Adaptive Signal Processing
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Lecture - 22
Lattice Filter

From what I have been doing because I have just come in the right time. We are dealing with Hilbert's space of random variables. All possible random variables with the inner product, 0 being random variable with the inner product given by the correlation, and in the general random variables are complex valued.

That is I am dealing with reverse Hilbert's space of what the field of complex numbers, scalars that will be using there in general complex. They are one particular problem will be now studying and that is a problem of linear prediction, this is related to optimal filter and adaptive filter as it will slowly you know I mean reveal. But we start with let us see what is, what is a linear prediction p th order linear prediction. Suppose if the optimal filter scenario we had this thing right.

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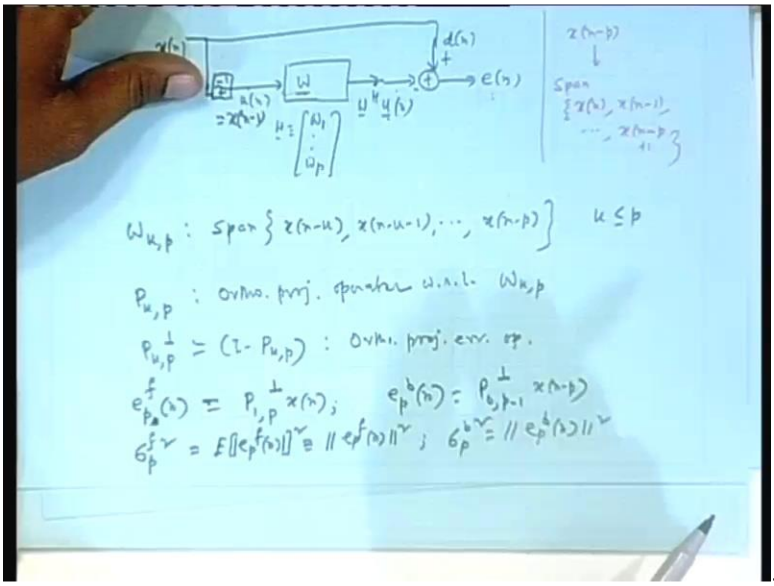

There is a sequence coming, we have a filter w, w vector. Output was w vector h, the input here. If the input you say U n then it is w 1 and there was a desired response. This is the error, the error was we took the variants of the error, which was the second order function of all the weights to minimize it. It gives only a quadratic function of the weights.

So, basically it is minima or maxima, and in this case is minima. You solve it you get a filter expression. And since, all the process has stationary jointly stationary and independently stationary also. The filter that comes out is independent of n optimal filtered, it does not depend on time index that we have seen in terms of filter expression is R inverse P and all that we have seen R is the auto correlation matrix of the input, P is the prosporation vector between d n and U n vector all those we know.

Now, we also know the actual meaning of these geometric interpretations that actually orthogonally projecting D n in the space span by U n, U n minus 1 up to some pass terms U n minus P projection is unique. This is that projection coefficient are given by this, they combine them you get the output, that output is filter output which will be nothing but combining linearly combining U n, U n minus 1 up to U n minus P by some coefficients.

So, that the error norm square is minimized, and that error individually orthogonal to each of U n, U n minus 1, that you have seen from our vector space point of view. We will consider now a particular case where this U n is coming from one signal x n with a delay and this is coming directly here. And how many, suppose we have got w equivalently, what w say? In this case, let put w 1 to w p not w 0, so, P coefficient.

So, U n is nothing but x n minus 1. What is this optimal filtering problem here then? You are estimating x n from what x n minus 1, x n minus 2 up to x n minus P. So, orthogonally projecting x n on the space span by x n minus 1, x n minus 2, up to x n minus P. So, what you get is the linear combination of the past three samples, which you say good estimate of current sample.

So, this estimate is called a linear prediction. That is the best possible in terms of vector space arrangement, in vector space notion, this is the best estimate. Because the corresponding error as the minimum norm square will be orthogonal to all these say x n minus 1, x n minus 2 up to x n minus P.

So, it is a special case of the general optimal filtering problem, where D n and input are generated from some x n. But obviously, x n is nothing but I mean this is our linear prediction problem, because what we are doing is we are trying to, I mean estimating x n as a past combination of as combination of past samples or whether you are projecting x of n under space span by x n minus 1, x n minus 2 dot, dot, dot, x n minus P.

The linear combination comes out here which is the prediction, it is the best prediction because that corresponding error I mean this corresponds to orthogonal projection. The error is perpendicular to this orthogonal to all these terms. I mean x n minus 1, x n minus 2, up to x n minus P, and therefore they will have minimum norm square, norm square means variants.

