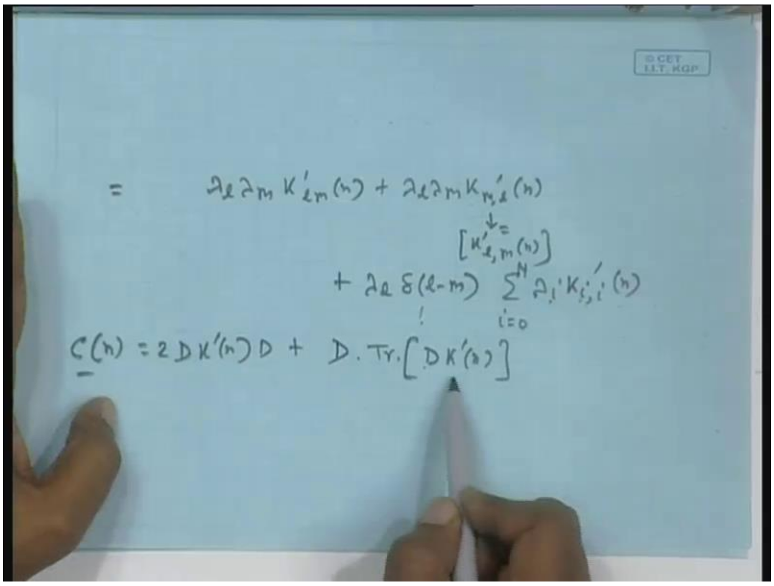

So, that means when you consider the matrix not just a element the entire matrix this will be D, let us see D k prime n D and twice of that because this two are same. So, twice D k prime n D plus you have got things here this is not difficult consider this is a scalar, this summation is a scalar independent of l and m. Now, you are finding out l m-th element this is just a scalar independent of l and m some quantity say 2, 3, 4 whatever keep it aside.

So, you have got only this delta, so it is not very easy it will contribute to what a matrix where only diagonal elements are nonzero l m-th element is given by this it will give rise to rise to a matrix where l m-th element is given by this. So, that means it will gives rise to what it will give rise to a diagonal matrix a diagonal matrix where. But, multiplied by lambda that will it will be D because l and m have to be same and there the element is, so this is 1 l and m only when l and m equal to 1 same. So, that means it is 1 multiplied by lambda l, so l, l-th element is lambda l that means this is a diagonal matrix this will give rise to D and this will give rise to what?

So, very good trace of either way you can write k prime n I told you trace of a b and trace of b are same and you can put the D here also, can you see this D k prime n. So, after all k prime n has those rows first row multiplied by lambda 0 second row multiplied by lambda 1 dot, dot, dot that is all and then you take the trace. So, first row multiplied by lambda 0 means, so that first diagonal element is multiplied by lambda 0, second diagonal element multiplied by lambda 1 and that is what you have. Here, lambda 0 first diagonal element lambda 1, second diagonal element like that of that matrix, so it is simply trace, so now let us collect the terms quickly because there are many 0 terms.

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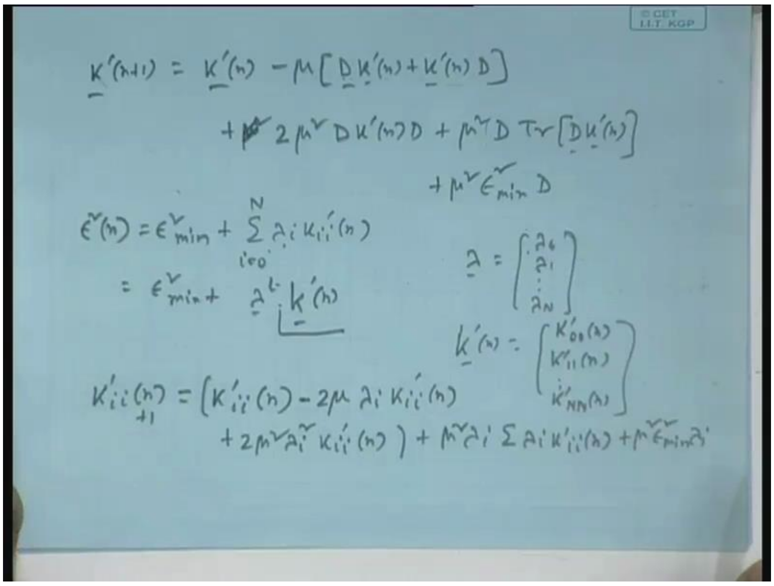

So, k prime n plus 1, we are finding out how the matrix evolves recursively there are so many terms minima is 0 and I put tick again those terms you know tick marks. So, there was k prime n, so k prime n then minus mu D k prime n and just a minute D k prime n.

Then the this term D k prime n and k prime n D both with mu and minus sign others are zeros and then I said to be done later this 1 and this 1 last one. So, the one which I left to be done later, now I know what it is there was a mu square, so I have to use that mu square also. So, plus mu square into this entire thing this entire thing I called C n, C n was C n was this, so mu square into this terms, so twice mu square D k prime n D plus mu square D trace and that last term. So, this last term mu square epsilon square min D this might look with frightening, but again with this all this become very simple.