In this case obviously, this combiner coefficients from stationarity, if you assume x n to be W ((Refer Time: 05:46)) stationarity and all that. We know from our, you know from our earlier analysis, what does this expression for W is that we know, that was the R inverse P. How R and P independent of n, how did that happen? Because of stationarity.

So, in this case, you know one thing that even though random variables here are x n minus 1, x n minus 2, up to x n minus P that all function of n. The thing that we are going to project that is x n that also a function of n, the combiner coefficients they are independent of n. So, either at n th index or at m th index or k th index, what you are going to do? It the same combiner of coefficient. That comes from stationarity that you all know there is no question. There is a combiner coefficients are the independent of n.

Now, keeping this in mind now let me introduce some notations and notions. Suppose, in that inverse space H, I consider a sub space W k p is a span of x n minus k, x n minus k minus 1 dot, dot, dot, x n minus p, and obviously k is less than equal to p. Some definition I am now giving you, starting from first sample x n minus k, to x n minus p, and k is of course less than equal to p otherwise it has no meaning. If we take that span call it W k p that is a sub space of the inverse space H.

So, this sub space as its own linear projection operator, orthogonal projection operator. I told you, for this sub space I mean giving this sub space take any vector of H, that is any random variable you can have an unique orthogonal position of that on this either operation is linear. Remember I proved early that operation is linear. So, that operator also is unique because it takes. If you take any vector, it gives only 1 output, 1 component uniquely.

So, that is a unique operator is not it. Any way. So, that I denoted as this P k p as the orthogonal projection operator with respect to the given W k p. But I told you often, instead of projection rather in the case of lattice in practice linear prediction and all, we will be more bothered about the error, error is orthogonal to the sub space we know. Error will be the orthogonal to the sub space, but that error I want to find out.

So, giving an external n number will say x not x or y whatever. P k p comma x, P k p working on y what is a y? It is a random variable that will give you the projection. Projection will be a best linear combination of this elements, but I want to find out the error, the error will be orthogonal to them, having the minimum norm square that error operator is what I will denote like this. Which is this actually y minus either identity operator, no matrix nothing just operator symbolically. What is, this is called Orthogonal projection error operator.

We have done all this in previous classes it will give you if you work on the external vector say y, this will give you the error, this is y minus the projection. At that will be orthogonal to all this fellows was spanning the space I. Of course, assuming that none of these samples are linearly related that is this sub spaces dimensional equal to the total number of entry here.

The samples of random process they are not in linearly related just a linear dependents relation that is always that is always they are implicit in all point being. In order to know linear relation linking some samples of a random process because in real life it hardly happens, this is the thing. Then I define two quantities, please concentrate on all this p th order p th order forward linear prediction error I will explain, please follow my notation e p f, p th order forward error, error means linear prediction error at index n.

It is nothing but if you consider this error, if will consider this error, can you tell me in terms of this notation what will that give. Now, in this case what is this error we are projecting x n orthogonally on the span by who x n minus 1 to x n minus p, that means k is 1.

So, the space is there is w 1 comma p. So, the corresponding projection is p 1 comma p, projection error is p 1 comma p perpendicular, that working on whom, who is being projected? x n. If instead of e p f n, I go for e p plus 1 f n. I would have to have one more term here x n minus p minus 1, one more term I have to bring in find out a projection.

And that is what I will be doing, but I will be there orthogonally decomposing the sub spaces this sub spaces into two, all those things I will do.

Now, this is forward prediction why this call forward prediction because very; obviously, from past we are looking at future prediction this is a normal thing you know it makes at least some sense. From past we are trying to, from past we are trying to find out the future. At least you get the coefficients use that use them separately to combine x n, x n minus 1 up to some term.

So, prediction is at x n plus 1 coefficients remains same that as some sense. But now I will brining doing another kind of prediction which does not appear to have any practical sense. But there is a key to everything that is called backward prediction. Backward prediction which predicting some past term from current from the future of the past term into the up to that current term.

Because, what is the use? The past term is already available with us. Exactly this is no I mean knowing to it, I agree there is no need to estimate, but still we will be doing it because of many reasons. One that if you are interested to find that projections for various values of p, 0, p equal to 1 p equal to you need that as a secondary composition you note those.

But the most fundamental reason, I will I will come into this, but first let me give you the definition I am turning over the page I hope you remember this notations. Please remember because I cannot go on showing it, but may be here only I can show. E p b n backward prediction p th order n th index. What we are doing here we are taking.

So, this candidate use see the notation when this e p f n, it should not get confused to the notation please see understand what for. First see whether it is f or p f means forward p means order n, n is a current index. That means, let x n has the guy to be projected or predicted x n and 1 whom is the projection x n minus 1 to x n minus p.

But if it is e p b n it is backward then take the past quantity what quantity n minus p, that you project on this p future once what is the p future once. X n you are started x n, x n minus 1 up to x n minus p plus 1, that you do not try to understand though what is the utility of that prediction, that we will see later and I use for this prediction I need this prediction we will put it that way.

Later you will see that this prediction actually will do grasmit orthogonalation of the vector space, that is later or else lattice will used. E p b n this means, that means, I am projecting this guy say x n minus p. On to whom? On the span of current 1 ((Refer Time: 14:57)) started current 1.

So, see how to use this rotation n first the backward prediction that means, I have to go to past term who is past term n is the current in its n minus p, x n minus p should be the term that should be it projected in to p future 1.What is the p future one is x n. So, in terms of our vector space notation, what is the space here n minus 0.

So, k equal to 0 and p minus 1 here w 0 comma p minus 1. See the notation what is the n minus k, it was general case n minus k p n minus p i 2. What is k here? k is 0, n minus 0. What is p here? p have n minus with in bracket p minus 1. In this notation it is w 0 comma p minus 1, which means p 0 comma p minus 1, which means p 0 comma p minus 1 here .

Do not get confused to this capital P and small p, that will be fine I hope you understand this prediction operator. This order if you want I can change this the small p to anything else, but I hope you can see this is that ok? That means this is equal to what, this is a error not projection, but the corresponding error. On this error are also random variables. So, I can have there variances also which all meet in my development.

So, I define the variants of this guy, my claim is that variants is independent of n. why? Simply what is this? This quantity is that prediction error means D n that is x n minus the projection x n minus the projection. What is that projection it is a linear combination of those elements x n minus 1, x n minus 2, up to x n minus p. What is the linear combiner of coefficient, we already know because of the stationarity they are independent of n.

For any general case you give a sequence here and you get some sequence something here, you get a linear combiner of coefficient of filter those are independent of n. That we have seen because of stationarity where ever I assume sationarity W s thing. That means, what is this quantity it is x n minus a linear combination of past terms x n minus 1 to x n minus p, but the combiner coefficients are constants independent of n that quantity at the ((Refer Time: 17:41)) is variant.

So, I will take the mod n square that the quantity multiplied by its conjugate and we are now take all the terms. You will get lot of correlation terms, but because of stationarity all n will go from everywhere. Are you following me? I am not saying this is very you should have to visualize, I am taking this is a x n minus linear combination of past term x n minus 1, x n minus 2, up to x n minus p linear combiner coefficients are independent of n that is we have seen we have found, the general case.

Now, this error I want to take this mod square or the error and multiplied by this conjugate and take expectation operation. There is a correlation that is the norm square definition in terms if inverse case. Multiply the random variable which is conjugate norm square take expected value take a expected value. The moment I take the expected value, that is a norm square, that is variants because of the stationarity of x n all those terms, multiplying terms they become independent of n. You get some elements, I mean where linear combiner coefficient will be present and correlation terms will be present, but nothing of n, that means, norm square of this guy that the variants of this guy also independent of n.

So, in the notation I can drop main that is what I am saying notational using sigma p if p n f goes as it is, but no n and I put a square because you know always like any Gaussian case in terms of sigma is a sigma square variants after all just for that I put 2, this 2 is the notation. And this is what? We can say E of square which is also equivalent to norm square of this guy, this is in vector space notation.

By the same logic, I can denote its, I can take it variants also, its variants also will be independent of n. After all what is e p b n we are giving here x n minus p, it here x n minus p that is all. But they join the stationary this input is stationary. Obviously, this filter will be independent of n and all that again, and the same logic works.

So, I give the notation sigma p b square which is this e p b n square that is a expected values of mod square of these. Can you see up to this? on the screen? up to this line? Now? Now, suppose you know somewhere order these two quantities p th order forward and backward prediction error, you know this. You know this quantities also and I say get me the same quantities at least to start with errors for p plus 1 th order recursively.

Giving this four quantities, that means, get me this two corresponding errors forward and backward for the next order p plus 1 th. And assume p 2 b, p 2 b at least equal to 1. So, that there is like you know here when you have p 1 comma p, in this small p is not 1, 1 comma 0 is meaningless here. You started x minus 1.

So, to start with assume small p 2 b equal to 1 or higher. I will have to take the case of p is equal to 0 also, that I will do later. That is at least first order prediction errors are giving to at least first order that is equal to 1 And you have to go for p plus 1, then p plus 2, and p plus 3, like that recursively. How to do that?