Now, remember this is how I am the seeing the dynamics of the matrix, but in that error analysis it is not the matrix entirely that was of interest was it was the only the diagonal elements. So, if you remember just for your this thing we had epsilon square n was epsilon square min, but there was an extra term what are the extra term lambda i k i i prime n. So, I am not interested in all the terms, I am only interested in that is, this is becomes this plus if you can call a vector lambda transpose into I am giving some definition now this is you know it is reworking i, i this is I am defining a vector.

Now, it is a just notation k prime n where lambda vector is lambda 0 lambda 1 dot, dot lambda n this notation and this k prime n. So, if you basically what is happening if you take all the diagonal entries put them in a vector form do not bring trace here. Now, this is if I take a vector because I am only interested about trace means again I have bring the matrix which one I am not dealing with this. Now, here, here, here, here only here this I am just writing in a compact form I am defining a lambda with lambda vector likely that what I am doing the diagonal entries. So, I am picking up from this matrix picking up and then putting them in a vector form that vector I am calling out lower case k prime n.

So, k prime n is nothing, but K prime 0, 0 n K prime 1, 1 n dot, dot, dot K prime n, n, n, basically what this k prime, I am interested in the diagonal matrix of this guy of this vector. So, I am taking all the diagonal entries 0, 0, 1, 1, 2, 2, 3, 3 like that putting in a vector form and corresponding Eigen values also I am writing lambda 0 lambda 1. Now, you see can I, now to write like this lambda 0 into this fellow lambda 1 into second fellow that is what you have here lambda 0 into k 0, 0 prime n lambda 1 into. So, I am just writing in a vector form there is nothing conceptual it is just notation some compact notation I do not want to get rid of the summation and all.

So, what is the quantity of interest either the diagonal entries as written here or this vector both same it consist of only this entries only not the entire matrix is of our interest I found the dynamics of the matrix. But, essentially other entries of the matrix do not matter to us only this vector, so therefore why not develop from this by very simple steps. So, a similar dynamic equation for the vector here only not on the all the elements of this non diagonal elements of the matrix, only for the diagonal elements this diagonal elements are put in a vector form.

Now, if you do that it is not very difficult you take a particular diagonal entry here k prime i i n scalar, I will do for a particular i and then put them in a vector form. So, that will give me the dynamics for this vector, sorry this is not n this is n plus 1 this is k prime i, i matrix. So, I am bothered about the i comma i-th element of the left hand side and, therefore the right hand side i, i-th element of this is again this minus mu now tell me what it is. So, i, i-th element of D into k prime n lambda 0 first row lambda one, first say or say lambda 0, 0-th row lambda 1 first row lambda 2 second row dot, dot.

So, lambda i, i-th row in that i-th row i-th column, so lambda i times k prime i this side also you will get the same thing because there you are hitting at the diagonal entry. So, you know lambda 0, 0-th column lambda 1 first column like that lambda i i-th column, so in that i-th column all that is multiplied by lambda i. Now, you go to the i-th row of that, so you get lambda i, so you are getting basically twice, I hope by this analysis you know you become somewhat expert in matrix manipulations.

So, this kind of things you know some statistical analysis as you go through this 2 mu [FL] what we had here lambda i. So, this quantity is coming back again this quantity then here also it will come please tell me what it will come here 2 mu square k prime n. So, first post multiplied by D all the first column with 0-th column lambda 0, first column multiplied by lambda 1 dot, dot, dot i-th column multiplied by lambda i, lambda i square should that be lambda i square.

Now, just a minute, yes 2 mu square lambda i square this one I did not work it out actually 2 mu square lambda I square same quantity this much I put separately they involve this k i i prime n here mu square. So, this is a scalar and D oh my god, so lambda i only this is the matrix is scalar, so i lambda i times this and this trace is what summation lambda I k prime i i n.

So, as I did before and after that this quantity mu square epsilon min lambda I from here epsilon by notation yes square is part of the notation fine. So, now you can take k i i prime n common, so you get 1 minus 2 mu lambda i plus 2 mu square lambda i square this, this, this you can take common k i i prime n. So, 1 minus 2 mu lambda I plus 2 mu square lambda I square this is dynamic constant you can call it rho, so now I am putting them in a vector form.

So, what dynamics comes up I will write that and call of today, sorry no I am bringing in this lower case that is why I am making it like this. So, the lower case means vector I told you lower case underscore is vector upper case under score is matrix that was the notation I told in the beginning. So, upper case, no underscoring means scalar upper case letter capital a capital b with underscore means matrix.

Student: In lower case and upper case of k is very similar

I am, I am trying my best to make it like that you see this you must grunt anyway this will not occur very frequently occur only for a while and go. So, this quantity 1 minus 2 mu lambda I plus 2 mu square I have to quick to wind of this quantity I call rho i a scalar a scalar number, oh my god sorry, here only I should have done this way. So, I do not think we have much time to deal quickly write down the result plus rest this quantity I will call rho i, rho i is a constant, but depends on i. So, that is why if you want to form the dynamics here also this will give rise to what I am putting them in a vector form.

So, this also vector form a scalar times a vector 1 element of the vector scalar times another element of the vector. So, this will be again k prime n vector, but this is diagonal matrix F can you see that some new diagonal matrix or may be what I do, I start from here in the next class because you know they have given the alarm bell.

So, there is no point in hurrying, in fact very little is left up at this I will have the dynamic equation and that equation will give us a stable solution under some conditions. So, that will give the stability for that error variance to grow, so I think I will just wind up in 10, 15 minute in the next class we will start from this equation mind you. So, I am still keeping this slide.

Thank you very much.