To start with forward prediction error, and to find out e p plus 1 f n what is the corresponding sub space then w 1 forward prediction. It means always started at 1, but go up to p plus 1, x n minus 1 dot, dot, dot, up to x n minus p minus 1, x n minus 1, it will go up to here like x n minus 1 dot, dot, dot, this term and 1 more term. This space, I try to write as a orthogonal decomposition in terms of orthogonal decomposition. So, that position on that can be written as a summation of two projection.

So, how you do that please see, I have given you the hint last time. This in general case, this is by last entry what I will do otherwise this was already there you have added 1 extra guy to this set what I will do? Space span by these I mean this w this span of total thing is what direct sum of space span by this much plus the span of this, because they are linearly independent. You remember that day I told alpha 1 to alpha p and there I will bring other vector beta over all space span by is what alpha 1 to alpha p direct sum that of beta, but they are not orthogonal.

By early to start with w is same as space span by x n minus 1 to x n minus p direct sum space span by this last guy, but they are not orthogonal. But what they will do this last guy I try to project orthogonally on this take the error, then this will be equivalent to space span by this set x n minus 1 to x n minus p, orthogonal sum the span of that error. Where those two space equivalent to all those things, we have seen in last time Remember?

Last time we have seen it was the projection, I mean this is containing that sub space and that sub space is containing this sub space. So, both are equivalent that kind of proof I gave you remember. I do the general case the alpha 1 to alpha p, and then beta is not in span of alpha 1 to alpha p direct sum beta. Then is same as I mean that is containing the space span by what, span of alpha 1 alpha p orthogonal sum projection of beta projection error respect to what projection of beta on the previous space.

Remember this? I did and I told in repeatedly this is what I will be using again and again. So, no question of re explaining it I will be simply orthogonally projecting this on the space span by this take the error. So, the entire space will be what is orthogonal sum of the space span by this much x n minus 1 to x n minus p.

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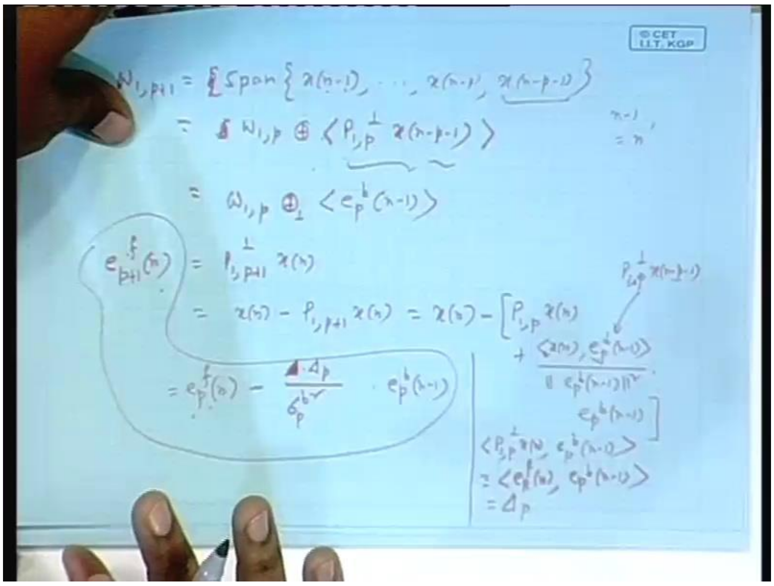

Then you can write on orthogonal sum and the span of the error that is W 1 p plus 1 which is span of and this new guy, this is same as you can write one line it is same as it is span of these direct sum span of this element. But that I am skipping, that is equivalent to what span of this fellow that is x n minus to x n minus p, which is nothing but W 1 comma p. Then orthogonal projection of this face that is p 1 p perpendicular span of this. And I told you when this is a single element instead of writing span of that element, I will use this notation remember this also I said when this is not inner product.

Inner product means after this there is a comma two elements. A comma b, x comma y, this is not an inner product I am using this two less than greater than symbols. This for 1 understand this is nothing but a new notation for indicating span of this single element. This notation I used in the last class to indicate. Instead of writing this span thing again and again whenever your single element is I said I will put to this two lines less than and greater than span and will mean the same thing span of this guy.

Now, question is what is this? Now, suppose n minus 1 you call n prime n minus 1 you call n prime that means, what this is x n prime. N prime minus 1 up to n prime minus p plus 1 and n prime minus p. So, this is projecting n prime minus p into its p future terms. And the error its p th order backward prediction error, but at what index of time, but n prime index of time what is the prime n minus 1.

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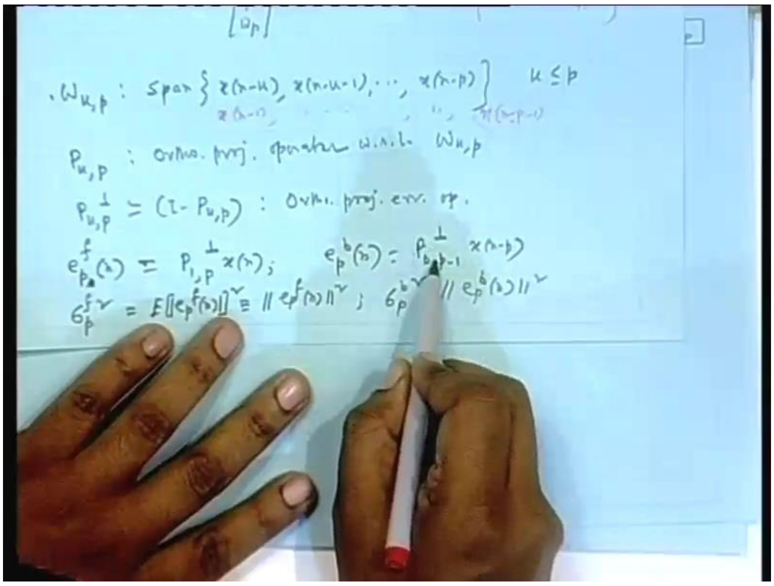

By this definition n prime minus 0, n prime minus 1 dot, dot, dot, n prime minus p plus 1. Those are the elements on that n prime minus p that is projected of the error. Not understanding? So, I am calling n minus 1 as n prime then your x n prime is what n prime minus 0 then n prime minus 1 dot, do,t dot, n prime minus p plus 1 and n prime minus p. This I am projecting on this part that is n prime minus p is projected on space span by what n prime minus 0 n prime minus 1 n prime p plus 1, this one. What is projected n prime minus p. So, what you get here e p b n prime an n prime is n minus 1.

So, this random sequence if is delayed by 1 that will what you get. This is a random variable at n equal to 0 n equal to 1 and equal to every adding every index is a random variable you can find the sequence of it. If you delay that you, then you get the next 1 n minus 1 that way. Are you getting me or not? That means, this is…

So, my purpose was to find out e p plus 1 f n right then you get me the prediction of error up to p th stage p th order why do not you get the same thing for p plus 1. And what is e p plus f n, it is p 1, this space index n with respect to n 1 means n minus 1, n minus 2, up to n minus p minus 1. This is what you have to find out there is a x n fellow has to as to be orthogonally projected on this total span, but this space is orthogonally decomposed as you get if you want you can put an orthogonal here, but I drop this, this two are mutually orthogonal right.

So, x n projected on this is, what is the total projection x n projected on this plus on this, but this is not projection this is error. So, I have to take this put this projection subtract it from x n into this what x n minus and what is this projection? Projection on this and projection on this, projection on this means I will go only up to this. x n projected on the how many parts only p parts x n minus 1 to x n minus p.

So, I will give the p th order for P x n order not p plus 1 th. This projection I can write as such a for your sake I do like you know step by step of 1 only this projection you can write as summation of two components projection on this, projection on this. Projection on this is what? plus projection on the orthogonal component, this you know projection on orthogonal component is what or any vector. Any vector projected on any other vector you know the what is that x projected on, say y projected on x.

Remember, I did this the that gross speed and all those things why inner product of this takes divided by norm square x, this times that vector x. I did all that keeping this in mind I did not do this just to teach you some mathematics. I told you repeatedly I will be using those things from now on, you follow this? Why we take divided by why inner product takes divided by norm squares there is a coefficients that times x.

That means x n what is the projection on this sub space, x n inner product with this guy, divided by norm squared of this guy, into this guy, into this guy. Now, x n minus this. This is the orthogonal this is the forward prediction error for p th order only. This is very easy to see this. This total space I am decomposition what this much span of this much plus another extra component coming with the projection.

So, x n projected on this remain the good hold p th order projection order only. In this there is no change nay that is this part and that you subtract from the next term you get the prediction error. But essential thing is here. Here you see the denominator, what is the denominator, norm square of backward prediction error. And I told you backward prediction error norm does not depend on the index because of stationarity. Did not I tell you this? This is independent of n only this p b j matter. So, this n square will have no meaning there.

So, the denominator will have sigma p b square. This quantity will be as it is. Here x n with this, and what is this quantity? This after all what, this a projection of this last fellow on the space span by this part. I will again write it that way p 1 comma p orthogonal x n minus p minus 1.

So, x n comma that. And remember I told you I showed you some fantastic thing last time that whenever you have an operator, you can interchange the operator. x n comma this projection of the projection error operator working on the another vector you can pull it you can repeat it on both or you can take it from here and push it here? You understand? What was the logic. Logic was x n can be written as what, x n itself can be written as what summation of two components projection on this space and the error.

Projection on this space and the error projection and this they are mutually orthogonal that goes only error part error part is again p 1 p orthogonal x n. So, p 1 comma p orthogonal x n. So, this operator gives repeated twice, then I can drop this operator why because this can be written as original vector x n minus p minus 1 minus the projection.

So, that projection is orthogonal with the error component coming from here. See this is repetition, this I explain. If you do not follow I cannot help I took strains to explain all these previous I am only reminding you I told you that time that please remember this always all throughout the course I will be making rapidly repeated use of this.

So, I will be brining it from here on top of the x n. The moment I bring p 1 p orthogonal on x n I get back this x n projected on this part and error, that is our good old p th order forward prediction error. So, inner product between p th order forward prediction error nth index p th order backward prediction error of n minus 1 th index, an inner product is correlation. The this inner product I will define as again this inner product. So, that will turn out to be this quantity. This quantity will be what? Inner product. In fact, I am not taking I will be only repeating this I am living as it is this is p 1 p orthogonal x n minus p minus 1 as it is this I can repeat on this.

So, this remains error and this comes under beta also. So, this becomes another error. So, this remains as it is which is equivalent to e p f n. Now, you see what is this inner product inner product means correlation between the two. What is the correlation, that is expected value of this into conjugate of this. What is this? This it is x n minus a linear combination of this past two elements, combiner coefficients of independent of n projection said here

And we are multiplying. So, again correlation terms will be free of n after you apply the e operator. So, this net quantity again will be independent of n because of stationarity. Are you following? e p f n is what x n minus a linear combination of this fellows if you have got an ((Refer Time: 37:05)) x n minus 1 x n minus p all that similarly kind of thing is will here again multiplying that two.

So, we have only you know meaning correlation terms all after e operation we will get correlation term they are all independent of n only that gap lag. So, this entire quantity is independent of n timing variant. I can give it a name p th order should not depend only on p, but not n I call you delta p. It is called partial correlation coefficient parcor, we will call it as parcor. Partial correlation coefficient parcor. So, this is the delta p.

So, you see this is one recursive relation, giving this fellow and this fellow that is p th order forward prediction error and the p th order for the past not even current. I can find out p plus 1 th order forward prediction error for the past for the current index. But I need to update this also, unless I update this how can I from here go to p plus 2 because that time I will be require e p plus 1 v n minus 1. Are you following me?

If I have to from giving these to I could obtain this. Then again how to get e p plus 2, that I have to give e p plus 1 f n, e p plus 1 b n minus 1. That means, I must update this in p also. I will apply the similar, absolutely similar philosophy, just little changing of terms I mean. Here the space consist of some terms there the space of consist of extra term and we will not consists the particular time like that philosophy is same. So, now I will go for e p plus 1 b n, this relation you remember this relation this is one half of the lattice filter lattice stage. Now, I will go for e p plus 1 b n.

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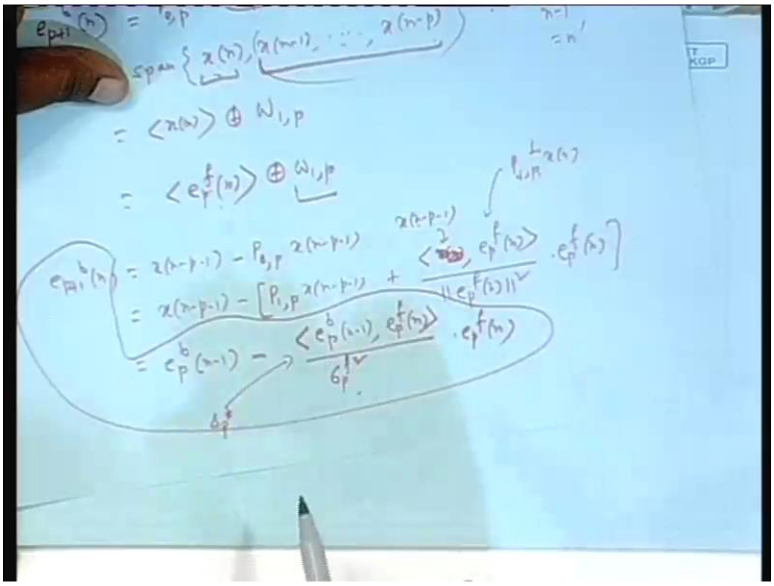

What is e p plus 1 b n in case forward error. e p b n means, go to the past term n minus p and then look at the p future terms including current. So, n minus 0 n minus 1 up to n minus p plus 1 this minus this on that I project x n minus p. So, now, I am going for this so; that means, I have to consider n minus p minus 1. This is how to consider this projected on from current that is 0 delay to, p th. N minus 0, n minus 1, up to n minus p. So, total p plus 1 terms. Those are the p plus 1 future terms if you are standing at these index. N minus p minus 1.

So, you will get a future is n minus p, then n minus p plus 1, then n minus p plus 2, dot, dot, dot, then n minus 2, n minus 1, n minus 0. This minus this on that I project this take the error that is my p plus 1 backward prediction error. What is the space here then w 0 p this is a projection, so here w 0 p. Here one term I will take out, I will take out 1 term as the new guy, which term? This term or this term, between that two, I will take out; obviously, first I will say direct sum this space is direct span of span of that single guy and the rest and then I will project that guy on to rest of the error.

So, I have an orthogonal decomposition ((Refer Time: 41:13)) here or here. Does not matters? It matters because my, this element is to the right of this. If I take this out I am at a loss there is s gap n minus p minus. I take this out and if I project this on this part I will get back my p th forward prediction error and all that n projected on this part n minus 1 to n minus p. This is same as what span of x n direct sums span of this rest. If you want you can write this way x n span of this remaining part, that is w 1 to p, w 1 to p and this x n. But this they are not orthogonal.

So, I projecting this x n orthogonally on this and take the error. So, this is the direct sum becomes the orthogonal sum if I project the x n on this part what is the error after all x n projected on what x n minus 1 to x n minus p this space of that the error forward prediction error. P th order that is now this fellow is to be projected on this together; that means, this on this because this is orthogonal decomposition.

And this fellow when projected on this part so that means, what have to do e p plus 1 b n. I repeat from here it means this error original fellow minus the projection, this projecting this guy on the space span by this. This projection subtracted from the original one. That is the error I am writing at this projection this projection can be written as what this much. Projection on this part projection on this part summation. First projection on this part, then here x n comma, mind you x n comma this, not this comma, this. Y is projected on x it is the inner product between y and x divided by mod x square not x and y. Two things are not same 1 is complex 1 is the conjugate of other equals comma equals p comma x n. No.

So, x n projected on this. So, x n with this guy inner product divided by norm square of this guy into this guy pretty simple projected on this x n with this by norm square divided by norm square into this guy. Now, here again n minus 1 if you call n prime, then look at only this part. After all p 1 comma p. What I am doing? I am taking one part here another part projection. Another projecting on that error, that error is this and next projection is what one projection I have to find out the on the space span, but the span by this.

But here if I take n minus 1 to be n prime then x n prime n prime minus 1 up to n prime minus p plus 1 and this guy is n prime minus p. So, n prime minus p is projected on the space span by the future ones. So, actually the p th order for backward friction error for an index n prime. And this becomes error when you back home please see this derivation.

So, you will see this is just in one case, one space get an external from the right side here from the left side it is just that nothing much. N prime n prime is n minus 1 times substituting it minus this again I will play the same trick here e p plus e p f n is p 1 p perpendicular x n p 1 p perpendicular x n. This I can repeat on this just a minute this is not x n this fellow is projected I made a mistake this is here you will have the guy who is projected is this guy nay. I am projecting this on the space on this two error.

So, this on this plus this on this, this on, this is the inner product between this guy and this guy divided by norm square of this guy and into this fellow. Because fellow who has projected is this fellow. So, I have to take the inner product between this fellow and this fellow take the divided by the norm square of this into this.

So, it is n minus p 1 p here and p 1 p perpendicular now you can repeat on this and what is p 1 p perpendicular. You have just now seen what is p 1 p p 1 p you get this space only 1 p nay p 1 p means x n minus 1 to x n minus p x n minus and x n minus. e p b n minus 1 this much on this we are projecting this taking the same thing p th order I mean, if you can call it n minus 1 to n prime n prime minus p n prime minus 0 n prime minus 1 to n prime minus p plus 1 that way.

So, p number of future term standing at when you are standing at, standing at n prime we are looking at p future terms on that you are projecting. So, p th order backward p x n error, but because we are projecting backward for using future we are projecting back. P th order backward prediction error for index n prime which is equal to n minus 1. That means this becomes inner product of divided by this norm square you know his is a variance of this and there is independent of n because of stationarity or that is using that notation into e p f n.

So, this is the another relation. So, these are the two relations, one was this. Here you call it delta p, what was delta p? e p f n and e p b n minus 1 they are inner product here this is reverse. So, it is delta p star, this quantity, this quantity is delta p star. We will show that sigma p f square and sigma p b square they are same we will prove you using induct stationarity will prove.

The numerator of this two quantities are different what is sigma p the variants of backward prediction error and this variants of forward prediction error of the same model, but they are same, that we will see. And the both real because variants after all both are real. So, only difference is in the numerator. Numerator is complex because inner product is not necessarily a real number which is x n. But once the conjugate of each other. So, that means, this over all multiplied in 1 recursion is the conjugate of the corresponding multiplied on the other recursion. I can draw a circuit.

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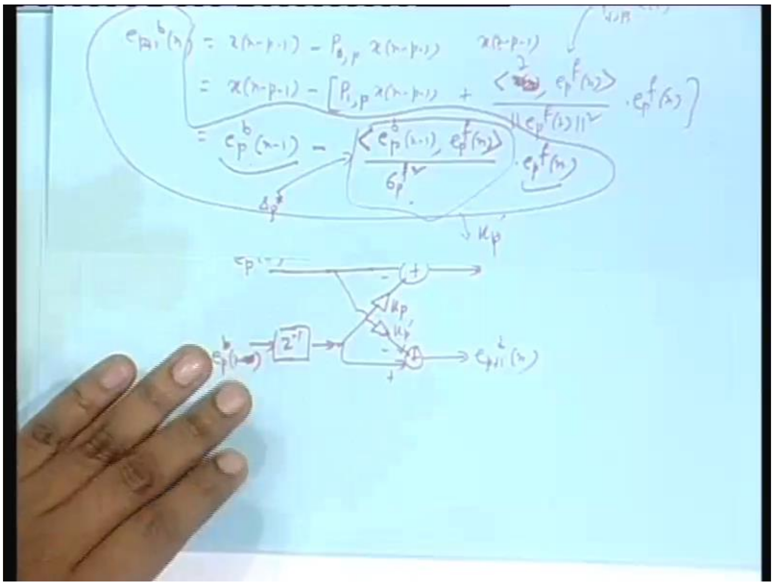

A stage now.

Student- Sir, all the properties of x n remainder is same in the reverse the picture, right? Then why do we have to prove that variant.

No, that is I mean you are saying intuitive there is no I mean I do not know how you can arrive that. If I lift the motion of stationarity they are not, but using the line of logic they are always it is only prove for stationarity. But why we are putting it because that should it applicable whether for stationary or non-stationary.

Student- But I think we have come till now.

That is okay, stationarity we have used square. I am making only independent of n that is all. Stationarity. So, first start with this guy forward prediction p plus 1 th order it takes e p f n, it takes e p f n, e p f n minus quantity this whole quantity I call it as k p. k p is called the reflection coefficient, k p reflection coefficient denominator is real numerator is in general complex k p. k p times this backward sorry this is n.

So, this is n minus 1, but independent of n, because of stationarity and you will get. This is for p greater than equal to 1 mind you usually tell p minimum value you take to be 1. What is work also called p equal to 0, that we will see and now this other 1 ep plus 1 bn again it uses the same 2 same 2 signals. e p f n, e p b n minus 1 that multiplies in a different way. This quantity I told the denominator are same, but I have not proved it yet.

So, this whole multiplier I call it k p prime now. Later I will show that 1 is a conjugate of that no point. So, now, you in give a new name. So, this entire quantity let us call k p prime. So, this will be this is one stage of lattice filter. Did I put here? And you can cascade it what you can cascade this is e plus e p plus 1 b n. This start with the delayed version there is delayed I am pull a delayed of the input if you take this and cascading it you will get the various orders.

We will show in the next class that both this variants are same then k p, k p prime are same the one is the conjugate of the other; obviously, if the denominators are same. Numerator are already conjugate of each other delta p and delta p star. So, together they are conjugate of each other because denominator is not only same they are real.

And then an important thing is the reflection coefficient the magnitude is always less than 1 that we will prove, using that we will show that if you can use the if you want to use the this lattice filter will develop the inter lattice filter for doing something like you know auto replacing modeling. You know something about this. Did I discuss in the course ((Refer Time: 53:50)) ar modeling and all that. I will do if you are in need of that also. So, their stability of the filter will be guaranteed because of this fact that refraction coefficient of magnitude is less than 1. We will get not only the cosalan filter and all those. Again DSP another thing will come, is it not? Just vector space you know they will be from now on. So, that is all for today. So, see you later.

Thank you.